Full Length Research Paper

Study on the approach of triangular fuzzy evaluation of logistics facility location based on supply chain management in China's companies

Zhang ShuShan

School of Business, Northeast Normal University, no.2555, JingYue Avenue, ChangChun, 130117, China. E-mail: zzbzss@sina.com. Tel: +8613019226185.

Accepted 15 March, 2012

Logistics facility location of supply chain (SC) is one of the important strategic problems in terms of systematic design of SC. Based on the needs of China's companies supply chain management (SCM), this paper aims at the defect of analytic hierarchy process (AHP) location approach, and makes supplement and improvement based on it, in consideration of practical application. It brings in the concept of triangular fuzzy number, proposes evaluation location approach in logistics network facilities of SC based on triangular fuzzy number, and introduces a new method, triangular fuzzy number comparison. The procedure of facility location is easy to program, spread and apply. Finally, the feasibility of such location approach is elaborated based on the application of constructing the evaluation index system of location and the location process of a certain group corporation's distribution center in China.

Key words: Logistics network, strategic facility location, triangular fuzzy number, evaluation.

INTRODUCTION

Network facility location of SC is one of the important strategic problems in terms of systematic design of SC. It can be viewed that through analysis that some scholars (Amiri, 2006; Klose et al., 2005; Bozkaya et al., 2010; Gong et al., 2003; Owen et al., 1998) do lots of researches about facility location, mainly focusing on quantitative modal analysis which emphasizes cost. Network facility location of SC can be influenced by a lot of factors (Sahin et al., 2007). It is a complex strategic decision-making process.

In addition, with the global integrative development, the environment for business management is changing all the time. In terms of strategic design like location, we should dynamically and comprehensively assess various factors including internal, external, short-term, and longterm aspects, to more scientifically determine the network facility location of supply chain (SC). Therefore, this paper takes the example of location by a distribution center of China's company, to comprehensively consider the factors that influence network facility location of SC. It applies quantitative approach combing with qualitative approach, and proposes facility comprehensive evaluation location approach of SC based on analytic hierarchy process (AHP) (Chang, 1996) of triangular fuzzy number.

AHP is a systematic evaluation analysis approach proposed by American operational researcher Professor A. L. Saaty in 1970s, which is done by combining qualitative analysis with quantitative calculation (Laarhoven et al., 1983). This arithmetic has already been applied in many fields due to the rationality and convenience of AHP (Zhu et al., 1997; Kahraman et al., 2003; Lee et al., 2008; Mikhailov et al., 2004), however, there are still defects on scaling procedure and judgement matrix.

Improved AHP arithmetic brings in fuzzy theory (Kaufman et al., 1991). The degree and aspects of using fuzzy theory (Jiang et al., 2002) is different, but there are



Figure 1. Triangular fuzzy number.

still several defects: it ignores that individual judgment has certain fuzzy property; it is difficult to simply and effectively combine qualitative and quantitative analysis; model structure and operating produce is not normative, and not good for programming and application.

Some researchers (Chen, 2001; Jahanshahloo et al., 2006; Bottani et al., 2006) propose some improved methods, Ertuğrul and Karakaşoğlu (2008) compare fuzzy AHP and fuzzy TOPSIS methods for facility location selection and the results, propose the fuzzy TOPSIS method of facility location.

This paper makes supplement and improvement about this approach, and proposes an application method of fuzzy AHPin facility comprehensive evaluation location approach of SC based on AHP approach of triangular fuzzy number.

TRIANGUANLAR FUZZY NUMBER

Definition 1: If fuzzy number P can be determined by (l, m, u), value of the membership function will be:

$$u_{p}(x) = \begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ \frac{u-x}{u-m} & m \le x \le u \\ 0 & \text{Otherwise} \end{cases}$$
(1)

We call P is triangular fuzzy number, written as P = (l, m, u). When l = m = u, P is an exact number. Distribution of the triangular fuzzy number is shown in Figure 1.

