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Full Length Research Paper

Optimal order quantity under advance sales and permissible delays in payments

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In order to attract more customers, it is a common practice for retailers to provide advance sales, for example, Maxim's Bakery in Hong Kong, Amazon.com, Movies Unlimited and Toys R Us. Similarly, suppliers often allow their retailers a permissible delay in payment in order to increase sales. Advance sales and trade credit policies provide numerous benefits for companies, including gaining additional discriminative customers and increased profit due to interest earned from payments received from committed customers prior to the start of the regular selling period. This article establishes an inventory model for retailers who simultaneously receive a permissible delay in payments from suppliers while offering advance sales to customers. We first present the model and then provide a simple method of obtaining the optimal order quantity and advance sales discount rate which achieves the maximum total profit per unit of time for the retailer. Finally, several numerical examples are used to illustrate the procedure.

Key words: Inventory, advance sales, trade credit, advance sales discount rate.

INTRODUCTION

Permissible delay is a common phenomenon in retailing, where a supplier permits the retailer a fixed time period to settle the total amount owed. This provides an advantage to the retailer as they can earn interest on the accumulated revenue received during the period of permissible delay. At the same time, permissible delay can also confer benefits to the supplier since the policy may attract new customers who consider it to be a type of price reduction.

Permissible delay in payments has been widely discussed in the literature. Chang et al. (2003) established an economic order quantity (EOQ) model for deteriorating items, in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Ouyang et al. (2006) developed a general EOQ model with trade credit for a retailer to determine the optimal shortage interval and replenishment cycle.

Goyal et al. (2007) introduced a new concept where the supplier charges the retailer progressive interest rates if the retailer exceeds the period of permissible delay, and established necessary and sufficient conditions for the unique optimal replenishment interval. Ho et al. (2008) proposed an integrated inventory model with retail price sensitive demand and trade credit financing. Chang et al. (2009b) formulated an integrated vendor–buyer inventory model with retail price sensitive demand, where the credit terms are linked to the order quantity. Chen and Kang (2010) developed integrated models with permissible delay in payments for determining the optimal replenishment time interval and replenishment frequency. There are also many relevant articles related to trade credit, including Goyal (1985), Dave (1985), Mandal and Phaujdar (1989), Aggarwal and Jaggi (1995), Hwang and Shinn (1997), Jamal et al. (1997), Liao et al. (2000), Sarker et al. (2000), Teng (2002), Huang (2003), Chang and Teng (2004), Chung and Liao (2004), Ouyang et al. (2005), Teng et al. (2005) and Chang et al. (2009a) and

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the research that they cite. However, none of the models presented in the above literature incorporates advance sales.

Along with environmental transformation and market competitiveness enhancement, advance sales have gradually become one of the newest sales models. Advance sales policies are widely used by retailers today, including Maxim's Bakery in Hong Kong, Amazon.com, Eslitebooks.com, Movies Unlimited, Toys R Us and Electronics Boutique. Customers who accept advance sales must prepay the entire discounted purchase amount prior to the regular sale season. Alternatively, customers can purchase the product at the regular price during the regular sale season.

Models have since been developed which incorporate advance sales policies. You (2006) addressed a service inventory control problem in which a firm sells products through an advance booking system, with the aim of optimizing product price to maximize the total expected profit.

You (2007) developed an advance sales system where a firm sells perishable inventory using a reservation system during the sales season over a limited planning time interval. You and Wu (2007) investigated the problem of ordering and pricing over a finite time planning horizon for an inventory system with advance sales and spot sales. They sought to develop a solution procedure to determine the optimal advance sales price, spot sales price, order size and replenishment frequency. Tsao (2009) considered retailer's promotion and replenishment policies with an advance sales discount under the supplier's and retailer's trade credits and presented an algorithm to simultaneously determine the optimal promotion effort and replenishment cycle time.

In this paper, we develop an inventory model where the supplier offers trade credit and the retailer provides advance sales. It is conceivable that customers who are unwilling to buy the product at the regular price may choose to do so with the price discount. Thus, by providing advance sales, the retailer is likely to gain additional demand during the advance sales period. Moreover, incorporating advance sales not only reduces financial risks, it also increases interest earned from payments received from committed orders prior to the regular sale season. Our aim is to determine the optimal advance sales discount rate and the optimal length of the regular selling period in order to maximize the total profit per unit of time.

