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Ranking DMUs based on efficiency stability

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Decision-making process is to find the best option from all of the feasible alternatives. Considering the efficiency interval, efficiency scores from optimistic and pessimistic points of view, all the possible evaluations can be illustrated. Therefore, Interval DEA models can be helpful for a decision maker needs all those possible efficiency values that reflect different perspectives. The mentioned upper and lower bound of efficiency interval is obtained from the optimistic and pessimistic viewpoints. As a matter of fact, it can be propounded that in assessing DMUs considering the mere optimistic efficiency score is not a acceptable criterion for ranking units and performance evaluation of them. Since, in the event that an entity has gain a high efficiency with a great risk it will not certainly have priority to a unit with relatively balancing efficiency, suitable confidence interval and a low risk. In this paper considering the above issues a method for ranking units based on efficiency intervals is presented. With an application the clarity of the proposed procedure will be demonstrated.

Key words: Data envelopment analysis, interval DEA models, ranking.

INTRODUCTION

Data envelopment analysis (DEA) is a non parametric technique for measuring and evaluating the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. In Classical DEA models inputs have to be minimized, outputs have to be maximized and units are assumed to operate under similar conditions. In accordance to the information about exiting data, DEA technique can estimate the efficiency frontier. If a DMU locates onto this frontier it is evaluated with the efficiency score of one and thus it is referred to as an efficient unit; otherwise if the correspondence efficiency score is less than one an it is referred to as an inefficient unit. While considering DEA technique it is possible to find targets and benchmark units for inefficient DMUs. In DEA the efficiency measure for a DMU is assumed as the maximum ratio of weighted sum of outputs to that of inputs. This maximum ratio, the efficiency value, is calculated from the optimistic viewpoint. Schaffnit et al. (1997) provided a paper in which a best practice analysis of a large Canadian bank has been presented. In their paper, Schaffnit et al, (1997), based on standard transaction and maintenance times, used DEA AR models with output multiplier constraints. Also a model which adds constraints on the input multipliers is used to find the cost efficient branches, and

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estimate “allocative” efficiency. Azizi (2011) in his paper confirms that the traditional DEA determines the best efficiency score based on which, DMUs are classified into optimistic efficient or optimistic non-efficient units, and the DEA-efficient DMUs determine the efficiency frontier. There is a comparable approach which uses the concept of inefficiency frontier for determining the worst relative efficiency score. In his paper, considered an integration of both efficiencies in the form of an interval. He emphesized that the obtained efficiency interval provides the decision maker with all the possible values of efficiency, which reflect various perspectives. Wang et al. (2007) stated that the worst relative efficiencies can be utilized for measuring the worst performances of DMUs, while efficiencies are measured within the range of greater than or equal to one. In their paper the efficiencies, corresponding to each DMU, are measured as an interval, whose upper bound is set to one and the lower bound is determined through introducing a virtual anti-ideal DMU. We discuss that we can have efficiency intervals consisting of the maximum and minimum ratios of weighted sum of outputs to that of inputs. In other words, the upper bound of efficiency interval is the efficicncy obtained from the optimistic view-point based on the same concept as in the conventional DEA. The correspondance lower bound is obtained from the pessimistic viewpoint by focusing on the inferior inputs and outputs. The great feature of considering both opti-mistic and pessimistic efficieny scores is that the efficiency interval can illustrate all the possible evaluations from various viewpoints.

In literature finding the lower bound of efficicny has been dealt with. jahanshahloo et al. (2010) provided a model for deriving the lower bound of efficiency. Entani and Tanaka (2006), while considering both the optimistic and the pessimistic viewpoints, proposed a DEA model with interval efficiencies and by adjusting corresponding given inputs and outputs they have improved the efficiency interval of a DMU. Entani and Tanaka (2006), for im-proving the lower bound of efficiency interval, have defined different target points for different DMUs. As they stated while the other presented interval DEA models cannot be used to measure the interval efficiency of a DMU with crisp data and can merely be utilized for interval data, their DEA model can be used for measuring the interval efficiency of a DMU with crisp, interval, fuzzy data or even with the mixture of those. In their paper Wang et al. (2005) studied how to conduct efficiency assessment using data envelopment analysis in interval and/or fuzzy input–output circumstances. The proposed interval DEA models are developed to measure the lower and upper bounds of the best relative efficiency of each DMU with interval input and output data. As discussed the obtained intervals are different from that formed by the worst and the best relative efficiencies of each DMU and this is a significant feature of the proposed model with which the models become more applicable.

