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Multi-objective project management by fuzzy integrated goal programming

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Regulating conflicting goals with the usage of all related resources through organization is the main work of project management (PM). In this paper, the issue of evaluating the conflicting goals tradeoffs of projects was to develop a plan that the decision-maker can use to shorten their total completion time and minimize the increasing project total cost. The study showed that the total project cost minimization problem and crashing cost minimization problem with reference to direct, indirect cost and relevant constraints can be solved simultaneously via the proposed fuzzy multi-objective linear programming (FMOLP) method. Next, considering its completion time in a suitable range, we are trying to find more efficient ways of utilizing the fuzzy set to solve fuzzy multi-objective PM decision problem, and the proposed approach applies the signed distance method to transform fuzzy numbers into crisp values. The proposed approach considers the imprecise nature of the input data by implementing the minimum operator and also assumes that each objective function has a fuzzy goal. In addition, the focus of this approach is minimizing the worst upper bound to obtain an efficient solution which is close to the best lower bound of each objective function. Moreover, for attaining our objective, at the end of this paper, a detailed numerical example will be presented to illustrate the feasibility of applying the proposed approach to actual PM decision problem. Furthermore, it was believed that this approach can be utilized to solve other multi-objective decision making problems in practice.

Key words: Project management, fuzzy set, fuzzy multi-objective linear programming.

INTRODUCTION

Fundamentally, something like the long period of time, the low duplication, the specific contract, the huge investment amount, much resources consumption and various kinds of work activities are significant characteristics of projects. Therefore, it is truly important for project managers to confirm the project completion that includes quality, effectiveness, the specified completion time and the allocated cost. Thus, the project managers must handle conflicting goals with the usage of all related resources through the organization in real-life situations. These conflicting goals need to be optimized simultaneously by project managers in the framework of fuzzy aspiration levels. Issues with proportional goal programming and fuzzy application have attracted the interest of more researchers and there are increasing papers dealing with these topics.

McCahon and Lee (1988) develop a comprehensive path analysis method devised in using fuzzy arithmetic and a fuzzy number comparison method to determine fuzzy project completion time and the degrees of

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criticality of each network path. Practically, total costs of projects are the sum of direct cost (labor, equipment, material and other cost related directly to projected activities), indirect cost (administration, depreciation, interest and other variable overhead cost) (Wang and Liang, 2004). The purpose of evaluating time-cost trade-offs is to develop a plan which the decision-maker (DM) can minimize the increase of project total cost and total crashing cost when shortening their total completion time.

Arikan and Gungor (2001) utilized fuzzy goal programming (FGP) approach to solve PM decision problems with two objectives: minimizing both completion time and crashing cost. Additional work such as Wang and Fu (1998) applied fuzzy mathematical programming to solve PM decision problems. These models aim to minimize total project cost and total crashing cost simultaneously.

In addition, Zadeh (1978) presented the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable. Since the expression of a possibility distribution can be viewed as a fuzzy set, possibility distribution may be manipulated by the combination rules of fuzzy sets and more particular of fuzzy restrictions (Dubois and Prade, 1988). Accordingly, Wang and Liang (2005) formulated a possibilistic programming model to solve fuzzy multi-objective project management (PM) problems with imprecise objective and constraints recently. Related works by Inuiguchi and Sakawa (1996), Hussein (1998), Tanaka and Guo (2000) on possibilistic programming linear method were applied to their decision making problems. Besides, the proposed possibilistic programming provides a more efficient way of solving imprecise PM problems and additionally, preserves the original linear model for all imprecise objectives and constraints with the proposed simplified weighted average, and fuzzy ranking techniques (Buckley, 1988; Lai and Hwang, 1992; Zadeh, 1978).

Furthermore, many PM decisions problems were almost assumed as the total completion time minimized with fuzzy linear membership function (Arikan and Gungor, 2001; Deporter and Eills, 1990; Wang and Liang, 2005) or the imprecise time adopted for triangular possibility distribution (Liang, 2008). Nevertheless, the imprecise project time is suited to the trapezoidal fuzzy numbers if the decision maker (DM) hopes to control the project completion time in a suitable range. When a project is extended beyond its normal completion time, the contractual penalty cost will be incurred. On the contrary, if a project is completed too fast before its completion time under normal conditions, much more crashing cost and float time will be incurred. This work applies the sign distances method to convert the fuzzy number into a crisp number. After the Center of Gravity was proposed, it is another new, easy and useful method for defuzzification. And its definition is more exact than the Center of Gravity (Yao and Wu, 2000).

In this paper, a fuzzy multi-objective linear programming (FMOLP) method is proposed to solve fuzzy multi-objective PM decision problems in uncertain environments. First, the proposed approach describes the problem, details the assumption, and formulates the problem. Second, it develops the interactive FMOLP model and procedure for solving PM problems. Next, it presents an example for applying the proposed approach to real PM decisions. And finally, it discusses the results of comparison for the practical application of the proposed approach and draws the conclusions.

FUZZY SET

A fuzzy set is an extension of a crisp set. A crisp set *A* can be define by using the membership method, which introduces a zero-one membership function for *A*, described by a characteristic function $\mu_{A(x)}$, where,

$$\mu_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases} \tag{1}$$

The crisp set *A*, whose characteristic function $\mu_{A(x)}$ is given in (1) which indicates *x* belonging to A and 0 indicates *x* do not belong to A, x∈X, X is universe of set. Now a set A was discussed, whose characteristic function takes value ranging from 0 to 1 (Klir and Yuan, 1995). When $\mu_{A(x_1)>}\mu_{A(x_2)}$ indicates that the degree of x_1 belong to A is bigger than the degree of x_2 belong to A. Now, the set A is a vague element membership relation. Then the set A is called as "Fuzzy Set" and its characteristic function is called as "membership function". The membership function of set $\tilde{A}$ can be illustrated in Equation 2 and expressed by

$$\mu_{\tilde{A}} : X \to \{0,1\}, \left(0 \le \mu_{\tilde{A}}(x) \le 1, x \in X\right) \tag{2}$$

A special fuzzy set $\tilde{A}$ defined in real line *R*, which is also universe of set *X* = *R*, and has three constraints: (1) $\tilde{A}$ is a normal fuzzy set; (2) A^{α} to all $\alpha \in (0,1)$ are closed intervals; (3) $\tilde{A}$ has a bounded support (Hsieh, 2005; Liang, 2008) are discussed.

Nowadays, two special types of fuzzy numbers are widely used: triangular fuzzy number and trapezoidal fuzzy number and are illustrated as follows.

Triangular membership function:

$$\mu_A(x) = \mu(x;a,b,c) = \begin{cases} \frac{(x-a)}{(b-a)}, & if\ a \le x < b \\ \frac{(c-x)}{c-b}, & if\ b \le x \le c \\ 0, & if\ x > c \vee x < a \end{cases} \tag{3}$$

Trapezoidal membership function:

$$\mu_A(x) = \mu(x;a,b,c,d) = \begin{cases} \frac{(x-a)}{(b-a)}, \ if\ a \le x < b \\ 1, \quad if\ b \le x \le c \\ \frac{(d-x)}{(d-c)}, \ if\ c < x \le d \\ 0, \ if\ x > d \lor x < a \end{cases} \tag{4}$$

Many ranking methods have been developed to transform fuzzy numbers into crisp values (Chen and Hwang, 1992). Yao and Wu (2000) proposed the signed distance method in A.C. 2000. It is better and more sensible than the Center of Gravity method.

