

*Full Length Research Paper*

# **A three-dimensional searched-based algorithm for an imperfect production System**

**Yu-Chung Tsao**

Sino-US Global Logistics Institute, Shanghai Jiao Tong University, Shanghai, China. E-mail: [yctsao@sjtu.edu.cn](mailto:yctsao@sjtu.edu.cn).  
Tel: +86-21-62932334.

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**This paper deals with the production and inventory problems under partial backlogging, trade credit and reworking of imperfect quality items. The objective is to determine the optimal production lot size and maximum backlogging level simultaneously while still minimizing the total cost. The problem is modeled as a four-branch function with dual variables. A three-dimensional searched-based algorithm was developed to solve the problem described. Numerical examples are given to illustrate the solution procedure and discuss the impact of various system parameters. The study concludes with a computational analysis that leads to variety of managerial insights.**

**Key word:** Optimization, imperfect production system, reworking, trade credit, partial backlogging.

## **INTRODUCTION**

Trade credit mostly exists between enterprises and it also affects the total cost. Because of the competitive market environment, suppliers usually delay the payment period to retailers in order to bring about further demand from retailers or increase their market share. Trade credit can decrease the buyers' pressure of inventory cost and increase their purchase willingness. In brief, the concept of trade credit is that a buyer receives goods instantaneously, but pays for them over a period of time. For example, Wal-Mart, the largest retailer in the world, uses trade credit as a larger source of capital than bank loans; trade credit for Wal-Mart is 8 times the amount of capital invested by shareholders.

Goyal (1985) was the first trade credit paper to examine the effect of the credit period on the optimal inventory policy. Over the years, a number of studies have been published that deal with the inventory problems under trade credit. From studies published within one year, Chen and Kang (2010) discussed the coordination between vendor and buyer considering trade credit and items of imperfect quality. Tsao (2010a) determined a two-phase pricing and inventory decisions for deteriorating and fashion goods under trade credit conditions. Kreng and Tan (2010) determined the optimal replenishment decisions under two levels of trade credit policy, if the purchaser's order quantity is greater than or

equal to a predetermined quantity. Tsao (2010b) considered multi-echelon multi-item channels subjected to supplier's credit period and retailer's promotional effort. Ouyang et al. (2010) used algebraic approach to determined optimal ordering cycle time under partially permissible delay in payments and inspection policy. Tsao (2010c) determined the optimal ordering policy under acceptance sampling plan and trade credit financing. Balkhi (2010) considered an economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case. Tsao and Sheen (2010) dealt with the dual problems of determining the ideal supplier credit period, and the best way for the retailer to make multi-item replenishment and pricing decisions, while still maximizing profits. The issue of trade credit is very popular to these field researchers.

Most of the trade credit papers considered EOQ model. So far, Chung and Huang (2003), Ouyang et al. (2006a), Huang (2007), Liao (2008), Teng and Chang (2009) and Chang et al. (2010) considered EPQ model under permissible delay in payments. All of them considered the perfect production system. This means that the production system did not produce defective products. However, in the real world cases, the product quality is not always perfect and usually depends on the state of the production process (Pearn et al., 2010). Hayek and

Salameh (2001) considered reworking imperfect items under permissible backlogging conditions. Sana (2010, a) developed a production-inventory model of imperfect quality products in a three-layer supply chain. Sana et al. (2010) developed an imperfect production process in a volume flexible inventory model. Sana (2010b) developed a production-inventory model with time-varying demand and imperfect production process. Sana (2010c) investigated an economic production lot size model in an imperfect production system in which the system may shift from an in-control state to out-of-control state. Sana and Chauduri (2010) considered an economic manufacturing quantity model in an imperfect production process. Sarkar et al. (2010a) developed a stock-dependent inventory model in an imperfect production process. Sarkar et al. (2010b) determined the optimal reliability, production lot size and safety stock in an economic manufacturing quantity model.

Lin (2010a, b) considered the defective items in different inventory models. Based on their great researches, Tsao et al. (2011) developed a production model under reworking of imperfect items and trade credit. When imperfect quality products are produced, they are reworked, which introduces a repair cost and a holding cost of imperfect quality products, leading to an increase in the total cost. It is essential to consider imperfect quality products when formulating a realistic production model.

When customer demand exceeds manufacturer supply, a shortage occurs. Some customers may buy a similar product from another manufacturer, while others are willing to wait for the original manufacturer because of factors such as brand loyalty, lower price, or the non-urgent nature of the demand, that is, partial backlogging. The backlogging rate may affect the maximum permissible backlogging level and consequently the total cost. Chang and Dye (2001) established an EOQ model with deteriorating items under the conditions of partial backlogging and trade credit and indicated that, shortages are neither completely backlogged nor completely lost. Ouyang et al. (2006b) established an EOQ model for deteriorating items under partial backlogging and trade credit. The paper extends the model of Tsao et al. (2010) to consider the partial backlogging situation. The problem becomes a four-branch function with two decision variables, which is not easy to solve in a short time.

This paper incorporates partial backlogging into the EPQ model under trade credit and reworking of imperfect quality items to arrive at a more realistic model. This is the first study to consider reworking of imperfect quality items, partial backlogging and trade credit, simultaneously. The objective is to determine the optimal production of lot size and maximum backlogging level simultaneously while still minimizing the total cost. The study develops a three-dimensional searched-based algorithm to solve the problem which has a four-branch function with two decision variables. The algorithm can be modified to

solve any problem with multi-branch function and multiple variables. Numerical experiment is given to illustrate the solution procedure and discuss the impact of various system parameters. These results should be a useful reference for managerial decisions and administrations.

