The purpose of this research is to analyze the performances of small and medium enterprises (SMEs) in the aftermath of the recent global crisis and the new problems associated to a weak banking system. SMEs from emerging economies have experienced critical situations both in the global crisis (2008-2010), but also in the context of the European sovereign - debt crisis (2010-2012), both characterized by multiple shocks for all type of businesses. In the case of SMEs the interest is to understand the mechanisms that influence the robustness and the resilience in turbulent periods but also the way of thinking the architecture of future investment strategies in order to realize an optimal development in the wake of crises. In addition, we should mention the impact of these turbulences on the psychology of owners/ investors with a new attitude/ preference for another set of businesses, characterized by lower risk. In this case it has become essential to observe a set of basic values such as profit margin per sold item and the market share. It is essential to maintain the market share in critical and turbulent period, because it is registered a decreasing demand for products and services and, at the same time, a fluctuation of prices. In the literature it is proved that investment strategies should consider a lot of critical technical, but also non-technical elements like the impact of the retail price on the supply in the context of the dynamics of the market, the quality and the specific features of the product in the context of the technological progress, financing solutions adapted to the product/ market. The external environment after crisis is essentially changed and the way of thinking strategies is also different. To better understand this new paradigm, very important for the strategy of SMEs, a simple but efficient quantitative methodology for understanding the dynamics of investments in turbulent periods is also presented.

Key words: small and medium enterprises (SMEs), investment strategy, performances and dynamics of firm, global crisis, critical and turbulent period.

INTRODUCTION

The analysis of small and medium enterprises (SMEs) performances is an important task for providing information on the real situation of SMEs and could offer a quick global dynamic picture of future evolutions, but also it offers effective policy making solutions. SMEs play a significant role in all economies as generator of employment and income, and drivers of innovation and growth, but also are more vulnerable to different type of shocks according to their less diversified portfolio, a weaker financial structure and a lower/no credit rating (Wiesner et al., 2007; Zahra et al., 2007). There is a diversity of SMEs and a critical criterion for development is based on their capacity to optimize the financing strategy (Biezma and San Cristobal, 2006; Hogan and Hutson, 2005) based on a better adaptability and flexibility. Concerning the SMEs, achieving the ability of resistance to impact has become essentially, as well as taking advantage of the favorable moments and opportunities, offered precisely by means of turbulences and the chaos within markets. In the actual context of the recovery after the crisis it is also essential to understand where it is situated the firm in the life cycle of their development and how to use different investment vehicles adapted to the future trends (Figure 1).
In the early stage of development, business angels (seed-financing BA), or angel groups/angel networks could support small projects/business difficult to be financed by other solutions (banking credit, private equity, capital market solutions), but in the actual environment this solution is not a reliable one). Venture capital (VC) could provide superior funding through participation in business risks and it also offers technical and especially managerial support.

The next step is represented by private equity funds (PEF) which represent collective investment vehicles capable to exploit the benefits of the high leverage investments in equity with the contribution of specialized intermediaries/investment funds. BA, VC, PEF signify those vehicles used regardfully in times of turbulences and chaos, since the perception of investors has been changed. Attitude towards investments that have carried risk would abolish the abilities of development, and the competition between markets might revive the entrepreneurial spirit in an innovative way.

The new strategic thinking should be more focused on the dynamics of the performances and this is difficult in the actual global context with uncertainties and high volatility. The global performance can be measured based on short term indicators (financial returns, profitability) and long term indicators (resources for future growth, customer’s satisfaction) measures but this could not offer a global recognized picture of the robustness of SMEs and this is not effective for comparisons (Rentizelas et al., 2007).

The spectrum of such issues should be subject to a new type of analysis, better focused on the capability to model the market share dynamics in different configurations and different macroeconomic environments.

