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Full Length Research Paper

Production planning modeling: A fuzzy multi objective fractional programming approach

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Production planning is one of the most important issues of managers in industry. Production managers face up with several goals that sometimes conflict with each other. Operations research techniques with considering the constraints, optimize organizational goals. Objectives of all these techniques are raising productivity in the organization. In this paper, the researchers present a multi-objective linear fractional model of production planning in wood and metal company. One of the difficulties in solving the multiple objective fractional problems is computational problem that arises from of variability, in the example of Charnes and Cooper (1961) methods. In this research, fuzzy approach issued to solve multiple objective fractional mathematical problems of Khavar-E-Miane Wood and Metal Company. First, with some assumptions, the fuzzy linear fractional goal programming method of Pal has been used to solve the problem of production planning. Then, the fuzzy method of Dutta is utilized to solve the problem. Comparison of results showed that both methods have identical solution.

Key words: Production planning, fuzzy goal programming, productivity, fuzzy fractional programming, wood and metal company.

INTRODUCTION

Nowadays, the most important issues considered by the industry managers is production planning. Manufacturers require a production policy to be globally competitive (Gwo and Shey, 2011). In production planning, managers, sometimes, may face up with goals to optimize inventory/sales, actual cost/standard cost, output/employee, etc., with respect to some constraints. Such types of problems are inherently multi objective fractional programming problems. Wide applications of fractional programming arise in different problems in operations research, for example, production, resource allocation (Chakraborty and Gupta, 2002), etc. Gilmore and Gomory (1963) discussed a stock cutting problem in paper industry and showed that under the given circumstances, it is important to minimize the ratio of wasted and used amount of raw material, instead of just minimizing the amount of wasted material.

Multi-objective linear programming is an extension of linear programming. It was introduced by Chaudhuri and De, (2011). The concept of multi-objective programming combined with fractional programming is an interesting area of research which incorporates many production planning applications (Luhandjula, 1984). In contrast to the single objective fractional programming (FP), multi objective fractional programming (MOFP) has not been extensively discussed (Rezaei and Davoodi, 2011), and in most of the MOFP approaches, the problems are converted into single objective FP problems and then solved, employing the method of Charnes and Cooper (1961) or Bitran and Novaes (1973). Most of these methodologies are computationally burdensome (Dutta et al., 1992). To overcome the computational difficulties of using conventional FP approaches to MOFP problems, the theory of fuzzy sets has been introduced in the field of FP (Pal et al., 2003). In particular, research into the

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application of fuzzy set theory in the area of production has been very successful (Wong and Vincents, 2011) and it represents an attractive tool to support the production planning (Dubois et al., 2002; Mula et al., 2010). Linguistic variable approach of Zadeh to fuzzified multi-objective linear fractional programming (FMOLFP) problem has been proposed by Luhandjula (1984). Luhandjula used linguistic approach to solve multi-objective linear fractional programming problem (MOLFPP) by introducing linguistic variables to represent linguistic aspirations of the decision maker. After representing imprecise aspirations of the decision maker by structured linguistic variables or converting the original problem via approximations or change of variables into a multiple objective linear programs, techniques of fuzzy linear programming has been used to reach a satisfactory solution. Dutta et al. (1992) reconsidered the problem of Luhandjula (1984) and pointed out some fallacies. One drawback of this approach is that the aggregation of membership functions is done with a compensatory operator which does not guarantee the efficiency of the optimal solution. Fallacies arise due to the fact that the transformation used in Luhandjula (1984) to transform the original problem fails to establish a one to one correspondence between the original feasible region and the transferred feasible region. Therefore, the original MOLFPP and the transformed MOLFPP are not equivalent (Dutta et al., 1992). Dutta et al. (1992) modified the linguistic approach of Luhandjula (1984) to solve MOLFPP. By constructing the desirable membership functions which combines the linguistic aspirations and also taking the view that the ratios (objective functions) should be close to the maximum value (maximum value of the numerator/minimum value of denominator), they proposed a "simple additive weighting" (SAW) model of MOLFPP in which the decision maker puts relative importance among the proximities by giving weights to the membership functions. Other approaches in this area have also been investigated (Sakawa and Kato, 1988). Thakr et al. (2009) provided a method to solve Fuzzy Linear Programming Problem (FLPP) where both the coefficient matrix of the constraints and cost coefficient are fuzzy in nature. Each problem, first converted into equivalent crisp linear problems, are then solved by standard optimization methods.

In the past few years, adaptation of existing multi objective programming methodologies (Romero, 1986) to fuzzy programming problems has been studied (Zimmermann, 1978, 1987). Among all the approaches developed so far, Goal Programming (GP) has appeared as a robust tool for solving multi objective fuzzy programming problems in (Petrovic and Akoz, 2007; Pal et al., 2003). In the GP model formulation of the problem, first, the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them.

