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Full Length Research Paper

A lot size model with random yields for investing to setup cost reduction under a limited capital budget

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In this paper, the study attempts to determine the optimal capital investment in setup cost reduction and optimal lot sizing policies for an economic order quantity (EOQ) model with random yields. The setup cost is treated as the function of capital expenditure in technology. The study shows that the expected total annual cost functions with capital investment is convex and develop a solution procedure to determine the optimal lot sizes and the capital investment in setup cost reduction with a limited capital budget. Finally, a numerical example is presented to illustrate the results and compared to the results without considering capital investment. These results evidently show that significant costs savings can be obtained through capital investment in setup cost reduction. In addition, the sensitivity analysis is also included.

Key words: Inventory, random yields, lot sizing, investment analysis.

INTRODUCTION

The random yield production or procurement problem has become an important research topic in production and inventory studies. In particular, several papers have reported the implications of yield randomness on lot sizing decisions. For an extensive review on many other lot sizing problems with random yields, interested readers are referred to the excellent paper by Yano and Lee (1995). Silver (1976) was one of the earliest authors that extended the classical economic order quantity (EOQ) model to include the case where the quantity received from the supplier does not necessarily match the quantity requisitioned. Kalro and Gohil (1982) extended Silver's model to include complete and partial backlogging of demands. Parlar and Berkin (1991) analyzed the supply uncertainty problem for a class of EOQ models. Recently, Yuanjie and Zhang (2008) studied a simply supply chain with one supplier and one retailer where there is random yield production and uncertain demand. Other related studies can be found in Shih (1980), Gerchak (1992), Parlar and Wang (1993), Parlar and Perry (1995), Erdem and Ozekici (2002).

In existing EOQ-type models, the yield distribution itself is assumed to be known and given. For example, the fraction distribution of defectives produced is fixed. However, to a certain extent, a firm may wish and be able to choose production processes, machines, and suppliers based on their yield distributions and associated costs. Such considerations are often mentioned in relation to modern manufacturing strategies (Spence, 1988). Some recent research has attempted to improve the production process by investment in modern production technology which can impact yield distribution. Cheng (1991) assumed that the unit production cost of an item increases with the yield rate. The optimal lot sizes and yield rate were obtained for this situation. Gerchak and Parlar (1990) considered the problem of jointly determining the yield variance and lot sizes when the yield variability could be reduced through appropriate investment. However, they did not investigate the advantages of capital investment in reducing setup cost. The benefits of reduced setups are well documented (Hong and Hayya, 1995). For example, faster changeovers have been associated with lower inventory, faster throughput, shorter lead time, improved quality, and lower unit cost. Much of the analytical work on setup

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reduction examined the benefits of reduced setups on inventory and setup cost. Recently, Lin and Hou (2005) extended the work of Noori and Keller (1986) and Gerchak (1992) to consider how the setup cost and yield standard deviation can be reduced through capital investments without a limited capital budget. Hou and Lin (2004) studied the effects of an imperfect production process on the optimal production run length when capital investment in process improvement is adopted. Hou (2007) extended the work of Hou and Lin (2004) to consider an economic production quantity model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure. However, the company has recognized the resources or budget allocated are usually limited. With a limited capital budget, the study would appropriately allocate capital budget to reduce setup cost and then determine the optimal lot sizing policies. As a result, the study considers a lot size model with random yields when the setup cost can be reduced through capital investment under a limited capital budget. Other several relationships between the amount of capital investment and setup cost level have been reported by many researches as in Porteus (1986), Billington (1987), Kim et al. (1992), Hwang et al. (1993), Hong (1997), Hofmann (1998), and Chung and Huang (1999).

Based on the above arguments, this article attempts to model the production process when yield variability can be reduced through investment in setup cost reduction under a limited capital budget. To the knowledge, no previous research has addressed such scenario. In this analysis, the study assumes that setup cost is the function of capital expenditure. The study shows that the objective function is convex. With this convexity, an iterative solution procedure is presented to find the optimal results. Therefore, the optimal capital investment and ordering policies that minimize the expected total annual costs for the system are appropriately determined. Finally, a numerical example is provided to illustrate the results obtained and assess the performance on cost savings by adopting capital investment under a limited capital budget. In addition, the sensitivity analysis is performed to investigate the effects of changing parameters values on the optimal solution of the system.