**METHODOLOGY**

**The modelling of the market share dynamics**

Using this model, although simplistic, has proven the advantage of focusing on maintaining the market quota, essential issues within SMEs in critical times of turbulences:

\[ \frac{d}{dt}(CP(t)) = f(PN(t), P(t); CP(t)) \]  

where \( CP(t) \) represents the market share of the firm, \( PN(t) \) the normal price of the item produced by the firm, \( P(t) \) the retail price of the product, a decision variable. In (1) there are considered the following assumptions:

1. The negative derivative condition (the case of absolute monopoly):

\[ f(PN(t), P(t); 1) < 0, \forall p \in \mathbb{R}^+ \]

2. The positive derivative condition (the case of market entry):

\[ f(PN(t), P(t); 0) > 0, \forall p \in \mathbb{R}^+ \]

3. The function of the market share variation increase toward the normal price:

\[ (\forall) p_1, p_2, \text{ with } p_1 > p_2 > 0 \Rightarrow f(p_1, P(t); CP(t)) > f(p_2, P(t); CP(t)) \]

4. The function of the market share variation decrease toward the retail price:

\[ (\forall) p_1, p_2, \text{ with } p_1 > p_2 > 0 \Rightarrow f(PN(t), p_1; CP(t)) < f(PN(t), p_2; CP(t)) \]

As opposed to the market share, the \( f \) function can experience
the following two scenarios:

1. Increasing as opposed to \( CP(t) \Rightarrow \) the “herding” effect occurs.

2. Decreasing as opposed to \( CP(t) \Rightarrow \) the “snob” effect occurs.

The set of objective of the firm dictates the type and the form of the function to be optimised. For example, when the market value is known, a version of the objective function is to maximise sales:

\[
\left( \max_{P(t)} \right) F = \left[ V(t) \cdot P(t) \cdot CP(t) \right] dt
\]

where \( V(t) \cdot \) estimated value of the entire quantity of products on the market at the \( t \) moment, while \( t_0 \) - initial moment and \( T \) - final moment.

The strategy for maintaining the market share in a turbulent/crisis period can be analysed by mixing two action courses the firm can opt for one action, based on technical elements, and another one, based on technical elements, and another one, that includes non technical elements.

The typical impact of the consumer behaviour effects in the market share dynamics

The use of the typical impact of the consumer behaviour is essential in turbulent periods and this information should also be integrated in the process of the design of different architectures of SME development. By demanding a minimum level of instruments in order to provide services himself, in the case of low income households, the consumer has to opt for at least one of the products, in a sufficient amount so as to help him provide services of the adequate quality and quantity. In this case, we have the asymptotic consumer behaviour, where the marginal utility of a product depends on the income level. The comparison between the two optimal problems are shown in equations 3 and 4 and it provides that in the two cases, \( \beta > 0 \), \( i = 1,2 \), \( \alpha > 0 \).

Let now the classical problem:

\[
\begin{align*}
\max & \quad U = x_1^\alpha \cdot x_2^\beta \\
V & = p_1 \cdot x_1 + p_2 \cdot x_2
\end{align*}
\]

\[
\begin{align*}
\max & \quad U = x_1^\alpha \cdot x_2^{f(V)} \\
V & = p_1 \cdot x_1 + p_2 \cdot x_2
\end{align*}
\]

The necessary conditions for the \( f(V) \) function are: continuity, positivity in the field it is defined for and the necessary limit conditions, \((1) \lim_{V \to 0} f(V) \to 0 \) and \((2) \lim_{V \to \infty} f(V) \to \beta \). The function can have a \( V \) threshold, as from which the firm is considering purchasing the second product/item, while below this level the firm is prompting the need to purchase the first product/item.

In a simple example of function can be the benchmark function, \( f(V) = 0, V < V_0 \) in the case of direct transition from substitutability to complementarity, but it can also have a continuous form of the type \( f(V) = \frac{\beta}{1 + \frac{V_0}{V}} \), with \( \gamma > 1 \) and \( \gamma \in \mathbb{R} \). It can be noted that, when \( V < V_0 \), the second product utility exponent becomes \( 0 \), which repositions the maximum towards the quasi-integral allocation of the income in order to obtain product 1. When the income \( V \) increases substantially, \( V >> V_0 \), the value of the \( f(V) \) function significantly approximates \( \beta \), and upon limit, \( \lim_{V \to \infty} f(V) = \beta \), which leads to a balanced structure of the consumption of the two product.

