African Journal of Business Management Vol. 6(10), pp. 3817-3821, 14 March, 2012
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM11.2623
ISSN 1993-8233 ©2012 Academic Journals

Full Length Research Paper

Replenishment lot sizing with an improved issuing policy and imperfect rework derived without derivatives

Singa Wang Chiu[1], Chia-Ling Chung[1], Kuang-Ku Chen[2] and Huei-Hsin Chang[3]*

[1]Department of Business Administration, Chaoyang University of Technology, Taichung 413, Taiwan.
[2]Department of Accounting, National Changhua University of Education, Changhua, Taiwan.
[3]Department of Finance, Chaoyang University of Technology, Taichung 413, Taiwan.

Accepted 23 November, 2011

This study uses an alternative approach to reexamine a replenishment lot size problem with discontinuous issuing policy and imperfect rework. A straightforward approach in terms of algebraic derivation is proposed instead of conventional method with the need of applying first-order and second-order differentiations to system cost function for proof of convexity before derivation of the optimal lot size. The research result obtained in this study is identical to that in Lee et al. (2011), where they adopted conventional method to solve the problem. The proposed algebraic approach is helpful for practitioners who may have insufficient knowledge of differential calculus to understand with ease such a real life vendor-buyer integrated problem.

Key words: Optimization, production-shipment system, random scrap rate, lot size, multi-distribution, algebraic approach, production planning.

INTRODUCTION

The most economical production lot size was first proposed by Taft (1918) to assist manufacturing firms in minimizing total production costs (it is also known as economic production quantity (EPQ) model). The EPQ model implicitly assumes that all items produced are of perfect quality. However, in real world production settings, due to different factors generation of nonconforming items seems inevitable. For this reason, many studies have been carried out during the past decades, to address the imperfect production and its related issues (Barlow and Proschan, 1965; Mak, 1985; Henig and Gerchak, 1990; Grosfeld-Nir and Gerchak, 2002; Chiu and Chiu, 2006; Jha and Shanker, 2009; Taleizadeh et al., 2010; Lodree et al., 2010; Chiu et al., 2010a-c; Ma et al., 2010; Saha et al., 2010; Sana, 2010; Mehdi et al., 2010; Wazed et al., 2010a-b; Kreng and Tan, 2010; Banerjee and Sharma, 2010; Chiu et al., 2011a).

Another unrealistic assumption of classic EPQ model is the continuous inventory issuing policy, for in vendor-buyer integrated production-shipment system, periodic deliveries instead of continuous policy is often used. Research has since been focused on addressing issues of various aspects of multi-deliveries in supply chain optimization (Goyal, 1977; Banerjee, 1986; Hahm and Yano, 1992; Viswanathan, 1998; Swenseth and Godfrey, 2002; Diponegoro and Sarker, 2006; Kim et al., 2008; Abolhasanpour et al., 2009; Chiu et al., 2009; Chen et al., 2010; Ye and Xu, 2010; Wong, 2010; Hsieh et al., 2010; Chiu et al., 2011b; Chen et al., 2011; Lee et al., 2011).

Lee et al. (2011) investigated the optimal replenishment lot size for a vendor-buyer integrated system with discontinuous issuing policy and imperfect rework. They employed the differential calculus to prove the convexity and derive the optimal production batch size for such a specific problem. Grubbström and Erdem (1999) presented an algebraic approach to solve the economic order quantity (EOQ) model with backlogging, without reference to the use of derivatives. Other studies that have applied the same (or similar) method include Cheng and Ting (2010), Chiu et al. (2010d). This paper applies the same algebraic approach to reexamine the problem studied by Lee et al. (2011). As a result, the optimal replenishment lot size and the long- run average cost function can all be derived without using differential

*Corresponding author. E-mail: chs@cyut.edu.tw.

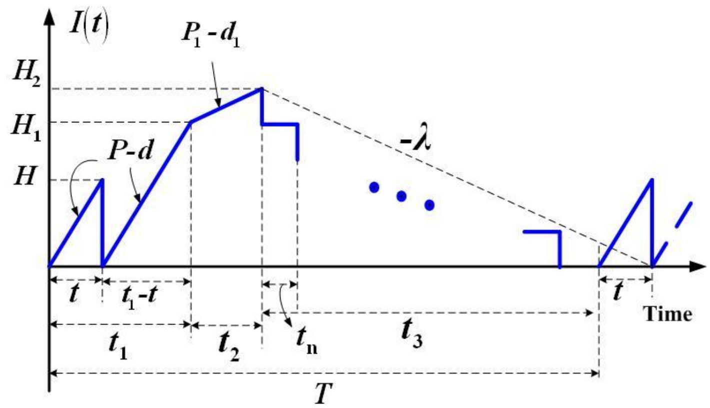

Figure 1. Producer's on-hand inventory of perfect quality items for the proposed model with discontinuous (n + 1) issuing policy and imperfect rework.

calculus.

