African Journal of Business Management Vol. 6(48), pp. 11758-11761, 5 December, 2012
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM12.572
ISSN 1993-8233 ©2012 Academic Journals

Full Length Research Paper

The chaotic long-run monopolistic competitor's output growth model

Vesna D. Jablanovic

Department of Economics, University of Belgrade, Faculty of Agriculture, Nemanjina 6, Belgrade, Serbia,
E-mail: vesnajab@ptt.rs. Tel: +3812198135. Fax: +381113161730.

Accepted 8 October, 2012

A monopolistically competitive market structure has some features of competition and some features of monopoly. Monopolistic competition has the following attributes: (i) many sellers; (ii) product differentiation; and (iii) free entry. In the long run equilibrium, price equals average total cost, and the firm earns zero economic profit. This paper want to show that the well known neoclassical microeconomics model on monopolistic competition implicitly has chaotic characteristics. The basic aim of this paper is to construct a relatively simple chaotic long-run monopolistic competitors's output growth model that is capable of generating stable equilibria, cycles, or chaos. A key hypothesis of this work is based on the idea that the coefficient, $\pi = \dfrac{d}{(\alpha-1)\, b\left(1+\dfrac{1}{e}\right)}$, plays a crucial role in explaining local stability of the monopolistic competitior's output, where, d – the coefficient of the marginal cost function of the monopolistic competitor, b- the coefficient of the inverse demand function, α - the coefficient of average cost growth.

Key words: Monopolistic competition, long-run, equilirbrium conditions, chaos.

INTRODUCTION

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1997), Grandmont (1985), Goodwin (1990), Medio (1993, 1996), Lorenz (1993) and Jablanovic (2011, 2012), among many others.

The basic aim of this paper is to construct a relatively simple chaotic long-run monopolistic competitors's output growth model that is capable of generating stable equilibria, cycles, or chaos.

THE MODEL

Monopolistic competition has characteristics of both competition and monopoly. Similar to competition, it has many firms, and free exit and entry. Similar to monopoly, the products are differentiated and each company faces a downward sloping demand curve. Monopolistic competition refers to a market situation with a relatively large number of sellers offering similar and differentiated products.

In the model of the monopolistically competitive firm take the inverse demand function

$$P_t = a - b\, Q_t \qquad (1)$$

Where P - monopolistic competitior's price; Q - monopolistic competitior's output; a, b - coefficients of the inverse demand function.

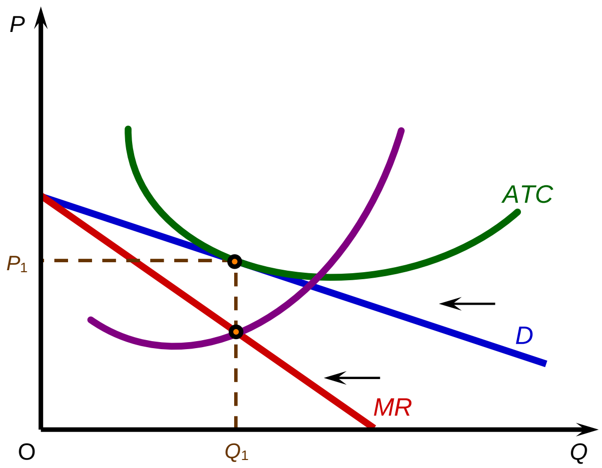

Figure 1. Long-run profit maximization by a monopolistically competitive firm.

Marginal revenue is

$$MR_t = P_t\left[1+\left(\frac{1}{e}\right)\right] \qquad (2)$$

Where MR - marginal revenue; P - monopolistic competitior's price; e - the coefficient of the price elasticity of demand (Figure 1).

Further, suppose the quadratic marginal-cost function for the monopolistically competitive firm is

$$MC_t = c + d\,Q_t + f\,Q_t^{\,2} \qquad (3)$$

where MC – marginal cost; Q – monopolistic competitior's output; c, d, f – coefficients of the quadratic marginal-cost function.

The long-run equilibrium of monopolistically competitive industry generates two equilibrium conditions. Firstly, a monopolistic competitor maximizes profit by producing the quantity at which marginal revenue equals marginal cost.