**TOWARD A NEW ARCHITECTURE OF THE INVESTMENT STRATEGY**

The use of the dynamic version of the Jorgenson model could be easily integrated in the designing process for re-configuring the strategy of the firm especially in the case we operate optimal investments (in order to maximize the profit) under equity capital depreciation. The analysis is based on the concept of dynamic models of the firm which are equipped with the constraints of the closed field type for decision variables in the form of: (Prelipcean et al., 2010)

1. Objective function:

\[
\begin{align*}
\max & \quad F(X,Y)
\end{align*}
\]

2. The differential equations system:

\[
\begin{align*}
\dot{X} & = FS(X,Y)
\end{align*}
\]

3. The restriction system:

\[
\begin{align*}
FX_i(X,Y) \leq X \leq FX_i(X,Y) \\
FY_i(X,Y) \leq Y \leq FY_i(X,Y)
\end{align*}
\]

where \( X \) = vector of the state variables, \( Y \) = vector of the decision variables, \( Opt\) = maximum or minimum, \( FS \) = the function of the differential equations system, \( FX_i, FY_i \) = restrictions for the state, decision variables.

The work hypotheses are:

1. The market chosen by the firm to sell its products is supposed to be in perfect competition, which means that
the price of the product \( p \) does not depend on the quantity supplied on the market (invariable).

2. The production function of the firm depends both on the level of workforce \( L(t) \), and on the level of capital \( K(t) \)
\[ \Rightarrow Q(t) = Q(K(t), L(t)) \] with strict growth and concavity properties in relation to both factors.

3. The initial marginal income of any production factor must overcome its marginal cost:
\[ \left\{ \begin{align*}
\frac{\partial (p \cdot Q(K(t), L(t)))}{\partial K(t)} & > c \cdot (i + a) \\
\frac{\partial (p \cdot Q(K(t), L(t)))}{\partial L(t)} & > w
\end{align*} \right. \] (8)

4. Investment funds \( I(t) \) are allocated from sales, at a unitary price \( c \), for wages respectively \( L(t) \) with the average monthly wage \( w \) (both markets for production factors are in perfect competition).

5. The need for investments arises, from the need to recover the depreciated capital at the rate \( a \), also, from the desire to increase the capital as long as it is lucrative.

6. For the appraisal comparison of the resources and income used at different moments in time, an updating approach is increasingly used in dynamic models that include the workforce. Now the model could be expressed in the following setting:

1. The objective function:
\[ \max_{I,L} \int_0^\infty e^{-\mu t} [p \cdot Q(K(t), L(t)) - w \cdot L(t) - c \cdot I(t)] dt \] (9)

2. The system of differential equations:
\[ \dot{K}(t) = I(t) - a \cdot K(t) \] (10)

3. with the following constraints:
\[ \left\{ \begin{align*}
0 & \leq K(t) \quad | \quad K(0) = K_0 \geq 0 \\
I_{\min} (t) & \leq I(t) \leq I_{\max} (t) \quad | \quad I_{\min} (t) < 0 \\
I_{\max} (t) & > 0
\end{align*} \right. \] (11)

Let now the Hamiltonian of the objective function:
\[ H(K(t), L(t), I(t), \Psi(t), t) = e^{-\mu t} [p \cdot Q(K(t), L(t)) - w \cdot L(t) - c \cdot I(t) + \lambda(t) \cdot (I(t) - a \cdot K(t))] \]