METHODS

In this study, an alternative approach is adopted to reexamine Lee et al.'s model (2011) as stated earlier. To ease the readability, this study adopts the exact notation as used in Lee et al. (2011). Description the model is as follows: consider a real life production system may produce x portion of random nonconforming items at a production rate d. Among nonconforming items, a θ portion is assumed to be scrap and the other (1 - θ) portion can be reworked at a rate P_1, within the same cycle when regular production ends. A θ_1 portion (where $0 \leq \theta_1 \leq 1$) of reworked items fails during rework and becomes scrap. The constant production rate P is larger than the sum of demand rate λ and production rate of defective items d. That is: $(P - d - \lambda) > 0$; where d can be expressed as $d = Px$. Let d_1 denote production rate of scrap items during rework process, then $d_1 = P_1\theta_1$.

Under the proposed n + 1 delivery policy, an initial installment of finished products is delivered to customer for satisfying the demand during producer's production uptime and rework time. Then, at the end of rework, when the rest of production lot is quality assured, fixed quantity n installments of finished products are delivered to customer at a fixed interval of time.

Cost variables include setup cost K per production run, unit production cost C, unit holding cost h, unit rework cost C_R, disposal cost per scrap item C_S, holding cost h_1 for each reworked item, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation includes:

Q = production lot size to be determined for each cycle.
t = the production time needed for producing enough perfect items for satisfying product demand during the production uptime t_1 and the rework time t_2.
t_1 = the production uptime for the proposed EPQ model.
t_2 = time required for reworking of defective items.
t_3 = time required for delivering the remaining quality assured finished products.
H = the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1 and rework time t_2.
H_1 = maximum level of on-hand inventory in units when regular production ends.
H_2 = the maximum level of on-hand inventory in units when rework process finishes.
T = cycle length.
t_n = a fixed interval of time between each installment of products delivered during t_3.
n = number of fixed quantity installments of the rest of finished lot to be delivered during t_3.
$I(t)$ = on-hand inventory of perfect quality items at time t.
φ = overall scrap rate per cycle (sum of scrap rates in t_1 and t_2).
$TC(Q)$ = total production-inventory-delivery costs per cycle for the proposed model.
E[$TCU(Q)$] = the long-run average costs per unit time for the proposed model.

Figure 1 depicts producer's on-hand inventory of perfect quality items (Lee et al., 2011). Again, for the purpose of easing readability, this paper adopted the same basic formulations as that in Lee et al. (2011). Total production-inventory-delivery cost per cycle $TC(Q)$ consists of variable manufacturing cost, setup cost, variable rework and disposal cost, the fixed and variable (n + 1) shipping cost, holding cost for perfect quality and nonconforming items during t_1 and t_2, holding cost for reworked items during t_2, and vendor's holding cost for finished goods during the delivery time t_3. Using the same formulation procedures, one has $TC(Q)$ as follows:

$$\begin{aligned} TC(Q) = {} & CQ + K + C_R\left[xQ(1-\theta)\right] + C_S\left[xQ\varphi\right] \\ & + (n+1)K_1 + C_T\left[Q(1-\varphi x)\right] \\ & + h\left[\frac{H}{2}(t) + \frac{H_1}{2}(t_1 - t) + \frac{H_2 + H_1}{2}(t_2) + \frac{dt_1}{2}(t_1)\right] \\ & + h_1\left[\frac{dt_1(1-\theta)}{2}(t_2)\right] + h\left[\left(\frac{n-1}{2n}\right)H_2 t_3\right] \end{aligned} \tag{1}$$