Property 1

For a triangular fuzzy number $\tilde{A} = (a, b, c) \in \Lambda$, the signed distance from $\tilde{A}$ to $\tilde{0}_1$ is defined as

$$d(\tilde{A},\tilde{0}_1) = \frac{1}{4}(a + 2b + c) \tag{5}$$

Property 2

For a trapezoidal fuzzy number $\tilde{A} = (p, q, r, s) \in \Lambda$, the signed distance from $\tilde{A}$ to $\tilde{0}_1$ is defined as

$$d(\tilde{A},\tilde{0}_1) = \frac{1}{4}(p + q + r + s) \tag{6}$$

Example

Let $\tilde{A}$ = (1, 6, 8), α-*cuts*, $0 \le \alpha \le 1$, then the signed distance from $\tilde{A}$ to $\tilde{0}_1$ is as follows:

$$d(\tilde{A},\tilde{0}_1) = \frac{1}{4}(1 + 12 + 8) = \frac{21}{4}.$$

Proof

From Zimmermann (1996), Yao and Wu (2000), for any $a \in R$, define the *signed distance* from a to 0 as $d_0(a, 0) = a$. If *a* >0, then the distance from *a* to 0 is $a = d_0(a, 0)$; if *a* <0, the distance from *a* to 0 is $-a = -d_0(a, 0)$. This is why $d_0(a, 0)$ is referred to as the signed distance from a to 0.

Let Λ be the family of all fuzzy sets $\tilde{C}$ defined on *R* with which the α-cut $C(\alpha) = [C_L(\alpha), C_U(\alpha)]$ exists for every $\alpha \in [0, 1]$, and both $C_L(\alpha)$ and $C_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$. Then, for any $\tilde{C} \in \Lambda$, we have

$$\tilde{C} = \cup_{0<\alpha<1} [C_L(\alpha)_\alpha, C_U(\alpha)_\alpha] \tag{7}$$

From this proof, the signed distance of two end points, $C_L(\alpha)$ and $C_U(\alpha)$, of the α-cut $C(\alpha) = [C_L(\alpha), C_U(\alpha)]$ of $\tilde{C}$ to the origin 0 is $d_0(C_L(\alpha), 0) = C_L(\alpha)$ and $d_0(C_U(\alpha), 0) = C_U(\alpha)$, respectively. The study define the signed distance from 0 to the interval $[C_L(\alpha), C_U(\alpha)]$ to be

$$d_0([C_L(\alpha), C_U(\alpha)], 0) = [d_0(C_L(\alpha), 0) + d_0(C_U(\alpha), 0]/2 = (C_L(\alpha) + C_U(\alpha))/2 \tag{8}$$

Since crisp interval $[C_L(\alpha), C_U(\alpha)]$ has a one-to-one correspondence with α-level fuzzy interval $[C_L(\alpha)_\alpha, C_U(\alpha)_\alpha]$, it is natural to define the signed distance from α-level fuzzy interval $[C_L(\alpha)_\alpha, C_U(\alpha)_\alpha]$ to $\tilde{0}_1$ as

$$d([C_L(\alpha), C_U(\alpha)], \tilde{0}_1) = d([C_L(\alpha)_\alpha, C_U(\alpha)_\alpha], 0) = (C_L(\alpha) + C_U(\alpha))/2 \tag{9}$$

Moreover, for $\check{C} \in \Lambda$, since the function (9) is continuous on $0 \le \alpha \le 1$, the integration to obtain the mean value of the signed distance are as follows can be used:

$$\int_0^1 d([C_L(\alpha)_\alpha, C_U(\alpha)_\alpha], \tilde{0}_1) d\alpha = 1/2 \int_0^1 (C_L(\alpha) + C_U(\alpha)) d\alpha \tag{10}$$

Thus, from equations (12) and (15), we have the following definition

$$d(\tilde{C},\tilde{0}_1) = \int_0^1 d([C_L(\alpha)_\alpha, C_U(\alpha)_\alpha], \tilde{0}_1) d\alpha = \frac{1}{2}\int_0^1 (C_L(\alpha) + C_U(\alpha)) d\alpha \tag{11}$$

According to equation (16), we obtain the following property 1.

PROPOSED METHOD

This section discusses the fuzzy multi-objective linear programming (FMOLP) for solving PM decision problems in a fuzzy environment. Firstly, it describes the problem, details the assumption and formulates the problem. Secondly, the method of fuzzier for imprecise cost is triangular possibility distribution whereas for imprecise time is trapezoidal fuzzy numbers. Thirdly, this work applies the sign distances method (Yao and Wu, 2000) to convert the fuzzy number into a crisp number. Finally, this paper will present a modified interactive fuzzy mathematic programming approach (El-Wahed and Lee, 2006) to determine the preferred compromise solution for the FMOLP problem.

Problem description, assumptions and notation

Assume that a project has n interrelated activities that must be executed in a certain order before the entire task can be completed in a fuzzy environment. In general, the environmental coefficients and related parameters are uncertain over the planning horizon. Accordingly, the incremental crashing costs for all activities, variable indirect cost per unit time, specified project completion time, and total budget are imprecise or/and fuzzy.

The problem focuses on the development of an interactive FMOLP approach with considering the DM hopes to control the project completion time in a suitable range to determining the right duration of each activity in the project, given a specified project completion time, the crash time tolerance for each activity and allocated total budget. The aims of this PM decision are to minimize simultaneously total project costs and total crashing costs. The original MOLP model proposed in this work is based on the following assumptions:

1. All of the objective functions and constraints are linear.
2. Direct costs increase linearly as the duration of an activity is reduced from its normal value to its crash value.
3. The normal time and shortest possible time for each activity and the cost of completing the activity in the normal time and crash time are certain over the planning horizon.
4. The indirect costs comprise two categories, that is, fixed costs and variable costs, and the variable cost per unit time is the same regardless of project completion time.
5. The decision maker (DM) has already adopted the pattern of triangular possibility distribution to represent the objectives of the imprecise total project cost, total crashing cost and related imprecise numbers except the specified completion time. It was adopted the trapezoidal fuzzy number.
6. The minimum operator is used to aggregate all fuzzy sets.

Assumptions 1, 2 and 3 imply that both the linearity and certainty properties must be technically satisfied in order to represent an optimization problem as a LP problem. For the sake of model facilitation, Assumption 4 represents that the indirect costs can be divided into fixed costs and variable costs. Fixed costs represent the indirect costs under normal conditions and remain constant regardless of project duration. Meanwhile, variable costs, which are used to measure savings or increases in variable indirect costs, vary directly with the difference between actual completion and normal duration of the project. Assumptions 4 concern the simplicity and flexibility of the model formulation and the fuzzy arithmetic operations. Assumption 5, addresses the effectiveness of applying triangular possibility distribution to represent imprecise objectives and related imprecise numbers except the specified completion time. In general, the project managers are familiar with estimating optimistic, pessimistic and most likely parameters from the use of the Beta distributions specified by the class PERT. The pattern of triangular distribution is commonly adopted due to ease in defining the maximum and minimum limit of deviation of the fuzzy number from its central value (Yang and Ignizio, 1991). And it applies trapezoidal fuzzy number to represent imprecise completion time with considering the specified range (Liang, 2008). Assumptions 5 and 6 convert the original MOLP problem into an equivalent ordinary single-objective LP form that can be solved efficiently by the standard simplex method.

The following notation is used.

