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Review

Progress and regression group performance in data envelopment analysis (DEA): An application of productivity Malmquist index

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In this paper, we extend the work of Camanho and Dyson (2006, 26:35-49) into a group of Malmquist productivity index for two time periods of measurement, based on the Malmquist index to enable the decision making units' internal efficiencies from those associated with their group characteristics. We construct an index reflecting the relative performance groups in two time periods, which can be decomposed into two indices, first for the comparison of efficiency changing and another index for comparison of technological change. We provide a computational method for extension of the Malmquist index group A in relation to group B of time t to time t+1. The utilization of the proposed approach is demonstrated with an illustrative example.

Key words: Data envelopment analysis (DEA), Malmquist productivity index, group efficiency.

INTRODUCTION

Malmquist productivity index measures the rate of progress and regression of a decision making unit of a collection of decision making units at a specific time compared with the previous time. In applying the index, there are two pre-suppositions (axioms) that should necessarily be considered in the application of the index. First, all the decision making units are in a community and all of them are compared with each other. Secondly, every decision making unit at the intersections of t and t+1 is active and the information of these two time intersections is at hand; therefore, it has a definition as previously stated. If one of these two axioms does not come true, the index cannot be applied traditionally. In this research, the first axiom is extended in a way that a collection of decision making units is extended to the collections of decision making units. In other words, suppose that all decision making units can be partitioned into sub-sets in a way that the members of each sub-set are true in a common property in the partitioning in a way that the members of other sub-sets do not enjoy the same property. Alternatively, if the decision making units can be partitioned into sub-sets with an index other than the input and output, Malmquist productivity index cannot calculate the progress and regression of the units. For example, if in a collection of bank branches there is another factor such as the rank of the bank or the geographical location of the bank (in a specific province, etc.) other than input and output indexes, we can partition bank branches into sub-sets belonging to a specific province.

Consequently, other dividing indexes can be considered too, all of which depend on the objective of the evaluation from the point of view of DM. In this research, it is intended to partition the collection of decision making units at least into three sub-sets in such a way as to be able to calculate the rate of these collections at the two times of t and t+1 in comparison with other groups. For more explanation, suppose that J= {1,…,n} is a group

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of decision making units and A,B and C are sub-sets of J in a way that:

$$A\neq\phi,\ B\neq\phi,\ C\neq\phi,\ A\cap B\neq\phi,\ A\cap C\neq\phi,\ B\cap C\neq\phi,\ A\cup B\cup C = J$$

Suppose that we decided to measure the rate of progress and regression of group A in comparison to B and C at two time intersections t and t+1 or to measure each of these groups. In this research, the mentioned subject has been extended in a way that the rate of progress and regression of each group (A, B and C) is calculated and measured in comparison to two other groups.

The Malmquist index was introduced by Caves et al. (1982). In this paper, we use data envelopment analysis (DEA) and Malmquist productivity index to develop measuring group performance on two times. The DEA is a nonparametric method for measuring the relative efficiency of a homogeneous set of DMUs with multiple inputs to produce multiple outputs. Usually, in DEA applications, DMUs activities display a large homogeneity. It is often important to investigate group form of DMUs from the set under analysis. Analysis by group was first introduced by Charnes et al. (1981). This theory has had little progress in recent years. Camanho and Dyson (2006) develop measure for group evaluation based on work developed by Fare and Grosskopf (1994). Jahanshahloo et al. (2011) apply for measuring human development index, measuring it with Malmquist productivity index group performance in DEA and for measuring group performance on interval data. They constructed the Malmquist productivity index as the geometric mean of two Malmquist productivity indexes of Caves et al. (1982) which are defined by a distance function D(.). The index applied for measurement of productivity changes over time. This index can be decomposed into two indexes:

a) an efficiency change index and b) a technological change index. The method of Camanho and Dyson makes comparisons relative to groups-specific frontiers only, without pooling the DMUs together to form a common frontier. Our method also uses the same assumption, but we develop their method into two periods. The Malmquist productivity index for group evaluation developed in this article can be multiplicatively decomposed into two indexes, one reflecting the within group efficiency spread in two time periods, and the other, reflecting the productivity between the frontiers groups in two time periods. We also present some numerical results of the proposed models for a small problem of bank branches which have three inputs and four outputs.