Notation

The mathematical model in this paper is developed on the following notation

p : Unit selling price
c : Unit purchase cost
h : Unit holding cost per unit of time excluding interest charges
s : Ordering cost per order
I_c : Interest charges per $ investment in stocks per unit of time
I_e : Interest earned per $ per unit of time
M : Permissible delay in settling account
t_p : Advance selling period
δ : Advance sales discount rate (all products are δ % off during the advance sale period) with $\delta \le 1 - c/p$, a decision variable
δ^* : Optimal discount rate during the advance sale period
T : Regular sale period, a decision variable
T^* : Optimal regular sale period
$Z(T,\delta)$: total profit per unit time, which is a function of T and δ
Z^* : maximum total profit per unit of time, i.e., $Z^* = Z(T^*,\delta^*)$

Assumptions

The mathematical model in this paper is developed on the following assumptions

1. The replenishment occurs instantaneously at an infinite rate.
2. Shortages are not allowed.
3. Customers who accept the advance sales offer must pre-pay for the committed orders prior to the start of the regular sale period.
4. No order cancellation or refund is permitted.
5. The demand rate, D, depends on the selling price, p, and the relationship between demand and price is linear and given by

$$D(p) = a - b\,p$$

where a and b are positive constants. We also assume that the demand rate is always positive. That is, $p < a/b$.

METHODOLOGY

The proposed model incorporates both advance selling and permissible delay policies. The supplier permits the retailer a fixed time period to settle the total account, while the retailer allows advance sales that induces customers to commit to their orders at a discounted price prior to the beginning of the regular sale season. Figure 1 displays the behavior of reservation level and inventory level over time. In the advance sale period $(0, t_p]$, all products are δ % off. The reservation level changes at a rate of $D(p - \delta p)$.

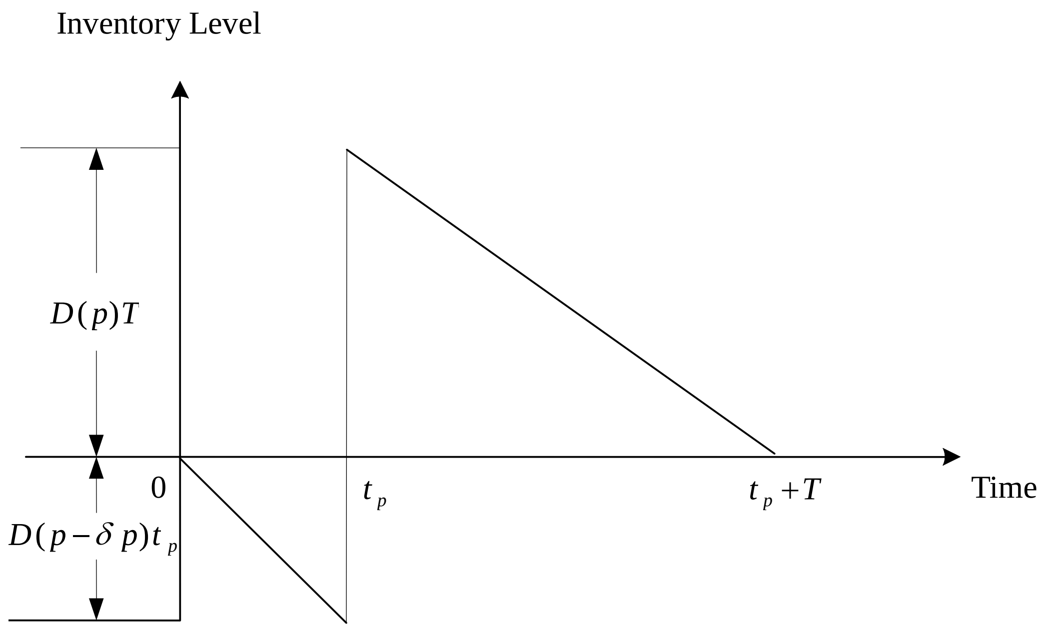

Figure 1. Advance sales offered by the retailer.

Moreover, during the period from time t_p to $t_p + T$, the inventory level changes at a rate of *D(p).*

The objective here is to maximize the retailer's total profit per unit of time. The total profit per unit of time of the retailer consists of the following elements:

1. sales revenue per unit of time

$= \{(1-\delta)\ pD(p-\delta p)\ t_p + pD(p)T\ \} / (t_p+T)$

$= \{(1-\delta)\ p[a-b(1-\delta)p]\ t_p + p(a-bp)T\ \} / (t_p+T)$,

2. Cost of placing an order per unit of time = $s / (t_p+T)$,

3. cost of purchasing per unit of time

$= \{cD(p-\delta p)\ t_p + cD(p)T\} / (t_p+T)$

$= \{c\,[a-b(1-\delta)p]\ t_p + c\,(a-bp)T\} / (t_p+T)$,

4. cost of carrying inventory (excluding interest payable) per unit of time

$= hD(p)\,T^2 / [2\,(t_p+T)]$

$= h\,(a-bp)\,T^2 / [2\,(t_p+T)]$,

5. Interest payable per unit of time for the items in stock; and

6. Interest earned per unit of time.

Regarding interest payable and earned (i.e., costs of (e) and (f)), we have two possible cases based on the values of T and *M*, namely, (i) $T \le M$ and (ii) $T \ge M$. These two cases are depicted in Figure 2.

