academicJournals

Vol. (7(41), pp. 4280-4287, 7 November, 2013
DOI: 10.5897/AJBM12.499
ISSN 1993-8233 © 2013 Academic Journals
http://www.academicjournals.org/AJBM

African Journal of Business Management

Full Length Research Paper

Cost, revenue and profit efficiency in supply chain

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Accepted 1st November, 2013

A supply chain is a set of suppliers, manufactures and distributors, which are linked together. The first chain is a supplier and the last one is customers. What is the most important in supply chain are high-quality products by the least cost and the most benefit. In this regard, each input and output and intermediate have unit price and unit cost information. The cost-minimization and the profit-maximization are a great importance for an effective management of supply chain. Data Envelopment Analysis (DEA) can be used to evaluate the variant types of efficiency such as technical efficiency, cost efficiency, revenue efficiency and profit efficiency. In this paper, we are going to evaluate cost, revenue and profit efficiency in a three-stage supply chain and a multi-Stage Supply chain. These models are illustrated by a numerical example. Finally, we compare the results of constant returns to scale (CRS) and variable returns to scale (VRS).

Key words: Data Envelopment Analysis (DEA), supply chain management (SCM), cost efficiency, revenue efficiency, profit efficiency.

INTRODUCTION

A supply chain is the combination of equipment, suppliers, manufactures, distributors, retailers and method of controlling inventory, purchasing and distribution, that it tends to improve the way your company finds raw materials it needs to produce a product or service and to deliver it to customers (Camm et al.,1997).

Supply chain effective management has been widely accepted as an important means for supplier or manufacturer or distributor, to obtain the best and high-quality products and services by the least cost and the most profit. Two major criteria are employed in the supply chain management, namely, the cost-minimization criteria (Camm et al., 1997) and the profit- maximization criteria (Cohen and Lee, 1989). The evaluation of the performance is a great importance and necessity for recognizing the aims of both cost-minimization and profit-maximization in supply chain management (SCM). In supply chain management (SCM), although decreasing the cost and increasing profit is very important, partnership is a significant factor for enhancing competitiveness. Data envelopment analysis (DEA) is one of the best ways for assessing the relative efficiency of a group of homogenous decision making units (DMUs) that use multiple inputs to produce multiple outputs, originated from the work by Charnes et al. (1978). DEA has been applied to evaluate the supply chain performance in several works such as, Chen and Liang (2006) and Chen et al. (2011) and so on. By using DEA, we can evaluate the variant types of efficiency when the information on prices and costs is available. Technology, cost, revenue and profit efficiency are the wheels that management is eager to know how extent and improve them. The

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traditional DEA models cannot be applied directly to the supply chain case because classical DEA treats each DMU, supply chain, as a black box and considers only the initial inputs from suppliers and final outputs of distributors. For the complex nature of supply chain, those intermediate products or linking activities are ignored. Thus, the type of inefficiencies and performance of supply chain cannot be determined. Then, several authors have attempted to account these links and consider supply chain as a network DEAby multi-stage (Chen et al., 2011). The network DEA model proposed by Lewise and Sexton (2004) has a multi-stage structure as an extension of the two-stage DEA model proposed in Sexton and Lewise (2003).This paper introduces models for evaluating the cost efficiency, revenue efficiency and profit efficiency in supply chain, and intermediate products have been used in these models.

The paper is divided as follows: Cost, revenue and profit efficiency models are presented in a three-stage supply chain. Also proposed models are developed in a multi-stage supply chain. The next section presents a numerical example that illustrates the proposed models. Finally, conclusions are given.

Background

Data envelopment analysis (DEA) is a technique used widely in the literature of the supply chain management. It is non-parametric method that uses mathematical programming techniques to evaluate the performance or relative efficiency of DMUs (Decision Making Unit) units (e.g., teams, people, branches of banks, hospitals, schools e.t.c.) in terms of multiple inputs and multiple outputs. The DEA focus was originally developed by Charnes et al. (1978). The techniques of DEA models have been further developed and expanded to a wide variety of applications in different contexts including education, health care, bank branches, education, armed forces, market research, management of supply chains, manufacturing, etc. (Charnes et al. 1994). It is not only used to evaluate efficiency, but also to compare each DMU to the best production units. According to Fare et al. (1994), the PPS (Production Possibility Set) is defined as the set of all inputs and outputs of a system in which inputs can produce outputs. DEA models can be input oriented and output oriented. Likewise, DEA models can address constant returns to scale (CRS) and variable returns to scale (VRS). DEA can calculate technical efficiency. Also DEA can evaluate types of inefficiency such as cost efficiency, revenue efficiency and profit efficiency when information on prices and costs are known exactly.

Technical efficiency

Technical efficiency depicts the capability of production units to transform inputs into outputs. Consider DMU_j(j=1,…,*n)* where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors ofDMU_jbe($X_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \in R^m, Y_j = (y_{1j}, y_{2j}, \dots, y_{sj}) \in R^s$)respectively and let$X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0$.

