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Full Length Research Paper

A supply chain model with defective items and disposal cost in a just-in-time (JIT) environment

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Previous studies in the issue of supply chain models with imperfect quality assumed the defectives could be sold at lower unit price by the end of the inspection process and the lot-for-lot delivery policy was employed. However, in practice, defective items may have no salvage value but may need additional disposal cost to process them. The beginning of the printed circuit board (PCB) industry in Taiwan provides a good example of such a situation. Besides, in a just-in-time (JIT) environment, the supplier splits the order quantity into small lot size and delivers them over multiple periods to response the buyer's need. Therefore, in this paper, a new mathematical model were developed, in which defective items are processed with disposal cost and lot-splitting (single setup, multiple delivers) policy is employed, to determine an optimal single-supplier- single-buyer inventory policy. The objective is to minimize the total joint annual cost incurred by the vendor and buyer in a JIT environment. A procedure free of employing convexity is developed to determine the order quantity, the number of deliveries and the shipping quantity. Numerical example was provided to illustrate the effectiveness of the proposed model. Based on the numerical example, sensitivity analysis was given to investigate the effects of the defective percentage on the optimal solution.

Key words: Supply chain, JIT, defective item, inventory.

INTRODUCTION

Since the advent of just-in-time (JIT) in the early 1980s, there have been numerous studies discussing JIT implementation. Recently researchers have extensively studied small lot sizing as a means of implementing successful JIT system (Kim and Ha, 2003). Pan and Liao (1989) were probably the first to use an economic order quantity (EOQ) model that delivered the order in multiple shipments to illustrate order splitting in the just-in-time (JIT) system. Ramasesh (1990) extended Pan and Liao's model (1989) to consider a multiple-shipment model with transportation cost. Kim and Ha (1997) proposed a model in which they considered the aggregate total relevant cost for both buyers and suppliers for frequent deliveries. Kim and Ha (2003) further proposed a new coordinated model that enhanced the linkage between the buyer and supplier. Van Nieuwenhuyse and Vandaele (2006) demonstrated that multiple shipments could increase the supplier's delivery reliability. However, one of the unrealistic assumptions in the above joint inventory models is that all produced units are of good quality. In fact, theprocess may deteriorate and thus, produce defects or imperfect quality items. Thus, recently, the inventory model with imperfect quality or defective has received the attention of researchers.

Although, there are many researchers (for example, Salameh and Jaber, 2000; Chan et al, 2003; Chang, 2004; Wee 2007, Maddah and Jaber, 2008) who focused on the imperfect quality issue, all of them optimized only one side (buyer) of the inventory pattern and neglected the entire inventory system pattern for both buyer and supplier. Huang (2004) was probably the first proponent to simultaneously consider the flawed items and joint buyer-supplier inventory. He pointed out that his approach had not been considered in previous research.

Later, Chung (2008) provided necessary and sufficient evidence for the existence of an optimal solution procedure for Huang's (2004) inventory model. More recently, Lin (2009) extended Huang's model into the case with inspection errors and returned cost. He also made a sensitivity analysis to investigate the effects of some parameters on the optimal solution. Note that all of the above papers discussed earlier assumed that the imperfect/defective quality item could be sold using a salvage value. However, imperfect/ defective items often have no salvage value and additional disposal cost must be paid to process them in many industries. The beginning of the printed circuit board (PCB) industry in Taiwan provides a good example. As a known fact, when PCB waste materials are burned using an improper process, the airborne chemicals and Dioxins will grievously harm health of human-beings by polluting the environment. Thus, the manufacturer must pay additional cost for processing PCB waste substrates by entrusting waste disposal to a specialized organization. More recently, the eco-conscious trend has garnered great concern and thus, the study believes that processing these wastes has no salvage value and also has great disposal costs. Unlike the traditional EOQ model with imperfect quality, this paper develops a model, in which a 100% screening process is performed, under defective item and disposal cost considerations. Based on the defective items with disposal cost consideration, this paper investigates the effects of a JIT lot-splitting (Single setup, multiple delivery) strategy on the integrated total relevant buyer- supplier cost by examining the optimal order lot size, the number of deliveries and the shipment quantity under the defective items with disposal cost consideration. An algorithm free of using convexity is also developed to determine the optimal solution.

