

Full Length Research Paper

A two-echelon inventory model for fuzzy demand with mutual beneficial pricing approach in a supply chain

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This paper develops a two-echelon inventory model with mutual beneficial pricing strategy with considering fuzzy annual demand; single vendor and multiple buyers in this model. The beneficial pricing strategy can benefit the vendor more than multiple buyers in the integrated system, when price reduction is incorporated to entice the buyers to accept the minimum total cost. Negotiation factors is very important in the in fuzzy model, it can balance the cost saving between the players. A numerical example with sensitivity analysis is provided to demonstrate the theory. Finally, this paper can prove that the price reduction mechanism is a mutual beneficial strategic partnership between the vendor and buyers.

Key words: Fuzzy annual demand, price reduction.

INTRODUCTION

In the supply chain management today, JIT requires cooperation between the buyer and the vendor, which is very helpful to form a special partnership between the buyer and the vendor. When the partnership between the buyer and the vendor becomes strong, it is very helpful in achieving tangible benefits for each other (Kelle et al., 2002). An effect supply chain network needs the close partnership between the buyers and the vendors. The concept of serial multi-echelon structures to determine the optimal policy was presented by Clark and Scarf (1960). Banerjee (1968) derived a joint economic lot size model for a single vendor and single buyer system where the vendor has a finite replenishment rate. Goyal (1988) generalized Banerjee's (1968) model by relaxing the assumption of the lot-for-lot policy of the vendor and showed that quantity per cycle being an integer multiple of the buyer's purchase quantity provides a lower or equal joint total relevant cost as compared to Banerjee's (1968) model.

One of the early authors who analyzed a vendor-oriented optimal quantity discount policy that maximized

the vendor's gain was Monahan (1984), but did so at no additional cost to the buyer. Lee and Rosenblatt (1986) extended Monahan's (1984) model and developed a new algorithm to solve the vendor's ordering and price discount policy. Lal and Staelin (1984) extended to handle variable ordering and shipping costs and situations where the seller faces numerous groups of buyers, each having different ordering policies. Weng and Wong (1993) considered the discount of the vendor's quantity from the perspective of reducing the vendor's operating cost and increasing the buyer's demand.

Weng (1995) developed a model for analyzing the impact of joint decision policies on channel coordination in a system including a supplier and a group of analogous buyers. A lot-for-lot joint pricing policy with price-sensitive demand was developed by Li et al. (1996). A lot-for-lot discount pricing policy for deteriorating items with constant demand rate developed by Wee (1998). Wang and Wu (2000) derived a combined discount pricing policy for a supplier to maximize its quantity discount obtained from many different buyers. Lu (1995) and Goyal (1995) derived the integrated model between the vendor and the buyers with unequal lot size. Looking back on the past research, none of them considered the general replenishing and pricing policies for an integrated supply chain system.

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Today, many researches pay much attention on the integration of the vendor and the buyers. Although the vendor has greater benefits than the buyers do, the buyers may have no much interest in cooperating. In order to let the buyers order more quantity, some incentive policy like price reduction may be a good policy in the integrated system. Most of the related researches assumed the average demand per year is fixed constant, but it is usually difficult for managers to set the demand as crisp values in reality. So, many researchers have been applying fuzzy demand theory and techniques to develop and solve production inventory problems. For example, Park (1987) considered fuzzy inventory costs by using arithmetic operations of the extension principle. Chen et al. (1996) fuzzified the demand, ordering cost, inventory cost, and backorder cost into trapezoidal fuzzy numbers in an EOQ model with backorder consideration. Mahata et al. (2005) investigated the joint economic lot size model as fuzzy values of the economic lot size model for purchaser and vendor. They find that the joint total relevant cost is slightly higher than in the crisp model after defuzzification. Wu and Yao (2003) models and investigate an integrated inventory model with backorder for fuzzy order quantity and fuzzy shortage quantity that these are a normal triangular fuzzy number.

This paper present a two-echelon fuzzy inventory model with mutual beneficial pricing strategy considering JIT concept and price reduction to the buyers for ordering larger quantity, and this model incorporates the fuzziness of annual demand. A numerical example is carried out to demonstrate the approach and the significance of considering the integration of supply chain network.