Notations and assumptions

The study defines the following notations to present the mathematical model in this paper:

1) D : Demand rate in units/per year.
2) Q : the quantity ordered, in units.
3) v : unit variable cost, in \$/unit.
4) r : inventory carrying charge, in \$/\$/per year
5) Y_Q : the quantity received given that Q units are ordered, is a random variable.
6) θ_U : the maximum available budget for investment in setup cost reduction.
7) θ_s : the available budget for investment in setup cost reduction, $0 \le \theta_s \le \theta_U$.
8) $f(\theta_s)$: the capital investment in setup cost reduction, defined as $f(\theta_s) = \dfrac{\theta_s}{\theta_U - \theta_s}$.
9) A_0 : the original setup cost.
10) A : the nominal setup cost per setup, a function of θ_s, with $A_0 = A(0)$.
11) A_L: the technological lower limit of setup cost when the capital investment $\theta_s = \theta_U$, that is, $A_L = A(\theta_U)$.
12) σ : the yield's standard deviation.
13) $E(Y_Q)$: the expected value of Y_Q given that Q units are ordered (that is, $E(Y_Q) = \mu Q$) .
14) σ_{Y_Q} : the standard deviation of Y_Q given that Q units are ordered, is proportional to Q (that is, $\sigma_{Y_Q} = \sigma Q$).
15) μ : the bias factor and $\mu = \dfrac{E(Y_Q)}{Q}$, represents the expected amount received as a proportion of the amount ordered.
16) i : cost of capital, in \$/\$/per year.
17) $f(\theta_s^*)$: the optimal capital investment in setup cost reduction.
18) Q^*: the optimal order quantity.

In addition, the following assumptions are used throughout this paper:

1) Demand is constant and deterministic.
2) The unit variable cost is independent of the quantity order.
3) The lead time is zero and independent of the quantity ordered.
4) The quantity received is a random variable depending upon the quantity ordered.
5) $A(\theta_s)$ is a continuously differentiable decreasing function and convex in capital expenditure θ_s .
6) For the sake of generality, the study assumes that $\mu > \sigma_0 \ge 0$.
7) The relationship between setup cost reduction and capital investment can be described using a logarithmic investment cost function under maximum available budget θ_U . That is, when the maximum available budget θ_U is utilized then the setup cost will decrease exponentially to a technological lower limit A_L . Hence, the capital

investment function in setup cost reduction, $f(\theta_s)$, can be stated as

$$f(\theta_s) = a\ln\left(\frac{A_0 - A_L}{A - A_L}\right) \quad \text{for } A_L < A \le A_0 . \qquad (1)$$

where $f(\theta_s) = \dfrac{\theta_s}{\theta_U - \theta_s}$ and $1/a$ is the fraction of the reduction in A per dollar increase in investment.

MATHEMATICAL MODEL

Based on the above notations and assumptions, the study has the expected relevant cost per unit time are

$$TC(Q) = \frac{DA_0}{\mu Q} + \frac{vrQ}{2\mu}\left[\sigma^2 + \mu^2\right] \qquad (2)$$

As it takes investment to reduce setup cost, the study should include an amortized investment cost in the proposed model. Therefore, the expected total annual costs of the system, $ETC(Q, \theta_s)$, are composed of equation (1) and the amortized total capital cost, $if(\theta_s)$, as follow:

$$ETC(Q, A) = \frac{DA}{\mu Q} + \frac{vrQ}{2\mu}\left(\sigma^2 + \mu^2\right) + if(\theta_s) \qquad (3)$$

Subject to $0 \le \theta_s \le \theta_U$

where $f(\theta_s)$ is based on Equation (1). Hence, Equation (3) can be stated as

$$ETC(Q, A) = \frac{DA}{\mu Q} + \frac{vrQ}{2\mu}\left(\sigma^2 + \mu^2\right) + ia\ln\left(\frac{A_0 - A_L}{A - A_L}\right) \qquad (4)$$

It is easy to show that ETC in Equation (4) is convex on A. To find the minimum cost, the partial derivatives of the $ETC(Q, A)$ can be evaluated as

$$\frac{\partial ETC(Q, A)}{\partial A} = \frac{D}{\mu Q} - \frac{ia}{A - A_L} = 0 \qquad (5)$$

From Equation (5), the optimal setup cost can be solved as

$$A^*(Q) = A_L + ia\frac{\mu Q}{D} \qquad (6)$$

Substituting Equation (6) into Equation (4) yields the following corresponding expected total annual cost expression, EAC:

$$EAC(Q) = \frac{DA_L}{\mu Q} + ia + \frac{vrQ}{2\mu}(\sigma^2 + \mu^2) + ia\ln\left(\frac{D(A_0 - A_L)}{ia\mu Q}\right) \qquad (7)$$

ANALYSIS AND SOLUTION PROCEDURE

This work aims to minimize the expected total annual cost $EAC(Q)$ defined in Equation (7) in $Q > 0$ and $A_L < A \le A_0$. Hence, the following results will be shown.