NOTATION AND ASSUMPTIONS

Notation

C, supplier's hourly setup cost; **D,** annual demand rate for buyer; **F,** fixed transportation cost per trip; **H_B,** holding cost /unit/year for buyer; **H_S,** holding cost /unit/year for supplier, $H_B > H_S$; **K,** order cost for buyer; **M,** annual production rate for supplier, M>D; **N,** number of deliveries per batch cycle (integer value); **Q,** order quantity for buyer; **R,** unit variable cost for processing and receiving per order; **S,** setup time/setup for supplier; **T,** planning horizon; **b,** the disposal cost for defective items per unit; **d,** the screening cost per unit; **q,** Delivery size per trip, q=Q/N; **x,** the screen rate, x >D; **p,** the defective percentage in Q; **f(p),** the probability density function of p.

Assumptions

(1) Supply chain system consists of a single supplier and a single buyer
(2) Demand for the item is constant over time
(3) Production rate is uniform and finite
(4) Successive shipments are scheduled so that the next delivery arrives at the buyer when the stock from the previous delivery has just been used up
(5) The number of perfect items is at least equal to the demand during screening time
(6) Time horizon is assumed to be infinite
(7) Shortages are not allowed

MATHEMATICAL MODEL

Referring to Kim and Ha (2003), the top half of Figure 1 in their work shows the buyer's inventory, while the bottom half displays the supplier's. However, in this paper, because defective items exist in each lot, their figure should be modified as Figure 1.

Let $TC(Q,N)_{buyer}$ and $TC(Q,N)_{supplier}$ denote the total cost for buyer and supplier, respectively. Note that $TC(Q,N)_{buyer}$ is the sum of the order cost, screening cost, receiving cost, and holding cost. The holding cost contains two parts: (1) the perfect items in each lot (2) and the defective items before they are screened out. Therefore, the total cost for the buyer can be written as follows:

$$TC(Q,N)_{Buyer} = K + dQ + RQ + \frac{H_B[Q(1-p)]^2}{2DN} + \frac{H_B pQ^2}{xN} \quad (1)$$

Let $TC(Q,N)_{supplier}$ be the sum of the setup cost, transportation cost, disposal cost and holding cost. Thus, the total cost for the supplier can be written as follows:

$$TC(Q,N)_{supplier} = CS + NF + bpQ + \frac{H_S Q^2}{D}\left[\frac{1}{2}\left((1-p) - \frac{D}{M}\right) + \frac{1}{N}\left(\frac{D}{M} - \frac{1-p}{2}\right)\right] \quad (2)$$

Where the expression for the holding cost is derived by David and Eben-Chaime (2003). Adding Equations (1) and (2) leads to the joint annual total cost function for the supplier and buyer. Therefore, the average cost per unit time can be given as follows:

$$TCU(Q,N)_{joint} = \frac{TC(Q,N)_{buyer} + TC(Q,N)_{supplier}}{T} \quad (3)$$

Because the replenishment cycle time is $T = Q(1-p)/D$, one can take the expected value of $TCU(Q,N)_{joint}$ with