Case 1: $T \le M$

In this case, the permissible payment time expires at or after the time at which the inventory is depleted completely. Thus, the retailer pays no interest for items in inventory. However, the retailer utilizes the sales revenue received during both the advance sale period and the permissible period to earn interest. Therefore, the interest earned per unit of time is:

$$pI_e\{D(p-\delta p)t_p^{\,2}/2 + D(p-\delta p)t_pM + D(p)T^2/2$$
$$+(M-T)D(p)T\} / (t_p+T)$$
=
$$pI_e\{[a-b(1-\delta)p]t_p^{\,2}/2 + [a-b(1-\delta)p]t_pM + (a-bp)T^2/2$$
$$+(M-T)(a-bp)T\} / (t_p+T)$$

Case 2: $T \ge M$

The retailer earns interest on sales revenue received during the advance sale period and the permissible period. Thus, the interest earned per unit of time is:

$$pI_e\{D(p-\delta p)\,t_p^{\,2}/2 + D(p-\delta p)\,t_pM + D(p)M^2/2\} / (t_p+T)$$
$$= pI_e\{[a-b(1-\delta)p]\,t_p^{\,2}/2 + [a-b(1-\delta)p]\,t_pM + (a-bp)M^2/2\} / (t_p+T)$$
.

On the other hand, after paying the total purchase amount to the supplier, the retailer still has some inventory on hand. Hence, for the items in inventory, the retailer faces a capital opportunity cost. The opportunity cost per unit of time is:

$$cI_cD(p)\ (T-M)^2 / [2\,(t_p+T)] =$$
$$cI_c(a-bp)\ (T-M)^2 / [2\,(t_p+T)].$$

Therefore, the total profit per unit of time of the retailer is:

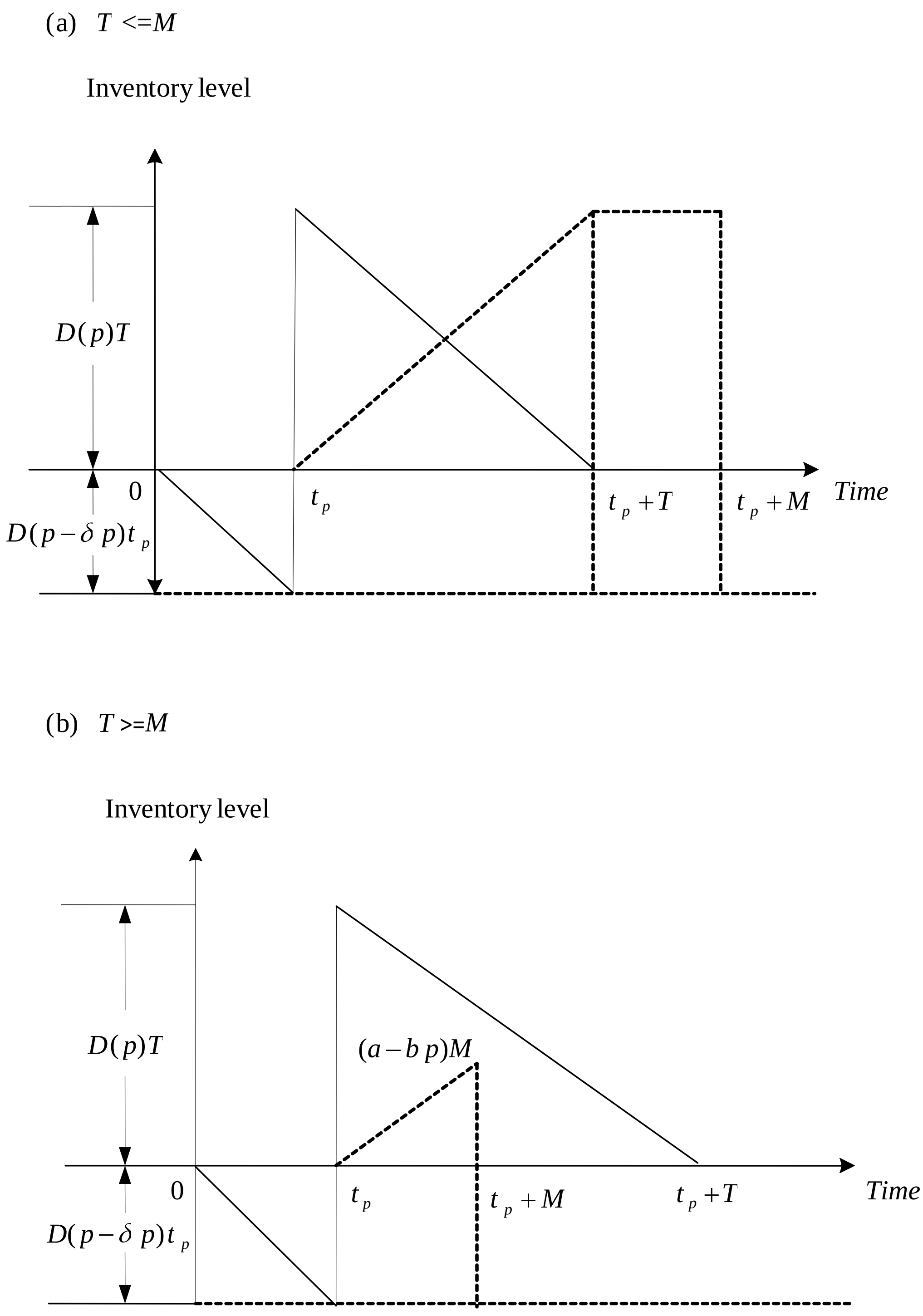

Figure 2. Graphical representation of inventory system.

$$Z(T,\delta)=\begin{cases} Z_1(T,\delta), & if\ T\le M, \\ Z_2(T,\delta), & if\ T\ge M, \end{cases} \tag{1}$$

where

$$\begin{aligned} Z_1(T,\delta) &= [a-b(1-\delta)p]\,[(1-\delta)p-c+pI_eM]-\{[\,b(p-c)-a \\ &+b\,(1-\delta)\,p]\,\delta\,pT+s+h\,(a-b\,p)\,T^2/2 \\ &-pI_e[a-b(1-\delta)p]\,(t_p{}^2-T^2)\big/2 \\ &+\delta b\,p^2 I_e(2M-T)T\big/2\}\big/(t_p+T). \end{aligned} \tag{2}$$

and

$$Z_2(T,\delta)$$
$$=[a-b(1-\delta)p]\,[(1-\delta)p-c+pI_eM]-\{[\,b(p-c)-a$$
$$+b\,(1-\delta)\,p\,]\,\delta\,pT+s+h\,(a-b\,p)\,T^2/2$$
$$+cI_c(a-bp)\,(T-M)^2\big/2-pI_e[a-b(1-\delta)p]\,(t_p{}^2-2MT)\big/2$$
$$-\,pI_e(a-b\,p)M^2\big/2\}\big/(t_p+T)\,. \qquad (3)$$

Note that $Z_1(M,\delta)=Z_2(M,\delta)$. Hence, for fixed δ, $Z(T,\delta)$ is continuous at point $T=M$.

RESULTS

Here, we present the solution procedure and determine the optimal solution to the two cases in discussed earlier. Our aim is to determine T^* and δ^* which maximize the total profit per unit of time $Z(T,\delta)$. Firstly, for fixed T, we take the first-order partial derivative of $Z(T,\delta)$ with respect to δ and derive

$$\frac{\partial\, Z\,(T,\delta)}{\partial\;\delta}=\frac{\partial\, Z_1(T,\delta)}{\partial\;\delta}=\frac{\partial\, Z_2(T,\delta)}{\partial\;\delta}$$

$$=\frac{p\,t_p}{t_p+T}\left[-2b\,p\delta-a+2b\,p-b\,c+\frac{b\,I_e p\,(t_p+2M)}{2}\right]. \quad (4)$$

Further, we let

$$f(\delta)=-2b\,p\delta-a+2b\,p-b\,c+\frac{b\,I_e p\,(t_p+2M)}{2}\,, \quad (5)$$

and find that

$$\frac{d\,f\,(\delta)}{d\,\delta}=-2\,b\,p<0\,. \qquad (6)$$

Therefore, $f(\delta)$ is a strictly decreasing function for $\delta\in[0,1-c/p]$. Moreover, we have

$$f(0)=-(a-bc)+\frac{b\,I_e p\,(t_p+2M)}{2}+2b(p-c)c\,, \qquad (7)$$

and

$$f\,(1-c/p)=-(a-bc)+\frac{b\,I_e p\,(t_p+2M)}{2}\,. \qquad (8)$$

For convenience, we let $A\equiv(a-bc)-b\,I_e p\,(t_p+2M)/2$.