## MODEL FORMULAION

The following notations are used in this study:

- $P$  : Production rate in units per unit time.
- $P_1$  : Rate of rework of imperfect quality items in units per unit time.
- $d$  : production rate of imperfect quality items in units per unit time.
- $C$  : production cost per item.
- $\lambda$  : Demand rate in units per unit time.
- $x$  : Percentage of imperfect quality items produced.
- $Q$  : Production lot size without backlogging (decision variable).
- $H$  : Inventory level when rework of imperfect quality items is completed.
- $H_1$  : Inventory level when original production is completed.
- $C_R$  : Repair cost per item of imperfect quality.
- $K$  : Setup cost for each production run.
- $h$  : Holding cost of perfect items per item per unit time.
- $h_1$  : Holding cost of the imperfect quality items being reworked.
- $S_p$  : Unit selling price per item of good quality.
- $M$  : The manufacturer's trade credit period offered by the supplier.
- $C_p$  : Unit purchasing cost per item.
- $I_e$  : Interest rate that can be earned per dollar.
- $I_p$  : Interest rate that can be charged per dollar.
- $w$  : Maximum backlogging level (*decision variable*).
- $\beta$  : Proportion of permissible backlogging.
- $r$  : Discount rate.

The mathematical model is developed under the following assumptions:

1. The demand rate and production rate are known constants.
2. The percentage of imperfect quality items produced is a known constant.
3. All imperfect quality items can be reworked.
4. Whenever a repair is completed, the product is added into the inventory of perfect quality items.
5. When shortages occur,  $\beta$  proportion of customers are

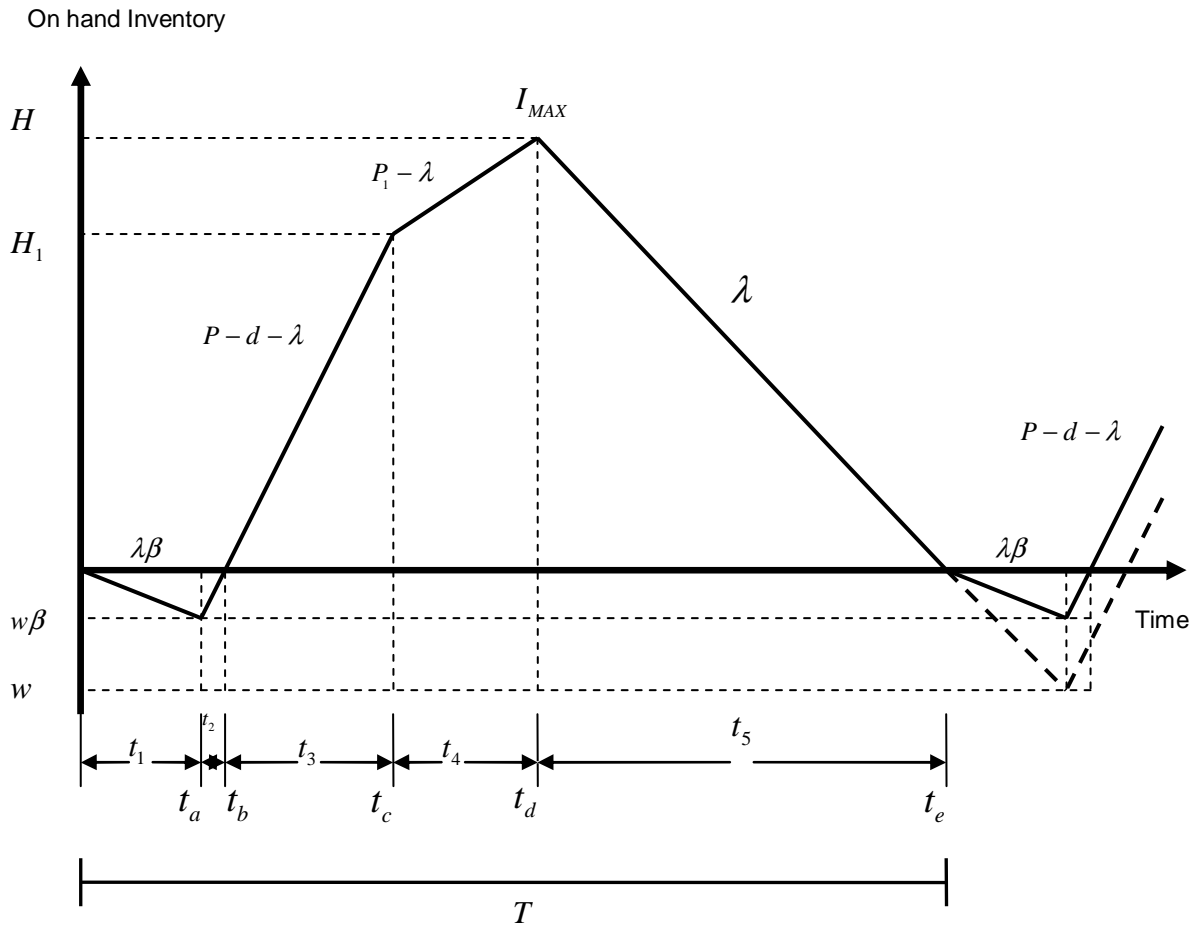


Figure 1. On-hand inventory of perfect quality item.

willing to wait to get the discount rate  $r$ .

6. The unit selling price of the products sold during the credit period is deposited in an interest bearing account with rate  $I_e$ . At the end of this period, the credit is settled and the company starts paying for the interest charges for the items in stock with rate  $I_p$ , where  $I_p > I_e$ .

On-hand inventory of perfect and imperfect quality items are shown in Figures 1 and 2, respectively.

The production rate of items of imperfect quality can be written as:

$$d = P \cdot x. \tag{1}$$

The production rate of good items is greater than or equal to the sum of the demand rate and the rate at which defective items are produced, that is,

$$P - d - \lambda \geq 0 \quad \text{or} \quad 0 \leq x \leq (1 - \frac{\lambda}{P}). \tag{2}$$

The cycle time is

$$T = t_1 + t_2 + t_3 + t_4 + t_5 \quad \text{and} \quad T = \frac{Q + w(1 - \beta)}{\lambda}. \tag{3}$$

The time of building up a backlogging level is

$$t_1 = \frac{w}{\lambda} \tag{4}$$

The time of eliminating the backlogging level is

$$t_2 = \frac{w\beta}{P - d - \lambda}. \tag{5}$$

The production time is

$$t_3 = \frac{H_1}{P - d - \lambda} = \frac{Q}{P} - \frac{w\beta}{P - d - \lambda}. \tag{6}$$