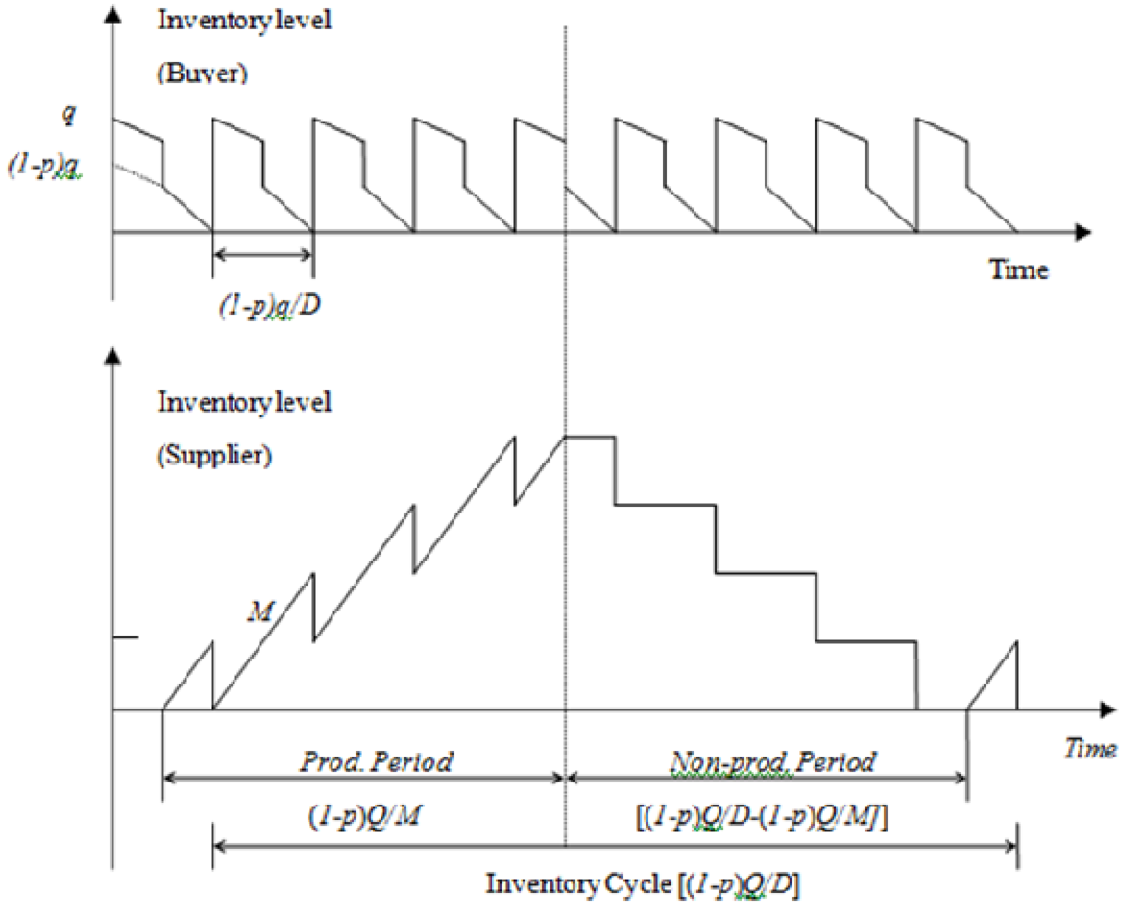

Figure 1. Time-weighted inventory for vendor and buyer.

respect to *p,* leading to:

$$ETCU(Q,N)=\frac{E\left[TCU(Q,N)_{joint}\right]}{E[T]}=\frac{D(K+CS+NF)}{QE_1}+\frac{D(d+bE[p])}{E_1}+\frac{H_B QE_2}{2NE_1}+\frac{RD}{E_1}$$
$$+\frac{H_S Q[(1-D/M)/2+(D/M-1/2)/N]}{E_1}+\frac{H_B QDE[p]}{xNE_1} \qquad (4)$$

Where $E_1=1-E[p],\ E_2=E\left[(1-p)^2\right]$.

Note that if all items are perfect (that is, no screening process is needed and no salvage value occurs), and Kim and Ha's (2003) model is a special case of the study model. However, it is not easy to show that the Hessian matrix in Equation (4) is positively definite. Therefore, an algorithm free of using convexity is developed to determine the optimal solution. This algorithm can further improve the flaws in Kim and Ha's (2003) solution procedure because they neglected the truth that Q and N are "simultaneously" obtained and misused Q, which may improperly obtained from Equation (4) in Kim and Ha's paper, as the optimal order quantity for the buyer.

THE SOLUTION PROCEDURE

In this section, the study develops a solution procedure, which is similar to that of Chung (2008), to find the optimal solution.