Taking into the randomness of defective rate x, one can use the expected values of x in cost analysis and with further derivation one obtains E[$TCU(Q)$] as follows [for detailed computations one can refer to Appendix in Lee et al. (2011)]:

$$E\left[TCU(Q)\right]=\frac{C\lambda}{1-\varphi E(x)}+\frac{\left[(n+1)K_1+K\right]\lambda}{Q\left[1-\varphi E(x)\right]}+\frac{C_{\mathrm{R}}E(x)(1-\theta)\lambda}{1-\varphi E(x)}+\frac{C_{\mathrm{S}}E(x)\varphi\lambda}{1-\varphi E(x)}+C_{\mathrm{T}}\lambda$$
$$+\frac{h_1 Q\lambda\left[E(x)\right]^2(1-\theta)^2}{2P_1\left[1-\varphi E(x)\right]}$$
$$+\frac{hQ}{2}\left[\begin{array}{l}\dfrac{2\lambda^3}{P^3\left[1-\varphi E(x)\right]}E\left(\dfrac{1}{1-x}\right)+\dfrac{4\lambda^3(1-\theta)}{P^2P_1\left[1-\varphi E(x)\right]}E\left(\dfrac{x}{1-x}\right)-\dfrac{\lambda^2}{P^2\left[1-\varphi E(x)\right]}\\ +\dfrac{2\lambda^3(1-\theta)^2}{PP_1^2\left[1-\varphi E(x)\right]}E\left(\dfrac{x^2}{1-x}\right)-\dfrac{2\lambda^2E(x)(1-\theta)}{PP_1\left[1-\varphi E(x)\right]}-\dfrac{\lambda\left[E(x)\right]^2(1-\theta)(1-\varphi)}{P_1\left[1-\varphi E(x)\right]}\\ -\dfrac{\lambda^2\left[E(x)\right]^2(1-\theta)^2}{P_1^2\left[1-\varphi E(x)\right]}+\left[1-\varphi E(x)\right]-\dfrac{\lambda\left[1-2\varphi E(x)\right]}{P\left[1-\varphi E(x)\right]}\\ -\left(\dfrac{1}{n}\right)\left[\begin{array}{l}\left[1-\varphi E(x)\right]-\dfrac{2\lambda}{P}-\dfrac{2\lambda E(x)(1-\theta)}{P_1}+\dfrac{2\lambda^2E(x)(1-\theta)}{PP_1\left[1-\varphi E(x)\right]}\\ +\dfrac{\lambda^2}{P^2\left[1-\varphi E(x)\right]}+\dfrac{\lambda^2\left[E(x)\right]^2(1-\theta)^2}{P_1^2\left[1-\varphi E(x)\right]}\end{array}\right]\end{array}\right] \tag{2}$$

The algebraic approach

Here, algebraic approach is employed to derive the optimal replenishment lot size and the optimal number of deliveries. It is noted that decision variable Q in Equation 2 has the forms of Q^{-1} and Q. Let π_1, π_2, and π_3 denote the following:

$$\pi_1=\frac{C\lambda}{1-\varphi E(x)}+\frac{C_{\mathrm{R}}E(x)(1-\theta)\lambda}{1-\varphi E(x)}+\frac{C_{\mathrm{S}}E(x)\varphi\lambda}{1-\varphi E(x)}+C_{\mathrm{T}}\lambda \tag{3}$$

$$\pi_2=\frac{\left[(n+1)K_1+K\right]\lambda}{\left[1-\varphi E(x)\right]} \tag{4}$$

$$\pi_3=\frac{h_1\lambda\left[E(x)\right]^2(1-\theta)^2}{2P_1\left[1-\varphi E(x)\right]}$$
$$+\frac{h}{2}\left[\begin{array}{l}\dfrac{2\lambda^3}{P^3\left[1-\varphi E(x)\right]}E\left(\dfrac{1}{1-x}\right)+\dfrac{4\lambda^3(1-\theta)}{P^2P_1\left[1-\varphi E(x)\right]}E\left(\dfrac{x}{1-x}\right)-\dfrac{\lambda^2}{P^2\left[1-\varphi E(x)\right]}\\ +\dfrac{2\lambda^3(1-\theta)^2}{PP_1^2\left[1-\varphi E(x)\right]}E\left(\dfrac{x^2}{1-x}\right)-\dfrac{2\lambda^2E(x)(1-\theta)}{PP_1\left[1-\varphi E(x)\right]}-\dfrac{\lambda\left[E(x)\right]^2(1-\theta)(1-\varphi)}{P_1\left[1-\varphi E(x)\right]}\\ -\dfrac{\lambda^2\left[E(x)\right]^2(1-\theta)^2}{P_1^2\left[1-\varphi E(x)\right]}+\left[1-\varphi E(x)\right]-\dfrac{\lambda\left[1-2\varphi E(x)\right]}{P\left[1-\varphi E(x)\right]}\\ -\left(\dfrac{1}{n}\right)\left[\begin{array}{l}\left[1-\varphi E(x)\right]-\dfrac{2\lambda}{P}-\dfrac{2\lambda E(x)(1-\theta)}{P_1}+\dfrac{2\lambda^2E(x)(1-\theta)}{PP_1\left[1-\varphi E(x)\right]}\\ +\dfrac{\lambda^2}{P^2\left[1-\varphi E(x)\right]}+\dfrac{\lambda^2\left[E(x)\right]^2(1-\theta)^2}{P_1^2\left[1-\varphi E(x)\right]}\end{array}\right]\end{array}\right] \tag{5}$$