Thus the profit-maximizing condition is that

$$MR_t = MC_t \qquad (4)$$

In a monopolistically competitive market, price exceeds marginal cost because profit maximization requires marginal revenue to equal marginal cost and because the downward-sloping demand curve makes marginal revenue less than the price. Equivalently, equation (5) expresses price directly as a markup over marginal cost, that is,

$$P_t = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \qquad (5)$$

The second condition, price (P) equal to average cost (ATC)

$$P_t = ATC_t \qquad (6)$$

means that each firm in the industry is earning only a normal profit. Economic profit is zero and there is no economic loss.

In accordance with (5) and (6) we obtain

$$ATC_t = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \qquad (7)$$

Further,

$$ATC_{t+1} = ATC_t + \Delta\, ATC, \qquad (8)$$

That is

$$(1-\alpha)\, ATC_{t+1} = ATC_t \qquad (9)$$

Substitution (6) and (7) in (9) gives

$$(1-\alpha)P_{t+1} = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \qquad (10)$$

Substitution (1) in (10) gives

$$(1-\alpha)\left(a-b\,Q_{t+1}\right) = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \qquad (11)$$

Firstly, it is supposed that a = 0 and c = 0. By substitution one derives:

$$Q_{t+1} = \frac{d}{b(\alpha-1)\left(1+\frac{1}{e}\right)}Q_t - \frac{f}{b\,(1-\alpha)\left(1+\frac{1}{e}\right)}Q_t^{\,2} \qquad (12)$$

Further, it is assumed that the long-run monopolistic competitors's output is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the long-run monopolistic competitors's output growth rate depends on the current size of the long-run monopolistic competitors's output , Q, relative to its maximal size in its time series Q^m. We introduce q as $q = Q / Q^m$. Thus q range between 0 and 1. Again we index q by t, that is, write q_t to refer to the size at time steps t = 0,1,2,3,....Now, growth rate of the long-run monopolistic competitors's output is measured as

$$q_{t+1} = \frac{d}{b(\alpha-1)\left(1+\frac{1}{e}\right)} q_t - \frac{f}{b\,(1-\alpha)\left(1+\frac{1}{e}\right)} q_t^{\,2} \qquad (13)$$

This model given by equation (13) is called the logistic model. For most choices of α , b, d, f, and e there is no explicit solution for (13). Namely, knowing α, b, d, f and e and measuring q_0 would not suffice to predict q_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (13) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of q_t. This difference equation (13) will posses a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point q_0 the solution is highly sensitive to variations of the parameters α, b, d, f and e; secondly, given the parameters α, b, d, f and e the solution is highly sensitive to variations of the initial point q_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

LOGISTIC EQUATION

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0,4], \quad z_t \in [0,1] \qquad (14)$$

is equivalent to the iteration of growth model (13) when we use the following identification:

$$z_t = \frac{(\alpha-1)\,f}{(1-\alpha)\,d} q_t$$

$$\pi = \frac{d}{(\alpha-1)\,b\left(1+\frac{1}{e}\right)} \qquad (15)$$

Using (13), and (15) we obtain

$$z_{t+1} = \frac{(\alpha-1)\,f}{(1-\alpha)\,d} q_{t+1} = \frac{(\alpha-1)\,f}{(1-\alpha)\,d}\left[\frac{d}{(\alpha-1)\,b\left(1+\frac{1}{e}\right)} q_t - \frac{f}{(1-\alpha)\,b\left(1+\frac{1}{e}\right)} q_t^{\,2}\right] =$$

$$= \frac{f}{(1-\alpha)\,b\left(1+\frac{1}{e}\right)} q_t - \frac{(\alpha-1)f^2}{(1-\alpha)^2\,b\,d\left(1+\frac{1}{e}\right)} q_t^{\,2}$$