Let N be fixed. Taking the derivative of $ETCU(Q,N)$ with respect to Q leads to:

$$\frac{dETCU(Q,N)}{dQ}=\frac{-D(K+CS+NF)}{Q^2E_1}+\frac{\Psi(N)}{2E_1} \qquad (5)$$

Where,
$\Psi(N)=[H_B(E_2+2DE[p]/x)-H_S(E_1-2\lambda)]/N+(E_1-\lambda)H_S$

Let

$$Q^*(N)=\sqrt{\frac{2DN(A+CS+NF)}{[H_B(E_2+2DE[p]/x)-H_S(1-2\lambda)]/N+(1-\lambda)H_S}} \quad (6)$$

Equation (6) will yield

$$\frac{dETCU(Q,N)}{dQ}\begin{cases}<0 & if \quad 0<Q<Q^*(N),\\ =0 & if \quad Q=Q^*(N),\\ >0 & if \quad Q>Q^*(N),\end{cases} \quad \text{(7a-c)}$$

Equations (7a - c) reveal that $ETCU(Q,N)$ is decreasing on $(0,Q^*(N)]$ and increasing on $[Q^*(N),\infty)$ if N is fixed. Therefore, when N is fixed, $ETCU(Q,N)$ will have the optimal solution at $Q=Q^*(N)$.

Substituting Equation (6) into Equation (4) and rearranging the result leads to

$$ETCU(N)=\left\{\sqrt{2D(A+CS+NF)\Psi(N)}+D(d+bE[p]+R)\right\}/E_1 \quad (8)$$

Ignoring the term independent of N and taking the square of $ETCU(N)$ in Equation (8), minimizing $ETCU(N)$ is equivalent to minimizing the following:

$$\varphi(N)=\frac{(K+CS)[H_B(E_2+2DE[p]/x)-H_S(E_1-2\lambda)]}{N}+FH_SN(E_1-\lambda) \quad (9)$$

If N is treated as a continuous variable, taking the derivative of $\varphi(N)$ with respect to N, the outcome is:

$$\frac{d\varphi(N)}{dN}=FH_S(E_1-\lambda)-\frac{(K+CS)[H_B(E_2+2DE[p]/x)-H_S(E_1-2\lambda)]}{N^2} \quad (10)$$

Furthermore, taking the derivative of $\Psi(N)$ with respect to N will lead to

$$\frac{d\Psi(N)}{dN}=\frac{-[H_B(E_2+2DE[p]/x)-H_S(E_1-2\lambda)]}{N^2} \quad (11)$$

Since $H_B>H_S$ and $(E_1-2\lambda)<1$, Eq. (11) is less than zero. It implies $\Psi(N)$ is decreasing on $[1,\infty)$, and
$\Psi(1)=H_B-H_S(E_1-2\lambda)+H_S(E_1-\lambda)>\Psi(N)>0$ (12)

Further, let

$$\Omega=\sqrt{\frac{(A+CS)[H_B(E_2+2DE[p]/x)-H_S(E_1-2\lambda)]}{FH_S(1-\lambda)}} \quad (13)$$

Thus, one has

$$\frac{d\varphi(N)}{dN}\begin{cases}<0 & if \quad 0<N<\Omega\\ =0 & if \quad N=\Omega\\ >0 & if \quad N>\Omega\end{cases} \quad \text{(14a-c)}$$

Equations (14a - c) imply the following Property holds.

Property 1: $\varphi(N)$ is decreasing on $(0,\Omega]$ and increasing on $[\Omega,\infty)$.

Now, let:

$$N_1^*=\lfloor\Omega\rfloor, \quad (15)$$

Where, $\lfloor\Omega\rfloor$ is the greatest integer less than or equal to Ω. Because the number of deliveries (N) is an integer, Property 1 and Equation (12) show that the optimal solution (N^*,Q^*) of $ETCU(Q,N)$ can be determined as follows:

$N^*=N_1^*$ or $N^*=N_1^*+1$ according to $\varphi(N^*)=\min\{\varphi(N_1^*),\varphi(N_1^*+1)\}$. Further, $Q^*=Q^*(N^*)$

This is then determined by Equation (6)

Furthermore, according to the same procedures suggested by Kim and Ha (2003), the minimum order quantity can be obtained easily as follows:

$$Q\geq\sqrt{\frac{2DN^*F}{H_B(E_2+2DE[p]/x)+H_S(2\lambda-E_1)}}=Q_{\min} \quad (16)$$

Therefore, if the order quantity is larger than $Q_{\min}$, the single-setup-multiple- delivery (SSMD) policy is superior to a single delivery policy.