Equations (7) and (8) become $f(0)=-A+2b(p-c)c$ and $f\,(1-c/p)=-A$, respectively. We then derive the following result:

Lemma 1: For any given *T*,

(a) if $A\ge 2b(p-c)c$, then $\delta^*=0$;

(b) if $A\le 0$, then $\delta^*=1-c/p$;

(c) If $0<A<2b(p-c)c$, then $\delta^*=\,[b\,p\,(4+2I_eM+I_e\,t_p)-2bc-2a]\big/(4b\,p)$.

The proof is given in Appendix A.

Note that $\delta^*=0$ implies that customers are willing to pay for their orders at the regular price prior to the beginning of the regular sale season. In addition, $\delta^*=1-c/p$ $((1-\delta^*)\,p=c)$ indicates that the retailer gains no profit during the advance sale period since the optimal unit selling price for the advance sale period, $(1-\delta^*)\,p$, is the same as the unit purchase price, c. In reality, it is unlikely for either of these two cases to occur. Hence, we focus on the case in which the optimal discount rate is $\delta^*=\,[b\,p\,(4+2I_eM+I_e\,t_p)-2bc-2a]\big/(4b\,p)$ (that is, $\delta^*>0$ and $(1-\delta^*)\,p>c$), which means that customers commit to their orders at a discounted price during the advance sale period. That is, $0<A<2b(p-c)c$.

Henceforth, we assume the condition $0<A<2b(p-c)c$ holds throughout the rest of this study. For fixed $\delta=\delta^*$, in order to find the optimal selling period T^*, we first take the first-order partial derivative of $Z_1(T,\delta^*)$ in Equation (2) with respect to T and obtain

$$\frac{d\,Z_1(T,\delta^*)}{d\,T}=\frac{1}{(t_p+T)^2}\Bigg\{\;s+[\,a-b(1-\delta^*)\,p-b(p-c)\,]\delta^*\,pt_p$$
$$-\frac{hT}{2}(a-b\,p)\;(2t_p+T)-\frac{p\,I_e}{2}(a-b\,p)(t_p+T)^2$$
$$-\frac{\delta^*\,b\,p^2\,I_e\,t_p}{2}(t_p+2M)\Bigg\} \qquad (9)$$

We let

$$G_1(T)=\;s+[\,a-b(1-\delta^*)\,p-b(p-c)\,]\delta^*\,pt_p-\frac{h}{2}(a-b\,p)\;(2t_p+T)T$$
$$-\frac{p\,I_e}{2}(a-b\,p)(t_p+T)^2-\frac{\delta^*\,b\,p^2\,I_e}{2}(t_p+2M)\;t_p\,, \quad (10)$$

and find that,

$$\frac{dG_1(T)}{dT} = -(h + pI_e)(a - bp)(t_p + T) < 0. \quad (11)$$

Hence, $G_1(T)$ is a strictly decreasing function for $T \in [0, M]$. Moreover, we obtain

$$G_1(0) = s - \frac{pI_e}{2}(a - bp)t_p^{\,2} - \frac{\delta^* b p^2 I_e}{2}(t_p + 2M)\, t_p + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p, \quad (12)$$

and

$$G_1(M) = s - \frac{h}{2}(a - bp)\,(2t_p + M)M - \frac{pI_e}{2}(a - bp)(t_p + M)^2 - \frac{\delta^* b p^2 I_e}{2}(t_p + 2M)\, t_p + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p. \quad (13)$$

For convenience, we let

$$\Delta_1 \equiv \frac{pI_e}{2}(a - bp)t_p^{\,2} + \frac{\delta^* b p^2 I_e}{2}(t_p + 2M)\, t_p - [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p, \quad (14)$$

and

$$\Delta_2 \equiv \frac{h}{2}(a - bp)\,(2t_p + M)M + \frac{pI_e}{2}(a - bp)(t_p + M)^2 + \frac{\delta^* b p^2 I_e}{2}(t_p + 2M)\, t_p - [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p \quad (15)$$

It is obvious that $\Delta_1 < \Delta_2$. Let T_1 denote the optimal value of T which maximizes $Z_1(T, \delta^*)$. We then derive the following result:

Lemma 2:

(a) If $s \le \Delta_1$, then $T_1 = 0$.

(b) If $s \ge \Delta_2$, then $T_1 = M$.