On the other hand, using (14), and (15) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = \left[\frac{d}{(\alpha-1)\,b\left(1+\frac{1}{e}\right)}\right]\left[\frac{(\alpha-1)f}{(1-\alpha)\,d}\right]q_t\left\{1-\left[\frac{(\alpha-1)f}{(1-\alpha)\,d}\right]q_t\right\}$$

$$= \frac{f}{(1-\alpha)\,b\left(1+\frac{1}{e}\right)} q_t - \frac{(\alpha-1)f^2}{(1-\alpha)^2\,b\,d\left(1+\frac{1}{e}\right)} q_t^{\,2}$$

Thus we have that iterating $q_{t+1} = \frac{d}{b(\alpha-1)\left(1+\frac{1}{e}\right)} q_t - \frac{f}{b\,(1-\alpha)\left(1+\frac{1}{e}\right)} q_t^{\,2}$ is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{(\alpha-1)\,f}{(1-\alpha)\,d} q_t$ and

$$\pi = \frac{d}{(\alpha-1)\,b\left(1+\frac{1}{e}\right)}.$$

It is important because the dynamic properties of the logistic equation (14) have been widely analyzed (Li and Yorke, 1975; May, 1976).

It is obtained that: (i) For parameter values $0 < \pi < 1$ all solutions will converge to z = 0; (ii) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on π; (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1)/\pi$; (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1)/\pi$; (v) For $3 < \pi < 4$ all solutions will continously fluctuate; (vi) For $3.57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

CONCLUSION

This paper suggests conclusion for the use of the simple chaotic model of a profit - maximizing monopolistic competitor in predicting the long-run fluctuations of the monopolistic competitor's output. The model (13) has to rely on specified parameters α, b, d, f and e, and initial value of the long-run monopolistic competitior's output, q_0. But even slight deviations from the values of parameters α, b, d, f and e, and initial value of the long-run monopolistic competitior's output, show the difficulty of predicting a long-term behavior of the long-run monopolistic competitior's output, q_0. A key hypothesis of this work is based on the idea that the coefficient $\pi = \frac{d}{(\alpha - 1) b \left(1 + \frac{1}{e}\right)}$ plays a crucial role in explaining local stability of the long-run monopolistic competitior's output where, d – the coefficient of the marginal cost function of the monopolistic competitor , b - the coefficient of the inverse demand function, α - the growth coefficient of the average cost. The quadratic form of the marginal cost function of the monopolistic competitor is important ingredient of the presented chaotic long-run monopolistic competitior's output growth model (13).

REFERENCES

Benhabib J, Day RH (1981). Rational choice and erratic behaviour. Rev. Econ. Stud. 48:459-471.

Benhabib J, Day RH (1982). Characterization of erratic dynamics in the overlapping generation model. J. Econ. Dyn. Control 4:37-55.

Day RH (1982). Irregular growth cycles. Am. Econ. Rev. 72:406-414.

Day RH (1983). The emergence of chaos from classica economic growth. Quart. J. Econ. 98:200-213.

Day RH (1997). Complex economic dynamics volume I: An introduction to dynamical systems and market mechanism. In: Discrete Dynamics in Nature and Society (pp. 177-178), 1. MIT Press.

Goodwin RM (1990). Chaotic economic dynamics. Oxford: Clarendon Press.

Grandmont JM (1985). On enodgenous competitive business cycles. Econometrica 53:994-1045.

Jablanovic V (2011). The chaotic Monopoly Price Growth Model. Chin. Bus. Rev. 10(11):985-990.

Jablanovic V (2012). Budget Deficit and Chaotic Economic Growth Models. Aracne editrice.S.r.l.

Li T, Yorke J (1975). Period three implies chaos. Am. Math. Monthly 8:985-992.

Lorenz EN (1963). Deterministic nonperiodic flow. J. Atmos. Sci. 20:130-141.

Lorenz HW (1993). Nonlinear dynamical economics and chaotic motion (2nd ed.). Heidelberg: Springer-Verlag.

May RM (1976). Mathematical models with very complicated dynamics. Nature 261:459-467.

Medio A (1993). Chaotic dynamics: Theory and applications to economics. Cambridge: Cambridge University Press.

Medio A (1996). Chaotic dynamics. Theory and applications to economics, Cambridge University Press, In De Economist 144(4):695-698.