BACKGROUND

Data envelopment analysis

Consider n decision making units (DMU_j : $j\in J=\{1,\ldots,n\}$) that each DMU_j is using inputs x_{ij} , i=1,…,m, to produce outputs y_{rj} , r=1,…,s. Let the input and output vectors for DMU_j be $X_j = (x_{1j},\ldots,x_{mj})^t$ and $Y_j = (y_{1j},\ldots,y_{sj})^t$, respectively. For DMU_j, it has been assumed that $X_j\geq 0, X_j\neq 0$ and $Y_j\geq 0, Y_j\neq 0$. The relative efficiency of the DMUp, $p\in J$ relative to the others is obtained from the following model. This model is called input-oriented CCR envelopment form:

$$\begin{aligned} & Min \quad \theta \\ & s.t \quad \sum_{j=1}^{n}\lambda_j x_{ij} \leq \theta x_{ip}, \qquad i=1,\ldots,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{rp}, \qquad r=1,\ldots,s, \\ & \lambda_j \geq 0, \qquad j=1,\ldots,n. \end{aligned} \quad (1)$$

θ^* is optimal value objective function model (1), and is called efficiency value of DMU_p. Obviously, $0<\theta^* \leq 1$. DMU_p is CCR-efficient (technical) if and only if $\theta^* = 1$. Otherwise, DMU_p is CCR-inefficient.

Review of program evaluation methods

Charnes et al. (1981) were the first to propose a method to separate the DEA efficiency measure into two components. The first one is only attributable to the context within which the DMU is required to operate (that is, associated with programs, policies or environmental conditions) and the other reflects internal managerial inefficiency (Charnes et al., 1981). Only a few theoretical development of program evaluation methods are reported (Elyasiani and Mehdian, 1992, 1995; Cummins et al., 1999; Golany and Storbeck, 1999; Brockett and Golany, 1996; Brockett et al., 1998). Cook et al. (1998) extended the methods for dealing with considering groups in DEA (Charnes et al., 1981).

Malmquist productivity index

Malmquist productivity index was introduced by Caves et al. (1982). Fare and Grosskopf (1994) constructed the DEA-based Malmquist productivity index as the geometric mean of two Malmquist productivity indexes which are defined by a distance function D(.) and refers to the technologies at time periods t and t+1. Fare and Grosskopf (1994) decompose Malmquist productivity index into two components, one measures the input technical efficiency change and the other measures technological change between two periods. Malmquist productivity index calculation requires two single period and two mixed period measures. The technical efficiency score for DMU_p in time period t and t+1 is found as optimal value to one of the following linear programming models:

$$D^{k'}(k) = \quad Min \quad \theta$$
$$s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{k'} \le \theta x_{ip}^{k}, \qquad i = 1,...,m,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^{k'} \ge y_{rp}^{k}, \qquad r = 1,...,s, \qquad (2)$$
$$\lambda_j \ge 0, \qquad j = 1,...,n.$$

Where k = t, t +1 and k' = t, t + 1. Fare and Grosskopf (1994) defined an input-oriented productivity index as the geometric mean of the two Malmquist indices developed by Caves et al. (1982), referring to the technologies at time periods t and t+1. For a particular DMU_p, $p \in J = \{1,\ldots,n\}$ is given as:

$$M^{t,t+1} = \left[\frac{D_p^t(t+1)}{D_p^t(t)} \cdot \frac{D_p^{t+1}(t+1)}{D_p^{t+1}(t)} \right]^{\frac{1}{2}} \qquad (3)$$

$M^{t,t+1}$ >1 indicates productivity gain, $M^{t,t+1}$<1 indicates productivity loss, and $M^{t,t+1}$=1 shows no change in productivity from time t to t+1.

MALMQUIST-BASED PERFORMANCE MEASURES FOR GROUPS OF DMUs OPERATING UNDER DIFFERENT CONDITION

Camanho and Dyson (2006) developed measures for the comparison of performance of groups of DMUs operating under different programs or environmental conditions. They proposed that it can be multiplicatively decomposed into an index reflecting the differences in efficiency spreads within each group and an index reflecting the differences in productivity between the groups best practice frontiers.