Then, achievement of the highest membership value (unity) to the possible extent of each of the fuzzy goal is considered. In the solution process, the under- and over-deviational variables of the membership goals associated with the fuzzy goals are introduced to transform the proposed model into an equivalent Linear Goal Programming (LGP) model to solve the problem efficiently in the decision situation.

The purpose of this paper is to describe, model and solve optimally, a problem of production planning in a real-life production system. In this paper, production planning in a wood and metal company have been modeled as a fuzzy linear fractional multiple goal programming problem with two fuzzy fractional goals. The problem, with some assumptions, is tailor-made to apply the fuzzy model of Dutta. Further in this paper, the fuzzy linear fractional goal programming model of Pal has been used to solve the problem of production planning. Then, results of both models are compared.

This paper provides an overview of the Dutta and Pal methods. The production planning problem in Khavar-E-Miane Furniture Co has also been presented. The modeling of the production planning problem as a MOLFPP has been described and the results of the computational experiment used to validate the procedure was presented. Finally, conclusions are made.

OVERVIEW OF THE FUZZY METHODS TO SOLVE MOLFPP

The general format of a classical multi objective linear fractional programming problem can be stated as:

$$\text{Optimize} \quad Z_k(X) = \frac{c_k X + \alpha_k}{d_k X + \beta_k}, \quad k = 1,2,...,K$$

$$\text{subject to} \quad X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b,\ X \ge 0,\ b \in R^m \right\}$$

$c_k, d_k \in R^n; \alpha_k, \beta_k$ are constants and $S \neq \phi$. It is customary to assume that $d_k X + \beta_k > 0, \forall X \in S.$

Dutta's method

The application of fuzzy set theory to this kind of problems is of recent origin (Dutta et al., 1992). Dutta et al. (1992) had improved upon Luhandjula's approach to solve such problems. Both workers used linguistic variables in their studies. The solution procedure according to Dutta et al. (1992) is as follows:

Step 1: Calculate for all i :

$$N_k^0 = \underset{x \in X}{Max}\, N_k(x) \quad where\ N_k(x) = c_k X + \alpha_k,$$
$$D_k^0 = \underset{x \in X}{Min}\, D_k(x) \quad where\ D_k(x) = d_k X + \beta_i. \qquad (2)$$

where N_k^0 and D_k^0, $\forall k=\overline{1,p}$ represent the maximal value of nominator $N_k(x)$ and the minimal value of denominator $D_k(x)$ on the set X.

Step 2: (a) choose thresholds p_k and s_k indicating the appropriate closeness to N_k^0 and D_k^0 for all k; (b) construct the membership function of goal k as given by Equations 3 and 4:

$$\mu_j^{N_k}=\begin{cases}0 & if \quad N_k(x)<p_k^j,\\ \dfrac{N_k(x)-p_k^j}{N_k^0-p_k^j} & if \quad p_k^j\le N_k(x)\le N_k^0,\\ 0 & if \quad N_k(x)>N_k^0.\end{cases} \tag{3}$$

$$\mu_j^{D_k}=\begin{cases}0 & if \quad D_k(x)>S_k^j,\\ \dfrac{S_k^j-D_k(x)}{S_k^j-D_k^0} & if \quad D_k^0\le D_k(x)\le S_k^j,\\ 0 & if \quad D_k(x)<D_k^0.\end{cases} \tag{4}$$

Step 3: Construct the problem (5) by giving appropriate (normalized) weights and solve it by any simplex procedure:

$$\begin{aligned}&Max\ \ v(\mu)=\sum\left(w_k\mu_k^{N_k}+w_k'\mu_k^{D_k}\right)\\ &s.t:\quad AX\le b,\ 0\le\mu_k^{N_k}\le 1\ ,\ 0\le\mu_k^{D_k}\le 1\ ,\ k=1,2,\dots,K\\ &X\ge 0\ ,\ \ \sum_{k=1}^{K}(w_k+w_k')=1\end{aligned} \tag{5}$$

where w_k and w_k', both positive, are the weights indicating the relative importance put by the decision maker among the proximities.

Pal's method

In MOFP, if an imprecise aspiration level is introduced to each of the objectives, then, these fuzzy objectives are termed as fuzzy goals. Let g_k be the aspiration level assigned to the k th objective $Z_k(X)$. Then, the fuzzy goals appear as:

$$\begin{aligned}&(a)\ Z_k(X)\underset{\sim}{>}g_k\ \ (for\ \max imizing\ \ Z_k(X));\\ &(b)\ Z_k(X)\underset{\sim}{<}g_k\ \ (for\ \min imizing\ \ Z_k(X)),\end{aligned} \tag{6}$$

where $\underset{\sim}{>}$ and $\underset{\sim}{<}$ Indicate the fuzziness of the aspiration levels, and is to be understood as "essentially more than" and "essentially less than" in the sense of Zimmermann (1978). Hence, the fuzzy linear fractional goal programming can be stated as follows:

$$\begin{aligned}&Find\ \ X\ \ so\ as\ to\\ &satisfy\ \ \ Z_k(X)\underset{\sim}{>}g_k,\quad k=1,2,\dots,k_1\\ &\qquad\qquad Z_k(X)\underset{\sim}{<}g_k,\quad k=k_1+1,\dots,K\\ &subject\ to\ \ AX\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b\\ &\qquad\qquad X\ge 0\end{aligned} \tag{7}$$