(i, j)	activity between event i and event j
$\tilde{Z}_1$	total project costs ($)
$\tilde{Z}_2$	total crashing costs ($)
D_{ij}	normal time for activity (i, j) (days)
d_{ij}	minimum crashed time for activity (i, j)
C_{Dij}	normal (direct) cost for activity (i, j) ($)
C_{dij}	minimum crashed (direct) cost for activity (i, j) ($)
$\tilde{k}_{ij}$	incremental crashing costs for activity (i, j)(representing the cost-time slopes) ($/day)
t_{ij}	duration time for activity (i, j) (days)(difference between normal time and crash time)
Y_{ij}	crash time for activity (i, j) (days)(difference between normal time and duration time)
E_i	earliest time for event i (days)
E_0	project start time (days)
E_{n}	project completion time (days)
T_0	project completion time under normal conditions (days)
$\tilde{T}$	specified project completion time (days)
C_I	fixed indirect cost under normal conditions ($)
$\tilde{m}$	variable indirect cost per unit time ($/day)
$\tilde{b}$	total allocated budget ($)
γ	cut level

Basic model

Two objective functions with minimizing total project costs and total crashing costs are simultaneously considered to develop the proposed multiple objectives linear programming (MOLP) model, as follows.

Minimize total project costs

$$Min\ \tilde{Z}_1 = \sum_i \sum_j C_{Dij} + \sum_i \sum_j \tilde{k}_{ij} Y_{ij} + C_I + \tilde{m}\left(E_n - T_o\right) \quad (12)$$

Where the terms

(1) $\sum_i \sum_j \tilde{C}_{Dij} + \sum_i \sum_j \tilde{k}_{ij} Y_{ij}$: total direct costs including total normal cost and total crashing cost, obtained using additional direct resources such as overtime, personnel and equipment.

(2) $C_I + \tilde{m}(E_n - T_o)$: indirect cost including those of administration, depreciation, financial and other variable overhead cost that can be avoided by reducing total project time.

(3) $\tilde{k}_{ij} = (\tilde{C}_{dij} - C_{Dij})/(D_{ij} - d_{ij})$: the analysis in this problem depends primarily on the cost-time slopes for the various activities.

Minimize total crashing costs

$$Min\ \tilde{Z}_2 = \sum_i \sum_j \tilde{k}_{ij} Y_{ij} \quad (13)$$

The time between event *i* and event *j*:

$$E_i + t_{ij} - E_j \le 0 \quad \forall i, \forall j \quad (14)$$

$$t_{ij} = D_{ij} - Y_{ij} \quad \forall i, \forall j \quad (15)$$

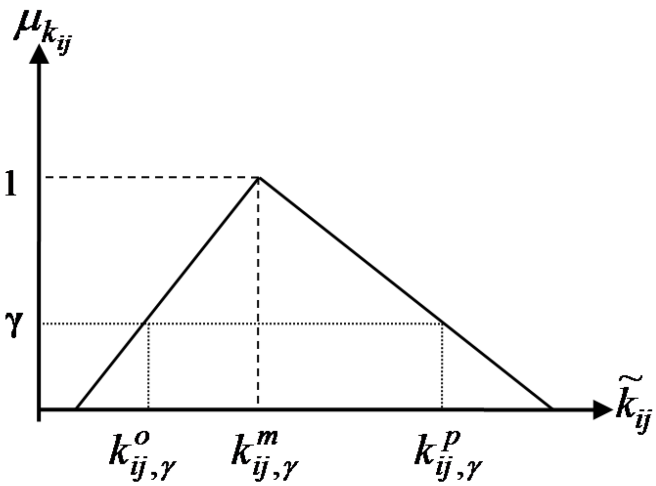

Figure 1. Membership function of $\tilde{k}_{ij}$

The crash time for activity (i, j):

$$Y_{ij} \le D_{ij} - d_{ij} \quad \forall i, \forall j \tag{16}$$

The project start time and total completion time is as follows:

$$E_0 = 0 \tag{17}$$

$$En \cong \tilde{T} \tag{18}$$

The total budget:

$$\tilde{Z}_1 \le \tilde{b}. \tag{19}$$

Non-negativity constraints on decision variables:

$$t_{ij}, Y_{ij}, E_i, E_j \ge 0 \quad \forall i, \forall j \tag{20}$$

In real-world situations, the incremental crashing costs for all activities in equation (12) and (19), the specified completion time for the project in equation (13) are often imprecise because some relevant information, such as the skills of the workers, law and regulations, available resources, and other factors, is incomplete or unavailable.

Model development

This work assumes that the decision maker (DM) has already adopted the pattern of triangular possibility distribution to represent the crashing cost, $\breve{k}_{ij}$, variable indirect cost per unit time, $\tilde{m}$, and total allocated budget, $\tilde{b}$, in the original fuzzy linear programming problem. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations. For instance, Figure 1 shows the distribution of the triangular fuzzy number $\breve{k}_{ij}$.

In practical situations, the decision maker (DM) can construct the triangular distribution of $\breve{k}_{ij}$ in objective (12) based on the following three prominent data: (1) the most pessimistic value (k^p_{ij}) that has a very low likelihood of belonging to the set of available values (possibility degree 0 if normalized); (2) the most likely value (k^m_{ij}) that definitely belongs to the set of available values (membership degree = 1 if normalized); and (3) the most optimistic value (k^o_{ij}) that has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized).

Similarly, the fuzzy data, $\tilde{m}$, in objective (12), $\tilde{b}$ in constraints (19), thus can be modeled using the distribution of triangular fuzzy number. Hence, the fuzzy data for $\breve{k}_{ij}$, $\tilde{m}$ and $\tilde{b}$ can be symbolized as follows:

$$\tilde{k}_{ij} = \left\{\left(\mathbf{y},\ \mu_{k_{ij}}(y)\right)\middle|\mathbf{y}\in\mathfrak{R}\right\} = (k^{o}_{ij,\gamma}, k^{m}_{ij,\gamma}, k^{p}_{ij,\gamma})^{\gamma} \tag{21}$$

$$\tilde{m} = \left\{\left(\mathbf{y},\ \mu_{m}(y)\right)\middle|\mathbf{y}\in\mathfrak{R}\right\} = (m^{o}_{\gamma}, m^{m}_{\gamma}, m^{p}_{\gamma})^{\gamma} \tag{22}$$

$$\tilde{b} = \left\{\left(\mathbf{y},\ \mu_{b}(y)\right)\middle|\mathbf{y}\in\mathfrak{R}\right\} = (b^{o}_{\gamma}, b^{m}_{\gamma}, b^{p}_{\gamma})^{\gamma} \tag{23}$$

And, this work assumes that the decision maker (DM) has already adopted the pattern of trapezoidal fuzzy numbers for $\tilde{T}$.

Figure 2 illustrates the membership function pattern of the trapezoidal fuzzy number $\tilde{T}$. The membership function $\mu_T(y) < \gamma$ implies that y has only a small likelihood of belonging to the set of available values.

The decision maker (DM) can construct the trapezoidal fuzzy number based on the four following prominent data (Liang, 2006): (1) the left main value $\underline{T}$: the lower bound that definitely belongs to the set of available values (grade of membership=1); (2) the right main value $\bar{T}$: the upper bound that definitely belongs to the set of available values (grade of membership=1); (3) the left spread $\underline{T} - \underline{\beta}^{\gamma}$: the lower bound of the set that has very little likelihood of belonging to the set of available values (grade of membership=0), and (4) the right spread $\bar{T} + \bar{\beta}^{\gamma}$: the upper bound of the set that has very little likelihood of belonging to the set of available values (grade of membership=0). Hence, the fuzzy data for $\tilde{T}$ can be symbolized as follows:

$$\tilde{T} = \left\{\left(\mathbf{y},\ \mu_{T}(y)\right)\middle|\mathbf{y}\in\mathfrak{R}\right\} = (\underline{T} - \underline{\beta}^{\gamma}, \underline{T}, \bar{T}, \bar{T} + \bar{\beta}^{\gamma})^{\gamma} \tag{24}$$

The membership values $\mu_T(y)$, $\mu_{kij}(y)$, $\mu_m(y)$, and $\mu_b(y)$ refers to the amount of information of y available to the DM. Practically, the DM can specify a cut level γ such that $\mu_T(y)<\gamma$, $\mu_{kij}(y)<\gamma$, $\mu_m(y)<\gamma$, and $\mu_b(y)<\gamma$ imply that y has only a small likelihood of belonging to the set of available values.