ASSUMPTIONS AND NOTATIONS

Assumptions

The proposed model in this paper is developed on the following assumptions:

- (a) The replenishment rate of the buyers is instantaneous, but the vendor’s replenishment rate is finite.
- (b) All buyers have constant demand rate.
- (c) All players have complete information between each other.
- (d) There are only single vendor and multiple buyers in this model.
- (e) Shortage is not allowed.
- (f) The vendor’s cycle time for each buyer is assumed the same for decreasing setup times.
- (g) Each buyer demand rate is normal triangular fuzzy numbers.

There are three scenarios in this model:

- 1. The first scenario: we neglect integration and price

reduction.

- 2. The second scenario: we consider the integration of the vendor and the buyers, but we don’t consider price reduction.

- 3. The final scenario: we consider the integration and price reduction of the vendor and the buyers simultaneously.

Notations

We defined the common parameters of the vendor and the buyers as follow: N : Number of buyers; d_j : Demand rate for buyer j, j=1, 2, 3...N; \tilde{d}_j : Fuzzy demand rate for buyer j, j=1, 2, 3...N; $\tilde{d}_j = (d_j - \Delta_{1j}, d_j, d_j + \Delta_{2j})$;

D : Total demand rate of all buyers; \square : Total fuzzy

demand rate of all buyers; $\tilde{D} = \sum_{j=1}^n \tilde{d}_j$; TC : Integrated total cost of the vendor and the buyers; R_{S_v} : Negotiation factor of cost saving between vendor and all players; R_{S_j} : Negotiation factor of cost saving between buyer j and all players.

The parameters of the vendor are as follow: R : Replenishment rate; C_v : Setup cost, \$ per cycle; C_{vb} : Fixed cost to process buyer’s order of any size; UC : Unit cost; FC_v : Percentage inventory carrying cost per time per unit dollar; I_{vj} : Average inventory level with respect to buyer j; I_{vi} : Average inventory level in scenario i; TC_v : Total cost for vendor; CS_v : Cost saving of TC_{v3} with respect to TC_{v1} .

The parameters of the buyers are as follow: OC_{bj} : Ordering cost for buyer j; P_{bj} : Unit purchased price for buyer j to the vendor; FC_{bj} : Percentage inventory carrying cost for buyer j per year per dollar; TC_b : Total cost for all buyers; CS_{bj} : Cost saving of TC_{b3j} with respect to TC_{b1j} for buyer j; CS_b : Cost saving of TC_{b3} with respect to TC_{b1} for all buyers.

The decision variables are: n_j : Number of deliveries from vendor to buyer j per cycle; P_{bj} : Unit purchased price for buyer j to the vendor; Q_{bj} : Lot size for buyer j.

PROPOSED MODEL WITH FUZZY DEMAND

By using Wee and Yang’s (2007) research, we can get a mathematic model of the buyers and the vendor

subscripted thus: The buyer j 's annual total cost (TC_{bij}) and all buyers' annual total costs (TC_{bi}) are:

$$TC_{bij} = \frac{d_j OC_{bj}}{Q_{bij}} + \frac{Q_{bij} P_{bij} FC_{bj}}{2} - (P_{b1j} - P_{bij}) d_j \tag{1}$$

$$TC_{bi} = \sum_{j=1}^N \frac{d_j OC_{bj}}{Q_{bij}} + \sum_{j=1}^N \frac{Q_{bij} P_{bij} FC_{bj}}{2} - \sum_{j=1}^N (P_{b1j} - P_{bij}) d_j \tag{2}$$

Since the vendor's cycle time interval $\frac{n_{ij} Q_{bij}}{d_j}$ for each buyer is assumed the same for decreasing setup times, the relationship between Q_{bij} and Q_{bi1} is as follows:

$$Q_{bij} = \frac{d_j n_{i1} Q_{bi1}}{n_{ij} d_1} \tag{3}$$

The vendor's average inventory level, I_{vi} is:

$$I_{vi} = \sum_{j=1}^N \frac{Q_{bij}}{2} \left[(n_{ij} - 1) \left(1 - \frac{D}{P} \right) + \frac{D}{P} \right] \tag{4}$$

The vendor's annual total cost is:

$$TC_{vi} = \frac{D \left(C_v + \sum_{j=1}^N n_{ij} C_{vb} \right)}{\sum_{j=1}^N n_{ij} Q_{bij}} + \sum_{j=1}^N I_{vij} P_v F_v + \sum_{j=1}^N (P_{b1j} - P_{bij}) d_j \tag{5}$$