Now, in the field of fuzzy programming, the fuzzy goals are characterized by their associated membership functions. The membership function μ_k for the k th fuzzy goal $Z_k\underset{\sim}{>}g_k$ can be expressed algebraically according to Tiwari et al. (1987) as:

$$\mu_k(X)=\begin{cases}1 & if\ \ Z_k(X)\ge g_k,\\ \dfrac{Z_k(X)-l_k}{g_k-l_k} & if\ \ l_k\le Z_k(X)\le g_k,\\ 0 & if\ Z_k(X)\le l_k\end{cases} \tag{8}$$

where l_k is the lower tolerance limit for the k th fuzzy goal. On the other hand, the membership function μ_k for the k th fuzzy goal $Z_k\underset{\sim}{<}g_k$ can be defined as:

$$\mu_k(X)=\begin{cases}1 & if\ \ Z_k(X)\le g_k,\\ \dfrac{u_k-Z_k(X)}{u_k-g_k} & if\ \ g_k\le Z_k(X)\le u_k,\\ 0 & if\ Z_k(X)\ge u_k\end{cases} \tag{9}$$

Where μ_k is the upper tolerance limit.

Now, in a fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible is actually represented by the possible achievement of their respective membership values to the highest degree.

Regarding this aspect of fuzzy programming problems, a GP approach seems to be the most appropriate for the problem considered in this paper.

Goal programming formulation

In fuzzy programming approaches, the highest degree of membership function is 1. So, as in Mohamed (1997), for the defined membership functions in (8) and (9), the flexible membership goals with the aspired level 1 can be presented as:

$$\frac{Z_k(X)-l_k}{g_k-l_k}+d_k^- - d_k^+ = 1, \qquad (10)$$

$$\frac{u_k-Z_k(X)}{u_k-g_k}+d_k^- - d_k^+ = 1. \qquad (11)$$

where $d_k^-(\geq 0)$ and $d_k^+(\geq 0)$ with $d_k^- d_k^+ = 0$ represent the under- and over-deviations, respectively, from the aspired levels.

In conventional GP, the under- and/or over-deviational variables are included in the achievement function for minimizing them and that depends upon the type of the objective functions to be optimized.

In this approach, only the under-deviational variable d_k^- is required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value.

Now, it can be easily realized that the membership goals in (10) and (11) are inherently nonlinear in nature and this may create computational difficulties in the solution process. To avoid such problems, a linearization procedure is presented in thus.

Linearization of membership goals

The k th membership goal in (10) can be presented as:

$$L_k Z_k(X) - L_k l_k + d_k^- - d_k^+ = 1, \text{ where } L_k = \frac{1}{g_k - l_k}$$

Introducing the expression of $Z_k(X)$ from (1), the afore goal can be presented as:

$$L_k(c_k X + \alpha_k) + d_k^-(d_k X + \beta_k) - d_k^+(d_k X + \beta_k) = L_k'(d_k X + \beta_k)$$

$$C_k X + d_k^-(d_k X + \beta_k) - d_k^+(d_k X + \beta_k) = G_k, \qquad (12)$$

Where $C_k = L_k c_k - L_k' d_k$, $G_k = L_k' \beta_k - L_k \alpha_k$.

Similar goal expressions for the membership goal in (11) can also be obtained. However, for model simplification, the expression in (12) can be considered as a general form of goal expression for any type of the stated membership goals. Now, using the method of variable change as presented by Kornbluth and Steuer (1981), the goal expression in (12) can be linearized as follows:

Letting $D_k^+ = d_k^+(d_k X + \beta_k)$, $D_k^- = d_k^-(d_k X + \beta_k)$, the linear form of the expression in (12) is obtained as:

$$C_k X + D_k^- - D_k^+ = G_k \qquad (13)$$

With $D_k^-, D_k^+ \geq 0$, $D_k^- \; D_k^+ = 0$ since $d_k X + \beta_k > 0, d_k^-, d_k^+ \geq 0$.