The objective function (12) and (13) in the original MOLP model formulated as mentioned have triangular possibility distributions. Geometrically, these two imprecise objectives are fully defined by two pairs of three prominent points (Z_1^o, 0), (Z_1^m, 1), (Z_1^p, 0), and (Z_2^o, 0), (Z_2^m, 1), (Z_2^p, 0). The imprecise objective of minimizing total project cost can be minimized by moving the three

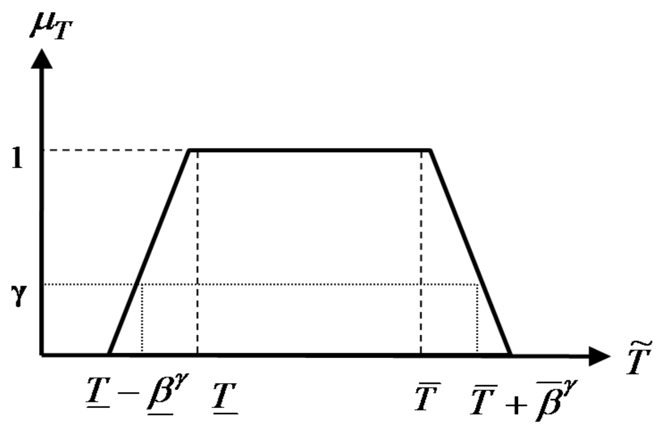

Figure 2. Membership function of $\tilde{T}$.

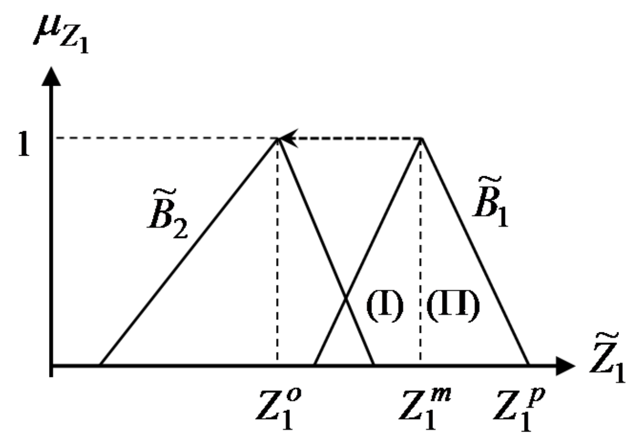

Figure 3. The strategy to minimize the imprecise objective function (Liang, 2008).

prominent points toward the left. Using Lai and Hwang's (1992) approach, the proposed approach substitute simultaneously minimizing Z_1^m, maximizing (Z_1^m - Z_1^o), and minimizing (Z_1^p - Z_1^m) for minimizing Z_1^m, Z_1^o and Z_1^p. And the imprecise objective of minimizing total crashing cost is the same.

The resulting six new objective functions still guarantee the declaration of moving the triangular distribution toward the left. Figure 3 illustrates the strategy for minimizing the imprecise objective function; that is, the auxiliary MOLP problem generated by this proposed approach comprises simultaneously minimizing the most likely value of imprecise total costs (Z_1^m), maximizing the possibility of obtaining lower total costs (region I of the possibility distribution in Figure 3) (Z_1^m - Z_1^o), and minimizing the risk of obtaining higher total costs (region II of the possibility distribution in Figure 3) (Z_1^p - Z_1^m). As indicated in Figure 3, possibility distribution $\tilde{B}_2$ is preferred to possibility distribution $\tilde{B}_1$. Expressions (25) – (27) list the results for the three new objective functions of total costs in Equation (12). Expressions (28) – (30) list the results for the three new objective functions of total crashing cost in Equation (13).

$$Min\ Z_{11} = Z_1^m = \sum_i \sum_j C_{Dij} + \sum_i \sum_j k_{ij}^m Y_{ij} + C_I + m^m \left(E_n - T_o \right) \quad (25)$$

$$Min\ Z_{12} = (Z_1^m - Z_1^o) = \sum_i \sum_j C_{Dij} + \sum_i \sum_j (k_{ij}^m - k_{ij}^o) Y_{ij} + C_I + (m^m - m^o) \left(E_n - T_o \right) \quad (26)$$

$$Min\ Z_{13} = (Z_1^p - Z_1^m) = \sum_i \sum_j C_{Dij} + \sum_i \sum_j (k_{ij}^p - k_{ij}^m) Y_{ij} + C_I + (m^p - m^m) \left(E_n - T_o \right) \quad (27)$$

$$Min\ Z_{21} = Z_2^m = \sum_i \sum_j k_{ij}^m Y_{ij} \quad (28)$$

$$Min\ Z_{22} = (Z_2^m - Z_2^o) = \sum_i \sum_j (k_{ij}^m - k_{ij}^o) Y_{ij} \quad (29)$$

$$Min\ Z_{23} = (Z_2^p - Z_2^m) = \sum_i \sum_j (k_{ij}^p - k_{ij}^m) Y_{ij} \quad (30)$$

Recalling equation (18) from the original MOLP model; $\tilde{T}$ is fuzzy and has trapezoidal distribution. It can be substituted by:

$$En \ \tilde{\leq}_R \ \tilde{T} \quad (31)$$

$$-En \ \tilde{\leq}_R \ -\tilde{T} \quad (32)$$

This work applies the signed distance method to convert $\tilde{T}$ into a crisp number. If the minimum acceptable membership degree, γ, is given, the auxiliary crisp inequality constraints can be presented as follows:

$$En \ \tilde{\leq}_R \ [\underline{T} + \underline{T} + \overline{T} + (\overline{T} + \overline{\beta}^{\gamma})]/4 \quad (33)$$

$$-En \ \tilde{\leq}_R \ -[(\underline{T} - \underline{\beta}^{\gamma}) + \underline{T} + \overline{T} + \overline{T}]/4 \quad (34)$$

Recalling equation (19) from the original MOLP model; $\tilde{k}_{ij}$, $\tilde{m}$, $\tilde{b}$ are fuzzy and have triangular distribution. This work applies the signed distance method to convert $\tilde{k}_{ij}$, $\tilde{m}$, $\tilde{b}$ into crisp number. If the minimum acceptable membership degree, γ, is given, the auxiliary crisp inequality constraints can be presented as follows:

$$\sum_i \sum_j C_{Dij} + \sum_i \left(\frac{k_{ij,\gamma}^o + 2k_{ij,\gamma}^m + k_{ij,\gamma}^p}{4} \right) Y_{ij} + C_I$$

$$+\left(\frac{m_\gamma^o+2m_\gamma^m+m_\gamma^p}{4}\right)\left(E_n-T_o\right)\leq\left(\frac{b_\gamma^o+2b_\gamma^m+b_\gamma^p}{4}\right) \quad (35)$$

The original MOLP model designed above is developed based on the fuzzy mathematical programming methods of Zimmermann (1976, 1978), Slowinski (1986), and signed distance method. The minimum operator presented by Bellman and Zadeh (1970) is used to aggregate fuzzy sets, and the original MOLP problem is then converted into an equivalent ordinary LP form.