In order to make comparison between two triangular fuzzy number, there is a following definition:

Definition 2: supposing $p_1 = (l_1, m_1, u_1) \cdot p_2 = (l_2, m_2, u_2)$ are two random triangular fuzzy numbers, the possibility of $p_1 \ge p_2$ will be:

$$W(p_{1} \ge p_{2}) = \begin{cases} (l_{1} - u_{2})/[(m_{2} - u_{2}) - (m_{1} - l_{1})]m_{1} < m_{2} \\ \\ 1 & m_{1} \ge m_{2} \end{cases}$$
(2)

Definition 3: Supposing the aggregate consisting of n + 1 triangular fuzzy numbers is $H = (p, p_1, p_2, \dots, p_n)$, the possibility of P will be:

$$V(p \ge p_1, p_2, \cdots, p_n) =$$

min { $V(p \ge p_1), V(P \ge p_2), \cdots, V(p \ge p_n)$ } (3)

Definition 4: Supposing the decision matrix $P = (p_{ij})_{n \times n}$, therein $p_{ij} = (l_{ij}, m_{ij}, u_{ij})$ is an triangular fuzzy number, and $0 \le l_{ij} \le m_{ij} \le u_{ij} \le 1, \forall i, j \in 1, 2, \dots, n$ If the matrix P satisfies:

(1)
$$l_{ii} = 0.5, m_{ii} = 0.5, u_{ii} = 0.5, \forall i;$$

(2) $l_{ij} + u_{ji} = 1, m_{ij} + m_{ji} = 1, u_{ij} + u_{ji} = 1, i \neq j, \forall i, j$

P will be called complementary judgement matrix of the triangular fuzzy number. The variable P_{ij} in the matrix indicates the degree that solution x_i prevails over solution

 x_j .

COMPREHENSIVE EVALUATION LOCATION APPROACH OF TRIANGULAR FUZZY NUMBER

To establish layer structure of the system

Layer analysis structure of factors that influence location will be determined, based on the analysis and conclusion of factors that influence network facility of SC, as well as certain principles for selection of evaluation index.

Steps for calculating evaluation index weight

Step 1: Constructing fuzzy judgement matrix: Based on the ration indication of triangular fuzzy number, the triangular fuzzy judgement matrix $P^k = [p_{ij}]_{n \times n}$ will be formed through comparison between a certain factor (principle) and relevant *n* variables in layer *k*. Therein variable $p_{ij} = (l_{ij}, m_{ij}, u_{ij})$ is a close span, which makes

 m_{ij} as the median. The evaluation $p = (l_{ij}, m_{ij}, u_{ij})$ will be given by the experts. l_{ij} represents the most conservative evaluation for the indexes given by the experts. m_{ij} represents the most possible evaluation for the indexes given by the experts. u_{ij} represents the most optimistic evaluation for the indexes given by the experts. Consequently, the primary evaluation matrix of triangular fuzzy number is formed.

Step 2: Integration of evaluation information: $p_{ij}^{t} = (l_{ij}^{t}, m_{ij}^{t}, u_{ij}^{t}), \quad i, j = 1, 2, \cdots, n, t = 1, 2, \cdots, T$ is the triangular fuzzy number given by expert t. The preference of each decision maker will be aggregated by simple average approach. The calculating formula is:

$$M_{ij}^{k} = \frac{1}{T} \otimes \left(p_{ij}^{1} \oplus p_{ij}^{2} \oplus \dots \oplus p_{ij}^{T} \right) = \frac{1}{T} \left(\sum_{i=1}^{T} l_{ij}^{i}, \sum_{i=1}^{T} m_{ij}^{i}, \sum_{i=1}^{T} u_{ij}^{i} \right)$$
(4)

Calculating the comprehensive triangular fuzzy number in layer k, and the comprehensive judgement matrix all factors in layer k to factor q in layer.

Step 3: Calculating the value of comprehensive importance degree: According to the calculating rule of the fuzzy number, and the formula,

$$S_{i}^{k} = \left[\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right] \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{n} l_{ij}, \sum_{i=1}^{n} \sum_{j=1}^{n} m_{ij}, \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij}\right]^{-1}$$
$$= \left[\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} m_{ij}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} l_{ij}}\right]$$
$$= \sum_{j=1}^{n} M_{ij}^{k} \otimes \left(\sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij}^{k}\right)^{-1}, \quad 1, 2, \cdots, n$$
(5)

Calculating normative fuzzy aggregate $S_1^k, S_2^k, \dots, S_n^k$, which respectively indicates the fuzzy comprehensive degree that all factors in layer k to factor q in layer k-1.