Then Equation 2 becomes

$$E\left[TCU(Q)\right]=\pi_1+Q^{-1}\pi_2+Q\pi_3 \tag{6}$$

or

$$E\left[TCU(Q)\right]=\pi_1+Q\left[\left(Q^{-1}\right)^2\pi_2+\pi_3\right] \tag{7}$$

With further rearrangements one has

$$E\left[TCU(Q)\right]=\pi_1+Q\left[\left(Q^{-1}\cdot\sqrt{\pi_2}\right)^2+\left(\sqrt{\pi_3}\right)^2-2\left(Q^{-1}\cdot\sqrt{\pi_2}\right)\left(\sqrt{\pi_3}\right)\right]$$
$$+Q\left[2\left(Q^{-1}\cdot\sqrt{\pi_2}\right)\left(\sqrt{\pi_3}\right)\right] \tag{8}$$

or

$$E\left[TCU(Q)\right]=\pi_1+Q\left[\left(Q^{-1}\cdot\sqrt{\pi_2}\right)-\left(\sqrt{\pi_3}\right)\right]^2+2\sqrt{\pi_2\cdot\pi_3} \tag{9}$$

One notes that E[$TCU(Q)$] is minimized, if the second term in Equation 9 equals zero. That is,

$$\left(Q^{-1}\cdot\sqrt{\pi_2}\right)-\left(\sqrt{\pi_3}\right)=0 \tag{10}$$

or

$$Q^*=\sqrt{\frac{\pi_2}{\pi_3}} \tag{11}$$

RESULTS AND DISCUSSION

Substituting Equations 4 and 5 in Equation 11 and with further derivations, one has

$$Q^*=\sqrt{\frac{2\lambda\left[(n+1)K_1+K\right]}{h\left\{\begin{array}{l}\dfrac{\lambda^2}{P}\left[\dfrac{2\lambda}{P^2}E\left(\dfrac{1}{1-x}\right)-\dfrac{1}{P}+\dfrac{(1-\theta)}{P_1}\left[\begin{array}{l}\dfrac{4\lambda}{P}E\left(\dfrac{x}{1-x}\right)-2E(x)\\ +\dfrac{2\lambda(1-\theta)}{P_1}E\left(\dfrac{x^2}{1-x}\right)\end{array}\right]\right]\\ -\dfrac{\lambda\left[E(x)\right]^2(1-\theta)}{P_1}\left[(1-\varphi)+\dfrac{\lambda(1-\theta)}{P_1}\right]+\left[1-\varphi E(x)\right]^2\\ -\dfrac{\lambda\left[1-2\varphi E(x)\right]}{P}\\ -\left(\dfrac{1}{n}\right)\left[\begin{array}{l}\left[1-\varphi E(x)\right]\cdot\left[\left[1-\varphi E(x)\right]-\dfrac{2\lambda}{P}-\dfrac{2\lambda E(x)(1-\theta)}{P_1}\right]\\ +\dfrac{2\lambda^2E(x)(1-\theta)}{PP_1}+\dfrac{\lambda^2}{P^2}+\dfrac{\lambda^2\left[E(x)\right]^2(1-\theta)^2}{P_1^2}\end{array}\right]\end{array}\right\}+\dfrac{h_1\lambda\left[E(x)\right]^2(1-\theta)^2}{P_1}}} \tag{12}$$