The Production Possibility Set $T_c$is defined as:

$$T_c = \{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, \dots, n\}$$

By the production possibility set above, the CCR model used to evaluate technical efficiency is as follows:

$$TE_o = min \quad \theta_o$$

Subject to$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}$ i=1,…,m,

$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}$ r=1,…, s,

$\lambda_j \geq 0$ j=1,…,n,

θfree.

The CCR assumes "constant returns to scale (CRS)", that is, the increase of investment by one unit generating output by one unit. The CRS assumption is appropriate when all DMUs are operating at an optimal scale. However, government regulations, constraints on finance and so on, may cause a DMU not to be operating at optimal scale. The use of the CRS specification when not all DMUs are operating at the optimal scale results in measures of technical efficiency (TE) that are confounded by scale efficiencies. In another model of DEA, the BCC model assumes "variable returns to scale", that is, the scale of output is varying. The use of the VRS specification permits the calculation of TE devoid of these scale efficiency effects. The CRS can be easily modified to account for VRS by adding the convexity constraint: $\sum_{j=1}^{n} \lambda_j = 1$. The efficiency value calculated in CCR is the "overall technical efficiency", whereas the efficiency value computed by BCC is "pure technical efficiency" (PTE). The former divided by the latter is "scale efficiency" (SE). It must be noted that TE and PTE are greater than zero and less or equal to one (Salgooghi et al., 2012).

Traditional DEA models deal with measurements of relative efficiency of DMUs regarding multiple-inputs and multiple outputs .By using these models internal linking activities are neglected. Network DEA model is dealt with intermediate products (Tone et al. 2009). Supply chain is like network DEA and supply chain has linking activities (Alfonso et al., 2010).

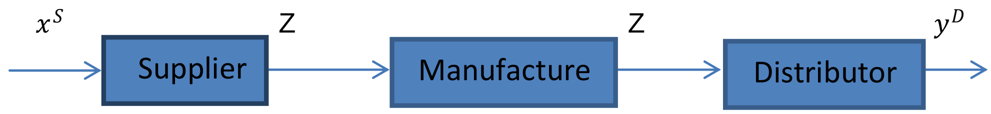

Figure 1. A three- stage supply chain.

According to Alfonso et al. suppose that n DMUs ($(j = 1, \dots, n)$ consist of k divisions $(k = 1, \dots, n)$ exist. Let x_j^k and y_j^k be the inputs and outputs to division k. Link leading from division k to division h is represented by (k, h).

There are k stages in supply chain. The Production Possibility Set is defined by (Alfonso et al., 2010):

P={$\left(x^k, y^k, z^{(p,k)}, z^{(k,q)}\right) \mid x^k \geq X^k\lambda^k, y^k \leq Y^k\lambda^k$, $z^{(p,k)}$=$Z^{(p,k)}\lambda^k$ (as inputs to k), $z^{(p,k)}$= $Z^{(p,k)}\lambda^p$ (as outputs from p) , $z^{(k,q)}$=$Z^{(k,q)}\lambda^q$(as inputs to q), $z^{(k,q)} = Z^{(k,q)}\lambda^k$(as outputs from k), $\lambda^k \geq 0$, $\lambda^p \geq 0$,$\lambda^q \geq 0$,}

Cost efficiency

Cost efficiency is defined as the effective choice of inputs vis a vis prices with the objective to minimize production costs, whereas technical efficiency investigates how well the production process converts inputs into outputs. It should be noted that DEA can also be used to measure cost efficiency (Rayeni and Saljooghi, 2012). Regarding this subject when input costs are available, there are two different situations: one common unit prices and costs for all DMUs and the other with different prices and costs from DMU to DMU. In this paper, different costs and prices are considered from DMU to DMU. Again, consider *n* decision making units (DMUs) with *m* inputs for producing *s* outputs. For DMU_j, an input–output bundle $(X = (x_1, x_2, \dots, x_n) \in R^{m\times n}$,$Y = (y_1, y_2, \dots, y_n) \in R^{s\times n}$), in which the inputs have costs $C = (c_1, c_2, \dots, c_n)$.Let us define another cost-based production possibility set P_c as:

$P_c = \{(\bar{x} \geq \bar{X}\lambda, y \leq Y\lambda, \lambda \geq 0\}$
Where $\bar{X} = (\bar{x}_1, \dots, \bar{x}_n)$ with $\bar{x}_j = \left(c_{1j}x_{1j}, \dots, c_{mj}x_{mj}\right)$. In order to obtain a measure of cost efficiency, when the input and output data are known and the prices differ from DMU to DMU, minimal cost model was proposed by Cooper et al. (2006) as follows:

$e\bar{x}^*$= min $e\bar{x}$
subject to $\bar{x} \geq \bar{X}\lambda$,
$y_o \leq Y\lambda$,
$\lambda \geq 0$

The CE measure is given by the ratio of the minimal cost value obtained from above model to the current cost at DMU_o as follows:

Cost Efficiency = α^* = $e\bar{x}_o^*$ / e$\bar{x}_o$

Cost efficiency model in three- stage supply chain

The three-stage supply chain is shown in Figure 1. Suppose that there are N supply chains, where stage S represents the supplier and the stage M represents a manufacturer and stage D represents a distributor. There are linking activities between stages of supply chain like that proposed linked by Tone et al. (2009) and Alfonso et al. (2010). The outputs of supplier are used by manufacturers. Also they produce inputs distributors. In this model inputs and input links are considered as variables. Each supply chain, SC_j, (j=1,2,…,N) has m inputs for the supplier, X_{ij}, (i=1,2,…,m), and K outputs from this supplier, Z_{kj}, (k=1,2,…,K), become the inputs to the manufacturer, And Z_{lj}, (l=1,2,…,L) become the outputs from manufacturer and the inputs for the distributor, The outputs from the distributor are represented Y_{rj}, (r=1,2,…,R).

Here input costs and input link costs are different from supply chain to supply chain. The cost efficiency are considered in supply chain based on excludes consideration of the unit input cost of supplier *C* =$(c_1, c_2, \dots, c_n)$, the unit input link cost of manufactured d^{MI}= $(d_1^{MI}, d_2^{MI}, \dots, d_K^{MI})$,the unit input link cost of distributor d^D = $(d_1^D, d_2^D, \dots, d_l^D)$. Production Possibility Set (PPS) is defined based on cost *C* =$(c_1, c_2, \dots, c_n)$ for the input supplier and cost d^{MI}= $(d_1^{MI}, d_2^{MI}, \dots, d_K^{MI})$, d^D = $(d_1^D, d_2^D, \dots, d_l^D)$ for input links as follows:

P_c = $\{(\bar{x}^S, y^D, z^S, z^{MO}, \bar{z}^{MI}, \bar{z}^D) \mid \bar{x}^S \geq \bar{X}^S\lambda^S, y^D \leq Y^D\lambda^D$,$z^S$=$Z^S\lambda^S$ (as output S), $\bar{z}^{MI}$= $\bar{Z}^{MI}\lambda^{MI}$ (as input M), z^{MO}=$Z^{MO}\lambda^{MO}$(as output M), $\bar{z}^D = \bar{Z}^D\lambda^D$(as input D), $\lambda^S \geq 0$, $\lambda^{MI} \geq 0$,$\lambda^{MO} \geq 0$,$\lambda^D \geq 0\}$ where $\bar{X}^S$= $(\bar{x}_1^S, \bar{x}_2^S, \dots, \bar{x}_n^S)$, $\bar{x}_j^S = \left(c_1 x_{1j}^S, \dots, c_m x_{mj}^S\right)$,$\bar{Z}^{MI}$= $(\bar{z}_1^{MI}, \dots, \bar{z}_n^{MI})$, ,$\bar{z}_j^{MI} = \left(d_1^{MI} z_{1j}^{MI}, \dots, d_k^{MI} z_{kj}^{MI}\right)$,

$\bar{Z}^D = (\bar{z}_1^D, \dots, \bar{z}_n^D)$, $\bar{z}_j^D = (d_1^D z_{1j}^D, \dots, d_l^D z_{lj}^D)$.

By the above Production Possibility Set (PPS), set P_c, α^* is obtained by the following LP problem. Also e is a row vector with all elements being equal to 1 and $(\bar{x}_o^{*S}, \bar{z}_o^{*MI}, \bar{z}_o^{*D}, \lambda^{*S}, \lambda^{*MI}, \lambda^{*MO}, \lambda^{*D})$ are the optimal solutions of the LP given below:

$e\bar{x}^{*S} + e\bar{z}^{*MI} + e\bar{z}^{*D}$ = min $e\bar{x}^S + e\bar{z}^{MI} + e\bar{z}^D$

Subject to $\bar{x}^S \geq \bar{X}^S \lambda^S$,
$y_o^D \leq Y^D \lambda^D$,
$\bar{z}^D = \bar{Z}^D \lambda^D$, (input to distributor),
$\bar{z}^{MI} = \bar{Z}^{MI} \lambda^{MI}$, (inputs to manufacturer),
$z_o^S = Z\lambda^S$, (as outputs from supplier),
$z_o^{MO} = Z^{MO} \lambda^{MO}$, (as outputs from manufacturer),
$\lambda^S \geq 0, \lambda^{MI} \geq 0, \lambda^{MO} \geq 0, \lambda^D \geq 0$.

Therefore, the cost efficiency, α^*, is defined as:

$\alpha^* = e\bar{x}_o^{*s} + e\,\bar{z}_o^{*MI} + e\bar{z}_o^{*D} \,/\, e\bar{x}_o^S + e\,\bar{z}_o^{MI} + e\bar{z}_o^D$.
DMU_o is the cost efficient if and only if α^*=1. Also is $0 \leq \alpha^* \leq 1$.
The cost efficiency models are defined for the supplier, manufacturer and distributor separately as follows:

Cost efficiency model in supplier

$e\bar{x}^{*S}$ = min $e\bar{x}^S$
subject to $\bar{x}^S \geq \bar{X}^S \lambda^S$,
$z_o^S = Z\lambda^S$, (as outputs from supplier),
$\lambda^S \geq 0$.