The original perfect quality items inventory level during a

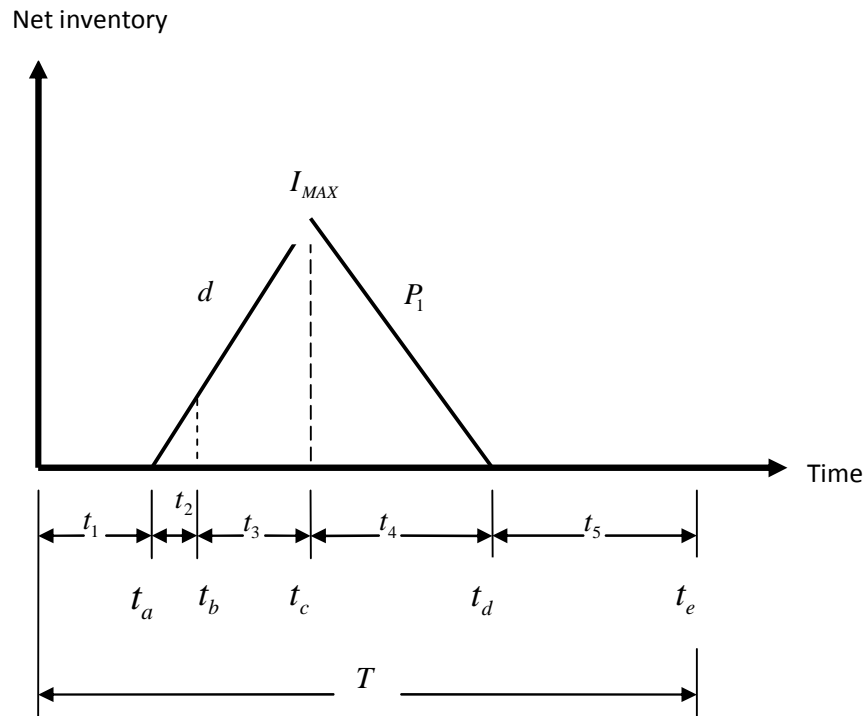


Figure 2. On-hand inventory of imperfect quality items.

cycle is

$$H_1 = (P - d - \lambda) \cdot \frac{Q}{P} - w\beta. \tag{7}$$

The rework time is

$$t_4 = \frac{d \cdot Q}{P \cdot P_1} = \frac{Q \cdot x}{P_1}. \tag{8}$$

The maximum inventory level is

$$H = Q \left[ 1 - \frac{\lambda(P_1 + d)}{PP_1} \right] - w\beta. \tag{9}$$

The time to consume  $H$  is

$$t_5 = \frac{H}{\lambda} = Q \left[ \frac{1}{\lambda} - \frac{P_1 + d}{PP_1} \right] - \frac{w\beta}{\lambda}. \tag{10}$$

Therefore,  $t_a = t_1 = \frac{w}{\lambda}$ ,  $t_b = t_1 + t_2 = \frac{w}{\lambda} + \frac{w\beta}{P - d - \lambda}$ ,

$$t_c = t_1 + t_2 + t_3 = \frac{w}{\lambda} + \frac{Q}{P},$$

$$t_d = t_1 + t_2 + t_3 + t_4 = \frac{w}{\lambda} + \frac{Q}{P} + \frac{Q \cdot x}{P_1} = \frac{w}{\lambda} + Q \left[ \frac{P_1 + d}{PP_1} \right],$$

$$t_e = t_1 + t_2 + t_3 + t_4 + t_5 = T = \frac{Q + w(1 - \beta)}{\lambda}.$$

The perfect quality items inventory level of production system can be described by the following differential equations:

$$\frac{dI_1(t)}{dt} = P - d - \lambda, \quad t_b \leq t \leq t_c \tag{11}$$

$$\frac{dI_2(t)}{dt} = P_1 - \lambda, \quad t_c \leq t \leq t_d; \tag{12}$$

$$\frac{dI_3(t)}{dt} = -\lambda, \quad t_d \leq t \leq t_e. \tag{13}$$

From the boundary conditions  $I_1(t_b) = 0$ ,  $I_2(t_d) = I_{MAX}$ , and  $I_3(t_e) = 0$ . We can solve differential equations as follows:

$$I_1(t) = (P - d - \lambda)(t - t_b), \quad t_b \leq t \leq t_c; \tag{14}$$

$$I_2(t) = I_{MAX} - (t_d - t)(P_1 - \lambda), \quad t_c \leq t \leq t_d; \quad (15)$$

$$I_3(t) = \lambda(t_e - t), \quad t_d \leq t \leq t_e. \quad (16)$$

From Equations 15 and 16 and the condition  $I_2(t_d) = I_3(t_d)$ , we obtain the maximum inventory level  $I_{MAX}$ :

$$I_{MAX} = \lambda(t_e - t_d). \quad (17)$$

Substituting Equation 17 into Equation 15, we obtain  $I_2(t) = \lambda(t_e - t_d) - (t_d - t)(P_1 - \lambda)$ . The imperfect quality items inventory level of production system can be described by the following differential equations:

$$\frac{dI_4(t)}{dt} = d, \quad t_a \leq t \leq t_c; \quad (18)$$

$$\frac{dI_5(t)}{dt} = -P_1, \quad t_c \leq t \leq t_d. \quad (19)$$

From boundary conditions  $I_4(t_a) = 0$  and  $I_5(t_d) = 0$ .

We solve the differential equations as follows:

$$I_4(t) = d(t - t_a), \quad t_a \leq t \leq t_c; \quad (20)$$

$$I_5(t) = P_1(t_d - t), \quad t_c \leq t \leq t_d. \quad (21)$$

The total annual cost consists of the following elements:

- a. Annual production cost =  $C \cdot \lambda$ .
- b. Annual repair cost =  $C_R \cdot \lambda \cdot x$ .
- c. Annual setup cost =  $K\lambda/[Q + w(1 - \beta)]$ .
- d. Annual holding cost =  $\frac{h\lambda}{Q + w(1 - \beta)} \cdot \left[ \frac{H_1 \cdot t_3 + (H_1 + H)}{2} \cdot t_4 + \frac{H \cdot t_5}{2} \right] + \frac{h_1 \lambda}{Q + w(1 - \beta)} \cdot \left[ \frac{d}{2} \cdot (t_2 + t_3)^2 + \frac{P_1 \cdot t_4^2}{2} \right]$ .
- e. The shortage cost for backlogged item =  $\frac{S_p \cdot r \cdot w \cdot (t_1 + t_2) \cdot \beta \cdot \lambda}{2[Q + w(1 - \beta)]}$ .
- f. Opportunity cost of lost sales =  $\frac{S_p w^2 (1 - \beta)(P - d + \lambda\beta)\lambda}{2\lambda(P - d - \lambda)[Q + w(1 - \beta)]}$ .
- g. There are four cases that are involved in interest charged and interest earned per year.

