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An integrated inventory models under cooperative and non-cooperative seller-buyer and vendor supply chain

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The first level of the supply chain basically contains three distinguished levels; these are single buyer (that is retailer), the second level vendor (that is warehouse) and single seller in the third level. The aim of this study was to investigate the cooperative of inventory models among the players in a three-level supply chain. In addition, the challenges involved in determining the optimal value of time interval between consecutive setups and orders in a coordinated inventory model were also taken into consideration. For this purpose, the researchers selected three types of non-cooperative models; nonetheless, the primary focus of this model is to reduce the coordinated total relevant cost to the minimum. This was followed by the implementation of recompense policy for the losses and profits of the coordinated inventory model proposed in the present study. The method which offers an optimal solution for the model developed in this study was obtained, whereas a numerical example was also given and demonstrated.

Key words: Three-level supply chain, inventory models, cooperative and non-cooperative models.

INTRODUCTION

It is important to note that the inventory decision model of supply chain has successfully been used in various organizations and companies worldwide for the past few years. In particular, the application of the inventory models in any supply chain offers a lot of benefits, specifically in term of achieving coordination. Several researchers (Chen et al., 2006; Dai et al., 2005; Esmaeili et al., 2009; Yang and Zhou, 2006), indicate that in a seller–buyer supply chain, there is a manufacturer who supplies products to a retailer in bulk (wholesale), and this retailer then retails them to consumers. Based on the literature reviewed, the seller has interchangeably been used to represent the supplier. Similarly, the term ‘retailer’ has commonly been used in place of ‘buyer’. For simplicity purposes, the terms, ‘organization buyer and seller’ are used in the present study. The supply chain also comprises of various facilities, whereby raw materials, intermediate products, or end products are produced, or processed, purchased, stored or sold. Apparently, one company alone cannot manage these facilities; hence, vendors, customers, third-party providers have to obtain them as divisions of other firms with which the company has business transactions with. Debates with respect to coordinated inventory models have consistently been argued in the following literature. Goyal and Gupta (1989) and Ben-Daya et al. (2008), have contributed to improve literature on cooperated inventory models. Goyal (1977) and Banerjee (1986) established cooperated inventory models, which came to be essential for development and extension of inventory models to decrease minimum inventory in the system and consequently, lead to lower costs. According to Goyal and Gupta (1989), applying the inventory decision models of players in a particular supply chain is a common method of obtaining good coordination. In fact, coordinating orders in a two-level [vendor–buyer(s)] supply chain has become the main issue discussed by various researchers in their studies (Boyaci and Gallego, 2002; Goyal, 2000; Gurnani, 2001; Hill, 1997; Jaber and Osman, 2006; Viswanathan and Wang, 2003; Weng,
Also developed an optimal pricing and ordering strategy for a deteriorating product to entice the buyer to accept the alliance. Yu et al. (2008) proposed a similar mathematical inventory model into account a cooperated supply chain of single deteriorating rate. The three-level supply chain comprising of a single seller, one warehouse and a single buyer was undertaken in the present study. Another major or primary aspect which was also taken into consideration in the present study was the coordinated analysis that was carried out between the seller, the warehouse and the buyer, and policies so as to motivate or promote a good coordination between various key players in a supply chain.

The remaining parts of the paper are arranged in the following manner: a discussion of the notation and assumptions which were underlying the models employed in the current study followed by the mathematical and solution models, whereby the coordinated model was also created. The expansion of the compensation policies for the benefits and losses using the proposed models was covered. This was followed by several computational results which include a number of numerical examples and discussion. Meanwhile, a conclusion and some suggestions for future work to be conducted in this area of study were put forward.

**NOTATION FORMULATION OF THE PROBLEM**

The notation and formulation utilized in the problem of the supply chain undertaken in the present study are introduced here. In more specific terms, it discusses several issues including decision variables, as well as the input parameters and assumptions underlying the three levels, namely the seller, vendor, and buyer. Please note the following:

- $a_w$ = warehouse's ordering cost per order
- $a_b$ = buyer's ordering cost per order
- $D$ = annual demand for the item
- $h_s$ = seller's holding cost per unit per unit time
- $h_w$ = warehouse's holding cost per unit per unit time
- $h_b$ = buyer's holding cost per unit per unit time
- $s$ = seller’s setup cost per setup
- $t_s$ = time interval between successive setups at seller
- $t_w$ = time interval between successive orders at warehouse
- $t_b$ = time interval between successive orders at buyer
- $u$ = positive integer number ($u > 1$)

Meanwhile, it is important to note the following assumptions which were used to come up with the proposed models in this study:

(a) The demand remained constant.
(b) Both shortages and backlogs were not allowed.
(c) The lead-time for the seller, warehouse, and buyers was set to either zero or replenishment was instantaneous.
(d) The interval time between the setups of the seller and the time interval between the orders of the warehouse is the integer multiple of the time interval between orders of the buyer ($u > 1$).
(e) The buyer has higher holding cost compared to that of the warehouse.
(f) A simple EOQ inventory model can be used to describe the
inventory policies of the seller and the buyer (Sarmah et al., 2006). Considering that the seller and buyer’s inventory models can be illustrated by uncomplicated EOQ, we can easily obtain the optimal policies. It is also crucial to highlight that unit costs would not have any impact on optimal policy as they remained constant; thus, they were excluded from the buyer’s, the warehouse’s and the seller’s cost functions. They were, however, applied in the numerical samples so as to evaluate or determine the vendor’s and the seller’s holding costs at the ends.

NON-COOPERATIVE MODEL

Using the non-cooperative structure, the seller, buyer and warehouse individual are considered here. In more specific terms, the interaction between the three as an individual model is considered, whereby the leader, as one of the participants, has the initiative and can enforce the strategy used on the follower who is another participant.

Independent model

Here, the non-cooperative model was formulated. It is important to note that in the independent individual model, the seller (e.g. supplier) and the ordering policies (e.g. warehouse and retailer) are independent parties. Meanwhile, the total cost for the non-cooperative model of the seller, the warehouse and the buyer can be obtained using the following formulas or equations:

\[
C_s(t_s) = \frac{s}{t_s} + \frac{Dt_s}{2}h_s, \quad t_s^* = \frac{2s}{\sqrt{D}h_s} \quad \text{(seller)}
\]

\[
C_s(t_s^*) = \sqrt{2Dh_s} \quad \text{(seller)}
\]

\[
C_s(u,t_w) = \frac{a_w}{t_w} + \frac{(u-1)Dt_w}{2}h_w, \quad C_w(u,t_w^*) = \frac{a_w}{t_w^*} \quad \text{(warehouse)}
\]

\[
C_b(t_b) = \frac{a_b}{t_b} + \frac{Dt_b}{2}h_b, \quad t_b^* = \frac{2a_b}{\sqrt{D}h_b} \quad \text{(buyer)}
\]

\[
C_b(t_b^*) = \sqrt{2Da_bh_b} \quad \text{(buyer)}
\]

Seller’s leader model

Regardless of the warehouse’s and the buyer’s orders, the high-technology products are made based on the seller’s decision. It is crucial to note that the leader model of the seller comprises of the seller (as the leader) and both the buyer and warehouse (as the followers). In addition, this particular model also contradicts with the buyer’s leader model. In this context, the warehouse and the buyer make decision or ordering policies based on the seller’s decision because the seller is a decision-maker. Hence, for the seller’s leader model, the total cost of the non-cooperative model can be obtained using the following equations:

\[
C_s(t_s) = \frac{s}{t_s} + \frac{Dt_s}{2}h_s, \quad t_s^* = \frac{2s}{\sqrt{D}h_s} \quad \text{(seller)}
\]

\[
C_s(t_s^*) = \sqrt{2Dh_s} \quad \text{(seller)}
\]

\[
C_s(u,t_w) = \frac{a_w}{t_w} + \frac{(u-1)Dt_w}{2}h_w, \quad C_w(u,t_w^*) = \frac{a_w}{t_w^*} \quad \text{(warehouse)}
\]

\[
C_b(t_b) = \frac{a_b}{t_b} + \frac{Dt_b}{2}h_b, \quad t_b^* = \frac{2a_b}{\sqrt{D}h_b} \quad \text{(buyer)}
\]

\[
C_b(t_b^*) = \sqrt{2Da_bh_b} \quad \text{(buyer)}
\]

This was particularly carried out to measure or calculate the optimal values of \(u\) and \(t_w^*\), so as the total cost per unit time could therefore be minimized. Meanwhile, the important or crucial condition \((u \geq 1\text{and } C_w(u,t_w^*) \geq 0)\) for the minimum of \(C_w(u,t_w^*)\) could be obtained using the following equation:

\[
\frac{\partial C_w(u,t_w)}{\partial t_w} = 0
\]
Whereby, solving Equation (10) would lead to:

\[ t_w^* = \sqrt{\frac{2a_w}{(u-1)Dw}} \]  

(11)

In addition, the existence of a unique optimal solution also needs to be studied. Hence, substituting or replacing Equation (11) with (8) can lead to the total cost per unit time for any given u.

\[ C_w(u) = \sqrt{2(u-1)Da_w Dw} \]  

(12)

\[ Z(u) \] was chosen so that the problem in the current study would correspond to the minimization of \[ Z(u) \] as follows:

\[ Z(u) = C_w(u)^2 = 2(u-1)Da_w Dw \]  

(13)

The aforementioned equation (that is Equation 13) is a linear increasing function that is dependent on u. The optimal minimum value of u is therefore equivalent to 1 all the time. In this context, the order quantity is directly delivered to the buyer by the seller.