Based on Bellman and Zadeh's concepts, fuzzy goals (*G*), fuzzy constraints (*C*), and fuzzy decisions (*D*), the fuzzy decision is defined as follows:

$$D = G \cap C \quad (36)$$

Next, this problem is characterized by the following membership function:

$$\mu_D(x)=\mu_G(x)\wedge\mu_C(x)=Min(\mu_G(x),\mu_C(x)) \quad (37)$$

Furthermore, the corresponding linear membership functions of the fuzzy objective functions of the auxiliary MOLP problem are defined by

$$\mu_{11}(Z_{11}(x))=\begin{cases}1, \text{ if } Z_{11}(x)<Z_{11}^{PIS}\\ \dfrac{Z_{11}^{NIS}-Z_{11}(x)}{Z_{11}^{NIS}-Z_{11}^{PIS}}, \text{ if } Z_{11}^{PIS}\leq Z_{11}(x)\leq Z_{11}^{NIS}\\ 0, \text{ if } Z_{11}(x)>Z_{11}^{NIS}\end{cases} \quad (38)$$

$$\mu_{12}(Z_{12}(x))=\begin{cases}1, \text{ if } Z_{12}(x)>Z_{12}^{PIS}\\ \dfrac{Z_{12}(x)-Z_{12}^{NIS}}{Z_{12}^{PIS}-Z_{12}^{NIS}}, \text{ if } Z_{12}^{NIS}\leq Z_{12}(x)\leq Z_{12}^{PIS}\\ 0, \text{ if } Z_{12}(x)<Z_{12}^{NIS}\end{cases} \quad (39)$$

The linear membership functions $\mu_{13}(Z_{13}(x))$ is similar to $\mu_{11}(Z_{11}(x))$. And The linear membership functions $\mu_{21}(Z_{21}(x))$, $\mu_{22}(Z_{22}(x))$ and $\mu_{23}(Z_{23}(x))$ are similar to $\mu_{11}(Z_{11}(x))$, $\mu_{12}(Z_{12}(x))$ and $\mu_{13}(Z_{13}(x))$.

Accordingly, the positive ideal solutions (PIS) and negative ideal solutions (NIS) of the six objective functions of the auxiliary MOLP problem can be specified as follows, respectively. And, a payoff table (Table 1) is constructed by using the solutions of single objective FMOLP model where Z_k is the original MOLP objective function *k*; Z_{kq}^f is the value of six new objective function *kq* at solution vector *f*, *k=1,* and *2; q=1,2,* and *3;f=1, 2 and 3*.

$$Z_{k1}^{PIS}=MinZ_k^m=Min_f(Z_{k1}^f),\ f=1,2,3. \quad (40a)$$

$$Z_{k1}^{NIS}=MaxZ_k^m=Max_f(Z_{k1}^f),\ f=1,2,3. \quad (40b)$$

$$Z_{k2}^{PIS}=Max(Z_k^m-Z_k^o)=Max_f(Z_{k1}^f),\ f=1,2,3. \quad (41a)$$

$$Z_{k2}^{NIS}=Min(Z_k^m-Z_k^o)=Min_f(Z_{k1}^f),\ f=1,2,3. \quad (41b)$$

$$Z_{k3}^{PIS}=Min(Z_k^p-Z_k^m)=Min_f(Z_{k1}^f),\ f=1,2,3. \quad (42a)$$

$$Z_{k3}^{NIS}=Max(Z_k^p-Z_k^m)=Max_f(Z_{k1}^f),\ f=1,2,3. \quad (42b)$$

Finally, the complete FMOLP model for solving PM decision problems can be formulated as follows:

Max β

s.t $\beta \leq \mu_{1q}(Z_{1q}(x))$ *q=1,2,3*

$\beta \leq \mu_{2q}(Z_{2q}(x))$ *q=1,2,3*

Equations (14)-(17), and (33)-(35)

$t_{ij}, Y_{ij}, E_i, E_j \geq 0\ \forall i, \forall j$

Where the auxiliary variable β is represent the overall degree of decision maker (DM) satisfaction with determined goal values.

After solving the single-objective LP problem to yield a compromise solution, the decision maker (DM) who is not satisfied with the initial solution can use modified El-Wahed and Lee's (2006) approach to change the model until a set of preferred satisfactory solution is found. In this paper, membership functions are determined for each objective function with considerable feedback (Table 1) in order to get the optimal solution which may lead to a preferred compromise solution corresponding to these aspiration levels.

(1) Let the optimal solution of objective functions of Z_{kq}, *k=1,* and *2; q=1* and *3* be Z_{kq}^*, *k=1,* and *2; q=1* and *3*. Compare each Z_{kq}^* with the existing Z_{kq}^{NIS} and apply the following rules to update the aspiration levels.

(1) If $Z_{kq}^* < Z_{kq}^{NIS}$, then replace Z_{kq}^{NIS} by Z_{kq}^*.

(2) If $Z_{kq}^* = Z_{kq}^{NIS}$, then keep these aspiration levels as they are.

(3) If $Z_{kq}^* = Z_{kq}^{PIS}$, then replace Z_{kq}^{PIS} by Z_{kq}^* and keep it/them until the solution procedure is terminated.

(2) Let the optimal solution of objective functions of Z_{kq}, *k=1,* and *2; q=2* be Z_{kq}^*, *k=1,* and *2; q=2*.

Compare each Z_{kq}^* with the existing Z_{kq}^{PIS} and apply the following rules to update the aspiration levels.

(1) If $Z_{kq}^{*} < Z_{kq}^{PIS}$, then replace Z_{kq}^{PIS} by Z_{kq}^{*}.

(2) If $Z_{kq}^{*} = Z_{kq}^{PIS}$, then keep these aspiration levels as they are.

(3) If $Z_{kq}^{*} = Z_{kq}^{NIS}$, then replace Z_{kq}^{NIS} by Z_{kq}^{*} and keep it/them until the solution procedure is terminated.

Furthermore, by considering these rules, the membership values and aspiration levels are updated to generate another optimal solution, and so on. The solution process terminates whenever one of the following criteria is satisfied:

(1) The decision maker (DM) accepts the modified solution and considers it the preferred compromise solution.
(2) The updated overall degree of decision maker (DM) satisfaction with determined goal value is lower than which the DM can accept.
(3) There is no significant improvement in the objective function values after additional modifications.
(4) The modification of the Z_{kq}^{PIS} or Z_{kq}^{NIS} leads to infeasible solution.

The solution procedure of the proposed interactive FMOLP approach for solving PM problems is as follows.

Step 1: Formulate the original MOLP model for solving project management (PM) decision problems according to equations (12) - (20).
Step 2: Model the fuzzy data using triangular and trapezoidal fuzzy numbers using equations (21) – (24).
Step 3: Develop the six new crisp objective functions of the auxiliary MOLP problem for the imprecise goal using Equations (25) – (30).
Step 4: Specify the inequality for the fuzzy constraints.
Step 5: Give the α-cut level, and then convert the imprecise constraints into crisp ones using signed distance method according to equations (33) – (35).
Step 6: Specify the linear membership functions for the six new objective functions, and then convert the auxiliary MOLP problem into an equivalent LP model using the minimum operator to aggregate fuzzy sets.
Step 7: Solve the ordinary LP model to deliver a set of compromise solutions. If the decision maker (DM) is dissatisfied with the initial solutions, the model must be modified until a set of preferred satisfactory solutions is obtained.