**Case 1:**  $t_b \leq M + t_b \leq t_c$

Annual interest charged:

$$IC_1 = \frac{C_p I_p}{T} \left[ \int_{M+t_b}^{t_c} ((P - d - \lambda)(t - t_b)) dt + \int_{t_c}^{t_d} (\lambda(t_e - t_d) - (t_d - t)(P_1 - \lambda)) dt + \int_{t_d}^{t_e} (\lambda(t_e - t)) dt + \int_{M+t_b}^{t_c} (d(t - t_a)) dt + \int_{t_c}^{t_d} (P_1(t_d - t)) dt \right]$$

$$= \frac{C_p I_p \lambda}{2[Q + w(1 - \beta)]} \left( -\frac{M^2 P^2 + Q^2}{P} + \frac{(Q - w\beta)^2}{\lambda} + M^2 \lambda - \frac{dw^2 \beta^2}{(d - P + \lambda)^2} + \frac{w\beta(w\beta - 2dM)}{P - d - \lambda} \right)$$

Annual interest earned

$$IE_1 = \frac{S_p (1 - r) I_e}{T} \left( \int_0^{t_a} \lambda \beta t dt \right) + \left( \lambda \beta t_a \frac{S_p (1 - r) I_e}{T} (M + t_b - t_a) \right) + \frac{S_p I_e}{T} \left( \int_{t_a}^{M+t_b} \lambda t dt \right)$$

$$= \frac{S_p I_e \lambda}{2[Q + w(1 - \beta)]} \left( \frac{w^2 \beta (1 - r)}{\lambda} + 2w\beta(1 - r) \left( M - \frac{w\beta}{P(x - 1) + \lambda} \right) + \lambda \left( -\frac{w^2}{\lambda^2} + \left( M + w \left( \frac{1}{\lambda} - \frac{\beta}{P(x - 1) + \lambda} \right) \right)^2 \right) \right)$$

**Case 2 :**  $t_c \leq M + t_b \leq t_d$ 

Annual interest charged

$$\begin{aligned}
 IC_2 &= \frac{C_p I_p}{T} \left[ \int_{M+t_b}^{t_d} (\lambda(t_e - t_d) - (t_d - t)(P_1 - \lambda)) dt + \int_{t_d}^{t_e} (\lambda(t_e - t)) dt + \int_{M+t_b}^{t_d} (P_1(t_d - t)) dt \right] \\
 &= \frac{C_p I_p \lambda ((d - P)(Q - w\beta) + \lambda(Q + MP - dM) - M\lambda^2)^2}{2\lambda(d - P + \lambda)^2 [Q + w(1 - \beta)]}.
 \end{aligned}$$

Annual interest earned

$$\begin{aligned}
 IE_2 &= \frac{S_p(1-r)I_e}{T} \left( \int_0^{t_a} \lambda\beta t dt \right) + \left( \lambda\beta t_a \frac{S_p(1-r)I_e}{T} (M + t_b - t_a) \right) + \frac{S_p I_e}{T} \left( \int_{t_a}^{M+t_b} \lambda t dt \right) \\
 &= \frac{S_p I_e \lambda}{2[Q + w(1 - \beta)]} \left[ \frac{w^2 \beta(1-r)}{\lambda} + 2w\beta(1-r) \left( M - \frac{w\beta}{P(x-1) + \lambda} \right) + \right. \\
 &\quad \left. \lambda \left( -\frac{w^2}{\lambda^2} + \left( M + w \left( \frac{1}{\lambda} - \frac{\beta}{P(x-1) + \lambda} \right) \right)^2 \right) \right].
 \end{aligned}$$

**Case 3:**  $t_d \leq M + t_b \leq t_e$ 

Annual interest charged

$$\begin{aligned}
 IC_3 &= \frac{C_p I_p}{T} \left[ \int_{M+t_b}^{t_d} (\lambda(t_e - t_d) - (t_d - t)(P_1 - \lambda)) dt + \int_{t_d}^{t_e} (\lambda(t_e - t)) dt + \int_{M+t_b}^{t_d} (P_1(t_d - t)) dt \right] \\
 &= \frac{C_p I_p \lambda ((d - P)(Q - w\beta) + \lambda(Q + MP - dM) - M\lambda^2)^2}{2\lambda(d - P + \lambda)^2 [Q + w(1 - \beta)]}.
 \end{aligned}$$

Annual interest earned

$$\begin{aligned}
 IE_3 &= \frac{S_p(1-r)I_e}{T} \left( \int_0^{t_a} \lambda\beta t dt \right) + \left( \lambda\beta t_a \frac{S_p(1-r)I_e}{T} (M + t_b - t_a) \right) + \frac{S_p I_e}{T} \left( \int_{t_a}^{M+t_b} \lambda t dt \right) \\
 &= \frac{S_p I_e \lambda}{2[Q + w(1 - \beta)]} \left[ \frac{w^2 \beta(1-r)}{\lambda} + 2w\beta(1-r) \left( M - \frac{w\beta}{P(x-1) + \lambda} \right) + \right. \\
 &\quad \left. \lambda \left( -\frac{w^2}{\lambda^2} + \left( M + w \left( \frac{1}{\lambda} - \frac{\beta}{P(x-1) + \lambda} \right) \right)^2 \right) \right].
 \end{aligned}$$