Consider the problem with fuzzy annual demand d_j by fuzzifying to a triangular fuzzy number \tilde{d}_j , where $\tilde{d}_j = (d_j - \Delta_{1j}, d_j, d_j + \Delta_{2j})$, $0 < \Delta_{1j} < d_j$, $0 < \Delta_{2j} < d_j$

and Δ_{1j}, Δ_{2j} are both determined by decision-makers. In this case, buyer j 's annual total cost and all buyers' annual total costs with fuzzy demand can be expressed as:

$$\square TC_{bij} = \frac{\square d_j OC_{bj}}{Q_{bij}} + \frac{Q_{bij} P_{bij} FC_{bj}}{2} - (P_{b1j} - P_{bij}) \square d_j \tag{6}$$

and

$$\square TC_{bi} = \sum_{j=1}^N \frac{\square d_j OC_{bj}}{Q_{bij}} + \sum_{j=1}^N \frac{Q_{bij} P_{bij} FC_{bj}}{2} - \sum_{j=1}^N (P_{b1j} - P_{bij}) \square d_j \tag{7}$$

Accordingly, the vendor's average inventory level and the vendor's annual total cost with fuzzy annual demand can

be expressed as:

$$\square I_{vi} = \sum_{j=1}^N \frac{Q_{bij}}{2} \left[(n_{ij} - 1) \left(1 - \frac{\square D}{P} \right) + \frac{\square D}{P} \right],$$

Where,

$$\square D = \sum_{j=1}^n \square d_j \tag{8}$$

and

$$\square TC_{vi} = \frac{\square D \left(C_v + \sum_{j=1}^N n_{ij} C_{vb} \right)}{\sum_{j=1}^N n_{ij} Q_{bij}} + \sum_{j=1}^N \square I_{vij} P_v F_v + \sum_{j=1}^N (P_{b1j} - P_{bij}) \square d_j \tag{9}$$

Where,

$$\square D = \sum_{j=1}^n \square d_j$$

Definition 1

From Kaufmann and Gupta (1991), Zimmermann (1996), Yao and Wu (2000), for any a and $0 \in R$, they define the signed distance from a to 0 as $d_0(a, 0) = a$. If $a > 0$, a is on the right hand side of origin 0; and the distance from a to 0 is $d_0(a, 0) = a$. If $a < 0$, a is on the left hand side of origin 0; and the distance from a to 0 is $-d_0(a, 0) = -a$. This is the reason why $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\square A$ defined on R , the α -cut t of $\square A$ is $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$, $0 \leq \alpha \leq 1$, and both $A_L(\alpha)$ and $A_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$. Then, for any $\alpha \in \Omega$, we have:

$$\square A = \bigcup_{0 \leq \alpha \leq 1} [A_L(\alpha)_\alpha, A_U(\alpha)_\alpha] \tag{10}$$

Besides, for every $\alpha \in [0, 1]$, the α -level fuzzy interval $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]$, has a one-to-one correspondence

with the crisp interval $[A_L(\alpha), A_U(\alpha)]$ that is, $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha] \leftrightarrow [A_L(\alpha), A_U(\alpha)]$ is one-to-one mapping. From Definition 1, the signed distance of two end points, $A_L(\alpha)$ and $A_U(\alpha)$ to 0 are $d_0(A_L(\alpha), 0) = A_L(\alpha)$ and $d_0(A_U(\alpha), 0) = A_U(\alpha)$, respectively. Hence, the signed distance of interval $[A_L(\alpha), A_U(\alpha)]$ to 0 can be represented by their average of $[A_L(\alpha), A_U(\alpha)]$. Therefore, the signed

distance of interval $[A_L(\alpha), A_U(\alpha)]$ to 0 can be represented as:

$$d_0([A_L(\alpha)_\alpha, A_U(\alpha)_\alpha], 0) = [d_0(A_L(\alpha), 0) + d_0(A_U(\alpha), 0)] / 2 = (A_L(\alpha) + A_U(\alpha)) / 2 \tag{11}$$

Further, because of the 1-level fuzzy point, \square_0 is mapping to the real number 0, the signed distance of $[A_L(\alpha)_\alpha, A_U(\alpha)_\alpha]$ to \square_0 can be defined as:

$$d_0([A_L(\alpha)_\alpha, A_U(\alpha)_\alpha], \square_0) = d_0([A_L(\alpha), A_U(\alpha)], 0) = [A_L(\alpha) + A_U(\alpha)] / 2 \tag{12}$$

Thus, from (11) and (12), since this function is continuous on $0 \leq \alpha \leq 1$ for $A \in \Omega$, we can use further equation to define the signed distance of A to \square_0 .