By transforming \( \Psi(t) = \lambda(t) \cdot e^{\mu t} \), the Hamiltonian becomes:
\[ \frac{H(\bullet)}{e^{\mu t}} = [p \cdot Q(K(t), L(t)) - w \cdot L(t) - c \cdot I(t)] + \Psi(t) \cdot (I(t) - a \cdot K(t)) \] (12)

Let
\[ H(\bullet) = H_{\text{ajun}} (\bullet), \]
\[ H_{\text{ajun}} (\bullet) = [p \cdot Q(K(t), L(t)) - w \cdot L(t) - c \cdot I(t)] + \Psi(t) \cdot (I(t) - a \cdot K(t)) \]
and the control variable \( \lambda(t) \) checks the dynamics equation
\[ \dot{\lambda}(t) = \frac{-\partial H(\bullet)}{\partial K}(t), \]
respectively \( \Psi(t) \) checks \( \Psi(t) = I \cdot \Psi(t) - e^{\mu t} \frac{\partial H(\bullet)}{\partial K}(t) \). In order to facilitate noting the formulas, the adjusted Hamiltonian will be used with the symbol \( H(\bullet) \) instead of the symbol \( H_{\text{ajun}} (\bullet) \).

Let define the Lagrangean \( \Lambda \) associated to the problem, by means of the multipliers \( \nu(t) \) associated to the state variable restriction \( (K(t)) \), respectively \( \mu_1 (t), \mu_2 (t) \) for the decision variable \( I(t) \):
\[ \Lambda(K(t), L(t), I(t), \nu(t), \mu_1 (t), \mu_2 (t)) = H(\bullet) + \mu_1 (t) \cdot (I(t) - I_{\min} (t)) + \mu_2 (t) \cdot (I_{\max} (t) - I(t)) + \nu(t) \cdot K(t) \]
\[ = p \cdot Q(K(t), L(t)) - w \cdot L(t) - c \cdot I(t) + \Psi(t) \cdot (I(t) - a \cdot K(t)) + \mu_1 (t) \cdot (I_{\max} (t) - I(t)) + \mu_2 (t) \cdot (I_{\min} (t) - I(t)) + \nu(t) \cdot K(t) \] (13)
The Kuhn-Tucker condition system requires that the partial derivatives of the Lagrangean in relation to the decision variables be null, along with the components related to the restrictions.

\[ \frac{\partial \Lambda (\bullet)}{\partial I}(t) = -c + \Psi(t) + \mu_1 (t) - \mu_2 (t) = 0 \] (14)
\[ \frac{\partial \Lambda (\bullet)}{\partial L}(t) = p \cdot \frac{\partial Q(\bullet)}{\partial L}(t) - w = 0 \] (15)
\[ \dot{\mu}_1 (t) \cdot (I_{\max} (t) - I(t)) = 0 \] (16)
\[ \dot{\mu}_2 (t) \cdot (I_{\max} (t) - I(t)) = 0 \] (17)
\[ \nu(t) \cdot K(t) = 0 \] (18)
\[ \mu_1 (t), \mu_2 (t), \nu(t) \geq 0 \] (19)
with the unknown elements \( I, L, \mu_1, \mu_2 \) and \( \nu \) corresponding to the 5 equations and the variables
\[ K(t) \text{ and } \Psi(t). \]

In the case of equations (16), (17), (19) there are the following possibilities:

1. \( \mu_1 = 0, \mu_2 = 0, \nu = 0 \) 2. \( \mu_1 > 0, \mu_2 = 0, \nu = 0 \)
2. \( \mu_1 = 0, \mu_2 > 0, \nu = 0 \)
3. \( \mu_1 > 0, \mu_2 > 0, \nu = 0 \) 5. \( \mu_1 = 0, \mu_2 = 0, \nu > 0 \)
4. \( \mu_1 > 0, \mu_2 > 0, \nu > 0 \)
6. \( \mu_1 > 0, \mu_2 = 0, \nu > 0 \)
7. \( \mu_1 = 0, \mu_2 > 0, \nu > 0 \) 8. \( \mu_1 > 0, \mu_2 > 0, \nu > 0 \)

The trajectories generated by the 5, 6, 7 and 8 versions are not applicable since they require that the level of capital be null during the entire time interval, which leads to an economic impossibility.