**Case 4:**  $T < M + t_b$ Annual interest charged=0.  
Annual interest earned

$$\begin{aligned}
 IE_4 &= \frac{S_p(1-r)I_e}{T} \left( \int_0^{t_a} \lambda \beta t dt \right) + \left( \lambda \beta t_a \frac{S_p(1-r)I_e}{T} (M+t_b-t_a) \right) + \frac{S_p I_e}{T} \left( \int_{t_a}^{t_e} \lambda t dt \right) + \\
 &\quad \left( \lambda(t_e-t_a) \frac{S_p I_e}{T} (M+t_b-t_e) \right) \\
 &= \frac{S_p I_e \lambda}{2\lambda(P(1-x)-\lambda)[Q+w(1-\beta)]} \left[ \begin{aligned} &P(x-1) \left( \frac{Q^2 - 2Q(w+w\beta+M\lambda)}{w\beta(w(1+r+\beta)+2Mr\lambda)} + \right) \\ &\lambda \left( \frac{Q^2 - 2Q(w+M\lambda)}{w\beta(w(1+r+\beta-2r\beta)+2Mr\lambda)} \right) \end{aligned} \right].
 \end{aligned}$$

Therefore, total annual cost  $TVC(Q)$  has four different as follows:

**Case 1:**  $t_b \leq M + t_b \leq t_c$

$$\begin{aligned}
 TVC_1(Q, w) &= C \cdot \lambda + C_R \cdot \lambda \cdot x + K\lambda/[Q+w(1-\beta)] + \\
 &\quad \frac{S_p \cdot r \cdot w \cdot (t_1+t_2) \cdot \beta \cdot \lambda}{2[Q+w(1-\beta)]} +
 \end{aligned}$$

$$\frac{h\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{H_1 \cdot t_3}{2} + \frac{(H_1+H)}{2} \cdot t_4 + \frac{H \cdot t_5}{2} \right] + \frac{h_1\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{d}{2} \cdot (t_2+t_3)^2 + \frac{P_1 \cdot t_4^2}{2} \right]$$

$$+ \frac{S_p w^2 (1-\beta)(P-d+\lambda\beta)\lambda}{2\lambda(P-d-\lambda)[Q+w(1-\beta)]} + IC_1 + IE_1; \quad (22)$$

$$\begin{aligned}
 TVC_2(Q, w) &= C \cdot \lambda + C_R \cdot \lambda \cdot x + K\lambda/[Q+w(1-\beta)] + \\
 &\quad \frac{S_p \cdot r \cdot w \cdot (t_1+t_2) \cdot \beta \cdot \lambda}{2[Q+w(1-\beta)]} +
 \end{aligned}$$

**Case 2:**  $t_c \leq M + t_b \leq t_d$

$$\frac{h\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{H_1 \cdot t_3}{2} + \frac{(H_1+H)}{2} \cdot t_4 + \frac{H \cdot t_5}{2} \right] + \frac{h_1\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{d}{2} \cdot (t_2+t_3)^2 + \frac{P_1 \cdot t_4^2}{2} \right]$$

$$+ \frac{S_p w^2 (1-\beta)(P-d+\lambda\beta)\lambda}{2\lambda(P-d-\lambda)[Q+w(1-\beta)]} + IC_2 + IE_2; \quad (23)$$

$$\begin{aligned}
 TVC_3(Q, w) &= C \cdot \lambda + C_R \cdot \lambda \cdot x + K\lambda/[Q+w(1-\beta)] + \\
 &\quad \frac{S_p \cdot r \cdot w \cdot (t_1+t_2) \cdot \beta \cdot \lambda}{2[Q+w(1-\beta)]} +
 \end{aligned}$$

**Case 3:**  $t_d \leq M + t_b \leq t_e$

$$\frac{h\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{H_1 \cdot t_3}{2} + \frac{(H_1+H)}{2} \cdot t_4 + \frac{H \cdot t_5}{2} \right] + \frac{h_1\lambda}{Q+w(1-\beta)} \cdot \left[ \frac{d}{2} \cdot (t_2+t_3)^2 + \frac{P_1 \cdot t_4^2}{2} \right]$$

$$+ \frac{S_p w^2 (1 - \beta)(P - d + \lambda \beta) \lambda}{2\lambda(P - d - \lambda)[Q + w(1 - \beta)]} + IC_3 + IE_3; \quad (24)$$

**Case 4:**  $T < M + t_b$

$$TVC_4(Q, w) = C \cdot \lambda + C_R \cdot \lambda \cdot x + K\lambda/[Q + w(1 - \beta)] + \frac{S_p \cdot r \cdot w \cdot (t_1 + t_2) \cdot \beta \cdot \lambda}{2[Q + w(1 - \beta)]} + \frac{h\lambda}{Q + w(1 - \beta)} \cdot \left[ \frac{H_1 \cdot t_3}{2} + \frac{(H_1 + H)}{2} \cdot t_4 + \frac{H \cdot t_5}{2} \right] + \frac{h_1 \lambda}{Q + w(1 - \beta)} \cdot \left[ \frac{d}{2} \cdot (t_2 + t_3)^2 + \frac{P_1 \cdot t_4^2}{2} \right] + \frac{S_p w^2 (1 - \beta)(P - d + \lambda \beta) \lambda}{2\lambda(P - d - \lambda)[Q + w(1 - \beta)]} + IC_4 + IE_4. \quad (25)$$

In Figure 1, the study assumes that the rate of rework of imperfect quality items is larger than the demand rate, that is,  $P_1 > \lambda$ . When  $P_1 < \lambda$  or  $P_1 = \lambda$  (as shown in Figures 3 and 4, respectively), following the same modeling steps mentioned above, we can obtain that results of  $P_1 < \lambda$  or  $P_1 = \lambda$  are the same as that of  $P_1 > \lambda$ . Therefore, the models (Equations 22, 23, 24 and 25) are the same no matter,  $P_1 > \lambda$ ,  $\lambda > P_1$  or  $P_1 = \lambda$ .