Theorem 1: $EAC(Q)$ is convex with respect to $Q > 0$.
Proof: Equation (7) yields

$$\frac{dEAC(Q)}{dQ} = \frac{-DA_L}{\mu Q^2} + \frac{vr}{2\mu}(\sigma^2 + \mu^2) - \frac{ia}{Q} \qquad (8)$$

and

$$\frac{d^2 EAC(Q)}{dQ^2} = \frac{2DA_L}{\mu Q^3} + \frac{ia}{Q^2} > 0 \qquad (9)$$

Thus, the study concludes that $EAC(Q)$ is convex on $Q > 0$.

Based on the convex nature of the function, $EAC(Q)$, the first-order condition for a minimum is given by $\frac{d}{dQ}EAC(Q) = 0$. Hence, the study can obtain

$$Q^* = \frac{ia\mu + \sqrt{(ia\mu)^2 + 2DA_L vr(\sigma^2 + \mu^2)}}{vr(\sigma^2 + \mu^2)} \qquad (10)$$

Notice that the optimal setup cost and lot size can be expressed in a closed form as shown in Equations (6) and (10), respectively. Thus, the optimal solution (Q^*, A^*) can be obtained easily by solving the above Equations (6) and (10). That is, the solution procedure which can initially use Equation (10) to get the optimal lot size Q^*, and then substituting Q^* into Equation (6) to find the optimal setup cost $A^*(Q^*)$. When no capital investment in setup cost reduction is made, then $A_L = 0$ and $A^* = A_0$. Hence, from Equation (6), the study has

$$ia = \frac{DA_0}{\mu Q} \qquad (11)$$

Thus, the corresponding expected total annual cost EAC in Equation (7) can reduce to the expression of Equation (2) which is the result of Silver (1976) and Gerchak and Parlar (1990). Therefore, the lot size model with random yields in this paper is an extension of both Silver (1976) and Gerchak and Parlar (1990). Next, substituting Equation (11) into Equation (10), the optimal lot size will reduce to

Table 1. The results for Example1.

Model type	Q^*	A^* $(f_k(\theta_s))$	$EAC(Q^*)$	EAC (%)
Model 1	30.135	24.701 (1895.485)	1103.999	52.67
Model 2	85.749	200 (0)	2332.381	---

EAC is defined the percentage of savings in *EAC* as compared with Model 2.

Table 2. Optimal results with changing the parameter A_L.

Model type	Changing the parameter A_L	Change in the parameter (%)	Q^*	A^* $(f_k(\theta_s))$	$EAC(Q^*)$	EAC (%)
	10	-50	22.255	13.472 (2081.225)	917.52	60.66
	15	-25	26.525	19.138 (1976.077)	1017.905	56.36
Model 1	20	0	30.135	24.701 (1895.485)	1103.999	52.67
	25	25	33.320	30.198 (1828.595)	1180.591	49.38
	30	50	36.202	35.647 (1770.386)	1250.246	46.40
Model 2	---	---	85.749	200 (0)	2332.381	---

$$Q^* = \sqrt{\frac{2DA_0}{vr(\sigma^2 + \mu^2)}} \qquad (12)$$

Therefore, the results for special case such as no capital investment in setup cost reduction are derived.

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Two different models used for comparison are specified as follows.

Model 1: lot size model with random yields and with capital investments (Equation (7)), and Model 2: lot size model with random yields and without capital investment (see Equation (2)).

The percentage of savings of expected total annual cost is defined by

$$\%EAC = \left[1 - \frac{EAC(Q^*)\text{ of model1}}{EAC(Q^*)\text{ of model 2}}\right] \times 100\%$$

To illustrate the above procedure, let us consider the following numerical example:

Example: D = 1000, i=0.15, A_0 = 200, σ = 1.2, μ = 2.0, v = 40, r = 0.25, a = 520, A_L = 20, $\theta_u = 2500$. Applying the proposed procedure as mentioned above, the study provides the results for the two models as in Table 1. From the results shown in Table 1, the study sees that significant savings of the expected total annual cost are achieved through capital investment. Here, the study gets the optimal setup cost A^* = \$24.701, the available budget for investment in setup cost reduction $\theta_s = 2498.682$. Then, the optimal capital investment in setup cost reduction, $f_k(\theta_s) =$ \$1895.485, namely, the optimal investment require \$1895.485 when the optimal setup cost A^* = \$24.701 and the optimal lot size Q^* = 30.135 are required. The corresponding expected total annual cost $EAC(Q^*)$ = \$1103.999, which is less than the cost when there is no capital investment for setup cost reduction. The percentage of expected total annual cost savings is given by 52.67%. That is, 52.67% of the expected total annual cost savings are relative to the model without capital investment in setup cost reduction.