Therefore, the cost efficiency in supplier is defined as:
$\alpha_S^* = e\bar{x}_o^{*S} / e\bar{x}_o^S$.

Cost efficiency model in manufacturer

$e\bar{z}^{*M}$ = min $e\bar{z}^M$
subject to $\bar{z}^{MI} = \bar{Z}^{MI} \lambda^{MI}$, (inputs to manufacturer),
$z_o^{MO} = Z^{MO} \lambda^{MO}$, (as outputs from manufacturer),
$\lambda^{MI} \geq 0$, $\lambda^{MO} \geq 0$.

Therefore, the cost efficiency in manufacturer is defined as:
$\alpha_M^* = e\,\bar{z}_o^{*MI} / e\bar{z}_o^{MI}$.

Cost efficiency model in distributor

$e\bar{z}^{*D}$ = min $e\bar{z}^D$
subject to $\bar{z}^D = \bar{Z}^D \lambda^D$, (input to distributor),

$y_o^D \leq Y^D \lambda^D$, $\lambda^D \geq 0$.

Therefore, the cost efficiency in distributor is defined as:

$\alpha_D^* = e\bar{z}_o^{*D} / e\bar{z}_o^D$

Revenue efficiency model in three- stage supply chain

In this section, revenue efficiency is represented in supply chain in which the prices play a role in the PPS on output. Hence price vector P = $(p_1, p_2, \dots, p_n)$ for the output distributor and price vectors $U^{MO} = (u_1^{MO}, u_2^{MO}, \dots, u_l^{MO})$ for output links of manufacture and price vectors $U^S = (u_1^S, u_2^S, \dots, u_k^S)$ for output links of supplier are considered. In this model outputs and output links are variables. PPS is as follows:

$P_R = \{(x^S, \bar{y}^D, \bar{z}^S, z^{MI}, \bar{z}^{MO}) \mid x^S \geq X^S\lambda^S, \bar{y}^D \leq \bar{Y}^D\lambda^D, \bar{z}^S = \bar{Z}^S\lambda^S$ (as output S), $z^{MI} = Z^{MI}\lambda^{MI}$ (as input M), $\bar{z}^{MO} = \bar{Z}^{MO}\lambda^{MO}$, $\lambda^S \geq 0, \lambda^{MI} \geq 0, \lambda^{MO} \geq 0, \lambda^D \geq 0\}$ where
$\bar{y}^D = (\bar{y}_1^D, \dots, \bar{y}_n^D)$, $\bar{y}_j^D = (P_1 y_{1j}^D, \dots, P_r y_{rj}^D)$,
$\bar{Z}^{MO} = (\bar{z}_1^{MO}, \dots, \bar{z}_n^{MO})$, $\bar{z}_j^{MO} = (u_1^{MO} z_{1j}^{MO}, \dots, u_k^{MO} z_{kj}^{MO})$,
$\bar{Z}^S = (\bar{z}_1^S, \dots, \bar{z}_n^S)$, $\bar{z}_j^S = (u_1^S z_{1j}^S, \dots, u_k^S z_{kj}^S)$.

By the above PPS, set P_R, β^*, is obtained as the following LP problem. Also e is a row vector with all elements being equal to 1 and $(\bar{y}_o^{*D}, \bar{z}_o^{*MO}, \bar{z}_o^{*S}, \lambda^{*S}, \lambda^{*MO}, \lambda^{*MI}, \lambda^{*D})$ are the optimal solutions of the LP given below:

$e\bar{y}^{*D} + e\bar{z}^{*MO} + e\bar{z}^{*S} =$ max $e\bar{y}^D + e\bar{z}^{MO} + e\bar{z}^S$
subject to $x_o^S \geq X^S\lambda^S, \bar{y}^D \leq \bar{Y}^D\lambda^D$,
$\bar{z}^S = \bar{Z}^S\lambda^S$, (as outputs from supplier),
$\bar{z}^{MO} = \bar{Z}^{MO}\lambda^{MO}$, (as outputs from manufacturer),
$z_o^{MI} = \bar{Z}^{MI}\lambda^{MI}$, (inputs to manufacturer),
$z_o^D = Z^D\lambda^D$, (input to distributor),
$\lambda^S \geq 0, \lambda^{MI} \geq 0, \lambda^{MO} \geq 0, \lambda^D \geq 0$.

Then, the Revenue Efficiency is defined as:

$\beta^* = e\bar{y}_o^D + e\bar{z}_o^S + e\bar{z}_o^{MO} \,/\, e\bar{y}_o^{*D} + e\,\bar{z}_o^{*S} + e\bar{z}_o^{*MO}$.
DMU_o is the revenue efficient if and only if β^*=1. Also is $0 \leq \beta^* \leq 1$.
In this case, also, the revenue efficiency models can be defined for the supplier, manufacturer and distributor, separately.