Step 4: According to formula (2), calculating the possibility of $S_i^k \ge S_j^k$, $V(S_i^k \ge S_j^k)$, $i, j = 1, 2, \dots, n$ $i \ne j$

Step 5: According to formula (3), calculating the possibility of $S_i^k \ge S_1^k, S_2^k, \dots, S_n^k$ for the importance of each index, that is:

$$d_q^k(x_i) = V(S_i^k \ge S_1^k, S_2^k \cdots, S_n^k), \quad i = 1, 2, \cdots, n$$

Step 6: Calculating the vector of monolayer index weight: The sequential weight vector of layer *k* in the location scheme can be regarded as $\widehat{W}_q^k = (d_q^k(x_1), d_q^k(x_2), \dots, d_q^k(x_n))^T$, according to the derived $d_q^k(x_i)(i = 1, 2, \dots, n)$. The sequential weight vector of layer *k* after normalization will be $U_q^k = W_q^k = (w_{1q}^k, w_{2q}^k, \dots, w_{nq}^k)^T$, which indicates the sequence of each factor in layer *k* to factor *q* in layer k - 1. (There in, $q = 1, 2, \dots, m$; *m* represents the module in layer k - 1).

Step 7: Calculating the vector of comprehensive index weight: If sequential vector to the general target by layer k-1 is $W^{k-1} = (w_1^{k-1}, w_2^{k-1}, \cdots, w_m^{k-1})^T$, the comprehensive sequence to the general target by all factors in layer k is shown as the following formula:

$$W^{k} = (w_{1}^{k}, w_{2}^{k}, \cdots, w_{n}^{k})^{\overline{r}} \overline{\int_{i=1}^{n} dU_{(x_{i})}^{k}} W^{k-1}$$
(6)

Therein:

$$U^{k} = \begin{bmatrix} U_{1}^{k}, U_{2}^{k}, \cdots & U_{m}^{k} \end{bmatrix} = \begin{bmatrix} w_{11}^{k} & w_{12}^{k} & \cdots & w_{1m}^{k} \\ w_{21}^{k} & w_{22}^{k} & \cdots & w_{2m}^{k} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1}^{k} & w_{n2}^{k} & \cdots & w_{nm}^{k} \end{bmatrix}$$

Comprehensive evaluation of the scheme

After the weight value that each evaluation corresponds to the general target is set, matrix R^k of the triangular fuzzy evaluation value will be calculated in consideration of the expert's opinions, adopting the triangular fuzzy number approach to score each evaluation index. The calculating procedure is similar as the way to determine index weight. Then the comprehensive evaluation value of each evaluation scheme will be calculated according to formula:

$$A^s = R^k_s \circ W^k \tag{7}$$

Therein: A^{s} ($s = 1, 2, \dots, n$) represents the location scheme n, and R_{s}^{k} ($k = 1, 2, \dots, n$) represents the comprehensive (weight mean of each expert's evaluation



Figure 2. System structure of the evaluation indexes for distribution center location.

result) vector of the triangular fuzzy evaluation that variables in layer k corresponds to the general target.

In order to make clear comparison between the triangular fuzzy numbers, this paper proposes a new comparison approach according to the distribution trait of triangular fuzzy number. The following formula can be used:

$$\overline{D} = \frac{\left(l_{ij} + 6m_{ij} + u_{ij}\right)}{8} \tag{8}$$

Calculating the mean, and make comparison between the triangular fuzzy numbers by making use of the mean. For comprehensive evaluation value of scheme A and scheme B, if $\overline{D}_A \succ \overline{D}_B$, scheme A is better, and scheme B is better in the opposite case; if $\overline{D}_A = \overline{D}_B$, scheme A is better in case of $m_{ij}^A \succ m_{ij}^B$, and scheme B is better in the opposite case. If the above method is invalid, A and B is regarded as equivalent. The decision makers could make the choice based on their psychological preference. If the decision maker is conservative, the value of l_{ij}^A and l_{ij}^B can be compared. When $l_{ij}^A \succ l_{ij}^B$, scheme A is better, and scheme B is better in the opposite case. If the

decision maker is optimistic, the value of u_{ij}^A and u_{ij}^B can be compared. When $u_{ij}^A \succ u_{ij}^B$, scheme A is better, and scheme B is better in the opposite case.

TEST OF THE APPROACH

This paper makes use of the comprehensive evaluation location approach of triangular fuzzy AHP to do applicable test about the approach, combing with the example of location of distribution center of the diary products for a diary group corporation.

Due to the limitation of the space, only 3 experts are chosen to do calculation about the scheme decision of distribution center location in this paper. The relationship among the factors in different layers that influence the decision are shown in Figure 2.