The trajectory generated by equation number 4 generates the following double parity: \( I_{\text{min}} = I(t) = I_{\text{max}}, \) but \( I_{\text{im}}, K_{\text{im}} \) \( \rightarrow \) the trajectory is not applicable.

**Trajectory 1** implies \( I_{\text{min}} < I(t) < I_{\text{max}} \)

and \(-c + \Psi(t) = 0 \Rightarrow \lambda(t) = \frac{c}{e^{at}} \Rightarrow \Psi(t) = c\)

\[ \Rightarrow \begin{cases} 
(i+a)\cdot c - p \cdot \frac{\partial Q(K,L)}{\partial K}(t) = 0 \\
p \cdot \frac{\partial Q(K,L)}{\partial K}(t) = w 
\end{cases} \]

\[ \Rightarrow \frac{\partial Q(K, f(K))}{\partial K}(t) = \left( i + a \right) \cdot c \\
L = f(K) 
\] (20)

The system equation provides as single solution the pair of constant values \((K^*, t^*).\) Since the level of capital is steady, its derivative is null and, therefore, the level of the investment is \(a \cdot K^*.\)

**Trajectory 2** leads to \( I(t) = I_{\text{min}} \Rightarrow K(t) = I_{\text{min}} - a \cdot K(t) \Rightarrow \) by solving the general equation, followed by the particular one \( K(t) = C \cdot e^{-at} + \frac{I_{\text{min}}}{a}. \) The value of the constant \(C\) is determined by means of the initial condition:

\[ K_0 = C \cdot e^{-a0} + \frac{I_{\text{min}}}{a} \Rightarrow K_0 = C + \frac{I_{\text{min}}}{a} \Rightarrow C = K_0 - \frac{I_{\text{min}}}{a}. \]

The trajectory of the capital variable will be:

\[ K(t) = \left( K_0 - \frac{I_{\text{min}}}{a} \right) \cdot e^{-at} + \frac{I_{\text{min}}}{a} \] \quad (21)

Note the downward slope of the capital which, for a long enough period of time would reach negative values (at the limit, when \( t \rightarrow \infty, K(t) \rightarrow \frac{I_{\text{min}}}{a} < 0 \) \( \Rightarrow \) the trajectory cannot be a final trajectory.

**Trajectory 3** is characterized by \( I(t) = I_{\text{max}} \Rightarrow K(t) = I_{\text{max}} - a \cdot K(t) \Rightarrow \) by solving the general equation, followed by the particular one \( K(t) = C \cdot e^{-at} + \frac{I_{\text{max}}}{a}. \)

The solution is similarly to Trajectory 2,

\[ K(t) = \left( K_0 - \frac{I_{\text{max}}}{a} \right) \cdot e^{-at} + \frac{I_{\text{max}}}{a} \]

At least one of these trajectories must be final; hence the transverse condition must be observed, \( \lim_{t \to \infty} \Psi(t) = \omega \) - finite. On the trajectory 1, \( \Psi(t) = c \) (the cost of the investment unit) – finite \( \Rightarrow \) trajectory 1 is a final trajectory. Along with trajectory 2, trajectory 3 cannot be final either, since the result of the differential equation of the adjunct variable \( \Psi(t) \) shows that it appears as \( \Psi(t) = C_1 \cdot e^{(i+a)t} + \frac{f \left( \frac{\partial Q}{\partial K} (t) \right)}{\partial K}. \)

As the derivative of the production function in relation to capital is a finite value (obtained from the strict growth and concavity hypothesis), it can be noted that – in this case \( \lim_{t \to \infty} \Psi(t) = \infty. \)

Trajectories 2 and 3 are not final because the firm cannot exist as a business entity without capital (Trajectory 2), and at some point, under perfect competition and an increasing and strictly concave production function, a maximum level investment no longer generates sufficient production growth in order to reach and exceed the required cost level, even more so since the moment the investment materializes outruns the production growth effect.