Profit Efficiency model in three- stage supply chain

To obtain the profit efficiency of DMU_o, the price vector P,

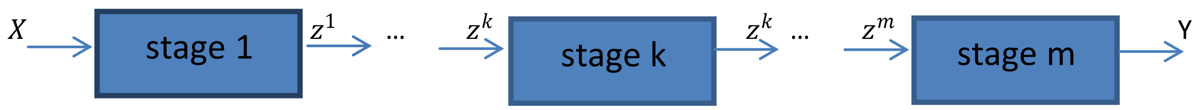

Figure 2. A multi – member supply chain.

U and cost vector C, d are utilized for inputs, input links, outputs and output links respectively. In this model inputs, outputs, input links and output links are variables. Then PPS is defined as follows:

$P_{PC}=\{(\bar{x}^S,\bar{y}^D,\bar{z}^S,\bar{z}^{MI},\bar{z}^{MO},\bar{z}^D)\,|\,\bar{x}^S \geq \bar{X}^S\lambda^S, \bar{y}^D \leq \bar{Y}^D\lambda^D,$ $\bar{z}^S = \bar{Z}^S\lambda^S$ (as output S), $\bar{z}^{MI} = \bar{Z}^{MI}\lambda^{MI}$ (as input M), $\bar{z}^{MO} = \bar{Z}^{MO}\lambda^{MO}(as\ output\ M), \bar{z}^D = \bar{Z}^D\lambda^D,\ \lambda^S \geq 0, \lambda^{MO} \geq 0, \lambda^{MI}, \lambda^D \geq 0\}$.

$\bar{X}^S = (\bar{x}_1^S, \bar{x}_2^S, \dots, \bar{x}_n^S)$, $\bar{x}_j^S = (c_1 x_{1j}^S, \dots, c_m x_{mj}^S)$,

$\bar{Z}^S = (\bar{z}_1^S, \dots, \bar{z}_n^S)$, $\bar{z}_j^S = (u_1^S z_{1j}^S, \dots, u_k^S z_{kj}^S)$,

$\bar{Z}^{MI} = (\bar{z}_1^{MI}, \dots, \bar{z}_n^{MI})$, $\bar{z}_j^{MI} = (d_1^{MI} z_{1j}^{MI}, \dots, d_k^{MI} z_{kj}^{MI})$,

$\bar{Z}^{MO} = (\bar{z}_1^{MO}, \dots, \bar{z}_n^{MO})$, $\bar{z}_j^{MO} = (u_1^{MO} z_{1j}^{MO}, \dots, u_k^{MO} z_{kj}^{MO})$,

$\bar{Z}^D = (\bar{z}_1^D, \dots, \bar{z}_n^D)$, $\bar{z}_j^D = (d_1^D z_{1j}^D, \dots, d_l^D z_{lj}^D)$,

$\bar{y}^D = (\bar{y}_1^D, \dots, \bar{y}_n^D)$, $\bar{y}_j^D = (P_1 y_{1j}^D, \dots, P_r y_{rj}^D)$.

By the set of P_{PC}, γ^* is obtained as the following LP problem. Also e is a row vector with all elements being equal to 1 and $(\bar{x}_o^{*S}, \bar{z}_o^{*MI}, \bar{z}_o^{*D}, \bar{y}_o^{*D}, \bar{z}_o^{*S}, \bar{z}_o^{*MO}, \lambda^{*S}, \lambda^{*D}, \lambda^{*MI}, \lambda^{*MO})$ are the optimal solutions of the LP given below:

$(e\bar{y}^{*D} + e\,\bar{Z}^{*S} + e\,\bar{Z}^{*MO}) - (e\bar{x}^{*S} + e\bar{Z}^{*MI} + e\bar{Z}^{*D})$ = max $(e\bar{y}^D + e\,\bar{Z}^S + e\,\bar{Z}^{MO})$ - $(e\bar{x}^S + e\bar{Z}^{MI} + e\bar{Z}^D)$

subject to $\bar{x}_o^S \geq \bar{X}^S\lambda^S = \bar{x}^S, \bar{y}_o^D \leq \bar{Y}^D\lambda^D = \bar{y}^D,$

$\bar{z}_o^S \leq \bar{Z}^S\lambda^S = \bar{z}^S$, (as outputs from supplier)

$\bar{z}_o^{MO} \leq \bar{Z}^{MO}\lambda^{MO} = \bar{z}^{MO}$, (as outputs from manufacture)

$\bar{z}_o^{MI} \geq \bar{Z}^{MI}\lambda^{MI} = \bar{z}^{MI}$, (as inputs to manufacture)