Consider ∂_A DMUs in group A, using inputs $X^A \in R_+^m$ to produce outputs $Y^A \in R_+^s$, and ∂_B DMUs in group B, using inputs $X^B \in R_+^m$ to produce outputs $Y^B \in R_+^s$. Input- output vector as (X_j^B, Y_j^B) for $j = 1,...,\partial_B$ represents the DMUs operating in group B. A similar notation is used for group A. $D^A(X_j^B, Y_j^B)$ represents the input distance function for a DMU in group B with respect to the frontier of group A. They define an overall measure for the comparison of performance between two groups of DMUs (groups A and B), associated with different programs, as follows:

$$I^{AB} = \left[\frac{(\prod_{j=1}^{\partial_A} D^A(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B} D^A(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}} \cdot \frac{\prod_{j=1}^{\partial_A} D^B(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}}{\prod_{j=1}^{\partial_B} D^B(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}} \right]^{\frac{1}{2}} \qquad (4)$$

Formula 4 decomposes into the following sub-components:

$$I^{AB} = \frac{\left[\prod_{j=1}^{\partial_A} D^A(X_j^A, Y_j^A)\right]^{\frac{1}{\partial_A}}}{\left[\prod_{j=1}^{\partial_B} D^B(X_j^B, Y_j^B)\right]^{\frac{1}{\partial_B}}} \cdot$$

$$\left[\frac{(\prod_{j=1}^{\partial_A} D^B(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A} D^A(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}} \cdot \frac{(\prod_{j=1}^{\partial_B} D^B(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D^A(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}} \right]^{\frac{1}{2}} \qquad (5)$$

The first ratio compares within-group efficiency spreads. The other ratio evaluates the productivity gap between the frontiers of the two groups. They set:

$$IE^{AB} = \frac{\left[\prod_{j=1}^{\partial_A} D^A(X_j^A, Y_j^A)\right]^{\frac{1}{\partial_A}}}{\left[\prod_{j=1}^{\partial_B} D^B(X_j^B, Y_j^B)\right]^{\frac{1}{\partial_B}}} \qquad (6)$$

And

$$IF^{AB} = \left[\frac{(\prod_{j=1}^{\partial_A} D^B(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A} D^A(X_j^A, Y_j^A))^{\frac{1}{\partial_A}}} \cdot \frac{(\prod_{j=1}^{\partial_B} D^B(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D^A(X_j^B, Y_j^B))^{\frac{1}{\partial_B}}} \right]^{\frac{1}{2}} \qquad (7)$$

A value of IE^{AB} less than one indicates that the efficiency spread is smaller in DMUs of group A than in those of group B. A value of IF^{AB} less than one indicates greater productivity of the frontier of group A compared to group B. To obtain a comparison between more than two groups, it is desirable that the index satisfy the circular relation of Frisch (1936), as follows:

$$I^{AB} \times I^{BC} = I^{AC} \qquad (8)$$

The circular relation for Formula 6 is always satisfied, as follows:

$$IE^{AB} \times IE^{BC} = IE^{AC} \quad (9)$$

In relation to the frontier productivity index in Formula 7, the circular relation is not always satisfied. When the ranking of interest involves N different groups, with ∂_i DMUs in each group i (i=1,…,N), The adjusted index for the comparison of frontier productivity between groups A and B is as follows:

$$IF_{adj}^{AB} = \left[\prod_{i=1}^{N} \frac{\left(\prod_{j=1}^{\partial_i} D^B(X_j^i, Y_j^i)\right)^{\frac{1}{\partial_i}}}{\left(\prod_{j=1}^{\partial_i} D^A(X_j^i, Y_j^i)\right)^{\frac{1}{\partial_i}}} \right]^{\frac{1}{N}} \quad (10)$$

This index satisfies the circular relation.

The Malmquist productivity index group A in relation to group B is calculated according to the following model:

$$\begin{aligned} D_B(X_p^A, Y_p^A) &= Min \quad \theta \\ s.t \quad \sum_{j \in B} \lambda_j X_j^B &\leq \theta X_P^A, \quad (11) \\ \sum_{j \in B} \lambda_j Y_j^B &\geq Y_p^A, \\ \lambda_j \geq 0 \quad & j \in B. \end{aligned}$$

According to their method, if group A at time t at PPS and DMUp of group B at time t+1 unit is under performance, then the model will be as follows:

$$\begin{aligned} D_R^{k'}(X_p^{k,s}, Y_p^{k,s}) &= Min \quad \theta \\ s.t \quad \sum_{j \in R} \lambda_j X_j^{k',R} &\leq \theta X_p^{k,s}, \quad (12) \\ \sum_{j \in R} \lambda_j Y_j^{k',R} &\geq Y_p^{k,s}, \\ \lambda_j \geq 0 \quad & j \in R. \end{aligned}$$