It is noted that Equation 12 is identical to Q^* in Lee et al. (2011), which was derived using the conventional differential calculus approach. Furthermore, in Equation 9 suppose the optimal replenishment lot size Q^* is used, the second term becomes zero; so the long-run average cost E[$TCU(Q^*)$] is

$$E\left[TCU\left(Q\right)\right]=\pi_1+2\sqrt{\pi_2\cdot\pi_3} \qquad (13)$$

Numerical example with further verification

The aforementioned results are verified by using the same numerical example as in Lee et al. (2011). Consider the following system parameters:

P = 60000 items per year
λ = 3400 items per year
x = a random scrap rate is assumed to be uniformly distributed over interval [0, 0.3]
θ = 0.1, the scrap rate during regular process
C = \$100 per item
K = \$20000 per production run
P_1 = 2,200, rate of rework
C_R = \$60 per item reworked
θ_1 = 0.1, the failure in rework rate
C_S = \$20, disposal cost for each scrap item
h = \$20 per item per year
h_1 = \$40 holding cost per item reworked
C_T = \$0.1 per item delivered
K_1 = \$4350 per shipment, a fixed cost.
h_2 = \$80 per item kept at the buyer's end, per unit time.

Suppose the proposed delivery policy has total shipments (n + 1) = 4 (as *Scenario* 2 in Lee et al. (2011)), from computations of Equations 12 and 13, one obtains the optimal replenishment lot size Q^* = 4271 and E[TCU(Q^*)] = \$441949.

One notes that both of the aforementioned results are identical to that were given in Lee et al. (2011). Furthermore, for computing the long-run average costs per unit time E[TCU(Q^*)], the proposed Equation 13 is much simpler than by the use of Equation 2.

Conclusions

Lee et al. (2011) used the conventional differential calculus method to derive the optimal replenishment lot size for a production system with discontinuous issuing policy and imperfect rework. This paper reexamines their problem by using an algebraic approach. Such a straightforward derivation allows practitioners who may not have sufficient knowledge of differential calculus to understand such a real life production system with ease.

ACKNOWLEDGEMENT

Authors thank National Science Council of Taiwan for supporting this research under Grant Number: NSC 99-2410-H-324-007-MY3.

REFERENCES

Abolhasanpour M, Khosrojerdi G, Sadeghi A, Lotfi E, Jafari AA, Arab S, Hojabri H, Kimiagari AM (2009) Supply chain planning with scenario based probabilistic demand. African J. Bus. Manage., 3(11): 705-714.
Banerjee A (1986) A joint economic-lot-size model for purchaser and vendor. Decis. Sci., 17(3): 292- 311.
Banerjee S, Sharma A (2010) Optimal procurement and pricing policies for inventory models with price and time dependent seasonal demand, Math. Comput. Model., 51(5-6): 700-714.
Barlow RE, Proschan F (1965) Mathematical Theory of Reliability. Wiley: New York.
Chiu SW, Chiu YSP (2006). Mathematical modeling for production system with backlogging and failure in repair. J. Sci. Ind. Res., 65(6): 499-506.
Chiu SW, Chen K-K, Lin H-D (2011a) Numerical method for determination of the optimal lot size for a manufacturing system with discontinuous issuing policy and rework. Int. J. Numer. Meth. Biomed. Eng., 27(10): 1545-1557
Chiu SW, Yang J-C, Wu M-F (2011b) An alternative approach for determining production lot size with repairable items and cost reduction delivery policy. Afr. J. Bus. Manage., 5(4): 1374-1377;
Chiu Y-SP, Chiu SW, Li C-Y, Ting C-K (2009). Incorporating multi-delivery policy and quality assurance into economic production lot size problem. J. Sci. Ind. Res., 68(6): 505-512.
Chen K-K, Chiu Y-SP, Hwang M-H (2010). Integrating a cost reduction delivery policy into an imperfect production system with repairable items. Int. J. Phys. Sci., 5(13): 2030-2037.
Chen K-K, Chiu SW, Yang J-C. (2011) Incorporating a cost lessening distribution policy into a production system with random scrap rate. Afr J. Bus. Manage., 5(5): 1927-1935.
Chiu Y-SP, Cheng F-T, Chang H-H (2010a). Remarks on optimization process of manufacturing system with stochastic breakdown and rework. Appl. Math. Lett., 23(10): 1152-1155.
Chiu Y-SP, Chen K-K, Cheng F-T, Wu M-F (2010b). Optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Comput. Math. Appl., 59(2): 919-932.
Chiu Y-SP, Ting C-K (2010c), A note on 'Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns', Eur. J. Oper. Res., 201(2): 641-643.
Chiu Y-SP, Chen K-K, Chang H-H (2010d). Solving an economic production lot size problem with multi-delivery policy and quality assurance using an algebraic approach. J. Sci. Ind. Res., 69(12): 926-929.
Diponegoro A, Sarker BR (2006). Finite horizon planning for a production system with permitted shortage and fixed-interval deliveries. Comput. Oper. Res., 33: 2387-2404.
Goyal SK (1977). Integrated inventory model for a single supplier-single customer problem. Int. J. Prod. Res., 15: 107-111.
Grosfeld-Nir A, Gerchak Y (2002). Multistage production to order with rework capability. Manage. Sci., 48(5): 652-664.
Grubbström RW, Erdem A (1999). The EOQ with backlogging derived without derivatives. Int. J. Prod. Econ., 59: 529-530.
Hahm J, Yano CA (1992). The economic lot and delivery scheduling problem: The single item case. Int. J. Prod. Econ., 28: 235-252.
Henig M, Gerchak Y (1990). Structure of periodic review policies in the presence of random yield. Oper. Res., 38(4): 634-643.
Hsieh T-P, Dye C-Y, Ouyang L-Y (2010) Optimal lot size for an item with partial backlogging rate when demand is stimulated by inventory above a certain stock level. Math. Comput. Model., 51(1-2): 13-32.
Jha JK, Shanker K (2009) Two-echelon supply chain inventory model with controllable lead time and service level constraint. Comput. Ind. Eng., 57(3): 1096-1104.
Kim S-L, Banerjee A, Burton J (2008). Production and delivery policies for enhanced supply chain partnerships. Int. J. Prod. Res. 46: 6207-6229.
Kreng VB, Tan S-J (2010) The optimal replenishment decisions under two levels of trade credit policy depending on the order quantity. Expert Syst. Appl., 37(7): 5514-5522.
Lee TJ, Chiu SW, Chang H-H. (2011). On improving replenishment lot size of an integrated manufacturing system with discontinuous issuing policy and imperfect rework. Am. J. Ind. Bus. Manage., 1(1): 20-29.
Lodree Jr. EJ, Geiger CD, Ballard KN (2010) Coordinating production and shipment decisions in a two-stage supply chain with time-sensitive demand. Math. Comput. Model., 51(5-6): 632-648.