Again, employing the same steps suggested by Kim and Ha (2003), the optimal delivery size q*, which remains the same over multiple deliveries, is obtained by dividing Q^* by N^*, as follows:

$$q^* = \frac{Q^*(N^*)}{N^*} \quad (17)$$

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

The proceeding developed model and theorem can be illustrated using the numerical example modified from Kim and Ha (2003). The parameters are as follows:

Production rate: $M = 19200$ units/year,
Demand rate: $D = 4800$ units/year,
Setup cost: $C = \$20$ /hour,
Ordering cost: $K = \$25$ /cycle,
Holding cost for supplier: $H_S = \$6$ /unit/year,
Holding cost for buyer: $H_B = \$7$ /unit/year,
Transportation cost: $F = \$50$ /trip,
Receiving cost: $R = \$1$ /unit.

Note that the supplier currently spends 6 hours with five workers to set up the production system. Thus, the one time setup cost is \$600 (\$20/hour × 5 worker × 6 hour). In addition, this study set the screening rate x = 87600 units/year, screening cost d = *\$0.5*/unit, disposal cost $b = \$10$ /unit, and assume that the defective percentage is uniformly distributed with *p.d.f.* as *f(p)* = 25, 0 ≤ p ≥ 0.04.. Then, one has $E[Y] = \int_0^{0.04} 25\, y dy = 0.02$. Using the procedure developed in Section 4, the study have the optimal order size Q^*=1137 and the optimal number of deliveries N^*=3. Thus, the delivery size is 379 units per trip. The expected total cost per year is \$15,577. Compared with the results provided by Kim and Ha (2003), the smaller the order quantity, the greater cost obtained in the study model with the same number of shipments. This result illustrates that optimal lot sizes are less than those in Kim and Ha (2003) if disposal cost occurs. This is because the buyer may spend extra cost to process the defective items in each cycle. Notice that, using Equation (16), the order quantity must be greater than 620 units. Otherwise, a single delivery policy would be preferable.

Comparison with Kim and Ha's model

Accounting for the defective quality items alters the minimum order lot size and the ratio of the modified number of deliveries. As discussed earlier, N^*, to Kim and Ha's number of delivers, $N^*_{K\&H}$, is given as:

$$\Psi = \frac{N^*}{N^*_{K\&H}} = \frac{\{N^* \mid \min[\varphi(N_1^*), \varphi(N_1^*+1)]\}}{\{N^*_{K\&H} \mid \min[\varphi_{K\&H}(N_1^*), \varphi_{K\&H}(N_1^*+1)]\}} \quad (20)$$

The ratio of the study order quantity discussed earlier, Q^*, to Kim and Ha's minimum order quantity, $Q^*_{K\&H}$, is given as:

$$\theta = Q^* / Q^*_{K\&H} = \sqrt{\Psi\left(\frac{K+CS+F\Psi N_{K\&H}}{K+CS+FN_{K\&H}}\right)\left(\frac{H_B + H_S[(2\lambda - E_1) + N_{K\&H}(E_1 - \lambda)]}{H_B(E_2 + 2DE[p]/x) + H_S[(2\lambda - E_1) + \Psi N_{K\&H}(E_1 - \lambda)]}\right)} \quad (21)$$

Again, the ratio of our minimum order quantity, $Q^{\min}$, to Kim and Ha's minimum order quantity, $Q^{\min}_{K\&H}$, is given as

$$\Phi = \frac{Q^{\min}}{Q^{\min}_{K\&H}} = \sqrt{\frac{\Psi[H_B + H_S(2\lambda - E_1)]}{H_B E_2 + H_S(2\lambda - E_1) + 2DH_B E[p]/x}} \quad (22)$$