Now, in making decision, minimization of d_k^- means minimization of $D_k^- / (d_k X + \beta_k)$, which is also a non-linear one. It may be noted that when a membership goal is fully achieved, $d_k^- = 0$ and when its achievement is zero, $d_k^- = 1$ are found in the solution. So, involvement of $d_k^- \leq 1$ in the solution leads to impose the following constraint to the model of the problem:

$$\frac{D_k^-}{d_k X + \beta_k} \leq 1$$

that is,

$$-d_k X + D_k^- \leq \beta_k$$

Here, on the basis of previous discussion, it may be pointed out that any such constraint corresponding to d_k^+ does not arise in the model formulation. Now, if the most widely used and simplest version of GP (that is minsum GP) is introduced to formulate the model of the problem under consideration, then the GP model formulation becomes:

$$\begin{aligned}
&\textit{Find } X \textit{ so as to} \\
&\textit{Minimize} \quad Z = \sum_{k=1}^{K} w_k^- D_k^- \\
&\textit{and satisfy} \quad C_k X + D_k^- - D_k^+ = G_k \\
&\textit{subject to } AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b \\
&\textit{and} \quad -d_k X + D_k^- \leq \beta_k, \\
&\qquad X \geq 0,\; D_k^-, D_k^+ \geq 0,\; k = 1, 2, \ldots, K.
\end{aligned} \qquad (14)$$

where Z represents the fuzzy achievement function consisting of the weighted under- deviational variables, where the numerical weights $w_k^-(\geq 0)$, $k = 1, 2, ..., k$ represent the relative importance of achieving the aspired levels of the respective fuzzy goals subject to the

Table 1. The brief production processes of wood and metal industry in Khavar-Miane furniture company.

Process (unit)	Object	Product (output)	Next workshop
Internal machine working	Setting square the wood, cutting in favourite length, cutting in favourable pieces, wood pieces, drilling and doubling the wood pieces, finishing and frezing	Wood pieces inside the furniture	Internal assembly
Internal assembly	Frezing (instrumenting), finishing and delicacy of internal pieces of the furniture, assemble of the furniture's internal pieces	Internal hank	Preparing the sub-surface
Preparing sub-surface	Cutting the cotton and sponge down, sponging	Internal hank covered by sponge	Covering
Covering	Springing and covering	Internal section of furniture	Final assembly
Cutting and swinging	Cutting the material and textile in required sizes, swinging the textile in needed forms	Furniture coverage	Covering
External machine working	Setting square the wood, cutting in favourite length, cutting in favorable pieces, wood pieces, drilling and doubling the wood pieces, finishing and frezing	Wooden pieces outside the furniture	External assembly
External assembly	Nailing the external corners, assemble of the external pieces.	External hank	Painting
Painting	Painting the external pieces of the furniture	External section of the furniture	Final assembly
Final assembly	Assemble of internal and external pieces, preparing the product for shipment	Various kinds of furniture	Shipment and marketing

constraints set in the decision situation. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed (1997) can be used to assign the values to $w_k^-,\ k = 1,2,...,k$. In the present formulation, w_k^- is determined as:

$$w_k^- = \begin{cases} \dfrac{1}{g_k - l_k} & for \max functiom \\ \dfrac{1}{u_k - g_k} & for \min functiom \end{cases} \quad (15)$$

The minsum GP method can then be used to solve the problem in (14).

METHODOLOGY

In this study, the mathematical methods have been used. Interview method has been applied to collect data related to benefit, cost, production capacity demand and inventory and the survey for modeling the company's production system. Generally, there are six necessary steps to apply the research model: definition of the problem, classification, modeling or formulating, solving, analysis of the sensitivity and validity of the model, and implementing. The same is true in the present research and it contained all aforementioned steps.

Wood and metal production system in a glance

The main raw materials of the wood and metal industry are: lumber, timber, textile, etc. Table 1 shows the brief production processes of this industry in Khavar-Miane Furniture Company. Also, we show the wood and metal industry's production process as in Figure 1.

Modeling of the production planning in Khavar-Miane wood and metal co as a MOLFPP

So far, we have got the production process of Khavar-Miane Company; now, we are going to introduce the company's production planning mathematical model. For the ease of modeling the production, each process mentioned earlier has been considered as a production workshop. In this model, one must note the chronology of the production processes and one to one or some to one relations among all or some unites and production activities. Therefore, the proper production planning model in such systems is multi-process or multi-product one.