 P_t^{ki} represents comparative triangular fuzzy judgement matrix of evaluation index. Therein *k* represents layer *k*, valued 0, 1, 2; *i* represents comparison target *i*, valued F_1, F_2, F_3, F_4 ; *t* represents expert *t*, valued 1, 2, 3. R_s^k (k = 2, s = 1,2,3) represents vector of the triangular fuzzy integration evaluation that variables in

0	DS	CE	PE	SE
DS	(0.5,0.5,0.5)	(0.2,0.5,0.6)	(0.4,0.6,0.7)	(0.4,0.6,0.8)
CE	(0.4,0.5,0.8)	(0.5,0.5,0.5)	(0.2,0.3,0.6)	(0.1,0.3,0.5)
PE	(0.3,0.4,0.6)	(0.4,0.7,0.8)	(0.5,0.5,0.5)	(0.2,0.4,0.5)
SE	(0.2,0.4,0.6)	(0.5,0.7,0.9)	(0.5,0.6,0.8)	(0.5,0.5,0.5)

Table 1. Triangular fuzzy judgement matrix with respect to O of expert P_1^1 .

Table 2. Triangular fuzzy judgement matrix with respect to O of expert P_2^1 .

SE
(0.4,0.7,0.9)
(0.4,0.6,0.8)
(0.3,0.5,0.6)
(0.5,0.5,0.5)

Table 3. Triangular fuzzy judgement matrix with respect to O of expert P_3^1 .

0	DS	CE	PE	SE
DS	(0.5,0.5,0.5)	(0.2,0.4,0.6)	(0.3,0.5,0.6)	(0.3,0.6,0.8)
CE	(0.4,0.6,0.8)	(0.5,0.5,0.5)	(0.2,0.4,0.5)	(0.4,0.6,0.7)
PE	(0.4,0.5,0.7)	(0.5,0.6,0.8)	(0.5,0.5,0.5)	(0.2,0.5,0.6)
SE	(0.2,0.4,0.7)	(0.3,0.4,0.6)	(0.4,0.5,0.8)	(0.5,0.5,0.5)

layer k corresponds to the general target (simple weighted mean of each expert's evaluation results), therein s represents the experts.

According to the research result from judgement matrix of evaluation index weight, matrix of relative importance evaluation for each index (only the data in principle layer is listed) and three schemes index evaluation value will be calculated. They are shown in Tables 1, 2, 3 and 4.

According to step 2 to 7, comprehensive sequence of index weight that each index corresponds to the general target can be calculated, the calculating result is as follows:

 $W_o^2 = U^2 W_o^1 =$ $\begin{pmatrix} 0.0898, 0.0703, 0.0528, 0.0676, 0.0980, 0.0953, \\ 0.0801, 0.1070, 0.0918, 0.0823, 0.072, 0.0930 \end{pmatrix}^T$

Then, the angular fuzzy comprehensive evaluation value for each location scheme can be calculated according to formula (7).

 $A^{1} = R_{1}^{2} \circ W_{o}^{2} = (0.26787, 0.43781, 0.64608)$ $A^{2} = R_{2}^{2} \circ W_{o}^{2} = (0.24881, 0.4327, 0.6209)$ $A^{3} = R_{3}^{2} \circ W_{o}^{2} = (0.28848, 0.4812, 0.66888)$

Respectively, calculating the mean of comprehensive evaluation value, according to formula (8).

$$\overline{D}_{1} = \frac{\binom{l_{ij} + 6m_{ij} + u_{ij}}{8}}{D_{2}} = 0.442601$$

$$\overline{D}_{2} = \frac{\binom{l_{ij} + 6m_{ij} + u_{ij}}{8}}{8} = 0.433239$$

$$\overline{D}_{3} = \frac{\binom{l_{ij} + 6m_{ij} + u_{ij}}{8}}{8} = 0.48057$$

It can be obviously seen from the calculating result: $\overline{D}_3 > \overline{D}_1 > \overline{D}_2$ that is, location scheme 3 is better than the other two schemes.

R	R_{1}^{2}	R_2^2	R_{3}^{2}
RP	(0.2,0.3,0.5)	(0.3,0.4,0.6)	(0.4,0.5,0.7)
AIL	(0.2,0.4,0.6)	(0.3,0.4,0.7)	(0.1,0.3,0.5)
CC	(0.3,0.6,0.8)	(0.3,0.5,0.7)	(0.3,0.4,0.5)
CV	(0.1,0.3,0.5)	(0.2,0.4,0.6)	(0.4,0.6,0.7)
С	(0.2,0.4,0.6)	(0.1,0.3,0.5)	(0.2,0.4,0.5)
SP	(0.4,0.5,0.7)	(0.3,0.6,0.7)	(0.2,0.5,0.7)
MS	(0.2,0.4,0.6)	(0.1,0.3,0.5)	(0.3,0.5,0.6)
PFV	(0.2,0.3,0.5)	(0.4,0.5,0.7)	(0.3,0.4,0.6)
NPOS	(0.4,0.6,0.8)	(0.2,0.4,0.6)	(0.3,0.6,0.8)
TI	(0.3,0.4,0.5)	(0.2,0.3,0.5)	(0.4,0.5,0.8)
PS	(0.4,0.5,0.8)	(0.4,0.6,0.8)	(0.4,0.7,0.9)
SCC	(0.3,0.6,0.9)	(0.2,0.5,0.6)	(0.2,0.4,0.7)