An example

Daya Technology Corporation was used as a case study demonstrating the practicality of the proposed methodology (Liang, 2008). Daya is the leading producer of precision machinery and transmission components in Taiwan. Currently, the deterministic CPM approach used by Daya suffers from the limitation owing to the fact that a decision maker (DM) does not have sufficient information related to the model inputs and related parameters. Alternatively, the proposed fuzzy multi-objective linear programming approach introduced by Daya can effectively handle vagueness and imprecision in the statement of the objectives and related parameters by using simplified triangular and trapezoidal distributions to model imprecise data.

The project management (PM) decision of Daya aims to simultaneously minimize total project cost and total crashing cost with considering control the project completion time between a suitable date in terms of direct costs, indirect costs, activity duration, and budget constraints. Table 2 lists the basic data of the real industrial case. Other relevant data are as follows: fixed indirect costs $12,000, saved daily variable indirect costs ($144, $150, $154), total budget ($40,000, $45,000, $51,000), and project completion time under normal conditions 125 days. The project start time ( ) is set to zero. The α-cut level for all imprecise numbers is specified as 0.5. The specified project completion time is set to (106, 112, 120, 123) days based on contractual information, resource allocation and economic considerations, and related factors. Figure 4 shows the activity on-arrow network. The critical path is 1 – 5 – 6 – 7 – 9 – 10 – 11.

The solution procedure using the proposed PLP approach for the Daya case is described as follows. First, formulate the original multi-objective FMOLP model for the PM decision problem according to Equations (12) to (20). Second, develop the six new objective functions of the auxiliary MOLP problem for the imprecise objective function (12) and (13) using Equations (25) to (30), and as follows:

$$Min\ Z_{11} = 24400 + (150Y_{12} + 180Y_{15} + 120Y_{24} + 130Y_{410} + 300Y_{56} + 150Y_{67} + 50Y_{79} + 125Y_{89} + 150Y_{910} + 100Y_{1011}) + 150\,E_{11} - 18750 \tag{43}$$

$$Min\ Z_{12} = (18Y_{12} + 16Y_{15} + 18Y_{24} + 18Y_{410} + 20Y_{56} + 14Y_{67} + 16Y_{79} + 14Y_{89} + 30Y_{910} + 20Y_{1011}) + 6\,E_{11} - 750. \tag{44}$$

$$Min\ Z_{13} = (14Y_{12} + 18Y_{15} + 8Y_{24} + 10Y_{410} + 24Y_{56} + 16Y_{67} + 8Y_{79} + 14Y_{89} + 10Y_{910} + 8Y_{1011}) + 4\,E_{11} - 500 \tag{45}$$

The Equations (28) to (30) are the same step.

Third, formulate the auxiliary crisp constraints using Equations (33) to (35) at γ = 0.5. The results are

$$E_{11} \tilde{\leq}_R [112+112+120+123]/4 = 116.75 \tag{46}$$

$$-E_{11} \tilde{\leq}_R -[106+112+120+120]/4 = -114.5 \tag{47}$$

$$24400 + (149.5Y_{12} + 180.25Y_{15} + 118.75Y_{24} + 129Y_{410} + 300.5Y_{56} + 150.25Y_{67} + 49Y_{79} + 125Y_{89} + 147.5Y_{910} + 98.5Y_{1011}) + 12000 + 149.5E_{11} - 18687.5 \leq 45250. \tag{48}$$

According to Equations (38) and (39), the corresponding linear membership functions of the six new objective functions can be defined. Additionally, specify the PIS and NIS of the imprecise/fuzzy objective functions with a payoff table (Table 3) in the auxiliary MOLP problem with Equations (40a) – (42b). Consequently, the equivalent ordinary LP model for solving the PM decision problem for the Daya case can be formulated using the minimum operator to aggregate fuzzy sets. Run this ordinary LP model by using Lingo computer software. The initial solutions are (37040.10, 37359.73, 37519.86), (1828.10, 2197.23, 2390.36), and the completion time is 116.75 days. Besides, if the DM is dissatisfied with the initial solutions, he may try to modify the results by adjusting the related parameters (PIS, NIS) until a set of preferred satisfactory solution is found. And, the decision maker (DM) hopes the updated overall

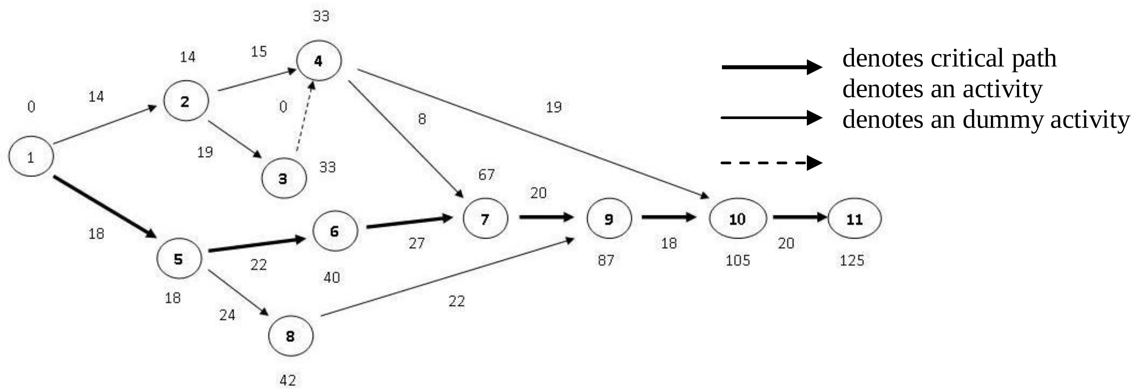

Figure 4. The project network of the Daya case (Liang, 2008).

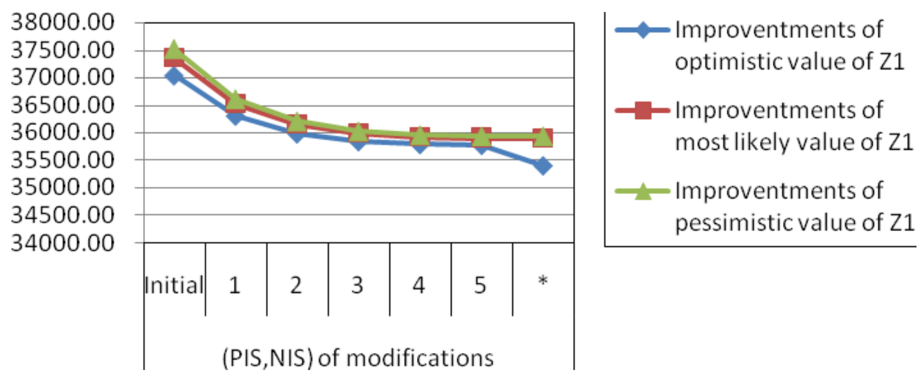

Figure 5. The triangular distribution of the total project costs.

degree of decision maker (DM) satisfaction with determined goal value which is not lower than 0.8. Hence, the improved solutions are (35779.52, 35901.90, 35939.52), (567.52, 739.40, 810.02), and the improved completion time is the same.

From the graphical representation in Figure 5, it is observed that the most optimistic value (Z_1^o), the most possible value (Z_1^m), and the most pessimistic value (Z_1^p) of total project cost gradually decrease and converge toward their ideal solutions Z_1^* with the modifications of PIS and NIS. And, the graphic variation of total crashing cost (Z_2) is the same. In summary, Table 4 lists initial and improved PM plans for the Daya case with the proposed FMOLP approach based on current information.