**SOLUTION APPROACH**

In this paper, the study considers that the manufacturer wants to determine the optimal production lot size  $Q^*$  and maximum backlogging level  $w^*$  to minimize the total cost  $TVC(Q, w)$ . The problem is to maximize

$$TVC(Q) = \begin{cases} TVC_1(Q, w) & \text{if } t_b \leq M + t_b \leq t_c \\ TVC_2(Q, w) & \text{if } t_c \leq M + t_b \leq t_d \\ TVC_3(Q, w) & \text{if } t_d \leq M + t_b \leq t_e \\ TVC_4(Q, w) & \text{if } T < M + t_b \end{cases} \text{ which is a}$$

four-branch function with two variables. Due to the very complex form of  $TVC(Q)$ , it is not easy to check the convexity from  $\frac{\partial^2 TVC(Q)}{\partial Q^2}$  directly. Instead, the study

develops an algorithm to find the optimal solution. From the optimization theorem in Calculus, the boundaries (endpoints) and the solutions of  $\frac{\partial TVC(Q, w)}{\partial Q} = 0$  and

$\frac{\partial TVC(Q, w)}{\partial w} = 0$  are candidates for optimal solution. We

developed a three-dimensional searched-based algorithm to find out  $Q^*$  and  $w^*$ . First, the study finds the extreme values and the boundary values of the four cases. If the

extreme value does not fit its restricted condition, it is excluded. Finally, we compare the boundary values and the extreme values that fit the restricted condition to determine the optimal solution. The algorithm is as follows:

**Three-dimensional Searched-based Algorithm :**

Step 1: Find the minimum point for Case 1.

Determine the extreme values by solving  $\frac{\partial TVC_1}{\partial Q} = 0$  and

$\frac{\partial TVC_1}{\partial w} = 0$  such that the Q satisfies  $t_b \leq M + t_b \leq t_c$  with respect to extreme value.

Let  $TVC_1(Q_1^*, w_1^*)$  associate with the minimum point, or any of boundary points, which gives the smallest value of  $TVC_1(Q, w_1)$ .

Step 2: Find the minimum point for Case 2.

Determine the extreme values by solving  $\frac{\partial TVC_2}{\partial Q} = 0$  and

$\frac{\partial TVC_2}{\partial w} = 0$  such that the Q satisfies  $t_c \leq M + t_b \leq t_d$  with respect to extreme value.

Let  $TVC_2(Q_2^*, w_2^*)$  associate with the minimum point, or any of boundary points, which gives the smallest value of  $TVC_2(Q, w_2)$ .

Step 3: Find the minimum point for Case 3.



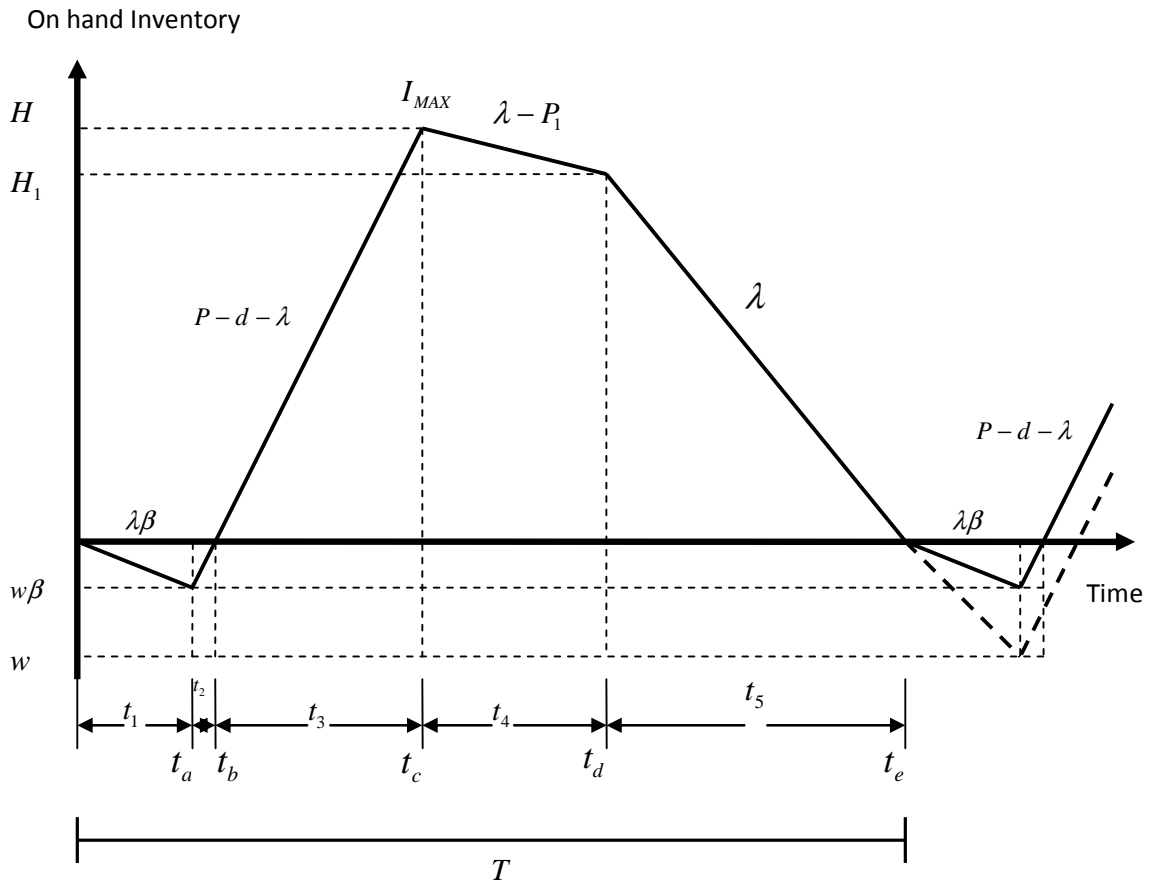


Figure 3. On-hand inventory of perfect quality items when  $P_1 < \lambda$ .

Determine the extreme values by solving  $\frac{\partial TVC_3}{\partial Q} = 0$  and  $\frac{\partial TVC_3}{\partial w} = 0$  such that the  $Q$  satisfies  $t_d \leq M + t_b \leq t_e$  with respect to extreme value.

Let  $TVC_3(Q_3^*, w_3^*)$  associate with the minimum point, or any of boundary points, which gives the smallest value of  $TVC_3(Q_3, w_3)$ .