Which R,S $\in$ {A,B} and k,k' $\in$ {t,t+1}. Since 2^4=16, therefore, we have 16 models of the afore type, that depends on times t , t+1 and groups A, B. Following the Malmquist index presented in Formula 3, and overall measure for the comparison of performance between two groups of DMUs presented in Formula 4, we define a measure for the comparison of performance between two groups of DMUs in two time periods, as follows:

$$I_{AB}^{t,t+1} = \left(\frac{\left(\prod_{j=1}^{\partial_A} D_A^t(X_j^{t,A}, Y_j^{t,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_A^t(X_j^{t,B}, Y_j^{t,B})\right)^{\frac{1}{\partial_B}}} \cdot \frac{\left(\prod_{j=1}^{\partial_A} D_A^{t+1}(X_j^{t+1,A}, Y_j^{t+1,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_A^{t+1}(X_j^{t+1,B}, Y_j^{t+1,B})\right)^{\frac{1}{\partial_B}}} \cdot \frac{\left(\prod_{j=1}^{\partial_A} D_B^t(X_j^{t+1,A}, Y_j^{t+1,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_B^t(X_j^{t,B}, Y_j^{t,B})\right)^{\frac{1}{\partial_B}}} \right)^{\frac{1}{2}}$$

$$\left(\frac{\left(\prod_{j=1}^{\partial_A} D_B^{t+1}(X_j^{t,A}, Y_j^{t,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_B^{t+1}(X_j^{t+1,B}, Y_j^{t+1,B})\right)^{\frac{1}{\partial_B}}} \cdot \frac{\left(\prod_{j=1}^{\partial_A} D_A^{t}(X_j^{t+1,A}, Y_j^{t+1,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_A^{t}(X_j^{t+1,B}, Y_j^{t+1,B})\right)^{\frac{1}{\partial_B}}} \cdot \frac{\left(\prod_{j=1}^{\partial_A} D_A^{t+1}(X_j^{t,A}, Y_j^{t,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_A^{t}(X_j^{t,B}, Y_j^{t,B})\right)^{\frac{1}{\partial_B}}} \right)^{\frac{1}{2}}$$

$$\left(\frac{\left(\prod_{j=1}^{\partial_A} D_B^{t}(X_j^{t,A}, Y_j^{t,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_B^{t}(X_j^{t+1,B}, Y_j^{t+1,B})\right)^{\frac{1}{\partial_B}}} \cdot \frac{\left(\prod_{j=1}^{\partial_A} D_B^{t+1}(X_j^{t+1,A}, Y_j^{t+1,A})\right)^{\frac{1}{\partial_A}}}{\left(\prod_{j=1}^{\partial_B} D_B^{t+1}(X_j^{t,B}, Y_j^{t,B})\right)^{\frac{1}{\partial_B}}} \right)^{\frac{1}{2}} \quad (13)$$

In the foregoing bracket, the first ratio, evaluating the mean distance with respect to the boundary of group A at time t and DMUs from group A at time t, is divided by the mean distance for DMU,s from group B at time t, and for DMU,s from group A at time t. The other ratio has the same interpretation. The fifth ratio evaluating the mean distance with respect to boundary of group A at time t, DMU,s for group A at time t+1, is divided by the mean distance for DMU,s of group B at time t+1, DMU,s from group B at time t+1.The other ratio has the same interpretation. Since there is no reasons for ranking of boundary for group A or B at time t and t+1, we choose the geometry means of the two. If the overall index value is less than unity, it indicates better performance in group

A than in group B of time t to time t+1. The overall Performance can be decomposed into two components:

$$I_{AB}^{t,t+1}=\left(\frac{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right).$$
$$\left(\frac{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right)$$
$$\left(\frac{(\prod_{j=1}^{\partial_A}D_B^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_B^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_B^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}\right)^{\frac{1}{2}}$$
$$\left(\frac{(\prod_{j=1}^{\partial_A}D_B^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}\cdot\frac{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B}D_A^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B}D_A^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right)^{\frac{1}{2}}$$
$$\left(\frac{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B}D_A^t(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B}D_A^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}.\right)^{\frac{1}{2}} \quad (14)$$

The first bracket compares the efficiency spread for groups at times t and t+1. The second bracket shows the productivity gap between the frontier of two groups at above time. The first bracket is called efficiency spread at two times and is denoted by $IE_{AB}^{t,t+1}$ and shown as follows:

$$IE_{AB}^{t,t+1}=\left(\frac{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right).$$
$$\left(\frac{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_B}D_B^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right) \quad (15)$$