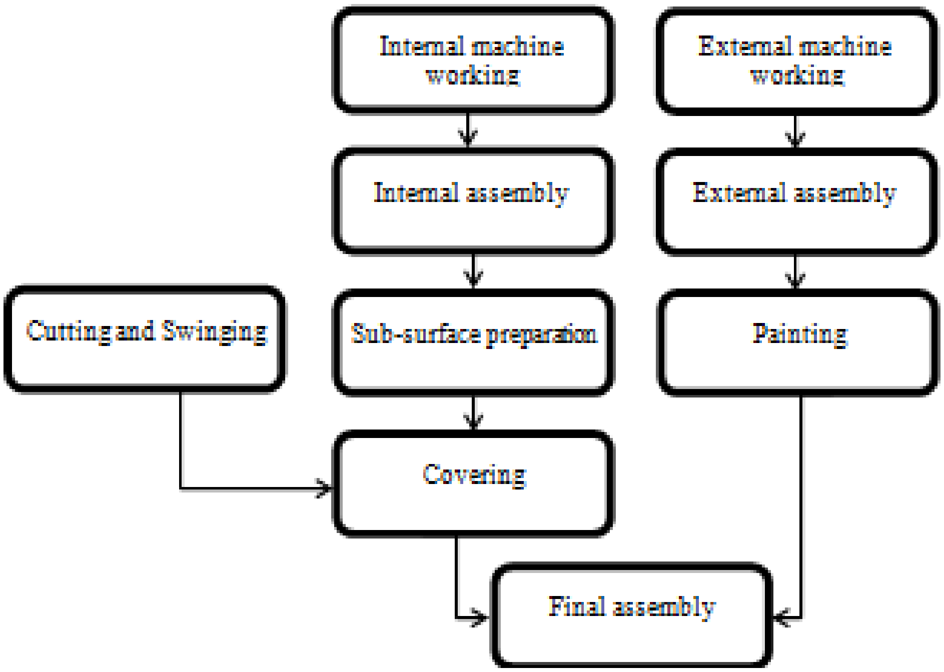

Figure 1. Wood and metal industry's production process.

Table 2. Characteristics of the decision variables.

Row	Description	Index
1	The number of production and assembly workshops in the factory	$i = 1, 2, \ldots, 9$
2	The type of manufactured furniture	$K = 1, 2, \ldots, 10$
3	Period of production	$T = 1, 2, \ldots, 60$

Presumptions applied in the model

Presumptions applied as the bases of the Khavar-Miane Company's production planning fractional mathematical model are as follows:

i. The model is a multi-product one.
ii. The model has been designed for day-periods and generally for a 2-month period.
iii. Supplies have been predicted for 10 kinds of furniture in all workshops (except workshop no. 9) and total calculated 80 kinds, at the beginning of the production planning period.
iv. Since the production of each workshop is based on the furniture, that is, when the workshop produces furniture it requires all components of a set of it, all variables and parameters unit are assumed one furniture set daily.

It must be noted that each workshop receives its input from the other workshop's output, then it processes and conducts some operations on the furniture and handed it over the next one; this procedures are repeated in all workshops but in 1, 5 and 6 workshops. Further, various components of this model will be introduced including decision variables, restrictions, parameters and the model's object functions.

Characteristics of the decision variables

Tables 2 and 3 shows the decision variables based on the defined characteristics.

Model's parameters

All the model's parameters have been provided in Table 4, including the workshops capacities, each furniture demand amount, benefit of a furniture set, the furniture cost. Later, a few of them will be entered in the model and solved in lingo software data aiming at solving the model.

System restrictions

Tables 5 and 6 reveal the system restrictions (four groups) related to capacity, balance, inventory at the end of the period and satisfying the orders. The balance restrictions have been specified in both workshops using dotted lines. The products at this stage are daily shipped and carried to sale-office in Tehran. So, the end-period inventory for the workshop 9 at *tth* day is considered as zero; also, with regard to the variables unit, that is, furniture set which is an integer, we may write:

Table 3. Decision variables.

Row	Description	Variable
1	The number of the k th kind product in t th period in i th workshop	x_{ikt}
2	Ending inventory of k th kind product in t th period in internal machine working	I_{ikt}
3	$D_k^+ = d_k^+(d_k X + \beta_k)$	D_k^+
4	$D_k^- = d_k^-(d_k X + \beta_k)$	D_k^-

Table 4. Model's parameters.

Row	Description	Parameters
1	Maximum capacity of t th day in i th workshop	$Ccap_{it}$
2	Minimum demand of k th kind furniture in m th month	D_{km}
3	Production Increment of an unit of k th kind furniture in i th period	c_{kt}
4	Production cost an unit of k th kind furniture in i th period	v_{kt}
5	The fix workshop cost during the two-month period	f_o
6	Confidence Store at the workshops for all furniture	$I_\circ$
7	Ideal dedicated levels	g_i
8	Lower limit of i th fuzzy object	l_i
9	The linguistic variables describing the proximity of $N_i(x)$ to N_i°	p_i
10	The linguistic variables describing the proximity of $D_i(x)$ to D_i°	s_i

$$x_{ikt}, I_{ikt} \geq 0, \text{int} \qquad i=1,2,\ldots,9 \quad k=1,2,\ldots,10 \quad , \quad t=1,2,\ldots,60$$