(c) If $\Delta_1 < s < \Delta_2$, then

$$T_1 = -t_p + \left\{ t_p^{\,2} + \frac{2s + 2b\delta^* p(\delta^* p - p + c)\, t_p - b\delta^* p^2 I_e (t_p + 2M) t_p}{(a - bp)(h + pI_e)} + \frac{2\delta^* p t_p - pI_e t_p^{\,2}}{h + pI_e} \right\}^{\frac{1}{2}}.$$

The proof is given in Appendix B

Subsequently, for fixed $\delta = \delta^*$, we take the first-order partial derivative of $Z_2(T, \delta^*)$ in Equation (3) with respect to T and obtain

$$\frac{d\, Z_2(T, \delta^*)}{d\, T} = \frac{1}{(t_p + T)^2} \Big\{ s + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p - \frac{hT}{2}(a - bp)\,(2t_p + T) - \frac{cI_c}{2}(a - bp)(T - M)(T + M + 2t_p) - \frac{pI_e}{2}[(a - bp)(t_p + M)^2 + \delta^* b p (t_p^{\,2} + 2t_p M)] \Big\} \quad (16)$$

We let,

$$G_2(T) = s + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p - \frac{hT}{2}(a - bp)\,(2t_p + T) - \frac{cI_c}{2}(a - bp)(T - M)(T + M + 2t_p) - \frac{pI_e}{2}[(a - bp)(t_p + M)^2 + \delta^* b p (t_p^{\,2} + 2t_p M)], \quad (17)$$

and find that,

$$\frac{dG_2(T)}{dT} = -(h + cI_c)(a - bp)(t_p + T) < 0. \quad (18)$$

Hence, $G_2(T)$ is a strictly decreasing function for $T \in [M, \infty)$. We obtain

$$\lim_{T\to\infty} G_2(T) = s + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p - \frac{h}{2}(a - bp)\lim_{T\to\infty}(2t_p T + T^2) - \frac{cI_c}{2}(a - bp)\lim_{T\to\infty}(T - M)(T + M + 2t_p) - \frac{pI_e}{2}[(a - bp)(t_p + M)^2 + \delta^* b p (t_p^{\,2} + 2t_p M)] = -\infty, \quad (19)$$

and

$$G_2(M) = s + [a - b(1-\delta^*)p - b(p-c)]\delta^* p t_p - \frac{h}{2}(a - bp)\,(2t_p + M)M - \frac{pI_e}{2}(a - bp)(t_p + M)^2 - \frac{\delta^* b p^2 I_e}{2}(t_p + 2M)\, t_p = G_1(M) = s - \Delta_2. \quad (20)$$

where Δ_2 is defined as above. Let T_2 denote the optimal value of T which maximizes $Z_2(T,\delta^*)$. We have the following result:

Lemma 3:

(a) If $s \le \Delta_2$, then $T_2 = M$.

(b) If $s > \Delta_2$, then

$$T_2 = -t_p + \left\{ t_p^{\,2} + \frac{c I_c (M+2t_p)M + 2\delta^* p t_p - p I_e (t_p + M)^2}{h + c I_c} + \frac{2s + 2\delta^* b p(\delta^* p - p + c)\, t_p - \delta^* b p^2 I_e (t_p + 2M) t_p}{(a - b p)(h + c I_c)} \right\}^{\frac{1}{2}}$$

.

The proof is given in Appendix C

Combining Lemmas 2 and 3, we obtain the following main result:

Theorem

For $0 < A < 2b(p-c)c$ (that is, $\delta^* = [b p\,(4 + 2I_e M + I_e t_p) - 2bc - 2a]/(4bp)$), we have:

(a) If $s \le \Delta_1$, then $T^* = 0$ and $Z^* = Z_1(0,\delta^*)$.

(b) If $\Delta_1 < s < \Delta_2$, then $T^* = T_1$ and $Z^* = Z_1(T_1,\delta^*)$, where T_1 is the same as Lemma 2

(c) If $s = \Delta_2$, then $T^* = M$ and $Z^* = Z_1(M,\delta^*) = Z_2(M,\delta^*)$.

(d) If $s > \Delta_2$, then $T^* = T_2$ and $Z^* = Z_2(T_2,\delta^*)$, where T_2 is the same as Lemma 3 (b).

Proof. This result immediately follows from Lemmas 2 and 3 and the fact that;

$$Z_1(M,\delta^*) = Z_2(M,\delta^*).$$

Once we obtain the optimal advance sales discount rate δ^* and the length of the regular sale period T^*, the optimal order quantity is as follows:

$$Q^* = D(p - \delta^* p)t_p + D(p)T^*$$
$$= [a - b(1-\delta^*)p]t_p + (a - b p)T^*$$

Numerical examples

The following numerical examples are given to illustrate the aforementioned solution procedure.