Therefore, depending on the level of the initial capital, the firm evolves on Trajectory 2 (when the level of the initial capital is higher than the value \( K^* \) which corresponds to the optimal level on Trajectory 1) or trajectory 3 (when the level of the initial capital is lower than \( K^* \)), and when the \( K^* \) level is reached, trajectory 1 is followed. The moment the trajectories switch is established by the parity between the capital evolution function and the optimal level to be reached, coinciding with the moment the marginal incomes of the
production factors equal the marginal costs:

\[ K(t) = \left( K_0 - \frac{I_{\text{extrem}}}{a} \right) e^{-at} + \frac{K_{\text{extrem}}}{a} K^* = \left( K_0 - \frac{I_{\text{extrem}}}{a} \right) e^{-at} + \frac{K_0^{\text{extrem}}}{a^2} = \frac{K^* - \frac{I_{\text{extrem}}}{a}}{K_0 - \frac{I_{\text{extrem}}}{a}} \]

\[ \Rightarrow -at = \ln \left( \frac{K^* - \frac{I_{\text{extrem}}}{a}}{K_0 - \frac{I_{\text{extrem}}}{a}} \right) \Rightarrow t = -\frac{1}{a} \ln \left( \frac{K^* - \frac{I_{\text{extrem}}}{a}}{K_0 - \frac{I_{\text{extrem}}}{a}} \right) = t_{\text{converg}} \]  

(22)

In conclusion, the switch from trajectory 2 and from trajectory 3

\[ t_{2_1} = -\frac{1}{a} \ln \left( \frac{K^* - \frac{I_{\text{min}}}{a}}{K_0 - \frac{I_{\text{min}}}{a}} \right) \]  

\[ t_{3_1} = -\frac{1}{a} \ln \left( \frac{K^* - \frac{I_{\text{max}}}{a}}{K_0 - \frac{I_{\text{max}}}{a}} \right) \]

The chart with the optimal evolution of the firm in the Jorgenson model can be synthesized in Figures 2 and 3.

CONCLUSIONS AND FUTURE WORK

In this paper a survey of the actual problems of SMEs in the context of the global turbulences is presented. A better understanding of the mechanisms that impact the dynamics of SMEs is essential in the design of investment strategies, especially in the actual context after the global crisis and the European sovereign debt crisis. The research work has demonstrated that, besides the existence of innovative tools drawn up in order to develop the SMEs, the attitude of investors towards risk has been significantly changed. Such climate of distrustfulness determines a reconfiguration of strategies, as well as the way of their establishment. A simple but efficient line would consist in focusing on that architecture able to offer a global picture, thus describing the status and position of the SMEs in a dynamic view. The analysis of the performances of SMEs in turbulent periods by using a simple but intuitive dynamic model offer a new way of thinking the future path of the business, but it also offer a new way of thinking the design of a special protection architecture against different shocks or contagion elements.

The models used in the analysis of the dynamics of the
firm are appropriate in the qualitative and quantitative appraisals, required both in normal and in crisis conditions. The most applied ones include the Jorgenson model, the Lesourne - Leban model, the Ludwig model and the Van Hilten model, with its extensions. The Jorgensen models focuses on designing the strategy of the firm for optimal investments when the capital of the firm depreciates, with the purpose of maximising the income of the company on an infinite time span. Certain extensions of the model are also presented with relevance for SMEs during turbulence or crisis situations. Future work should be more focused on the possibilities to incorporate this type of modelling into the analysis of the impact of self-financing strategies in the context of the recent transformation of the investment attitude for SMEs. The interest is also to analyse how to incorporate new strategies based on investment vehicles or investment funds in the context of maintaining the critical market share in turbulent/crisis periods in order to preserve the existence of the business.

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