Here the aim is to determine the upper and lower bounds of efficiency for ranking DMUs. Since, investigation and consideration of all accomplishment and failure factors can result in alternatives that help in decision making. As a matter of fact it can be propounded that in assessing DMUs considering the mere optimistic efficiency scores is not a sufficient criterion for performance evaluation and ranking of units. Since, in the event that an entity has gained a high efficiency score with a great risk it will not certainly have a priority to an entity with relatively balancing efficiency score, suitable confidence interval and a low risk. Thus, for ranking entities both efficiency and stability should be considered.

The paper unfolds as follows: First, some preliminaries about lower and upper efficiency bounds will be discussed; then the procedure for ranking units, considering these bounds, will be explained and the result are gathered and examined. Section 4 concludes tha paper.

Application

In this competitive world, considering a factor directly influenced the competition, has a fundamental importance. Doubtlessly, taking a reliable action with an acceptable assurance interval is better than that with a high risk. What will be discussed here is ranking DMUs based on stability interval, from the optimistic and pessimistic viewpoints, in efficiency evaluation. Thus, at first some preliminaries about the lower and upper efficiency bounds and then ranking due to these efficiency bounds will be discussed.

Preliminaries

The relative efficiency can be acquired from various viewpoints. In this section we have briefly reviewed the Interval DEA model which yields the efficiency interval, (Wang et al., 2005). By efficiency interval correspondance upper and lower bounds are considered. These intervals are acquired by solving two optimization problems. While the efficiency of a DMU is calculated from the optimistic viewpoint relative ratio is maximized with respect to input and output of the other DMUs. When optimistic viewpoint is considered, corresponding relative ratio of the under evaluation unit is minimized. As stated in jahanshahloo et al. (2010) evaluations from the optimistic and pessimistic viewpoints, respectively, will yield the upper and lower bounds of efficiency interval. The conventional DEA is regarded as the evaluation from the optimistic viewpoint; the upper bound of efficiency interval for DMU_o can be obtained through solving conventional model, called

CCR, Charnes et al. (1978). Considering the original CCR model formulated as a fractional programming problem, the problem to acquire the upper bound of efficiency interval is formulated as follows:

$$\begin{aligned} \max \quad & \frac{U^t Y_o / V^t X_o}{Max\{U^t Y_j / V^t X_j, \quad j = 1,...,n\}} \\ s.t. \quad & V \geq 0, \\ & U \geq 0, \end{aligned} \tag{1}$$

where x_j and y_j are the given input and output vectors of DMU_j, j=1,...,n, are all semipositive. Also v and u are the input and output weight vectors. Thus, there exists n $DMUs$ with m inputs and s outputs. It should be noted that the denominator in (1) plays an important role of normalizing efficiency value. The ratio of weighted sum of outputs to that of inputs for DMU_o is compared to the maximum ratio of all $DMUs$. In CCR model, the ratios of weighted sum of outputs to that of inputs for all $DMUs$ are constrained to be less than or equals one for normalization. The linear counterpart of (1) is the following model. Thus, the upper bound of efficiency interval is obtained through solving the basic DEA model denoted as the following LP problem:

$$\begin{aligned} \max \quad & U^t Y_o \\ s.t. \quad & V^t X_o = 1, \\ & U^t Y_j - V^t X_j \leq 0, \quad j = 1,...,n, \\ & U \geq 0, \quad V \geq 0. \end{aligned} \tag{2}$$