RESULTS OF COMPARISON

Several significant management implications regarding the practical application of the proposed approach are as follows. From Table 5 applying LP-1 to minimize the total project cost, the optimal value of total project cost and crashing were $35,900 and $1,075. Applying LP-2 to minimize the total crashing cost, the optimal value of total project cost and crashing were $39,322.5 and $0. Alternatively, using the PLP model developed by Liang with linear membership function to simultaneously minimize total project cost and completion time obtains total project cost, Z_1 = (35868.00, 36012.50, 36057.50) and total crashing cost, Z_2 = (656.00, 850.00, 928.00), and the overall degree of decision maker (DM) satisfaction is 0.8325. It reveals that the proposed FMOLP solutions are a set of more efficient solutions, by comparison with the optimal objective value obtained by the ordinary single-objective LP model and Liang (2008). The most important advantage of the proposed FMOLP approach is if the DM is dissatisfied with the initial solutions, the model can be modified during the solution procedure, until a set of preferred satisfactory solutions is obtained. Figure 6 shows the change in triangular possibility distributions of total project costs (Z_1) for the Daya case. As indicated in Figure 6, improved solutions are preferred to initial solutions. Besides, Table 6 reveals that conflicts exists among results of sensitivity analysis of minimizing total project

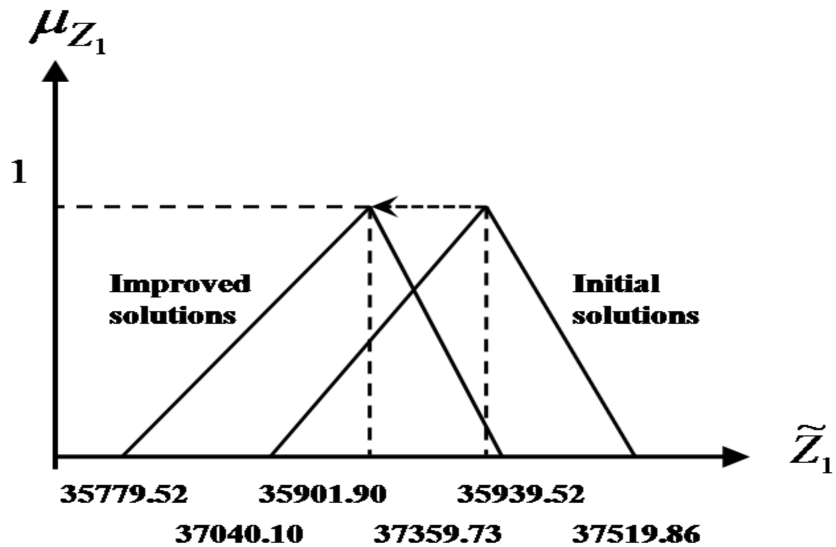

Figure 6. The triangular distribution of the total project costs.

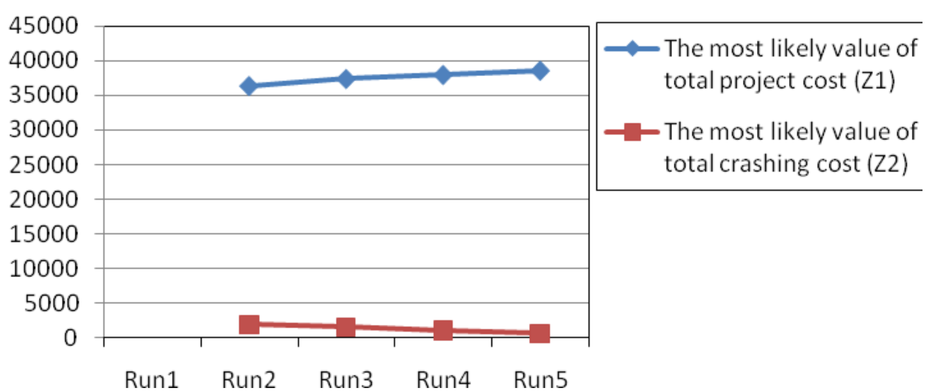

Figure 7. The most likely value of objectives of analyzing sensitivity for varying the project completion time.

cost and minimizing project completion time for varying project completion time. Accordingly, while project completion time increases, the project total cost increases because total indirect cost increase significantly. Figure 7 plots the most likely value of total project cost and total crashing cost versus the project completion time. In practice, the crashing cost such as overtime, personnel and equipment will decrease when the project completion time increases.

Table 7 compares the fuzzy multi-objective linear programming (FMOLP) model presented in this work to the FLP, FGP, MFOLP, and MFGP models. To summarize, several significant characteristics distinguish the proposed model from the other models. Firstly, the proposed model meets the requirements for actual application because it simultaneously minimizes total project cost and total crashing cost. Secondly, the proposed approach yields a preferred efficient solution and the DM's overall levels of satisfaction. If the solution is L=1, then all of the fuzzy objective and constraints are fully satisfied; if 0<L<1, then all of the fuzzy objective and constraints are satisfied at the given L; if L=0, then none of the fuzzy objective and constraints is satisfied. Thirdly, the proposed model sets up a systematic framework that facilitates the decision-making process, enabling the DM interactively to modify the membership grades of the objectives until a set of preferred satisfactory solutions is obtained. Finally, the proposed model yields more wide-ranging decision information than other models. It provides more information on alternative crashing strategies in terms of direct cost, indirect cost, specified project completion time and allocated total budget.

Table 1. The corresponding PIS and NIS for the fuzzy objective functions.

Objective functions	Min $Z_{11} = Z_1^m$	Max $Z_{12} = (Z_1^m - Z_1^o)$	Min $Z_{13} = (Z_1^p - Z_1^m)$	Min $Z_{21} = Z_2^m$	Max $Z_{22} = (Z_2^m - Z_2^o)$	Min $Z_{23} = (Z_2^p - Z_2^m)$	(PIS, NIS)
Z_{11}	Z_{11}^1	Z_{11}^2	Z_{11}^3	---	---	---	$(Z_{11}^{PIS}, Z_{11}^{NIS})$
Z_{12}	Z_{12}^1	Z_{12}^2	Z_{12}^3	---	---	---	$(Z_{12}^{PIS}, Z_{12}^{NIS})$
Z_{13}	Z_{13}^1	Z_{13}^2	Z_{13}^3	---	---	---	$(Z_{13}^{PIS}, Z_{13}^{NIS})$
Z_{21}	---	---	---	Z_{21}^1	Z_{21}^2	Z_{21}^3	$(Z_{21}^{PIS}, Z_{21}^{NIS})$
Z_{22}	---	---	---	Z_{22}^1	Z_{22}^2	Z_{22}^3	$(Z_{22}^{PIS}, Z_{22}^{NIS})$
Z_{23}	---	---	---	Z_{23}^1	Z_{23}^2	Z_{23}^3	$(Z_{23}^{PIS}, Z_{23}^{NIS})$

Data source: This research reorganization.

Table 2. Summarized data n the Daya case (in US dollar).

(i, j)	D_{ij} (days)	d_{ij} (days)	C_{Dij} ($)	C_{dij} ($)	k_{ij} ($/days)
1-2	14	10	1000	1600	(132, 150, 164)
1-5	18	15	4000	4540	(164, 180, 198)
2-3	19	19	1200	1200	---
2-4	15	13	200	440	(112, 120, 128)
4-7	8	8	600	600	---
4-10	19	16	2100	2490	(112, 130, 140)
5-6	22	20	4000	4600	(280, 300, 324)
5-8	24	24	1200	1200	---
6-7	27	24	5000	5450	(136, 150, 166)
7-9	20	16	2000	2200	(34, 50, 58)
8-9	22	18	1400	1900	(111, 125, 139)
9-10	18	18	700	1150	(120, 150, 160)
10-11	20	18	1000	1200	(80, 100, 108)

Data source: Liang (2008).