Ma W-N, Gong D-C, Lin GC (2010). An optimal common production cycle time for imperfect production processes with scrap. Math. Comput. Model., 52(5-6): 724-737.
Mak KL (1985). Inventory control of defective products when the demand is partially captive. Int. J. Prod. Res., 23(3): 533-542.
Mehdi R, Nidhal R, Anis C (2010). Integrated maintenance and control policy based on quality control. Comput. Ind. Eng., 58(3): 443-451.
Saha A, Roy A, Kar S, Maiti M (2010) Inventory models for breakable items with stock dependent demand and imprecise constraints. Math. Comput. Model., 52(9-10): 1771-1782.
Sana SS (2010) Demand influenced by enterprises' initiatives - A multi-item EOQ model of deteriorating and ameliorating items. Math. Comput. Model., 52(1-2): 284-302.
Swenseth RS, Godfrey RM (2002). Incorporating transportation costs into inventory replenishment decisions. Int. J. Prod. Econ., 77(2): 113-130.
Taleizadeh AA, Wee HM, Sadjadi SJ (2010). Multi-product production quantity model with repair failure and partial backordering. Comput. Ind. Eng., 59(1): 45-54.
Taft EW (1918). The most economical production lot. Iron Age, 101: 1410–1412.
Viswanathan S (1998). Optimal strategy for the integrated vendor-buyer inventory model. Eur. J. Oper. Res., 105: 38-42.
Wazed MA, Ahmed S, Nukman Y (2010a). Impacts of quality and processing time uncertainties in multistage production system. Int. J. Phys. Sci., 5(6): 814-825.
Wazed MA, Ahmed S. Yusoff N (2010b). Impacts of common processes in multistage production system under machine breakdown and quality uncertainties. Afr. J. Bus. Manage., 4(6): 979-986.
Wong J-T (2010). The distribution processing and replenishment policy of the supply chain under asymmetric information and deterioration: Insight into the information value. Expert Syst. Appl., 37(3): 2347-2353.
Ye F, Xu X (2010) Cost allocation model for optimizing supply chain inventory with controllable lead time. Comput. Ind. Eng., 59(1): 93-99.