Table 4. Performance score of every index given by three experts $R_1^2 R_2^2 R_3^2$.

Therein: Social environment--SE; performance forecast--PE; competition environment--CE; demand situation— DS; regional population--RP; average income level--AIL; consumption custom--CC; Consumption Volume per capita—CV; Competitors--C; supplement products--SP; market share--MS; passenger flow volume per day--PFV; net profit of sales--NPOS; transportation infrastructure--TI; policy support--PS; social culture condition— SCC.

CONCLUSIONS

Network facility location of SC (factory, logistic center, warehouse, supply outlet, distribution outlet, etc.) is an important problem concerning with strategic decision making. It will directly influence relevant cost and success of SC operation.

This paper proposes an application approach in network facility location of SC based on TFAHP, which standardizes the evaluation process. It is easy to program, spread and apply, as well as increase the efficiency and scientific property of strategic facility location of SC. The proposed model can be an effective evaluation approach for solving the problem of logistics facilities location of SC.

REFERENCES

- Amiri A (2006). Designing a distribution network in a supply chain system: Formulation and efficient solution procedure. Eur. J. Oper. 171(2):567-576.
- Bottani E, Rizzi A (2006). A fuzzy TOPSIS methodology to support outsourcing of logistics services. Supply Chain Manag. 11 (4):294-308.
- Bozkaya B, Yanik S, Balcisoy S (2010). A GIS-Based Optimization Framework for Competitive Multi-Facility Location-Routing Problem. Networks Spat. Econ. 10(3):297-320.
- Chang DY (1996). Applications of the extent analysis method on fuzzy AHP. Eur. J. Oper. 95(3):649-655.
- Chen CT (2001) A fuzzy approach to select the location of the distribution center. Fuz. Set. Syst. 118(1):65-73.

- Ertuğrul İ, Karakaşoğlu N (2008). Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection. Int. J. Adv. Manuf. Technol. 39:783-795.
- Gong YC, Guo XF, Cai TJ, Li WJ (2003). Research on the choosing model of physical distribution sites and its arithmetic. Chin. J. Hig. Tran. 4:123-126.
- Jahanshahloo GR, Hosseinzadeh LF, Izadikhah M (2006). Extension of the TOPSIS method for decision-making problems with fuzzy data. Appl. Math. Comput. 181:1544-1551.
- Jiang YP, Fan ZP (2002). A practical ranking method for reciprocal judgement matrix with triangular fuzzy numbers. Syst. Eng. 3:89-92.
- Kahraman C, Cebeci U, Ulukan Z (2003). Multi-criteria supplier selection using fuzzy AHP. Logist. Inf. Manag. 16(6):382-394.
- Kaufman A, Gupta MM (1991). Introduction to fuzzy arithmetic: theory and application. Nueva York: Van Nostrand Reinhold, pp. 22-30.
- Klose A, Drexl A (2005). Facility location models for distribution system design. Eur. J. Oper. 162(1):4-29.
- Laarhoven VPJM, Pedrycz W (1983). A Fuzzy Extension of Saaty's Priority Theory. Fuz. Set. Syst. 11(3):199-227.
- Lee AHI, Chen WC, Chang CJ (2008). A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan. Expert. Syst. Appl. 34(1):96-107.
- Mikhailov L, Tsvetinov P (2004). Evaluation of services using a fuzzy analytic hierarchy process. Appl. Soft. Comput. 5:23-33.
- Owen SH, Mark S, Daskin (1998). Strategic facility location: A review. Eur. J. Oper. 111(3):423-447.
- Sahin G, Süral H (2007). A review of hierarchical facility location models. Comput. Oper. 34(8):2310-2331.
- Zhu KJ, Zhang XL, Xiao LJ (1997). The Method and Application of Fuzzy AHP. Sys. Eng. Theor. Appl. 12:64-69.