The value of $IE_{AB}^{t,t+1}$ lesser than one shows that the efficiency spread in DMU's for group A is less than group B at the times t and t+1. The second bracket shows the productivity gap between the frontier of two groups at times t and t+1. The index for distance measurement between the best-practice frontier of groups A and B at times t and t+1 is stated as:

$$IF_{AB}^{t,t+1}=\left(\frac{(\prod_{j=1}^{\partial_A}D_B^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^t(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_B^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}\cdot\frac{(\prod_{j=1}^{\partial_A}D_B^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A}D_A^{t+1}(X_j^{t,A},Y_j^{t,A}))^{\frac{1}{\partial_A}}}\right)^{\frac{1}{2}}$$

$$\left(\frac{(\prod_{j=1}^{\partial_A} D_B^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}{(\prod_{j=1}^{\partial_A} D_A^t(X_j^{t+1,A},Y_j^{t+1,A}))^{\frac{1}{\partial_A}}}.\frac{(\prod_{j=1}^{\partial_B} D_B^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D_A^t(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}.\frac{(\prod_{j=1}^{\partial_B} D_B^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D_A^{t+1}(X_j^{t,B},Y_j^{t,B}))^{\frac{1}{\partial_B}}}\right)^{\frac{1}{2}}$$

$$\left(\frac{(\prod_{j=1}^{\partial_B} D_B^t(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D_A^t(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}.\frac{(\prod_{j=1}^{\partial_B} D_B^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}{(\prod_{j=1}^{\partial_B} D_A^{t+1}(X_j^{t+1,B},Y_j^{t+1,B}))^{\frac{1}{\partial_B}}}.\right)^{\frac{1}{2}} \quad (16)$$

The value of $IF_{AB}^{t,t+1}$ lesser than one shows that the frontier of group A in relation to group B at the times t and t+1 has had more productivity. The state case was the comparison of only for two groups. In general form the comparison of more than two groups is important. It is desirable that the overall index satisfies the circular relation of Frisch (1936). We state that the efficiency spread index in circular relation satisfies as:

$$IE_{AB}^{t,t+1} \times IE_{BC}^{t,t+1} = IE_{AC}^{t,t+1}. \quad (17)$$

So, it can compare the efficiency spread index for two groups at times t and t+1 , $IE_{AC}^{t,t+1}$ and it can be obtained the above with comparison of these groups at the times t and t+1 to any other group that is considered a reference. The frontier productivity index does not always satisfy in circular relation. Fare and Grosskopf (1996) proved that Malmquist's index satisfies in transition relation, if and only if it illustrates the technology Hicks (Charnes et al., 1981). That is, varying of technology in total time transport the frontier to same rate. This kind of technology is the most constrained and is not observed in the real world applications. Like groups execution text, we are not interested in constraint on group frontier in the afore index making. However, a strong executive ranking is obtained only when the productivity index frontier satisfies circular relation. For this reason, the adjusted index which is denoted by $adIF_{AB}^{t,t+1}$ is defined as follows:

$$adIF_{AB}^{t,t+1} = \left[\prod_{i=1}^{N}\left(\frac{(\prod_{j=1}^{\partial_i} D_B^t(X_j^{t,i},Y_j^{t,i}))^{\frac{1}{\partial_i}}}{(\prod_{j=1}^{\partial_i} D_A^t(X_j^{t,i},Y_j^{t,i}))^{\frac{1}{\partial_i}}}.\frac{(\prod_{j=1}^{\partial_i} D_B^{t+1}(X_j^{t+1,i},Y_j^{t+1,i}))^{\frac{1}{\partial_i}}}{(\prod_{j=1}^{\partial_i} D_A^{t+1}(X_j^{t+1,i},Y_j^{t+1,i}))^{\frac{1}{\partial_i}}}\right)\left(\frac{(\prod_{J=1}^{\partial_I} D_B^t(X_j^{t+1,i},Y_j^{t+1,i}))^{\frac{1}{\partial_i}}}{(\prod_{j=1}^{\partial_I} D_A^t(X_j^{t+1,i},Y_j^{t+1,i}))^{\frac{1}{\partial_i}}}.\frac{(\prod_{j=1}^{\partial_i} D_B^{t+1}(X_j^{t,i},Y_j^{t,i}))^{\frac{1}{\partial_i}}}{(\prod_{j=1}^{\partial_i} D_A^{t+1}(X_j^{t,i},Y_j^{t,i}))^{\frac{1}{\partial_i}}}\right)\right]^{\frac{1}{N}} \quad (18)$$