Example 1: This example is based on real data provided by a retailer in Taiwan in relation to one of their items. The supplier offers a permissible delay period of 30 days (*M* = 1 month). The retailer offers customers an advance sales period of 30 days ($t_p = 1$ month). The interest earned per \$ per year is 12% (interest rate per month, $I_e =$ 0.01000) and the interest charged per \$ investment in stocks per year is 10 % (interest rate per month, $I_c =$ 0.008333). In addition, $p =$ \$280 / unit, $c =$ \$182 / unit, a = 800, b = 2.5, $h = \$30/\text{unit}/\text{month}$ and $s = \$500$/order.

Under these parameters, we find that $0 < A = 334.5 < 2b(p-c)c = 89180$; hence, $\delta^* = 0.1111$. Further, since $s = 500 < \Delta_1 = 2558.03$, from the Theorem, we obtain the optimal value of T as $T_1 = 0$. Thus, the optimal solution is $(T^*, \delta^*) = (0, 0.1111)$ and the maximum total profit per unit of time is $Z^* = Z_1(T^*, \delta^*) =$ 12138. The optimal order quantity is $Q^* \approx 177.75$ units.

Example 2: We consider the same parameters as in Example 1, except with the following: $I_e = 0.00100$ and p = \$260 / unit. Under these parameters, we find that $0 < A = 344.025 < 2b(p-c)c = 70980$; hence, $\delta^* = 0.0354$. Further, since $\Delta_1 = 230.87 < s = 500 < \Delta_2 = 7039.37$, from the Theorem, we obtain the optimal value of T as $T_1 = 0.0576$. As a result, the optimal solution is $(T^*, \delta^*) = (0.0576, 0.0354)$, the maximum total profit per unit time is $Z^* = Z_1(T^*, \delta^*) = 11477.4$ and the optimal order quantity is $Q^* \approx 181.63$ units.

Conclusion

In this article, we considered an inventory model with price-dependent demand. In our model, the retailer provides advance sales whereby customers can commit orders at a discounted price prior to the beginning of the regular sale season. Further, the supplier allows the retailer a specified credit period to settle the balance during which no interest accrues. Advance sales offers two benefits. Firstly, the retailer can gain additional demand by implementing advance sales. Moreover, advance sales increases the amount of interest earned since interest is earned on payments received from advance sales orders prior to the regular sale season. We provide a Theorem to determine the optimal advance sales discount rate and the optimal length of the regular sale period for which the total profit per unit of time is maximized. Finally, numerical examples were given to illustrate the solution procedure.

In the future research, our model can be extended in several ways. For instance, it could be of interest to consider the situation where the retailer determines when to start advance sales. In addition, the model may be generalized to the price-dependent demand in which price is a decision variable.

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APPENDIX

Appendix A:

The proof of Lemma 1.

(a) Since $f(\delta)$ is a strictly decreasing function for $\delta \in [0, 1-c/p]$, $f(0) = -A + 2b(p-c)c \le 0$ implies $f(\delta) \le 0$ for all $\delta \in [0, 1-c/p]$. Therefore, $\dfrac{\partial Z(T,\delta)}{\partial \delta} = \dfrac{p\, t_p\, f(\delta)}{t_p + T} \le 0$. Hence, for a given T, $Z(T,\delta)$ is a decreasing function of δ. That is, a smaller value of δ results in a larger value of $Z(T,\delta)$. Thus, the maximum value of $Z(T,\delta)$ occurs at the boundary point $\delta = 0$ that is, $\delta^* = 0$.
(b) Conversely, $f(1-c/p) = -A \ge 0$ implies $f(\delta) \ge 0$ for all $\delta \in [0, 1-c/p]$ since $f(\delta)$ is a strictly decreasing function for $\delta \in [0, 1-c/p]$. Thus, $\dfrac{\partial Z(T,\delta)}{\partial \delta} = \dfrac{p\, t_p\, f(\delta)}{t_p + T} \ge 0$, which means, for a given T, $Z(T,\delta)$ is an increasing function of δ. That is, a larger value of δ leads to a larger value of $Z(T,\delta)$. Therefore, the maximum value of $Z(T,\delta)$ occurs at the boundary point $\delta = 1 - c/p$ i.e., $\delta^* = 1 - c/p$.
(c) $0 < A < 2b(p-c)c$ implies $f(0) > 0$ and $f(1-c/p) < 0$. Since $f(\delta)$ is a strictly decreasing function in $\delta \in [0, 1-c/p]$, by Intermediate Value Theorem, we can find a unique value δ such that $f(\delta) = 0$. Consequently, the point δ which satisfies $f(\delta) = 0$ not only exists but also is unique. Solving $f(\delta) = 0$ (that is, $\dfrac{\partial Z(T,\delta)}{\partial \delta} = 0$), we can obtain the optimal solution of δ, which is given by $\delta^* = [b\, p\, (4 + 2 I_e M + I_e t_p) - 2bc - 2a]/4bp$.