Step 4: Find the minimum point for Case 4.

$$\text{Let } TVC(Q^*, w^*) = \text{Min}\{TVC_1(Q_1^*, w_1^*), TVC_2(Q_2^*, w_2^*), TVC_3(Q_3^*, w_3^*), TVC_4(Q_4^*, w_4^*)\}.$$

**EMPIRICAL RESULTS**

This section presents a numerical study to illustrate the

Determine the extreme values by solving  $\frac{\partial TVC_4}{\partial Q} = 0$  and  $\frac{\partial TVC_4}{\partial w} = 0$  such that the  $Q$  satisfies  $T < M + t_b$  with respect to extreme value.

Let  $TVC_4(Q_4^*, w_4^*)$  associate with the minimum point, or any of boundary points, which gives the smallest value of  $TVC_4(Q_4, w_4)$ .

Step 5:

proposed solution approach and provide quantitative insights. The goals of the numerical study in this study are as follows:

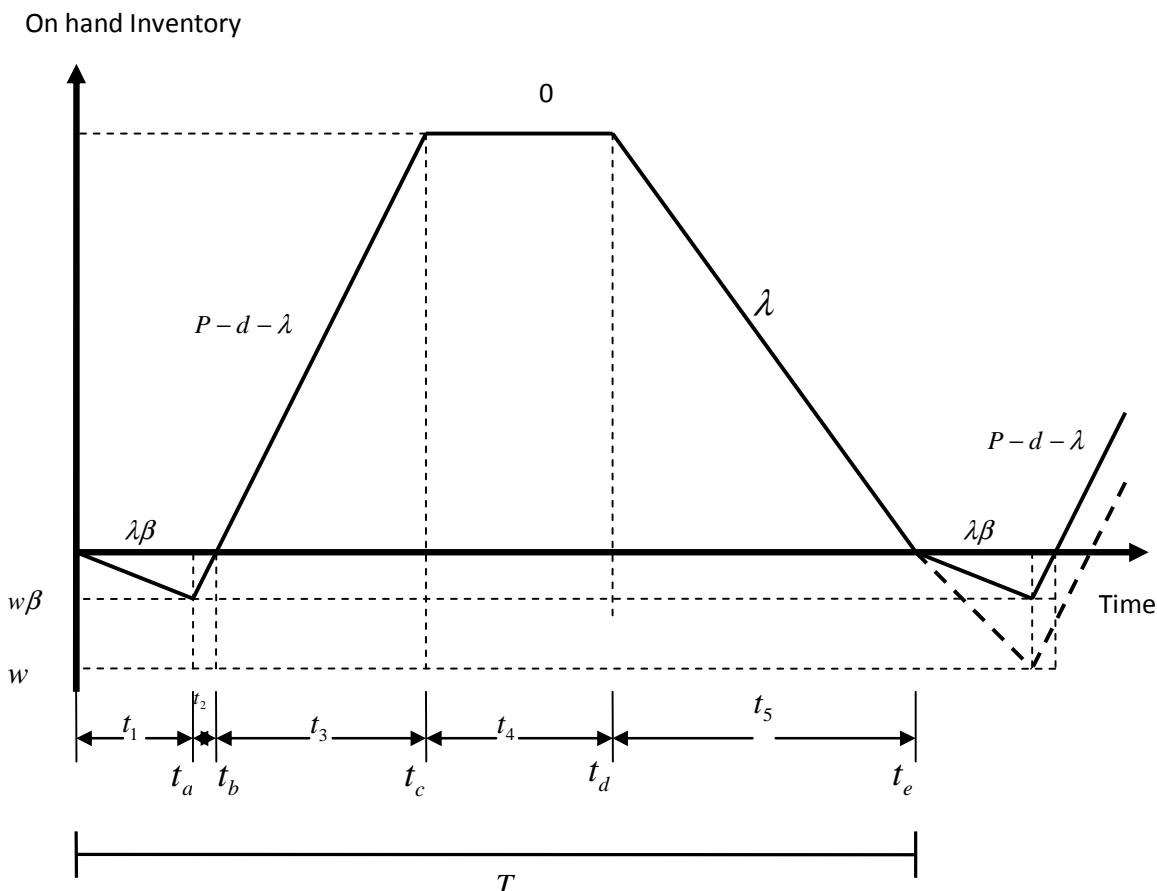


Figure 4. On-hand inventory of perfect quality items when  $P_1 = \lambda$ .

1. To illustrate the procedures of the solution approach.
2. To discuss the impacts of different values of parameters on decisions and cost and use different values of parameters to show more examples rather than one case.

Most values of the parameter setting follow the examples of Hayek and Salameh (2001) and Tsao et al. (2011). The study considers the following case for empirical illustration: A manufacturer producing a product which has a constant demand rate of 400 units per year. The machine used to manufacture this item has a production rate of 1600 units per year. The production cost per item is \$104. The machine setup cost is \$1500. The holding cost per perfect quality item is \$20 per year, and the holding cost per imperfect quality item is \$22 per year. The repair cost per imperfect quality item is \$8. The imperfect quality items are reworked at a rate of 700 units per year. The percentage of imperfect quality items produced is 5%. The production rate of imperfect quality items is 80. The manufacturer's trade credit period offered by the supplier is 0.1 year. The annual interest rate that can be earned per dollar is 0.1. The annual interest rate that can be charged per dollar is 0.15.

The unit cost per item is \$120. The unit selling price per item of good quality is \$180. The discount rate for the customers that are willing to wait for the next replenishment is 30%. The proportion of permissible backlogging is 90%.

The results computed using three-dimensional searched-based algorithm is as follows. The optimal solution in Case 1 is  $(Q_1^*, w_1^*) = (160, 0)$  and its total cost is 47432.2; the optimal solution in Case 2 is  $(Q_2^*, w_2^*) = (150.725, 0)$  and its total cost is 46948.1; the optimal solution in Case 3 is  $(Q_3^*, w_3^*) = (252.89, 98.55)$  and its total cost is 45435.7; the optimal solution in Case 4 is  $(Q_4^*, w_4^*) = (40, 0)$  and its total cost is 56700.6. Therefore, the optimal solution is  $(Q^*, w^*) = (Q_3^*, w_3^*) = (252.89, 98.55)$  and  $TVC(Q^*, w^*) = TVC_3(Q_3^*, w_3^*) = 45435.7$ . A graphic illustration of  $TVC_3(Q_3, w_3)$  is shown in Figure 5.