The model's object functions

The main goal of the company is maximizing the benefit. We are to express it as productivity, we present the company's goal as fraction (10), where the numerator is the benefit of all productions and the denominator is the total costs (the cost of total productions + the fix workshop cost):

$$MaxZ_1(x) = \frac{\sum_{k=1}^{K}\sum_{t=1}^{T} c_{kt} x_{9kt}}{\sum_{k=1}^{K}\sum_{t=1}^{T} v_{kt} x_{9kt} + f_o} \tag{16}$$

Moreover, each workshop has an exclusive store and product components are accumulated at the workshop corner; despite the vast space, no capacity restriction has been considered in this regard. Since the extra ordinary inventory imposes the capital ression on the company, reduction of this parameter is vital. On the other hand, since there is no guarantee for extra-production in other workshops, extra production is possible and a result of high inventory and capital recession are possible. Since we do not access precise information about the capital recession cost, the fraction 11 attempts to prevent production increase and reduce the workshops inventory:

$$MaxZ_2(x) = \frac{\sum_{k=1}^{K}\sum_{t=1}^{T} x_{9kt}}{\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} I_{ikt} + I_\circ} \tag{17}$$

Designing the production planning fuzzy-linear mathematical model for Khavar-Miane company

Here, we attempt to solve and compare the company's model using two different models of Pal and Dutta, aiming at providing the possibility of measuring the modeling validity and the applied methods in solving them.

Linearization of the fractional model based on the Pal et al.'s (2003) model

In Pal et al.'s (2003) model formulation of the problem, first, the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then, achievement of the highest membership value (unity) to the possible extent of each of the fuzzy goals is considered. In the solution process, the under- and over-deviational variables of the membership goals associated with the fuzzy goals are introduced to transform the proposed model into an equivalent LGP model to solve the problem efficiently in the decision situation. First, the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them as follows:

Table 5. System restrictions.

Row	Balance constraints	Constraint	Workshop
1	$\sum_{t=1}^{T} x_{1kt} - \sum_{t=1}^{T} x_{2kt} = I_{1kt}$	$\sum_{k=1}^{K} x_{1kt} \le Ccap_{1t}$	Internal machine working
2	$x_{2kt} \le I_{1kt} + x_{1kt}$	$\sum_{k=1}^{K} x_{2kt} \le Ccap_{2t}$	Internal assembly
3	$\sum_{t=1}^{T} x_{2kt} - \sum_{t=1}^{T} x_{3kt} = I_{2kt}$ $x_{3kt} \le I_{2kt} + x_{2kt}$	$\sum_{k=1}^{K} x_{3kt} \le Ccap_{3t}$	Sub-surface preparation
4	$\sum_{t=1}^{T} x_{3kt} - \sum_{t=1}^{T} x_{4kt} = I_{3kt}$ $x_{4kt} \le I_{3kt} + x_{3kt}$	$\sum_{k=1}^{K} x_{4kt} \le Ccap_{4t}$	Covering
5	$\sum_{t=1}^{T} x_{5kt} - \sum_{t=1}^{T} x_{4kt} = I_{5kt}$ $x_{4kt} \le I_{5kt} + x_{5kt}$	$\sum_{k=1}^{K} x_{5kt} \le Ccap_{5t}$	Cutting and swinging
6	$\sum_{t=1}^{T} x_{6kt} - \sum_{t=1}^{T} x_{7kt} = I_{6kt}$	$\sum_{k=1}^{K} x_{6kt} \le Ccap_{6t}$	External machine working
7	$x_{7kt} \le I_{6kt} + x_{6kt}$	$\sum_{k=1}^{K} x_{7kt} \le Ccap_{7t}$	External assembly
8	$\sum_{t=1}^{T} x_{7kt} - \sum_{t=1}^{T} x_{8kt} = I_{7kt}$ $x_{8kt} \le I_{7kt} + x_{7kt}$	$\sum_{k=1}^{K} x_{8kt} \le Ccap_{8t}$	Painting

Table 6. Constraints of final assembly workshop.

Row	Balance constraint	Demand constraint	Capacity constraint	Workshop
9	$\sum_{t=1}^{T} x_{4kt} - \sum_{t=1}^{T} x_{9kt} = I_{4kt}$ $x_{9kt} \le I_{4kt} + x_{4kt}$ $\sum_{t=1}^{T} x_{8kt} - \sum_{t=1}^{T} x_{9kt} = I_{8kt}$ $x_{9kt} \le I_{8kt} + x_{8kt}$	$\sum_{t=1}^{T} x_{9kt} \ge D_{km}$	$\sum_{k=1}^{K} x_{9kt} \le Ccap_{9t}$	Covering Final assembly Painting

$$\mu_2 = \frac{\dfrac{\sum_{k=1}^{K}\sum_{t=1}^{T} x_{9kt}}{\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} I_{ikt} + I_{\circ}} - l_2}{g_2 - l_2} \qquad l_2 \le Z_2(x) \le g_2 \tag{19}$$