Next, defuzzify $\square TC_{bi}$ and $\square TC_{vi}$ by using the signed distance method. From Definition 1, the signed distance of $\square TC_{bi}$ and $\square TC_{vi}$ to \square_0 is given by:

$$d(\square TC_{bi}, \square_0) = \sum_{j=1}^N \frac{d(d_j, \square_0) OC_{bj}}{Q_{bij}} + \sum_{j=1}^N \frac{Q_{bij} P_{bij} FC_{bj}}{2} - \sum_{j=1}^N (P_{b1j} - P_{bij}) d(d_j, \square_0) \tag{13}$$

$$d(\square TC_{vi}, \square_0) = \frac{d(D, \square_0) \left(C_v + \sum_{j=1}^N n_{ij} C_{vb} \right)}{\sum_{j=1}^N n_{ij} Q_{bij}} + \sum_{j=1}^N I_{vij} P_v F_v + \sum_{j=1}^N (P_{b1j} - P_{bij}) d(d_j, \square_0) \tag{14}$$

where $D = \sum_{j=1}^n d_j$, $d(d_j, \square_0)$ the signed distance of fuzzy number d_j to \square_0 by Appendix, that is:

$$d(d_j, \square_0) = \frac{1}{4} \left[(d_j - \Delta_j) + 2d_j + (d_j + \Delta_j) \right] = d_j + \frac{1}{4} (\Delta_{2j} - \Delta_{1j}) \tag{15}$$

Substituting the result of (14) into (15) and (13), we have:

$$TC_{bi} \equiv d(\square TC_{bi}, \square_0) = \sum_{j=1}^N \frac{OC_{bj}}{Q_{bij}} \left(d_j + \frac{1}{4} (\Delta_{2j} - \Delta_{1j}) \right) + \sum_{j=1}^N \frac{Q_{bij} P_{bij} FC_{bj}}{2} - \sum_{j=1}^N (P_{b1j} - P_{bij}) \left(d_j + \frac{1}{4} (\Delta_{2j} - \Delta_{1j}) \right) \tag{16}$$

$$TC_{vi} \equiv d(\square TC_{vi}, \square_0) = \frac{\sum_{j=1}^N \left(d_j + \frac{1}{4} (\Delta_{2j} - \Delta_{1j}) \right) \left(C_v + \sum_{j=1}^N n_{ij} C_{vb} \right)}{\sum_{j=1}^N n_{ij} Q_{bij}} + \sum_{j=1}^N I_{vij} P_v F_v + \sum_{j=1}^N (P_{b1j} - P_{bij}) \left(d_j + \frac{1}{4} (\Delta_{2j} - \Delta_{1j}) \right) \tag{17}$$

Where TC_{bi} is regard as total cost for all buyers in the i-th scenario in the fuzzy sense; TC_{vi} is regard as total cost of the vendor in the i-th scenario in the fuzzy sense.

DISCUSSION FOR THREE SCENARIOS

We follow Wee and Yang's (2007) model to make the discussion of three scenarios.

Scenario 1: Integration and price reduction are not considered

By buyer viewpoint, the buyers have the priority to make the first-step decision. The related costs without integration are as follows:

$$TC_{b1}^* = \underset{all\ Q_{b1j},\ n_{1j}}{Minimize} \sum_{j=1}^N TC_{b1j} \tag{18}$$