One issue needs to be mentioned here is that from among n units, there exists one unit with the efficiency score of 1 which guarantees that model (1) provides relative efficiency. But this is not true for the case of obtaining the lower bound of efficiency (pessimistic viewpoint). Therefore, as Entani et al. (2006) have presented, by minimizing the objective function in (1) with respect to the weight variables, the lower bound of efficiency interval is obtained by following problem:

$$\begin{aligned} \min \quad & \frac{U^t Y_o / V^t X_o}{Max\{U^t Y_j / V^t X_j \mid j = 1,...,n\}} \\ s.t. \quad & V \geq 0 \\ & U \geq 0, \end{aligned} \tag{3}$$

As stated in Entani et al. (2006), the optimal objective value of this model is obtained with inferior inputs and outputs of DMU_o. Therefore, it can be said that it is the evaluation from the pessimistic viewpoint. The efficiency interval denoted as $[\theta_o^{l*}, \theta_o^{u*}]$ illustrates all the possible evaluations from various viewpoints.

Another model which was proposed by Jahanshahloo et al. (2010) for assessment of the lower bound of efficiency is as follows which correct the shortcoming of the model proposed by Entani Et al. (2006). In their paper they have discussed that there is no guarantee to have relative efficiency, thus they have suggested the following model:

$$\begin{aligned} \min \quad & U^t Y_o \\ s.t. \quad & V^t X_o = 1, \\ & U^t Y_j - V^t X_j \leq 0, \quad j = 1,...,n, j \neq o \\ & U^t Y_p - V^t X_p = 0, \\ & U \geq 0, \quad V \geq 0. \end{aligned} \tag{4}$$

To evaluate DMU_o this model should be solved in turn for all units. It should be noted that this model may be infeasible for some units. At the end collecting the results the lower bound of efficiency will be resulted from the following formula:

$$\theta_o^{l*} = Min\{1, Min\{\theta_{oj}^l \mid j = 1,...,n, j \neq o\}\}$$

Data

In this competitive world, considering a factor directly influenced the competition, has a major importance. Investigation and consideration of all accomplishment and failure factors, can result in alternatives which help in making decision. What has achieved a special importance for making decision is reliability and stability of a DMU under various circumstances (optimistic and pessimistic). Doubtlessly, taking a reliable action with an acceptable assurance interval is better than that with a high risk. What will be discussed here is ranking DMUs based on stability from the optimistic and pessimistic viewpoints in efficiency evaluation. As a matter of fact it can be propounded that in assessing DMUs considering mere optimistic efficiency score is not a criterion for ranking and performance evaluation of units. Since, in the event that an entity has gain a high efficiency score with a high risk it will not certainly have priority to that of with relatively balancing efficiency score, suitable confidence interval and a low risk. Thus, for ranking entities both efficiency and stability should be considered. These circumstances

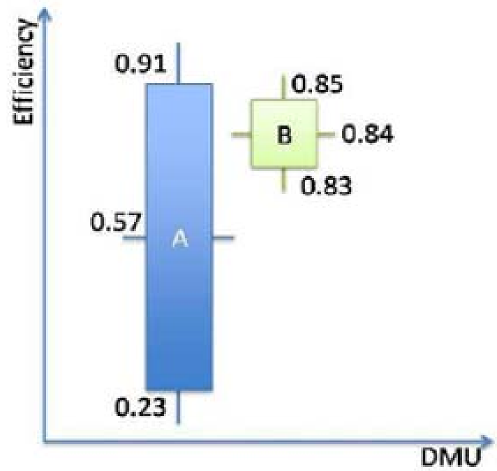

Figure 1. Efficiency interval.

Table 1. Inputs and outputs.

	O1	O2	O3	O4	O5	I1
Mean	0.2695	0.5904	0.9950	0.1158	0.2614	31.83847
Variance	0.0844	0.1851	0.0002	0.0567	0.1059	38.11880

are schematically depicted in Figure 1.