Then, we change the above fuzzy objects into the goal objects accounting to the Pal approach and achieve Equations 20 and 21:

$$\sum_{k=1}^{K}\sum_{t=1}^{T}(c_{kt} x_{9kt}) - g_1\left(\sum_{k=1}^{K}\sum_{t=1}^{T} v_{kt} x_{9kt} + f_o\right) - D_1^{+} + D_1^{-} = 0 \tag{20}$$

$$\sum_{k=1}^{K}\sum_{t=1}^{T} x_{9kt} - g_2\left(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} I_{ikt} + I_{\circ}\right) - D_2^{+} + D_2^{-} = 0 \tag{21}$$

Based on the aforementioned equations, the system restrictions of Equations 22 and 23 will be added to the model:

$$-(g_1 - l_1)\left(\sum_{k=1}^{K}\sum_{t=1}^{T} v_{kt} x_{5kt} + f_0\right) + D_1^{-} \le 0 \tag{22}$$

$$-(g_2 - l_2)\left(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} I_{ikt} + I_{\circ}\right) + D_2^{-} \le 0 \tag{23}$$

Since this model is aimed at maximizing both membership functions, we can minimize the sum of the weighted negative deviations from membership functions. Therefore, the fuzzy mathematical model of production planning in Khavar-Miane Company based on Pal et al.'s (2003) model is given in Table 7.

Table 7. Fuzzy fractional model of production planning in Khavare-Miane Furniture Co.

Fuzzy method of Dutta	Fuzzy method of Pal
$Maximize\ v(\mu)=w_1\mu_1+w_2\mu_2+w_3\mu_3+w_4\mu_4$ $such\ that:$ $\sum_{t=1}^{T}x_{1kt}-\sum_{t=1}^{T}x_{2kt}=I_{1kt}$ $x_{2kt}\le I_{1kt}+x_{1kt}$ $\sum_{t=1}^{T}x_{6kt}-\sum_{t=1}^{T}x_{7kt}=I_{6kt}$ $x_{7kt}\le I_{6kt}+x_{6kt}$ $\sum_{t=1}^{T}x_{3kt}-\sum_{t=1}^{T}x_{4kt}=I_{3kt}$ $x_{4kt}\le I_{3kt}+x_{3kt}$ $\sum_{t=1}^{T}x_{5kt}-\sum_{t=1}^{T}x_{4kt}=I_{5kt}$ $x_{4kt}\le I_{5kt}+x_{5kt}$ $\sum_{t=1}^{T}x_{4kt}-\sum_{t=1}^{T}x_{9kt}=I_{4kt}$ $x_{9kt}\le I_{4kt}+x_{4kt}$ $\sum_{t=1}^{T}x_{7kt}-\sum_{t=1}^{T}x_{8kt}=I_{7kt}$ $x_{8kt}\le I_{7kt}+x_{7kt}$ $\sum_{t=1}^{T}x_{8kt}-\sum_{t=1}^{T}x_{9kt}=I_{8kt}$ $x_{9kt}\le I_{8kt}+x_{8kt}$ $\sum_{t=1}^{T}x_{9kt}\ge D_{km}$ $\sum_{k=1}^{K}x_{ikt}\le Ccap_{it}$ $\sum_{t=1}^{T}I_{9kt}=0$ $\mu_1=\frac{\sum_{k=1}^{K}\sum_{t=1}^{T}c_{kt}x_{kt}-p_1}{N_1^{\circ}-p_1}$ $\mu_2=\frac{\sum_{k=1}^{K}\sum_{t=1}^{T}x_{kt}-p_2}{N_2^{\circ}-p_2}$ $\mu_3=\frac{S_1-(\sum_{k=1}^{K}\sum_{t=1}^{T}v_{kt}x_{kt}+f_{\circ})}{S_1-D_1^{\circ}}$ $\mu_4=\frac{S_2-\left(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}I_{ikt}+I_{\circ}\right)}{S_2-D_2^{\circ}}$ $X\ge 0,\ 0\le\mu_i\le 1\ \ i=1,2,3,4$	$find\ X\ so\ as\ to:$ $Minimize\ Z=W_1^{-}D_1^{-}+W_2^{-}D_2^{-}$ $satisfy:$ $\sum_{k=1}^{K}\sum_{t=1}^{T}(c_{kt}x_{9kt})-g_1(\sum_{k=1}^{K}\sum_{t=1}^{T}v_{kt}x_{9kt}+f_o)-D_1^{+}+D_1^{-}=0$ $\sum_{k=1}^{K}\sum_{t=1}^{T}x_{9kt}-g_2\left(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}I_{ikt}+I_{\circ}\right)-D_2^{+}+D_2^{-}=0$ $subject\ to:$ $\sum_{t=1}^{T}x_{1kt}-\sum_{t=1}^{T}x_{2kt}=I_{1kt}$ $x_{2kt}\le I_{1kt}+x_{1kt}$ $\sum_{t=1}^{T}x_{6kt}-\sum_{t=1}^{T}x_{7kt}=I_{6kt}$ $x_{7kt}\le I_{6kt}+x_{6kt}$ $\sum_{t=1}^{T}x_{2kt}-\sum_{t=1}^{T}x_{3kt}=I_{2kt}$ $x_{3kt}\le I_{2kt}+x_{2kt}$ $\sum_{t=1}^{T}x_{3kt}-\sum_{t=1}^{T}x_{4kt}=I_{3kt}$ $x_{4kt}\le I_{3kt}+x_{3kt}$ $\sum_{t=1}^{T}x_{5kt}-\sum_{t=1}^{T}x_{4kt}=I_{5kt}$ $x_{4kt}\le I_{5kt}+x_{5kt}$ $\sum_{t=1}^{T}x_{4kt}-\sum_{t=1}^{T}x_{9kt}=I_{4kt}$ $x_{9kt}\le I_{4kt}+x_{4kt}$ $\sum_{t=1}^{T}x_{7kt}-\sum_{t=1}^{T}x_{8kt}=I_{7kt}$ $x_{8kt}\le I_{7kt}+x_{7kt}$ $\sum_{t=1}^{T}x_{8kt}-\sum_{t=1}^{T}x_{9kt}=I_{8kt}$ $x_{9kt}\le I_{8kt}+x_{8kt}$ $\sum_{t=1}^{T}x_{9kt}\ge D_{km}$ $\sum_{k=1}^{K}x_{ikt}\le Ccap_{it}$ $\sum_{t=1}^{T}I_{9kt}=0$ and $-(g_1-l_1)(\sum_{k=1}^{K}\sum_{t=1}^{T}v_{kt}x_{5kt}+f_0)+D_1^{-}\le 0$ $-(g_2-l_2)(\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}I_{ikt}+I_{\circ})+D_2^{-}\le 0$ $D_1^{-},D_2^{-},D_1^{+},D_2^{+}\ge 0$ $x_{ikt},I_{ikt}\ge 0,\text{int}\quad k=1,2,\ldots,10\ ,\ t=1,2,\ldots,60\ ,\ m=1,2.$