In addition, sensitivity analysis is performed to investigate the performance on cost savings when capital investment in setup cost reduction is adopted under a limited capital budget. Here, the effects of changing the parameters A_L, a, μ and σ on the optimal lot sizing policies and the expected total annual cost savings are studied. Whenever one parameter is changing by some percentage, all other parameters are remained at their original values. Investigation has been done for both positive and negative changes of these parameters. The obtained results are summarized in Tables 2 to 5. The following inferences can be obtained from the sensitivity analysis based on Tables 2 to 5.

(1) Table 2 shows that the lot sizes and expected total annual cost decreases as lower limit of setup cost (A_L) decreases. Furthermore, the expected total annual cost savings significantly increases as A_L decreases. This means the more capital investment is required to achieve the smaller setup cost as A_L decreases which results in the more considerable cost savings.

Table 3. Optimal results with changing the parameter *a*.

Model type	Changing the parameter *a*	Change in the parameter (%)	Q^*	$A^*\ (f_k(\theta_s))$	$EAC(Q^*)$	EAC (%)
	260	-50	28.588	22.230 (1141.664)	948.844	59.32
	390	-25	29.352	23.434(1544.077)	1029.991	55.84
Model 1	520	0	30.135	24.701(1895.485)	1103.999	52.67
	650	25	30.937	26.033(2207.249)	1172.567	49.73
	780	50	31.757	27.431(2486.081)	1236.698	46.98
Model 2	---	---	85.749	200(0)	2332.381	---

Table 4. Optimal results with changing the parameter μ .

Model type	Changing the parameter μ	Change in the parameter (%)	Q^*	$A^*\ (f_k(\theta_s))$	$EAC(Q^*)$	EAC (%)
	1.0	-50	43.812	23.417(2061.335)	1378.202	40.91
	1.5	-25	36.247	24.241(1949.049)	1184.043	49.23
Model 1	2.0	0	30.135	24.701(1895.485)	1103.999	52.67
	2.5	25	25.483	24.969(1866.640)	1063.860	54.39
	3.0	50	21.943	25.135(1849.603)	1041.067	55.36
Model 2	---	---	85.749	200(0)	2332.381	---

Table 5. Optimal results with changing the parameter σ .

Model type	Changing the parameter σ	Change in the parameter (%)	Q^*	$A^*\ (f_k(\theta_s))$	$EAC(Q^*)$	EAC (%)
	0.6	-50	34.078	25.316(1831.551)	1017.627	56.37
	0.9	-25	32.263	25.033(1860.014)	1054.916	54.77
Model 1	1.2	0	30.135	24.701(1895.485)	1103.999	52.67
	1.5	25	27.917	24.355(1935.242)	1162.694	50.15
	1.8	50	25.758	24.018(1977.092)	1229.013	47.31
Model 2	---	---	85.749	200(0)	2332.381	---

(2) Table 3 shows that both the lot sizes and setup cost decrease as parameter *a* decrease. This implies that the optimal lot sizes depend on how costly it is to reduce the setup cost. Moreover, the expected total annual cost savings significantly increases as a decrease.

(3) From Table 4, the study can see that when the mean yield μ increases, both the optimal lot sizes and expected total annual cost decrease. This implies that a reduction in the average proportion of defectives results in lower values of lot sizes and expected total annual cost. In other words, improving quality will cause lower values of lot sizes and expected total annual cost.

(4) From Table 5, the study can see that *Q* is decreasing in σ . In other words, the required lot sizes will increase if the yield variability can also be reduced through capital investment. In addition, the expected total annual cost savings significantly increases as σ decreases. Hence, the study may further consider the optimal investment allocation between yield variability and setup cost reductions in future study.

Conclusions

Since appropriate investments in modern production technology are an important strategy in manufacturing, the study attempts to model the production process by which yield variability can be reduced through investment in setup cost reduction. In the analysis, the study assumes that the relationship between setup cost reduction and capital investment can be described using a logarithmic investment cost function under maximum available budget. To explore lot sizing policies, the expected total annual cost function with capital investment was formulated. The study showed that the cost function is convex and developed a solution procedure to determine the optimal lot sizes and setup cost. Therefore, the optimal capital investment could be appropriately determined. Finally, a numerical example is provided to illustrate the results and evaluate the effects of utilizing capital investment. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the model is carried out. It should be emphasized that these results evidently show that significant cost savings can be achieved by investing in setup cost reduction. This approach is consistent with the JIT manufacturing philosophy which calls for reducing setup cost to achieve the inventory reductions.

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