$\bar{z}_o^D \geq \bar{Z}^D\lambda^D = \bar{z}^D$, (as inputs to distributor)

$$\lambda^S \geq 0, \lambda^{MO} \geq 0, \lambda^D \geq 0, \lambda^{MI} \geq 0$$

Then, based on an optimal solution, the profit efficiency can be defined in the form of the ratio by:

$$\gamma^* = \frac{\{(e\bar{y}_o^D + e\,\bar{Z}_o^S + e\bar{Z}_o^{MO}) - (e\bar{x}_o^S + e\,\bar{Z}_o^{MI} + e\bar{Z}_o^D)\}}{\{(e\bar{y}_o^{*D} + e\,\bar{Z}_o^{*S} + e\bar{Z}_o^{*MO}) - (e\bar{x}_o^{*S} + e\,\bar{Z}_o^{*MI} + e\bar{Z}_o^{*D})\}}.$$

A multi- stage supply chain (cost efficiency, revenue efficiency, profit efficiency)

In general, a supply chain possibly consists of several members, such as suppliers, manufacturers, distributors, retailers and so on (Figure 2). Hence, the above models are developed for a multi-stage supply chain.

The cost efficiency, revenue efficiency and profit efficiency can be evaluated for multi-member supply chain by solving the following models:

Cost efficiency

$e\bar{x}^* + \sum_h e\bar{z}^{*(k,h)}$ = min $e\bar{x} + \sum_h e\bar{z}^{(k,h)}$

subject to $\bar{x} \geq \bar{X}\lambda^1$, (as input stage 1),

$y_o \leq Y\lambda^m$, (as output stage m),

$\bar{z}^{(k,h)} = \bar{Z}^{(k,h)}\lambda^h$, (as inputs to stage h) ∀ (k,h),

$z_o^{(k,h)} = Z^{(k,h)}\lambda^k$, (as output from stage k) ∀ (k,h),

$\lambda^1 \geq 0, \lambda^m \geq 0, \lambda^h \geq 0, \lambda^k \geq 0.$

$Z^{(k,h)}$: linking activities from stage k to stage h.

The cost efficiency is defined as:

CE = ($e\bar{x}_o^* + \sum_h e\bar{z}_o^{*(k,h)}$) / ($e\bar{x}_o + \sum_h e\bar{z}_o^{(k,h)}$).

Revenue efficiency

$e\bar{y}^* + \sum_k e\bar{z}^{*(k,h)}$ = max $e\bar{y} + \sum_k e\bar{z}^{(k,h)}$

subject to $x_o \geq X\,\lambda^1$

$\bar{y} \leq \bar{Y}\lambda^m$

$\bar{z}^{(k,h)} = \bar{Z}^{(k,h)}\lambda^k$, (as outputs from stage k) ∀ (k,h),

$z_o^{(k,h)} = Z^{(k,h)}\lambda^h$, (as inputs to stage h) ∀ (k,h),

$\lambda^1 \geq 0, \lambda^m \geq 0, \lambda^h \geq 0, \lambda^k \geq 0.$

RE = (e$\bar{y}_o + \sum_k e\bar{z}_o^{(k,h)}$)/(e$\bar{y}_o^* + \sum_k \bar{z}_o^{*(k,h)}$).

Profit efficiency

(e$\bar{y}^* + \sum_k e\bar{z}^{*(k,h)}$) – ($e\bar{x}^* + \sum_h e\bar{z}^{*(k,h)}$) = max (e$\bar{y} + \sum_k e\bar{z}^{(k,h)}$) – ($e\bar{x} + \sum_h e\bar{z}^{(k,h)}$)

subject to $\bar{x}_o \geq \bar{X}\lambda^1 = \bar{x}$,

$\bar{y}_o \leq \bar{Y}\lambda^m = \bar{y}$,

$\bar{z}^{(k,h)} = \bar{Z}^{(k,h)}\lambda^k \geq \bar{z}_o^{(k,h)}$, (as outputs from stage k) ∀ (k,h),

$\bar{z}^{(k,h)} = \bar{Z}^{(k,h)}\lambda^h \leq \bar{z}_o^{(k,h)}$, (as inputs to stage h) ∀ (k,h),

$\lambda^1 \geq 0, \lambda^m \geq 0, \lambda^h \geq 0, \lambda^k \geq 0.$

Based on an optimal solution, the profit efficiency can be

Figure 3. The cost efficiency for supplier, manufacturer and distributor.

Table1. Inputs, outputs and links.

DMU	X1	X2	Y1	Y2	Z1	Z2	Z3	Z4
SC1	5	4	25	30	5	9	10	7
SC2	3	6	30	20	8	9	1	4
SC3	4	1	18	15	7	12	3	2
SC4	6	2	22	12	6	10	9	13
SC5	5	6	23	24	12	8	7	15
SC6	9	4	21	16	3	6	12	17
SC7	10	3	14	18	9	10	8	12

Table 2. Unit input cost, unit price, unit input link cost, unit output link price.