It can easily be shown that $IF_{AB}^{t,t+1}$ satisfies the circular relation. Therefore, Malmquist productivity index satisfies in circular relation and can be used for greater performance of two groups at times t and t+1:

$$adI_{AB}^{t,t+1} = IE_{AB}^{t,t+1} \times adIF_{AB}^{t,t+1} \quad (19)$$

THE EVALUATION OF BANK BRANCHES PERFORMANCE

Here, we consider bank branches in three provinces (three groups) - West Azarbaijan, Kerman and Khozestan - that are called groups A, B and C, respectively. These regions include the following number of branches: West Azarbaijan: 26, Kerman: 39, Khozestan: 48. Therefore, groups A, B and C have 26, 39 and 48 DMUs, respectively. Each DMU has three inputs: demands (I_1), dividends (I_2) and employees (I_3) and four outputs: commissions received (O_1), received interest (O_2), total value of deposits (O_3) and facilities (O_4). In our experimental applications, all regions have a number of branches relatively high in relation to the number of inputs and outputs. Summary statistics for inputs and outputs in each region in time t are given in Table 1 while for inputs and outputs in each region in time t+1 are given in Table 2.

We employ Malmquist index to evaluate two groups A and B in two times where group A (West Azarbaijan

Table 1. Summary statistics for inputs and outputs of regions in time t.

Variable	West Azarbaijan		Kerman		Khozestan	
	SD	Mean	SD	Mean	SD	Mean
I1	6050049031	4503887054	8380053604	3255250518	27688602126	8554404376
I2	786029413.9	876003835.8	683408312.3	1127357522	680637125.8	1148414607
I3	9.4881583	17.88461538	8.365735821	16.41025641	6.035449387	12.77083333
O1	41831424	79461833	64638765.21	53120728.64	146416180.4	146159151.2
O2	494193552	408015258	1815843254	532238395.5	858213325	371935831.5
O3	29986985478	48782623679	24516267870	53982517815	31955006077	58161583497
O4	23021403039	32070650036	45221940265	44538372498	33613391771	32100380031

province) includes 26 DMUs and group B (Kerman province) includes 39 DMUs. Each DMU contains three inputs and four outputs in any time. Table 3 shows the results of comparison of productivity between two regional groups A and B in two times t, t+1.

Table 3 shows the results of comparison of productivity between two regional performance group A and B in times t, t+1. From Table 3, it can be concluded that the productivity in Kerman province (group B) is greater than the in West Azarbaijan province (group A) in times t, t+1. Note that the element below the diagonal of the matrix is the inverse of the value in the upper part of the matrix.

Table 4 shows the results of comparison of efficiency spread between West Azarbijan province and Kerman province in times t, t+1. Table 4 shows the results of comparison of efficiency spread between groups A and B in times t, t+1. Table 4 shows that since the value of $IE_{AB}^{t,t+1}$ is greater than 1, the efficiency spread in DMU's for group A is greater than group B in the times t and t+1.

Table 5 shows the results of comparison of frontier productivity between West Azarbaijan province and Kerman province in times t, t+1. Table 5 shows the results of comparison of frontier productivity between West Azarbaijan province and Kerman province in times t, t+1. Since the value of $IF_{AB}^{t,t+1}$ is smaller than 1, Table 5 shows that frontier productivity in DMU's for West Azarbaijan province is less than Kerman province in the times t and t+1.

To obtain a performance ranking of all groups, the adjusted index ($adIF_{AB}^{t,t+1}$) should be used. Comparison of the three regions using index efficiency spread is reported in Table 6. Figure 1 shows the results of data taken from the first line in Table 6, using West Azarbaijan as the reference.

According to Table 6, we conclude that Khozestan has the smallest efficiency spread, Kerman has second rank and west Azarbaijan has the largest efficiency spread. Note that the element below the diagonal of the matrix is the inverse of the value in the upper part of the matrix. Comparison of the three regions using adjusted index ($adIF_{AB}^{t,t+1}$) is reported in Table 7.