Table 8. Outputs of Pal and Dutta's model associated with object values.

Variable	Profit (F_1)	Cost (F_2)	Production rate (F_3)	Inventory (F_4)
Values (RS)		0.2377028E+10		80.00000
Object values	0.6410434E+09	$\frac{F_1}{F_2}=.269$	6000.000	$\frac{F_3}{F_4}=75$
Membership degrees		$\mu_1=\frac{.269-.1}{.241}=.701$		$\mu_2=\frac{75-2.234}{72.766}=1$

Linearization the fraction model based on the Dutta et al.'s (1992) model

In this model, we define membership function for the numerator and denominators of the two fractional goals identified. Once membership functions are defined, Dutta et al. (1992) give their simple additive weighting model to optimize functions. Table 7 shows the company's production planning fuzzy mathematical model based on the Dutta et al.'s (1992) model. Of course, in this method, the weight of the numerator and denominator goals (w_i) is achieved based on the experts' opinion.

RESULTS AND DISCUSSION

Outputs of Pal and Dutta's models

The Pal model contains 10268 decision variables (of which 10200 variables are integers) and 10173 restrictions. Dutta model contains 10268 decision variables (of which 10200 variable are integers) and 10777 restrictions. Both models were solved using the Lingo software. The values are similar in both model and the results have been presented in Table 8.

Khavar-Miane Company's benefit during the last 2 months was 618046940 and 2418491500 respectively. The results of the model suggest that the company's benefit increased about 22996460 and the total costs decreased about 41463500 unite.

Comparison of the objects in the two models

As seen, the values of μ_1 and μ_2 are equal in both models; thus, they provide us with the same results.

Values of the decision variables

The results of Pal and Dutta's models for the workshops showed that the decision variables are different in models; so the model's optimum answer related to the company is a multiple optimum one.

Conclusions

In this research, we studied the wood and metal industry and its dominating condition of decision-making beside the library studies. In this regard, a mathematical method was designed in proportion to simple space and the real needs which is in form of a multi-product, multi-stage and multi-period production system. The research's mathematical model is a multi object one; so, applying fuzzy theories and the Pal and the Dutta fuzzy methods in solving the multi-object model, we may overcome the calculation difficulties rooted in variable changes in previous methods. The values of the decision variables have been presented in attachments 1 and 2; the answer of the model is a multiple optimum. It is worth noting that the results of the object values are equal in both fuzzy models; also, this model has a specific situation with multiple optimums, the values of the decision variables differ in both models. Applying this model, managers achieve multiple objects and will be able to plan production in their companies so that production combination facilitates the promotion of the productivity. Also, other companies' manager may use this model with regard to their goals; it is enough to make some minor changes in it.

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