Subject to
$$Q_{b1j} = \frac{\tilde{d}_j n_{1j} Q_{b11}}{n_{1j} \tilde{d}_1}$$

$$TC_{v1}^* = \underset{all\ Q_{b1j},\ n_{1j}}{Minimize} TC_{v1} \tag{19}$$

and

$$TC_1^* = TC_{v1}^* + TC_{b1}^* \tag{20}$$

Scenario 2: The integration of the vendor and all buyers without price reduction

The purchased unit cost for each buyer in scenario 2 is assumed to be the same as that in scenario 1. The purpose of integration is to minimize the integrated total cost through information and profit sharing. The optimal value of the integrated total cost in scenario 2 is:

$$TC_2^* = \underset{all\ Q_{b2j},\ n_{2j}}{Minimize} (TC_{v2} + TC_{b2}) \tag{21}$$

Subject to
$$Q_{b2j} = \frac{\tilde{d}_j n_{2j} Q_{b21}}{n_{2j} \tilde{d}_1}$$

Scenario 3: The integration of the vendor and all buyers with price reduction

The discount price of the j-th buyer, P_{b3j} is smaller than P_{b1j} or P_{b2j} . Let buyer j's cost saving be defined as the

difference between TC_{b3j} and TC_{b1j} , and all buyers' cost saving be the difference between TC_{b3} and TC_{b1} , one has:

$$Cs_{bj} = TC_{b1j} - TC_{b3j} \tag{22}$$

and

$$Cs_b = TC_{b1} - TC_{b3} \tag{23}$$

Their relationship is defined as:

$$Rs_{bj} = \frac{Cs_{bj}}{Cs_v + Cs_b} \tag{24}$$

and

$$Rs_v = \frac{Cs_v}{Cs_v + Cs_b} \tag{25}$$

Where Rs_{bj} and Rs_v are the negotiation factors and
$$Rs_v + \sum_{j=1}^N Rs_{bj} = 1$$

$$TC_3^* = \sum_{j=1}^N \underset{all\ Q_{b3j},\ n_{3j}}{Minimize} (TC_{b3j} + TC_{v3j}) \tag{26}$$

Subject to constraints (3), (24) and (25). Substituting (3) into (26), TC_3 is function of Q_{b31} , n_{3j} and P_{b3j} . For each integer n_{3j} , one can solve Q_{b31} by satisfying the following condition:

$$\frac{\partial TC_3}{\partial Q_{b31}} = 0 \tag{27}$$

Substituting Q_{b31} from (27) into (24) and (25), each P_{b3j} can be derived by solving the simultaneous equations of (24) and (25). $TC_3(n_{3j})$ is the integrated total cost in scenario 3 and a function of variable n_{3j} . The optimal solution of n_{3j} can be derived to satisfy the following condition:

$$TC_3(n_{3j} - 1) \geq TC_3(n_{3j}) \leq TC_3(n_{3j} + 1) \tag{28}$$

It is noted that the variables P_{b3j} , Q_{b3j} and n_{3j} are optimized jointly with constraints (3), (24) and (25).

Table 1. The optimal solutions in various scenarios.

Scenario i	1	2	3
P_{bi1}	25	25	23.264
P_{bi2}	25	25	23.221
n_{i1}	3	1	1
n_{i2}	4	2	1
Q_{bi1}	97	302	286
Q_{bi2}	145	302	572
TC_i	4744	4,304	4,199
$TC_i - TC_1$	---	-440	-545
TC_{bi}	1,208	1,759	844
$TC_{bi} - TC_{b1}$	---	+551	-364
TC_{bi1}	500	838	318
TC_{bi2}	708	921	526
TC_{vi}	3,537	2,546	3,355
$TC_{vi} - TC_{v1}$	---	-991	-182

Thus, we can use the following procedure to find the optimal values of Q and n for fuzzy annual demand.

For $j= 1, 2, \dots, N$.

Step 1: Obtain Δ_{1j} and Δ_{2j} from the decision-makers.

Step 2: Use Q_{b31} from (27) to determine RS_{bj} and RS_v .

Step 3: Compute P_{b3j} from equation (24) and (25).

Step 4: Compute n_{3j} by inequality (28).

The $TC_3^*(Q_{b3j}, n_{3j}, P_{b3j})$ is the optimal joint total expected annual cost.