As it can be seen unit A with optimistic efficiency score of 0.91 is better than unit B with optimistic efficiency score of 0.85. But unit B, while it is considered from pessimistic viewpoint, has the efficiency score of 0.83 which is much better than that of unit A which is 0.23. Thus, in regarding the provided ranking method, ranking with efficiency intervals, unit B has a better rank order than that of unit A.

The important issue needs to be mentioned here is that the in efficiency scores of any two different units are different from each other. As stated in DEA literature it is not possible, for two different in efficient units, to have the same efficiency score. If this happens, by considering more decimal numbers it can be seen that the efficiency scores are different. This issue is true while the assessment is considered from either optimistic or pessimistic points of view. The only case where the efficiency scores are equal is for efficient units, with the efficiency score of one. But while supper efficiency scores of two different units are considered, the corresponding values are different from each other.

Considering the obtained upper and lowe bounds for efficiency, ranking units based on average of these score may be satisfactory. It should be noted that it is possible that the averages of two differedt pairs of numbers be the same. But the acquired efficicncy scores are real numbers and the case where the averages of two differedt pairs of real numbers be the same is very rare. This procedure is performed for ranking 1816 bank branches. Hence, through these branches, 100 units have been randomly selected. In this application one input and five outputs have been considered as listed in the following tables (Tables 1-3). In these tables mean, variance, ranges and related frequencies are listed.

RESULTS

As discussed, for ranking DMUs both optimistic and pessimistic viewpoints should be considered. The important issue is the stability of that unit with high average efficiency score which results in a better rank for the unit under assessment. In doing so, both efficiency scores, optimistic and pessimistic, are calculated and listed in Table 4.

As existing in literature, one of the famous ranking methods is supper efficicny which is provided by Anderson and Peterson (1993). Due to the existence of units with the upper and lower bound of efficiencies which are equal to 1, the supper efficiency in both pessimistic and optimistic viewpoints has been calculated.

As you can see unit 40 has the upper supper efficiency score of 1.325 and the correspondence lower supper

Table 2. Data description.

O1		O2		O3	
Range	**Frequency**	**Range**	**Frequency**	**Range**	**Frequency**
0.008-0.132	42	0.046-0.165	34	0.86-0.878	1
0.132-0.256	27	0.165-0.184	1	0.878-0.896	1
0.256-0.38	10	0.184-0.303	8	0.896-0.914	2
0.38-0.504	6	0.303-0.422	4	0.914-0.932	3
0.504-0.628	2	0.422-0.541	1	0.932-0.950	23
0.628-0.752	1	0.541-0.66	52	0.950-0.968	60
0.752-0.876	3	0.66-0.779	42	0.986-0.986	5
0.872-1	9	0.779-1	52	0.986-1	5

Table 3. Data description.

O4		O5		I1	
Range	**Frequency**	**Range**	**Frequency**	**Range**	**Frequency**
0-0.125	71	0.0005-0.13	56	19.58-22.54	2
0.125-0.25	8	0.13-0.255	17	22.54-25.5	17
0.25-0.375	1	0.255-0.38	2	25.5-28.46	21
0.375-0.5	3	0.38-0.405	1	28.46-31.42	8
0.5-0.625	1	0.405-0.53	7	31.42-34.38	10
0.625-0.75	1	0.53-0.655	2	34.38-37.34	14
0.75-0.875	5	0.655-0.78	1	37.34-40.3	22
0.875-1	10	0.78-1	14	40.3-43.29	6

Table 4. Ranks.