Figure 2 shows the results of data taken from the first line in Table 7, using West Azarbaijan as the reference. Table 7 shows the results of the productivity ranking of all regional frontiers obtained using the index $adIF_{AB}^{t,t+1}$, described in Formula 18. Table 7 shows the results of the productivity ranking of all region frontiers using the adjusted index ($adIF_{AB}^{t,t+1}$). The element below the diagonal of the matrix is the inverse of the value in the upper part of the matrix. According to Table 7, we conclude that Khozestan has the smallest productivity, Kerman is second ranked and west Azarbaijan has the largest productivity. The values of the index ($IF_{AB}^{t,t+1}$) and adjusted index ($adIF_{AB}^{t,t+1}$) are different. For example, $IF_{AB}^{t,t+1} = 0.545452$ (Table 6) but $adIF_{AB}^{t,t+1} = 0.169813$ (Table 7). Comparison of the three regions using adjusted index is reported in Table 7. Figure 3 shows the results of data taken from the first line in Table 8, using West Azarbaijan as the reference.

The results reported in Table 8 and Figure 3 shows that Kerman is the best performing region, west Azarbaijan has the second rank and Khozestan has the third rank. According Table 8, the circularity of this adjusted index can be easily proovec $adI_{AB}^{t,t+1} \times adI_{BC}^{t,t+1} = adI_{AC}^{t,t+1}$, that is, 2.264216 × 0.119528 = 0.270636.

CONCLUSION

The purpose of this study was to develop the Malmquist productivity index to DMUs for groups in two times. Group of decision making units has been decomposed into three mutually exclusive groups and their efficiency has been compared with each other. For this propose, the definition of Malmquist productivity index have been extended, which can be used for evaluating progress and regress of each group. Malmquist productivity index has been decomposed into two components, the first

Table 2. Summary statistics for inputs and outputs of regions in time t+1.

Variable	West Azarbaijan		Kerman		Khozestan	
	SD	Mean	SD	Mean	SD	Mean
I1	6062939238	4518819284	1096200492	3257756257	28759657952	9125818835
I2	955611536	1107426478	764019245.1	1383837396	858257527.3	1443808131
I3	9.226040455	17.73076923	8.883422053	16.46153846	6.049563804	12.66666667
O1	44643020.51	90885314.42	65483752	53120729	167796545.6	168264810.7
O2	643560622.1	575948265.3	1819119839	5.91E+08	947753471	502343524.8
O3	26297861876	46742882972	24669680502	53603120092	29722048023	2.72627E+12
O4	22688523676	33619251620	48122674779	46460190602	33108695485	33276016181

Table 3. Comparison of regional performance $I_{AB}^{t,t+1}$ between groups A and B in times t, t+1.

Groups A/B	West Azarbaijan	Kerman
West Azarbaijan	1	2.307651
Kerman	0.433341	1

Table 4. Comparison of efficiency spread ($IE_{AB}^{t,t+1}$) between groups A and B in times t, t+1.

Groups A/B	West Azarbaijan	Kerman
West Azarbaijan	1	4.230714
Kerman	0.236367	1

Table 5. Comparison of frontier productivity ($IF_{AB}^{t,t+1}$) between groups A and B in times t, t+1.

Groups A/B	West Azarbaijan	Kerman
West Azarbaijan	1	0.545452
Kerman	1.833342	1

Table 6. Index $IE_{AB}^{t,t+1}$ for comparison between three groups.

A/B	West Azarbaijan	Kerman	Khozestan
West Azarbaijan	1	4.230708	6.008659
Kerman	0.236367	1	1.420249
Khozestan	0.166426	0.704102	1

component is given by $IE_{AB}^{t,t+1}$ which compares the efficiency changing. The value of $IE_{AB}^{t,t+1}$ lesser than one shows that the efficiency spread in DMU's for group A is less than group B in times t and t+1. The second

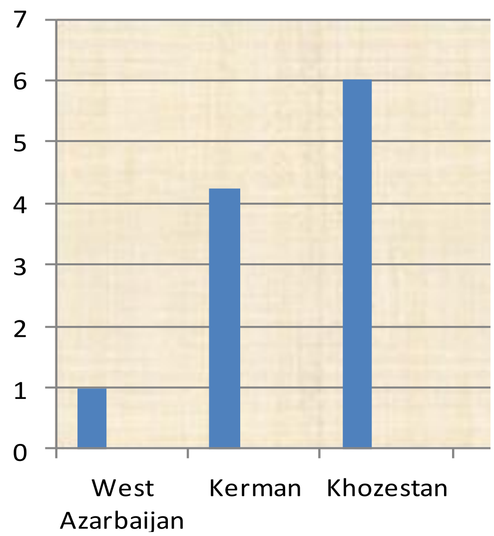

Figure 1. Index for the comparison of efficiency spread, using West Azarbaijan as the reference region.