Meanwhile, the time interval between the successive orders for the buyer is \( t_b \), whereas the time interval between the successive setups and orders for the seller and the warehouse are \( t_s = 3t_b \), and \( t_w = 3Dw \), respectively. The completed lot is then delivered to the warehouse at the end of the time interval for the seller. In the warehouse, it sends directly the buyer’s order quantity \( Q \) to the buyer (which is the same to cross-docking, that is, a process where a product is exchanged between trucks to enable each of the truck sending to a buyer’s store has products from different sellers) at the beginning of the time interval. Meanwhile, more than twice of the minimum total cost of the cooperative model could be achieved by replacing \( t_b \) into Equation (15), as follows:

\[ C_{co}(u) = \sqrt{\frac{2D(h_b + Dw + h_w + h_s)(u a_b + a_w + s)}{u}} \]  

(19)

The optimal value of \( u \) was another objective to be achieved in this study. Based on the essential condition of \( \{u = u^*: L(u^*) \leq L(u^* - 1) \text{ and } L(u^*) \leq L(u^* + 1)\} \), \( L(u) \) was defined in the present study, as follows:

\[ L(u) = C_{co}(u)^2 = \frac{(h_b - h_w)(s + a_w)}{u} + u a_b (h_s + h_w) \]  

(20)

Accordingly, the following should be obtained:

\[ \frac{(h_b - h_w)(s + a_w)}{u} + u a_b (h_s + h_w) \]  

(21)

Note that Equation 21 was used to obtain the following condition:

\[ u^*(u^* - 1) \leq \frac{(h_b - h_w)(s + a_w)}{a_b (h_s + h_w)} \leq u^*(u^* + 1) \]  

(22)

In order to determine \( u^* \) and \( t_b^* \), the optimal value of inequality (22) needs to be obtained first. Hence, if \( u \) is greater than 1 in Equation
Table 1. The buyer’s optimal value of $u_{t}$ and the non-cooperative models of the warehouse and seller.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Seller (year)</th>
<th>Warehouse</th>
<th>Buyer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>0.1333</td>
<td>0.1333, $u=1$</td>
<td>0.0471</td>
</tr>
<tr>
<td>Seller’s leader</td>
<td>0.1333</td>
<td>0.1333, $u^*=1$</td>
<td>0.1333</td>
</tr>
<tr>
<td>Buyer’s leader</td>
<td>0.0471</td>
<td>0.0471, $u=1$</td>
<td>0.0471</td>
</tr>
</tbody>
</table>

Table 2. The total cost incurred for the non-cooperative models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sellers annual cost ($)</th>
<th>Warehouse’s annual cost ($)</th>
<th>Buyer’s annual cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>9000</td>
<td>2250.56</td>
<td>3181.98</td>
</tr>
<tr>
<td>seller’s leader</td>
<td>9000</td>
<td>2250.56</td>
<td>5061.51</td>
</tr>
<tr>
<td>buyer’s leader</td>
<td>14328.47</td>
<td>6369.42</td>
<td>3181.98</td>
</tr>
<tr>
<td>Total cost</td>
<td>(In)4432.54</td>
<td>(seller) 16312.07</td>
<td>(buyer) 23879.87</td>
</tr>
</tbody>
</table>

22. $u^* = u$ should therefore be set. Otherwise, this should be set, $u^* = 1$, whereas the optimal value for Equation 18 should be determined.

COMPENSATION POLICY

As a result of coordination, it has been stated that some players may benefit from it, while others may suffer a financial loss. On the contrary, several other researchers, such as Munson and Rosenblatt (2001) have reported that coordination may lead to reduction in the total cost of a supply chain. Therefore, all the parties (namely, the seller, the warehouse and the buyer) should share the net benefit in some fair procedures. For this reason, the compensation policy of Goyal’s (1989) method, was proposed for the three-level supply chain, whereby benefits and losses are shared between the parties involved. In this study, E was obtained using the following equation:

$$E_s + E_w + E_b = 1$$  \hspace{1cm} (23)

$$E_s = \frac{C_s(t_s^*)}{C_s(t_s^*) + C_u(u^*, t_u^*) + C_b(t_b^*)}, \text{Cost of seller}$$  \hspace{1cm} (24)

$$E_w = \frac{C_u(u^*, t_u^*)}{C_s(t_s^*) + C_u(u^*, t_u^*) + C_b(t_b^*)}, \text{Cost of warehouse}$$  \hspace{1cm} (25)

$$E_b = \frac{C_b(t_b^*)}{C_s(t_s^*) + C_u(u^*, t_u^*) + C_b(t_b^*)}, \text{Cost of buyer}$$  \hspace{1cm} (26)