DMU	SC1	SC2	SC3	SC4	SC5	SC6	SC7
C1	543.9 $	1257.9 $	417.9 $	512.9 $	856.2 $	286.85 $	450 $
C2	168 $	835.8 $	2097.7 $	560.32 $	1020.3 $	1400.5 $	1300 $
P1	10093.5 $	6854 $	3500.8 $	4500 $	2800.9 $	3700.5 $	1600.25 $
P2	6050.9 $	8961.7 $	3462.9 $	4800.3 $	5024.6 $	3900.4 $	3400.9 $
d1	1206.3 $	8073.7 $	853.8 $	565 $	1200 $	1232.6 $	354.87 $
d2	400 $	1500 $	1683 $	300 $	1235 $	1480 $	1450.9 $
d3	6783.3 $	5450.4 $	2436 $	2450 $	3500 $	3400 $	4538.9 $
d4	4864 $	6791.4 $	2951.7 $	1200 $	2300 $	1580 $	1390.3 $
u1	8400 $	6927.9 $	627.9 $	459.9 $	1035 $	1600.5 $	1290.4 $
u2	278 $	1293.8 $	3085.9 $	1400 $	1850.9 $	1760.4 $	1640.9 $
u3	5000.9 $	4360 $	1263.9 $	1000 $	1700 $	4000.6 $	2500.8 $
u4	1000.9 $	5693.7 $	1895.8 $	1200 $	4500.6 $	3580.1 $	2680.9 $

defined in the form of the ratio by:

$$PE = \{ (e\bar{y}_o + \sum_k e\bar{z}_o^{(k,h)}) - (e\bar{x}_o + \sum_h e\bar{z}_o^{(k,h)}) \} / \{ (e\bar{y}_o^* + \sum_k e\bar{z}_o^{*(k,h)}) - (e\bar{x}_o^* + \sum_h e\bar{z}_o^{*(k,h)}) \}.$$

Numerical example

We present an illustrative example to describe the cost efficiency, revenue efficiency and profit efficiency in a typical three-member supply chain process. The cost efficiency for supplier, manufacturer and distributor is calculated, separately (Figure 3). The price is introduced for produced goods that are just for outputs and the cost is introduced for inputs and inside activities which are as inputs for each stage. The cost can include stock costs in depot, shipment cost, insurance cost, private cost and etc. Table 1 represents the data for 7 supply chains (furniture produce).Table 2 consists of unit input cost, unit price, unit input link cost, unit output link price. Suppliers are the trunk of a tree, manufacturers are furniture factory and distributors consist of wholesale and retailers. There

Table 3. Cost efficiency, CRS.

DMU	α	α_S	α_M	α_D
SC1	0.343	1	1	0.259
SC2	0.213	0.383	0.061	0.405
SC3	1	1	1	1
SC4	0.553	0.791	1	0.429
SC5	0.596	1	1	0.358
SC6	0.391	1	0.870	0.228
SC7	0.396	0.779	0.503	0.299

Table 4. Cost efficiency, VRS.

DMU	α	α_S	α_M	α_D
SC1	1	1	1	1
SC2	0.597	1	0.270	1
SC3	1	1	1	1
SC4	0.779	0.912	1	0.716
SC5	1	1	1	1
SC6	0.501	1	1	0.348
SC7	0.604	0.788	0.716	0.537

Table 5. Revenue efficiency, CRS.

DMU	β	β_S	β_M	β_D
SC1	0.804	0.630	0.353	1
SC2	1	1	1	1
SC3	0.443	1	0.035	1
SC4	0.149	0.256	0.139	0.145
SC5	0.193	0.323	1	0.137
SC6	0.163	0.146	1	0.099
SC7	0.128	0.262	0.315	0.083

Table 6. Revenue efficiency, VRS.

DMU	β	β_S	β_M	β_D
SC1	0.975	0.783	0.979	1
SC2	1	1	1	1
SC3	1	1	1	1
SC4	0.582	0.360	0.580	0.624
SC5	0.880	0.406	1	1
SC6	0.864	0.270	1	1
SC7	0.431	0.542	1	0.302

are two inputs to the first stage (supplier) such as X1, X2 (wood, labor) and costs C1,C2 (shipment cost, labor's salary), are consumed to generate outputs such as Z1, Z2 (small and large lumbers) with costs of transport and insurance). In the second stage (manufacturer), Z1, Z2 are used to generate outputs Z3, Z4 (table and chair) with costs of transport and insurance), in the third stage (distributor), Z3,Z4 are consumed to generate outputs Y1,Y2 (total seller chair and partial seller table). Table 3 reports the cost efficiency in three- members supply chains in CRS. Supply chains (SC1,SC3,SC5,SC6) are cost efficient in supplier, (SC1,SC3,SC4,SC5) are cost efficient in manufacturer and (SC3) cost efficiency in distributor. Also SC3 is cost efficient because in each stage is cost efficient in constant returns to scale. Also SC1 and SC3 are cost efficient in VRS in Table 4. Tables 5 represents SC2 is revenue efficient in CRS. In Table 6 SC2 and SC3 are revenue efficient in VRS. In the first stage (supplier), two inputs such as X1, X2 (wood, labor) are consumed to generate outputs such as Z1, Z2 (small lumbers and large lumbers). In the second stage (manufacturer), Z1, Z2 are used to generate outputs Z3, Z4 (table and chair), in the third stage (distributer), Z3, Z4 are consumed to generate outputs Y1, Y2 (total seller chair and partial seller table).