Table 3. The corresponding PIS and NIS for the fuzzy objective functions.

Objective functions	Min Z_{11}	Max Z_{12}	Min Z_{13}	Min Z_{21}	Max Z_{22}	Min Z_{23}	(PIS, NIS)
Z_{11}	35900*	39332.5	35900	---	---	---	(35900, 39332.5)
Z_{12}	86	492.5*	122	---	---	---	(492.5, 86)
Z_{13}	51	353	37.5*	---	---	---	(37.5, 353)
Z_{21}	---	---	---	737.5*	4170	737.5	(737.5, 4170)
Z_{22}	---	---	---	135.5	542*	171.5	(542, 135.5)
Z_{23}	---	---	---	84	386	70.5*	(70.5, 386)

Note: '*' denotes the optimal value with the single goal LP model.

$Z_1^* = (Z_{11}^* - Z_{12}^*, Z_{11}^*, Z_{11}^* + Z_{13}^*)$; $Z_2^* = (Z_{21}^* - Z_{22}^*, Z_{21}^*, Z_{21}^* + Z_{23}^*)$

Table 4. FMOLP solutions for the Daya case.

Initial solutions		
Y_{ij} **(days)**	t_{ij} **(days)**	E_i **(days)**
Y_{12}=4, Y_{15}=0, Y_{23}=0, Y_{24}=2, Y_{47}=0, Y_{410}=3, Y_{56}=0, Y_{58}=0, Y_{67}=0, Y_{79}=4, Y_{89}=0.94, Y_{910}=3, Y_{1011}=2	t_{12}=10, t_{15}=18, t_{23}=19, t_{24}=13, t_{47}=8, t_{410}=16, t_{56}=22, t_{58}=24, t_{67}=27, t_{79}=16, t_{89}=21.06, t_{910}=15, t_{1011}=18	E_1=0, E_2=10, E_3=59, E_4=59, E_5=18, E_6=40, E_7=67, E_8=61.94, E_9=83, E_{10}=98, E_{11}=116.75
(PIS, NIS)	Objective values	
Z_{11}=(35900, 39332.50), Z_{12}=(492.50, 86), Z_{13}=(37.50, 353), Z_{21}=(737.50, 4170), Z_{22}=(542, 135.50), Z_{23}=(70.50, 386)	$\tilde{Z}_1$ = (37040.10, 37359.73, 37519.86) $\tilde{Z}_2$ = (1828.10, 2197.23 , 2390.36) Completion time = 116.75 days.	β = 0.5747

Improved solutions		
Y_{ij} **(days)**	t_{ij} **(days)**	E_i **(days)**
Y_{12}=0, Y_{15}=0, Y_{23}=0, Y_{24}=0, Y_{47}=0, Y_{410}=0, Y_{56}=0, Y_{58}=0, Y_{67}=0, Y_{79}=4, Y_{89}=0, Y_{910}=2.27, Y_{1011}=1.99	t_{12}=14, t_{15}=18, t_{23}=19, t_{24}=15, t_{47}=8, t_{410}=19, t_{56}=22, t_{58}=24, t_{67}=27, t_{79}=16, t_{89}=22, t_{910}=15.73, t_{1011}=18.01	E_1=0, E_2=14, E_3=59, E_4=59, E_5=18, E_6=40, E_7=67, E_8=61, E_9=83, E_{10}=98.73, E_{11}=116.75
(PIS, NIS)	Objective values	
Z_{11}=(35900,35920.49), Z_{12}=(126.10, 86.00), Z_{13}=(37.50, 38.78), Z_{21}=(737.5, 758.00) Z_{22}=(175.60, 135.50), Z_{23}=(70.50, 71.78)	$\tilde{Z}_1$ = (35779.52, 35901.90, 35939.52) $\tilde{Z}_2$ = (567.52, 739.40, 810.02) Completion time = 116.75 days.	β = 0.9072

Note: $\tilde{Z}_1 = (Z_{11} - Z_{12}, Z_{11}, Z_{11} + Z_{13})$; $\tilde{Z}_2 = (Z_{21} - Z_{22}, Z_{21}, Z_{21} + Z_{23})$.

Table 5. Comparison of solutions.

Item	**LP-1**	**LP-2**	**Liang (2008)**	**The proposed FMOLP approach**
Objective function	Min Z_1	Min Z_2	Max β	Max β
β	100%	100%	83.25%	90.72%
$\tilde{Z}_1$ *($)*	35900.00	35900.00	(35868.00, 36012.50, 36057.50)	(35779.52, 35901.90, 35939.52)
$\tilde{Z}_2$ *($)*	1075.00	737.50	(656.00, 850.00, 928.00)	(567.52, 739.40, 810.02)
Completion time (days)	114.50	116.75	116.75	116.75

CONCLUSIONS

This work presents a fuzzy multi-objective linear programming (FMOLP) approach for solving project management (PM) decision problems in a fuzzy environment. It provides a systematic framework that facilitates decision making, enabling a decision maker (DM) to interactively modify the imprecise data and parameters until a set of satisfactory compromise solution is obtained. Presenting a fuzzy multi-objective linear

Table 6. Results of sensitivity analysis for varying the project completion time.

Item	Run 1	Run 2	Run 3	Run 4	Run 5
E_n (days)	107	114	121	126	132
β		0.5459433	0.4824047	0.3258309	0.1583232
$\tilde{Z}_1$ ($)	Infeasible	(36378.12, 37046.05, 37206.21)	(37369.05, 37676.65, 37821.67)	(37928.14, 38214.09, 38341.17)	(38529.20, 38789.06, 38909.66)
$\tilde{Z}_2$ ($)	Infeasible	(1922.12, 2296.05, 2500.21)	(1545.05, 1876.65, 2037.67)	(1096.13, 1364.09, 1497.17)	(689.20, 889.06, 969.66)

Table 7. The comparisons of five PM decision models.

Factor	FLP (Wang and Fu, 1998)	FGP (Arikan and Gungor, 2001)	MFOLP (Liang, and Wang, 2003)	MFGP (Wang and Liang, 2004)	The proposed FMOLP approach
Objective function	Single	Multiple	Multiple	Multiple	Multiple
Objective property	Fuzzy	Fuzzy	Fuzzy	Fuzzy	Fuzzy
Constraint property	Deterministic / Fuzzy	Deterministic	Deterministic	Deterministic	Deterministic / Fuzzy
DM's overall level of satisfaction	Not considered	Considered	Considered	Considered	Considered
Main consideration	Cost	Time / cost	Time / cost / resource	Time / cost	Time / cost
Decision parameter	Deterministic / Fuzzy	Deterministic	Deterministic	Deterministic	Fuzzy
Direct and crashing cost	Not considered	Considered	Considered	Considered	Considered
Indirect cost	Not considered	Not considered	Considered	Considered	Considered
Specified completion time	Not considered	Not considered	Not considered	Not considered	Considered

Data source: This research reorganization.

programming methodology with considering completion time in a suitable range for multi-objective project management (PM) decisions is the main contribution. It is critical that the objective values which are satisfied should often be imprecise as the cost coefficients and parameters are imprecise and such imprecision always exists in real-world PM decisions.

Computational methodology developed here can easily be extended to any other situations and can handle the realistic PM decisions. Future research may apply the time value of money to project total cost, direct cost, indirect cost, crashing cost and allocated budget, etc. Finally, this case only involves about hundreds of decision variables and parameters in model test, the decision maker can formulate this proposed approach in solving large scale project management (PM) problems of industrial cases.

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