EXPERIMENTS AND DISCUSSION

This study explores how cooperative and non-cooperative models will influence the supply chain behavior, as well as performance in each level of the supply chain. This research also examines the case originally to justify the proposed cooperated inventory policy. Several numerical samples are presented here. These are given so as to demonstrate several crucial aspects of the three-level supply chain, namely one buyer, one vendor (warehouse), and one seller which have been discussed previously. Note that Examples 1, 2 and 3 given shortly, demonstrate the non-cooperative and cooperative models, respectively. It is also important to highlight that in the 3 examples given subsequently, the following have been set: $D = 15000 \text{ unit/year}$, $s = $ 600/\text{setup, a_s = $ 300/order, a_b = $ 75/order, h_s = $ 4.5 }$

$/\text{unit/year, h_w = $ 4.5/unit/year, and h_b = $ 4.5/unit/year.}$

Based on the data presented in Tables 1, 2 and 3, the total cost against the buyer’s leader model could be lowered by $6,451.45 (that is 37%), if the three-level supply chain was coordinated, we can reduce (approximately 37%). Hence, sharing of the benefits is apparently required and necessary. The numerical examples, which are specifically meant to illustrate several crucial aspects of the compensation policy, such as information sharing, are also given in this paper. The optimal values applying the compensation policies and the cooperative and non-cooperative inventory policy for the seller, warehouse and the buyer are summarized in Table 4. Cooperative inventory models after compensation policy outperform the other level of the supply chain. The total cost of the cooperative inventory models is reduced by 26.89, 27.21, and 27.15% and the total cost is reduced by 3,853.99, 1,733.47, and $ 864.01 as
Table 3. The total cost incurred for the cooperative models.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Seller</th>
<th>Warehouse</th>
<th>Buyer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time interval</td>
<td>0.129</td>
<td>0.129</td>
<td>0.0430</td>
</tr>
<tr>
<td>Annual cost</td>
<td>$9004.91</td>
<td>$5212.44</td>
<td>$3211.07</td>
</tr>
<tr>
<td>Total cost</td>
<td>-</td>
<td>-</td>
<td>$17428.42</td>
</tr>
</tbody>
</table>

Table 4. Information sharing and compensation policy.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Seller</th>
<th>Warehouse</th>
<th>Buyer</th>
</tr>
</thead>
<tbody>
<tr>
<td>E*Total cost ($)</td>
<td>0.601*17428.42</td>
<td>0.266*17428.42</td>
<td>0.133*17428.42</td>
</tr>
<tr>
<td>Non-Cooperative (Buyer's leader)</td>
<td>0.0471</td>
<td>0.0471</td>
<td>0.0471</td>
</tr>
<tr>
<td>Time interval</td>
<td>14328.47</td>
<td>6369.42</td>
<td>3181.98</td>
</tr>
<tr>
<td>Total cost ($)</td>
<td>10474.48</td>
<td>4635.95</td>
<td>2317.97</td>
</tr>
</tbody>
</table>

Conclusion

The problem pertaining to warehouse and the three seller–buyer supply chain management has been taken into consideration and discussed in this paper. At the same time, both the non-cooperative and cooperative models have also been given, whereas the inventory model used to determine the optimal value of time interval and the integer number of both the cooperative and non-cooperative model has also been developed. As for the total cost per unit time, it was found that the compensation policy, that is specifically information sharing, has successfully obtained improved results compared to the non-cooperative models in this study. In particular, the cooperative policy used to achieve the total cost per unit time, was found to have significantly reduced it, than that which was obtained using the non-cooperative models. The numerical examples have been applied to demonstrate applied theory. Meanwhile, the effects of both the time interval and total cost of the model on the decisions of the seller, the warehouse and the buyer have also been studied through the same analysis. This is where future work lies in, as there are a lot of aspects which could be investigated into. Among others, other types of compensation policy, specifically the price quantity discount policy can be developed in future studies, whereas the model proposed in the present study can be further extended whereby a multiple three-level supply chain is employed.

REFERENCES