DMU	U.S.E	L.S.E	Mean	R1	R2	DMU	U.S.E	L.S.E	Mean	R1	R2
40	2.325	1.000	1.663	1	1	61	0.688	0.442	0.565	51	51
59	1.416	1.000	1.208	2	2	70	0.726	0.404	0.565	40	52
89	1.149	0.985	1.067	3	3	17	0.598	0.528	0.563	76	53
54	1.131	1.000	1.066	4	4	47	0.715	0.401	0.558	45	54
30	1.063	0.990	1.027	5	5	98	0.630	0.483	0.556	67	55
87	0.950	0.940	0.945	6	6	43	0.652	0.457	0.554	63	56
8	0.924	0.898	0.911	9	7	62	0.708	0.395	0.551	48	57
46	0.883	0.863	0.873	12	8	25	0.694	0.403	0.549	50	58
91	0.926	0.817	0.872	8	9	58	0.661	0.431	0.546	57	59
90	0.911	0.788	0.849	10	10	60	0.614	0.473	0.544	71	60
31	0.849	0.830	0.840	15	11	65	0.655	0.431	0.543	60	61
4	0.911	0.732	0.821	11	12	33	0.612	0.455	0.533	72	62
93	0.818	0.787	0.802	20	13	20	0.654	0.412	0.533	61	63
77	0.933	0.659	0.796	7	14	52	0.665	0.401	0.533	56	64
34	0.826	0.705	0.765	18	15	53	0.641	0.402	0.522	65	65
13	0.765	0.728	0.746	29	16	56	0.595	0.448	0.521	78	66
92	0.833	0.655	0.744	17	17	57	0.683	0.355	0.519	53	67
38	0.752	0.731	0.742	32	18	10	0.672	0.360	0.516	54	68
72	0.823	0.624	0.723	19	19	23	0.628	0.397	0.513	68	69

Table 4. cont'd

9	0.814	0.633	0.723	22	20	84	0.636	0.384	0.510	66	70
64	0.867	0.574	0.721	14	21	99	0.565	0.440	0.503	93	71
73	0.881	0.554	0.717	13	22	27	0.657	0.345	0.501	58	72
95	0.802	0.631	0.716	24	23	51	0.624	0.377	0.500	69	73
5	0.733	0.655	0.694	36	24	63	0.571	0.423	0.497	88	74
1	0.701	0.683	0.692	49	25	50	0.653	0.336	0.494	62	75
80	0.838	0.537	0.687	16	26	94	0.523	0.460	0.492	98	76
96	0.687	0.661	0.674	52	27	71	0.620	0.362	0.491	70	77
86	0.815	0.526	0.671	21	28	81	0.611	0.365	0.488	73	78
67	0.801	0.532	0.667	25	29	19	0.597	0.371	0.484	77	79
66	0.729	0.599	0.664	38	30	78	0.605	0.358	0.482	74	80
7	0.732	0.582	0.657	37	31	12	0.591	0.351	0.471	79	81
82	0.800	0.494	0.647	26	32	55	0.576	0.364	0.470	84	82
24	0.809	0.484	0.646	23	33	48	0.599	0.338	0.468	75	83
88	0.709	0.561	0.635	47	34	97	0.581	0.347	0.464	82	84
41	0.796	0.458	0.627	27	35	37	0.589	0.337	0.463	80	85
42	0.652	0.584	0.618	64	36	6	0.568	0.357	0.463	89	86
26	0.777	0.445	0.611	28	37	68	0.583	0.340	0.462	81	87
18	0.752	0.462	0.607	33	38	32	0.578	0.341	0.460	83	88
3	0.757	0.444	0.601	30	39	29	0.567	0.349	0.458	91	89
39	0.719	0.481	0.600	44	40	11	0.575	0.339	0.457	85	90
100	0.752	0.444	0.598	31	41	36	0.572	0.335	0.453	87	91
35	0.752	0.437	0.594	34	42	75	0.551	0.353	0.452	95	92
85	0.749	0.428	0.588	35	43	21	0.574	0.327	0.450	86	93
49	0.666	0.498	0.582	55	44	83	0.566	0.334	0.450	92	94
28	0.726	0.431	0.578	41	45	69	0.562	0.328	0.445	94	95
22	0.724	0.431	0.578	42	46	15	0.567	0.312	0.440	90	96
2	0.723	0.431	0.577	43	47	44	0.531	0.304	0.418	96	97
14	0.714	0.440	0.577	46	48	79	0.523	0.305	0.414	99	98
16	0.726	0.417	0.572	39	49	76	0.525	0.299	0.412	97	99
74	0.656	0.486	0.571	59	50	45	0.514	0.291	0.403	100	100