Unit input cost of supplier is $C = (c_1, c_2)$, the unit input link cost of manufacture is $d^{MI} = (d_1^{MI}, d_2^{MI})$,the unit input link cost of distributor is $d^D = (d_3^D, d_4^D)$.Price vector P = (p_1, p_2) is for the output distributor and price vectors $U^{MO} = (u_1^{MO}, u_2^{MO})$ is for output links of manufacture and price vectors $U^S = (u_3^S, u_4^S)$ is for output links of supplier.

Supply chains (SC1,SC3,SC5,SC6) are cost efficient in supplier, (SC1,SC3,SC4,SC5) are cost efficient in manufacture and (SC3) is cost efficient in distributor. Also SC3 is cost efficient because it is cost efficient in each stage by the constant returns to scale assumption. Also SC1 and SC3 are cost efficient in VRS. Also they are seen as $\alpha_{(VRS)} \geq \alpha_{(CRS)}, \beta_{(VRS)} \geq \beta_{CRS}$ in Tables 3, 4, 5 and 6. Also SC2 is revenue efficient because it is revenue efficient in each three-stage by the constant returns to scale assumption. SC2 and SC3 are revenue efficient in VRS.

Conclusion

In this work, we have proposed the cost efficiency, revenue efficiency and profit efficiency in a three-stage supply chain and multi-stage supply chain. All models are considered by constant returns to the scale assumption (CRS).Then cost efficiency model has yielded under variable returns to scale (VRS) in numerical example. There is at least one supply chain cost efficient or revenue efficient in each columns of observed table. A

supply chain is cost efficient or revenue efficient if and only if it is efficient in all stages. There are 7 supply chains (furniture produce) in numerical example. Suppliers are the trunk of a tree, manufacturers are furniture factory and distributors are consisting of, wholesale and retailers. By the way supply chains (SC1,SC3,SC5,SC6) are cost efficient in supplier, (SC1,SC3,SC4,SC5) are cost efficient in manufacturer and (SC3) is cost efficiency in distributor. Also SC3 is cost efficient because it is cost efficient in each stage by the constant returns to scale assumption. Also SC1 and SC3 are cost efficient in VRS. Also they are seen that$\alpha_{(VRS)} \geq \alpha_{(CRS)}$ and$\beta_{VRS} \geq \beta_{CRS}$ in Tables 3, 4, 5 and 6. Also SC2 is revenue efficient because it is revenue efficient in each three-stage in CRS. SC2 and SC3 are revenue efficient in VRS.
Future research subjects are included:

1-Scale and allocative efficiencies in SCCR (supply chain CCR) and SSBM (supply chain SBM).
2-Consider the decomposition of cost, revenue and profit efficiency in the presence of radial models and non-radial models.
3-Consider multi objective models for overall cost efficiency or revenue efficiency and associated weights for each stage (supplier, manufacture, distributor).

ACKNOWLEDGEMENT

The authors deeply appreciate all the comments and suggestions that are useful for improving the presentation of this paper.

REFERENCE

Alfonso E, Kalenatic D, Lopez C (2010). Modeling the synergy level in a vertical collaborative supply chain through the IMP interaction model and DEA framework.
Camm JD, Chorman TE, Dull FA, Evans JR, Sweeney DJ, Wegryn GW (1997). Blending OR/MS, judgment, and GIS: restructuring P&G's supply chain. Interfaces 27(1):128-142.
Charnes A, Cooper WW, Rhodes E (1978).Measuring the efficiency of decision making units. Eur. J. Oper. Res. 2(6):429-444.
Charnes A, Cooper WW, Levin A, Seiford L (1994). Data Envelopment Analysis, theory, methodology, and applications.Bostone. Kluwer Academic.
Chen Y, Liang L (2006). A DEA game model approach to supply chain efficiency.Annals Operations Research. 145(1):5-13.
Chen C, Hong Y (2011). Network DEA model for supply chain performance evaluation. European Journal of Operational Research. 213:147-155.
Cohen MA, Lee HL(1989).Resource deployment analysis of global manufacturing and distribution networks. J. Manuf. Oper. Manage. 2:81-104.
Cooper WW, Seiford LM, Tone K (2005). Introduction to Data Envelopment Analysis and its uses.
Fare R, Grosskopf S, Lovell C (1994). Production frontiers. New york. Cambridge University Press.
Lewis HF, Sexton TR (2004). Network DEA: Efficiency analysis of organizations withcomplex internal structure. Comput. Oper. Res. pp.1365-1410.
Rayeni MM, Saljooghi F (2012). Equitable allocation of fixed costs and its effect on technical and cost efficiency: Case study in universities. Afr. J. Bus. Manage. pp.1263-1269.
Sexton TR, Lewis HF (2003). Two-stage DEA: An application to major league baseball. J. Prod. Anal. pp.227-249.
Tone K, Tsutsui M (2009). Network DEA: a slacks-based measure approach. Eur. J. Oper. Res. 197:243-252.