Table 7. Adjusted index ($adIF_{AB}^{t,t+1}$) for comparison between three groups.

A/B	West Azarbaijan	Kerman	Khozestan
West Azarbaijan	1	0.535186	0.045041
Kerman	1.868551	1	0.084160
Khozestan	22.201994	11.882129	1

component is given by the relation $IF_{AB}^{t,t+1}$ which compares the technological change. The value of $IF_{AB}^{t,t+1}$ lesser than one shows that the frontier of group A in relation to group B in times t and t+1 has had more productivity. Conventionally, the Malmquist productivity index is used to compare the productivity of a certain DMU in two different time periods that any two DMUs can be used in general. This extended method may be used

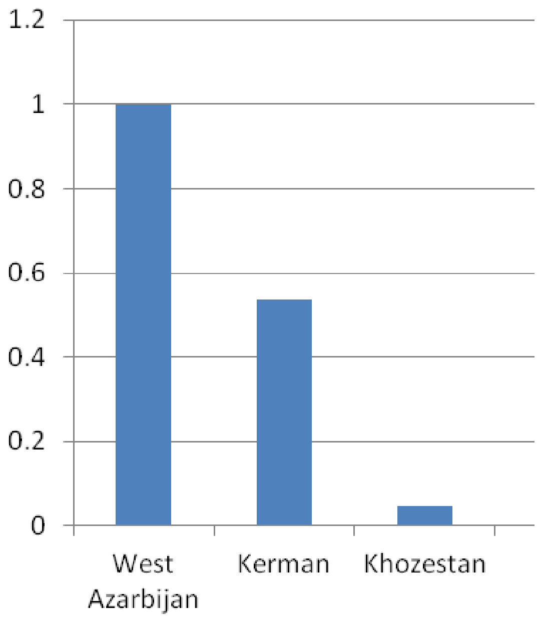

Figure 2. Adjusted index for the comparison of productivity, using West Azarbaijan as the reference region.

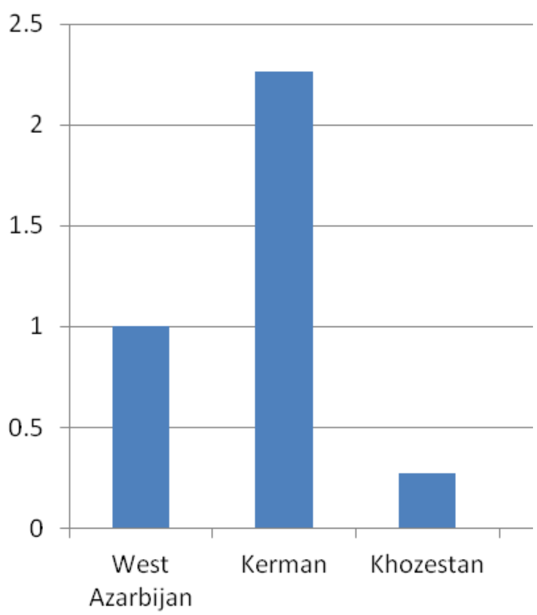

Figure 3. Adjusted index for the ranking of regional performance.

Table 8. Adjusted index for comparison between three groups.

A/B	West Azarbaijan	Kerman	Khozestan
West Azarbaijan	1	2.264216	0.270636
Kerman	0.441654	1	0.119528
Khozestan	3.695000	8.366241	1

for ranking DMUs for implementing the suggested method; the input oriented model may be used with constant return to scale technology. The Malmquist productivity index does not enjoy the transition relation, but the adjusted index, that is $IF_{AB}^{t,t+1}$ enjoys the transition relation.

The method is used for a set of branches of commercial banks and the results show the validity of the method. The tables indicate more efficiency spread in West Azarbijan province than in Kerman province whereas, its productivity frontier is smaller than Kerman province. Hence, Kerman province possesses more productivity than West Azarbijan. To obtain performance ranking of all groups (more than two groups), the adjusted productivity index was used, the results of which were presented in Table 6. This table shows that Khozestan province has a low productivity frontier in comparison with the rest.

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