NUMERICAL EXAMPLE

The preceding theory can be illustrated by the following numerical example. One vendor and two buyers annual demand rate $d_1 = 250, d_2 = 500$ units per year, vendor’s replenishment rate: $R = 12,000$ units per year; buyers’ ordering cost $OC_{b1} = \$ 100, OC_{b2} = \$ 100$; buyers’ percentage carrying cost per year per dollar $FC_{b1} = 0.2, FC_{b2} = 0.2$ buyers’ purchased unit price before price discount: $P_{b11} = P_{b12} = \$ 25, P_{b21} = P_{b22} = \$ 25$ vendor’s setup cost $CV = \$ 2,000$; vendor’s fixed cost to process buyer’s order of any size $C_{vb} = \$ 100$; vendor’s percentage carrying cost per year per dollar $FC_v = 0.2$ vendor’s unit cost $U_c = \$ 20$; negotiation factors: $RS_v =$

$1/3, RS_{b1} = 1/3, RS_{b2} = 1/3$. The optimal solutions in various scenarios are discussed in Table 1 (Wee and Yang 2007).

In Table 1, we got the following information: scenario 3 has the lowest price than scenarios 1 and 2. Scenario 3 has the highest lot size than scenarios 1 and 2 and scenario 3 has the lowest total cost than scenarios 2 and 1. As such, scenario 3 (the integration and price reduction of the vendor and the buyers simultaneously) is the best solution in the 3 scenarios. By revising scenario 3, we discussed scenario 3 with fuzzy annual demand in this paper. For the model proposed in scenario 3, solve for the optimal unit purchased price for buyer j in scenario 3:

P_{b3j} , and find the optimal lot size for buyer j in scenario3: Q_{b3j} , and number of deliveries from vendor to

buyer j per cycle in scenario 3: n_{3j} in the fuzzy sense for various given sets of $(\Delta_{1j}, \Delta_{2j})$, $j = 1,2$. Note that in practical situations, Δ_{1j} and Δ_{2j} are determined by the decision-makers due to the uncertainty of the problem. The results are summarized in Tables 2 and 3 and the total cost of buyer and vendor are summarized in Table 4.

Furthermore, Tables 2 and 3 lists the results of the fuzzy case results with those of the crisp one; Table 4 lists the result of total cost in scenario 3 which is with the fuzzy annual demand. The optimal unit purchased price for buyer P_{b3j} , $j = 1, 2$ and the optimal lot size for buyer Q_{b3j} $j = 1,2$ can be derived easily from Wee and Yang (2007) using the classical optimization technique.

Consequently, we have $P_{b31}^{**} = 23.264, P_{b32}^{**} = 22.221$ unit price $Q_{b31}^{**} = 286, Q_{b32}^{**} = 572$ and $TC_3^{**} = 4198.74$. Then

Table 2. Optimal solutions for the model with fuzzy demand of the buyer 1.

d_1	P_{b31}^*	Q_{b31}^*	n_{31}	$V_{P_{b31}}(\%)$	$V_{Q_{b31}}(\%)$
(200,250,400)	23.374	298	1	0.0047	0.0420
(225,250,475)	23.471	310	1	0.0089	0.0839
(50,250,450)	23.264	286	1	0.0000	0.0000
(25,250,275)	22.989	258	1	-0.0118	-0.0979
(100,250,300)	23.138	272	1	-0.0054	-0.0490

Table 3. Optimal solutions for the model with fuzzy demand of the buyer 2.

d_2	P_{b32}^*	Q_{b32}^*	n_{32}	$V_{P_{b32}}(\%)$	$V_{Q_{b32}}(\%)$
(475,500,725)	23.320	597	1	0.0043	0.0437
(450,500,950)	23.406	620	1	0.0080	0.0839
(50,500,950)	23.221	572	1	0.0000	0
(50,500,550)	22.974	517	1	-0.0106	-0.0962
(275,500,575)	23.108	545	1	-0.0049	-0.0472

Table 4. Total cost in scenario 3.

TC_{b31}	TC_{b32}	TC_{b3}	TC_{V3}	TC_3^*	$V_{TC_3}(\%)$
341.68	560.33	902.01	3522.15	4424.16	0.0537
365.68	591.55	957.23	3683.63	4640.86	0.1053
318.76	526.15	844.91	3353.83	4198.74	0.0000
268.44	454.73	723.17	3094.63	3817.8	-0.0907
293.12	490.55	783.67	3085.29	3868.96	-0.0785