Appendix B:

The proof of Lemma 2.

(a) Since $G_1(T)$ is a strictly decreasing function for $T \in [0, M]$, $G_1(0) = s - \Delta_1 \le 0$ implies $G_1(T) \le 0$ for all $T \in [0, M]$. Thus, we obtain $\dfrac{d\, Z_1(T,\delta^*)}{d\, T} = \dfrac{G_1(T)}{(t_p + T)^2} \le 0$. Therefore, $Z_1(T,\delta^*)$ is a decreasing function of T, that is, a smaller value of T causes a larger value of $Z_1(T,\delta^*)$. Hence, the maximum value of $Z_1(T,\delta^*)$ occurs at the boundary point $T = 0$. That is, $T_1 = 0$.
(b) Conversely, $G_1(M) = s - \Delta_2 \ge 0$ implies $G_1(T) \ge 0$ for all $T \in [0, M]$ because $G_1(T)$ is a strictly decreasing function for $T \in [0, M]$. Thus, $\dfrac{d\, Z_1(T,\delta^*)}{d\, T} = \dfrac{G_1(T)}{(t_p + T)^2} \ge 0$. Therefore, $Z_1(T,\delta^*)$ is a increasing function of T, which means a larger value of T causes a larger value of $Z_1(T,\delta^*)$. Hence, the maximum value of $Z_1(T,\delta^*)$ occurs at the boundary point $T = M$. That is, $T_1 = M$.
(c) $\Delta_1 < s < \Delta_2$ implies $G_1(0) > 0$ and $G_1(M) < 0$. Since $G_1(T)$ is a strictly decreasing function in $T \in [0, M]$, by the Intermediate Value Theorem, we can find a unique value T such that $G_1(T) = 0$. Consequently, the point T which satisfies $G_1(T) = 0$ not only exists but also is unique. Solving $G_1(T) = 0$ (that is, $\dfrac{d\, Z_1(T,\delta^*)}{d\, T} = 0$), we obtain the

optimal value of T as $T_1 = -t_p + \left\{ t_p^{\;2} + \dfrac{2s + 2b\delta^* p(\delta^* p - p + c)\, t_p - b\delta^* p^2 I_e (t_p + 2M) t_p}{(a - b\,p)(h + p\,I_e)} + \dfrac{2\delta^* p t_p - p I_e t_p^{\;2}}{h + p\,I_e} \right\}^{\frac{1}{2}}$.

Appendix C

The proof of Lemma 3

(a) Since $G_2(T)$ is a strictly decreasing function for $T \in [M, \infty)$, $G_2(M) = s - \Delta_2 \le 0$ implies $G_2(T) \le 0$ for all $T \in [M, \infty)$.Thus, we have $\dfrac{d\, Z_2(T, \delta^*)}{d\, T} = \dfrac{G_2(T)}{(t_p + T)^2} \le 0$. Therefore, $Z_2(T, \delta^*)$ is a decreasing function of T, which means a smaller value of T causes a larger value of $Z_2(T, \delta^*)$. Hence, the maximum value of $Z_2(T, \delta^*)$ occurs at the boundary point $T = M$ i.e., $T_2 = M$.

(b) Since $G_2(T)$ is a strictly decreasing function for $T \in [M, \infty)$, $G_2(M) > 0$ (i.e., $s \le \Delta_2$) and $\lim_{T \to \infty} G_2(T) < 0$, by the Intermediate Value Theorem, we can obtain a unique value T such that $G_2(T) = 0$. Consequently, the point T which satisfies $G_2(T) = 0$ not only exists but also is unique. Solving $G_2(T) = 0$ (i.e., $\dfrac{d\, Z_2(T, \delta^*)}{d\, T} = 0$), we obtain the optimal value of T which is given by:

$$T_2 = -t_p + \left\{ t_p^{\;2} + \frac{c I_c (M + 2t_p) M + 2\delta^* p t_p - p I_e (t_p + M)^2}{h + c I_c} + \frac{2s + 2\delta^* b\, p(\delta^* p - p + c)\, t_p - \delta^* b\, p^2 I_e (t_p + 2M) t_p}{(a - b\,p)(h + c\,I_c)} \right\}^{\frac{1}{2}}.$$