efficiency is 1. Certainly, this unit can be introduced as a pioneer. This is, the same circumstances for unit 59 and 54. But the major point is that unit 89 with the lower and upper bounds of 1.0149 and 0.985, respectively, has gained a better rank order in comparison to that of unit 54. It is worth mentioning that the reason is due to the priority in the optimistic case with the extent of 0.018 and the inferiority in the pessimistic case in the extent of 0.015. Thus it gains a better rank order. This issue is set up for units 46 and 91 in an other manner. Unit 46 has more stable and appropriate efficiencies, with the lower and upper bounds of 0.863 and 0.883 respectively, and these two for unit 91 is 0.817 and 0.926. It is evident that it has a more confidential performance relative to unit 91 thus it gains more reliability. Since in pessimistic viewpoint it has superiority with the extent of 0.046 to that of unit 91 and it has higher average efficiency.

Moreover, according to what has been provided in Table 4 a fundamental difference in unit 42 can be seen which is, this unit has improved its status 28 levels while considering conventional rank order; that means considering just the upper bounds. Also, unit 91 and 1 have improved their statuses, respectively, and they have witnessed 25 and 24 level raises; thus they have gained better ranks. Furthermore, units 27, 10 and 57 have witnessed 14 level drops in their corresponding ranks and these units are of the most unstable units. As you can see the upper and lower efficiency bounds correspond to unit 27 are, respectively, 0.657 and 0.343. It should be noted that in the case of pessimistic viewpoints the efficiency of this unit has been reduced to half. Thus, it gains the rank of 72th. It should be mentioned that in the conventional ranking method it has the rank order of 58.

In the Table 4 the results of ranking DMUs are gathered.

Under the column U.S.E and L.S.E, the upper supper efficiency and the lower efficiency are listed, respectively. Under the column named R1 DMUs are ranked according to the upper bound of efficiency and under the column R2 they are ranked according to the upper and lower bounds of efficiency.

In Table 4, there is a major rank difference between these two ranking methods. For instance, considering unit 40 there is no rank difference. But for unit 8, this deference is +2 which means the first rank, based on the upper bound, is better than that of the second one, which is based on the upper and lower efficiencies. Moreover, this difference for unit 91 is -1 which means the second rank, which is based on the upper and lower efficiencies, is better than that of the first one, based on the upper bound.

Conclusion

DMUs can be relatively evaluated from various viewpoints and as a result the efficiency scores are acquired as intervals. Considering a factor directly influenced the competition in this competitive world, has a fundamental significance. Thus, Investigation and consideration of all factors correspond to accomplishment and failure can lead to different alternatives which help making decision. This has achieved a special importance for making decision is reliability and stability of a DMU under various circumstances (optimistic and pessimistic). In this paper an approach has been proposed for ranking according to the efficiency intervals while optimistic and pessimistic efficiency scores are considered. Since, in the event that an entity has gained a high efficiency score with a great risk certainly it will not have priority to a unit with relatively balancing efficiency score, suitable confidence interval and a low risk. Thus, for ranking entities both efficiency and stability should be considered. The proposed ranking method in this paper does not suffer from ranking non extreme efficient units, as most of rankig methods do, and moreover it is always feasible. Besides, through finding lower efficiency it can be found that to what extent a unit can risk. In this method the case where a unit operates badly can also be distinguished. In this case corresponding lower efficiency will decrease a lot. Also, if the lower bound of efficiency is acceptable this means that the under assessment unit, in most of the situations, performs well. One more issue that needs to be mentioned is that in this method the other units do not affect the rank order of the under evaluation unit, as in supper efficicny method in which the new efficicnt frontier constructed throuh the remaining units and the under evaluation unit has been compared to this frontier. Moreover, the proposed ranking method is merely affected from the corresponding efficiency bounds.

Conflict of interest

Author(s) have not declared any conflit of interest

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