Then, the relative variation between fuzzy case and crisp one for the optimal unit purchased price and the optimal lot size and the optimal total cost can be measured by $V_{P_{b3j}}(\%) = (P_{b3j}^* - P_{b3j}^{**}) / P_{b3j}^{**} \times 100\%$ and $V_{Q_{3j}}(\%) = (Q_{b3j}^* - Q_{b3j}^{**}) / Q_{b3j}^{**} \times 100\%$ $j = 1,2$; $V_{TC_3}(\%) = (TC_3^* - TC_3^{**}) / TC_3^{**} \times 100\%$, respectively, as reported in the last two columns of Tables 2 and 3, and the last column of Table 4. From Tables 2, 3 and 4, we observe that:

(1) when $\Delta_{1j} < \Delta_{2j}$ for $j = 1,2$, as $(\Delta_{21} - \Delta_{11})$ increase up to 0 from 200; $(\Delta_{22} - \Delta_{12})$ increase up to 0 from 400 both $V_{P_{b3j}}$ and $V_{Q_{b3j}}$ $j=1,2$ and V_{TC_3} , increase simultaneously. As $(\Delta_{2j} - \Delta_{1j})$ for $j = 1, 2$ increase, $V_{P_{b3j}}, V_{Q_{b3j}}, V_{TC_3}$ increase.

(2) when $\Delta_{1j} > \Delta_{2j}$, $j = 1,2$ then we have $d(\bar{d}_j, \bar{\theta}_1) < d_j$, $j = 1,2$. In this case, $P_{b31}^* < P_{b31}^{**}$ $Q_{b31}^* < Q_{b31}^{**}$ and

$P_{b32}^* < P_{b32}^{**}$, $Q_{b32}^* < Q_{b32}^{**}$ and $TC_3^* < TC_3^{**}$ which result in $V_{P_{b3j}} < 0$, $V_{Q_{b3j}} < 0$ $j=1,2$ and $V_{TC_3} < 0$. Further, as the value $(\Delta_{2j} - \Delta_{1j})$ $j = 1,2$ decreases, both $V_{P_{b3j}}$ $V_{Q_{b3j}}$ $j=1,2$ and V_{TC_3} decrease, which means the smaller the difference between Δ_{1j} and Δ_{2j} the smaller the variation of the solutions between fuzzy case and crisp case.

(3) when $\Delta_{11} = \Delta_{21} = 200$, $d(\bar{d}_1, \bar{\theta}_1) = d_1 = 250$; $\Delta_{12} = \Delta_{22} = 500$, $d(\bar{d}_2, \bar{\theta}_1) = d_2 = 500$ In this case, the solutions of the fuzzy case are identical to those of the crisp case, and hence $V_{P_{b3j}} = 0$, $V_{Q_{b3j}} = 0$ $j=1,2$ and $V_{TC_3} = 0$.

From the example, although we can not ascertain which of the solution is better, the major advantage of the fuzzy model is that the uncertainty of the real situation is obtained well than the crisp model. In addition, the

decision-makers can use the solution which derived from the fuzzy model to perform sensitivity analysis, and to examine the effects of uncertainties.

CONCLUSION

In this paper, we discussed the proposed model with fuzzy annual demand. Uncertainties of annual demand are interested in real supply chain inventory systems. However, we don't pay much attention to this in past study, and it is because there may be a lack of historical data to estimate the annual demand. In this situation, using a crisp value is not appropriate. The proposed model of Wee and Yang (2007) is worthwhile to be reconsidered and we provide an alternative approach. This paper proposes a fuzzy model for two-echelon inventory problem. For the fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimation of total profit per unit time in the fuzzy sense and then the corresponding optimal P_{bj} and Q_{bj} are derived to minimize the total cost. Additionally, the proposed fuzzy model can be reduced to a crisp problem and the optimal lot size and price in the fuzzy sense can be reduced to that of the classical two-echelon inventory model. Although we are not sure the solution obtained from the fuzzy model is better than the solution of the crisp one, the fuzzy approach has the advantage that keeps the uncertainties which always correspond with the real situations better than the crisp one does. Furthermore, the inventory problem in the real situation can be properly solved with this proposed fuzzy model.