It is interesting to investigate the influences of the interest charged  $I_p$ , interest earned  $I_e$  and percentage of imperfect quality items  $x$  on the production decisions

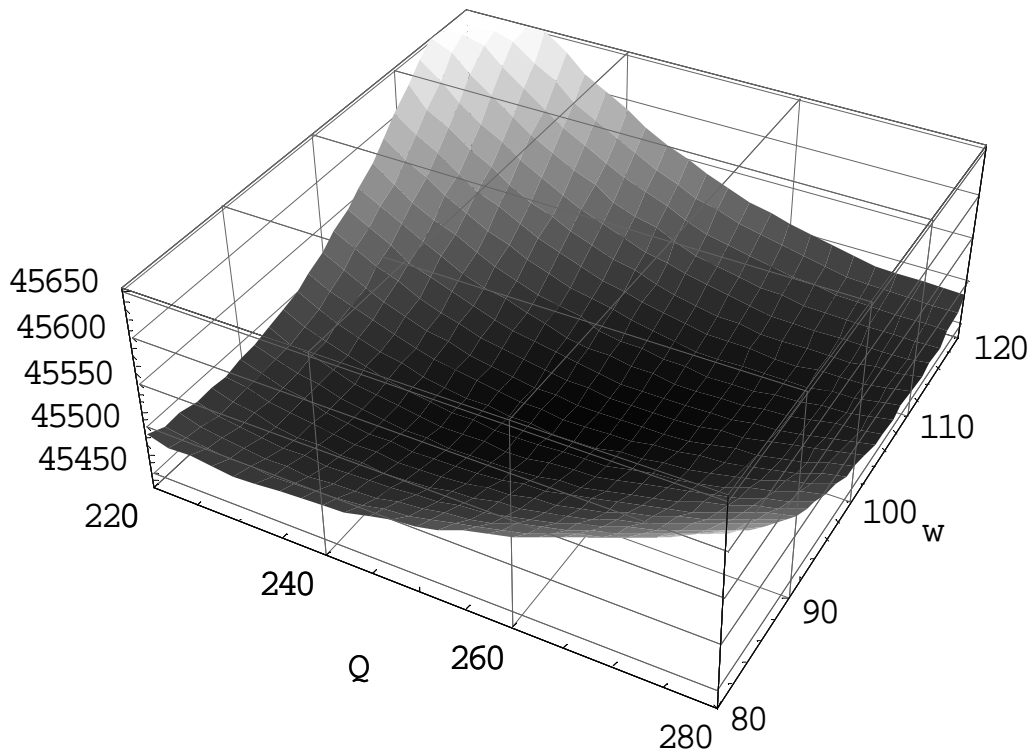


Figure 5. Graphic illustration of  $TVC_3(Q_3, w_3)$ .

Table 1. Effect of  $I_p$  and  $I_e$  on  $Q$  and  $w$ .

$(Q, w)$		$I_p$			
		0.125	0.150	0.175	0.200
$I_e$	0.090	(256.984,93.130)	(250.721,94.296)	(245.383,95.323)	(240.776,96.233)
	0.095	(257.969,95.436)	(251.783,96.585)	(246.512,97.545)	(241.964,98.489)
	0.100	(258.996,97.825)	(252.890,98.955)	(247.689,99.946)	(243.204,100.824)
	0.105	(260.070,100.301)	(254.047,101.410)	(248.920,102.383)	(244.499,103.243)

Table 2. Effect of  $I_p$  and  $I_e$  on  $TVC$ .

$TVC(Q, w)$		$I_p$			
		0.125	0.150	0.175	0.200
$I_e$	0.090	45,494.1	45,550.3	45,599.8	45,643.7
	0.095	45,439.9	45,493.8	45,541.3	45,583.4
	0.100	45,384.0	45,435.7	45,481.1	45,521.3
	0.105	45,326.3	45,375.7	45,419.0	45,457.4

and total cost. The effects of  $I_p$  and  $I_e$  on  $(Q, w)$  and  $TVC$  are given in Tables 1 and 2, respectively. We can observe that  $Q$  will decrease (negative effect) as  $I_p$  increases, while  $w$  and  $TVC$  will increase (positive effect)

as  $I_p$  increases. If the interest rate that can be charged increase, it is reasonable that the company decreases the production lot size to reduce the interest charges for the items in stock. On the other hand, both  $Q$  and  $w$  will increase (positive effect) and  $TVC$  will decrease (negative

**Table 3.** Effect of  $x$  on  $Q$  and  $TVC$ .

	$x$				
	0.03	0.04	0.05	0.06	0.07
$Q$	252.795	252.842	252.890	252.939	252.990
$w$	99.180	99.068	98.955	98.838	98.720
$TVC(Q, w)$	45,372.5	45,404.1	45,435.7	45,467.2	45,498.8

effect) as  $I_e$  increases. If the interest rate that can be earned increase, it is reasonable that the company increases the production lot size to take advantage of the larger interest earned. The effects of  $x$  on  $Q$ ,  $w$  and  $TVC$  are shown in Table 3. As  $x$  increases,  $Q$  and  $TVC$  will increase (positive effect) while  $w$  will decrease (negative effect). If the percentage of imperfect quality items increase, the manufacturer should increase production lot size to satisfy customer demand.

## CONCLUSION

This study considers the production problem under trade credit, partial backlogging and reworking of imperfect quality items. We extend the traditional production model by considering reworking of imperfect quality items, partial backlogging and trade credit to cope with more realistic situation. The objective is to determine the optimal production lot size and maximum backlogging level simultaneously while still minimizing the total cost. Three-dimensional searched-based algorithm is developed to solve the problem described. Some numerical analyses were used to show the influences of the interest charged, interest earned and the percentage of imperfect quality items on the production decisions and total cost.

The contributions of this paper to the literature and managerial decision making are as follows. First of all, this is the first study to consider reworking of imperfect quality items, partial backlogging and trade credit simultaneously. Secondly, the study provides an easy-to-use the algorithm named three-dimensional searched-based algorithm to solve the problem described. Thirdly, it verifies the impacts of the interest charged, interest earned and the percentage of imperfect quality items on the production decisions and total cost. These results should be a useful reference for managerial decisions and administrations.

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