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REFERENCE

- Banerjee A (1968). A joint economic lot size model for purchaser and vendor. *Dec. Sci.*, 17 (3): 292-311.
- Clark AJ, Scarf H (1960). Optimal policies for a multi-echelon inventory problem. *Manage. Sci.*, 6(4): 475-490.
- Chen SH, Wang CC, Ramer A (1996). Backorder fuzzy inventory model under function principle. *Inform. Sci.*, 95(1-2): 71-79.
- Goyal SK (1988). A joint economic lot size model for purchaser and vendor: a comment. *Dec. Sci.* 19(1): 236-241.
- Goyal SK (1995). A one-vendor multi-buyer integrated inventory model: a comment. *Eur. J. Oper. Res.*, 82(1): 209-210.
- Kaufmann A, Gupta MM (1991). *Introduction to Fuzzy Arithmetic: Theory and Applications*, Van Nostrand Reinhold, New York
- Kelle P, Pawlowski S, Akbulut A (2002). The influence of quantitative and organizational factors on the cooperation in supply chain. 12th International Working Seminar on Production Economics, Igls/Innsbruck Austria.
- Lal P, Staelin R (1984). An approach for developing an optimal discount pricing policy. *Manage. Sci.*, 30(12):1524-1539.
- Lee LE, Rosenblatt MJ (1986). A generalized quantity discount pricing model to increase supplier's profits. *Manage. Sci.*, 32(9): 1177-1185
- Lu L (1995). A one-vendor multi-buyer integrated inventory model. *Eur. J. Oper. Res.*, 81(2): 312-323.
- Li SX, Hung Z, Ashley A (1996). Inventory channel coordination and bargaining in a manufacturer-retailer system. *Ann. Oper. Res.*, 68(1): 47-60.
- Mahata GC, Goswami A, Gupta DK (2005). A joint economic lot size model for purchaser and vendor in fuzzy sense". *Comput. Math. Appl.*, 50 (10-12): 1767-1790
- Monahan JP (1984). A quantity discount-pricing model to increase vendor profits. *Manage. Sci.*, 30(6): 720-726.
- Park KS (1987). Fuzzy-set theoretic interpretation of economic order quantity, *IEEE Transactions on Systems. Man Cybernetics. SMC*, 17(6): 1082-1084.
- Weng ZK, Wong RT (1993). General models for the supplier's all-unit quantity discount policy. *Nav. Res. Log.*, 40(7): 971-991.
- Weng ZK (1995). Channel coordination and quantity discount. *Manage. Sci.*, 41(9): 1509-1522.
- Wee HM (1998). Optimal buyer-seller discount pricing and ordering policy for deteriorating items. *Eng. Econ.*, 43(2): 151-168.
- Wang Q Wu Z (2000). Improving a supplier's quantity discount gain from many different buyers. *IIE Trans.*, 32(11): 1071-1079.
- Wee HM, Yang PC (2007). A mutual beneficial pricing strategy of an integrated vendor-buyers inventory system. *Int. J. Adv. Manuf. Tech.*, 34(1-2): 179-187.
- Wu KJ, Yao S (2003). Fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity. *Eur. J. Oper. Res.*, 150 (2): 320-352.
- Yao JS, Wu K (2000). Ranking fuzzy numbers based on decomposition principle and signed distance, *Fuzzy Sets Sys.*, 116(2): 275-288.
- Zimmermann HJ (1996). *Fuzzy Set Theory and its Application*, 3rd ed., Kluwer Academic publishers, Dordrecht.

APPENDIX

For a fuzzy set $\bar{A} \in \Omega$ and $\alpha \in [0,1]$, the $\alpha \in cut$ of the fuzzy set \bar{A} is;

$$A(\alpha) = \{x \in \Omega \mid u_A(x) \geq \alpha\} = [A_L(\alpha), A_U(\alpha)],$$

where $A_L(\alpha) = a + (b-a)\alpha$ and $A_U(\alpha) = c - (c-b)\alpha$.

From Definition 1, we obtain the following equation. The signed distance of \bar{A} to $\bar{\theta}_1$ is defined as:

$$d(\bar{A}, \bar{\theta}_1) = \int_0^1 d([A_L(\alpha), A_U(\alpha)], \bar{\theta}_1) d\alpha = \frac{1}{2} \int_0^1 (A_L(\alpha), A_U(\alpha)) d\alpha$$

So this equation is:

$$a(\bar{A}, \bar{\theta}_1) = \frac{1}{2} \int_0^1 [A_L(\alpha), A_U(\alpha)] d\alpha = \frac{1}{4}(2b+a+c)$$