Further, the change in the expected total cost per unit time, Ω, is determined as:

$$\Omega = ETCU(Q^*, N^*) - TC(Q^*_{K\&H}, N^*_{K\&H}) = D(K+CS)\left[\frac{1-\theta E_1}{\theta E_1 Q_{K\&H}}\right] + DFN_{K\&H}\left(\frac{\Psi - \theta E_1}{\theta E_1 Q_{K\&H}}\right) + RD\left(\frac{1}{E_1} - 1\right)$$
$$+ \frac{D(d + bE[p])}{E_1} + \frac{H_B DE[p] Q_{K\&H}}{\Psi x E_1 N_{K\&H}} + \frac{H_B Q_{K\&H}}{2}\left[\frac{E_2\theta - \Psi E_1}{E_1 \Psi N_{K\&H}}\right] + \frac{(2\lambda - E_1) H_S Q_{K\&H}}{2}\left[\frac{\theta - \Psi E_1}{E_1 \Psi N_{K\&H}}\right] + \frac{H_S(E_1 - \lambda) Q_{K\&H}(\theta - 1)}{2} \quad (23)$$

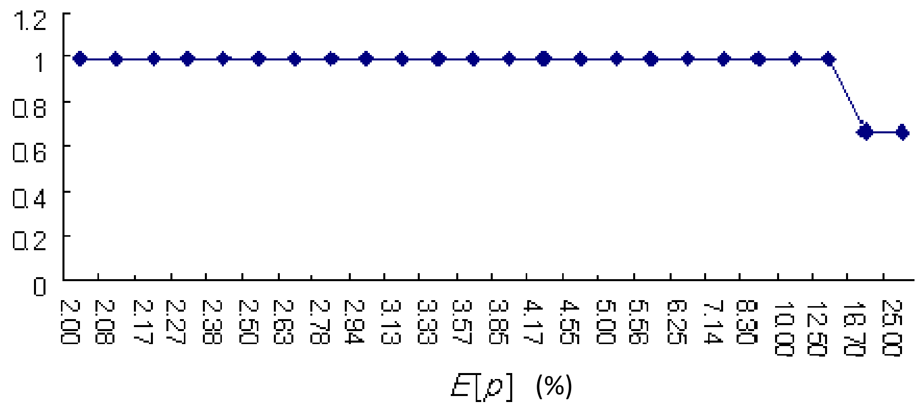

Figure 2. The behavior of the ratio of Ψ when the study model compared to Kim and Ha's model for the increase in $E[p]$.

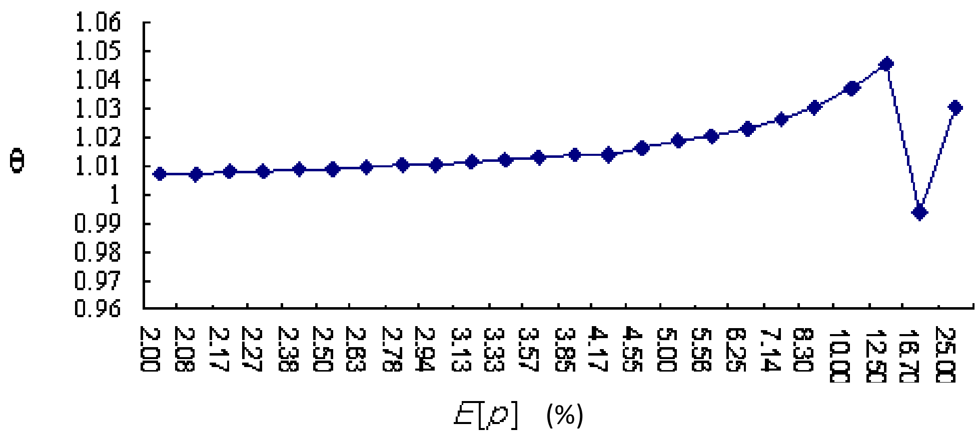

Figure 3. The bahaviour of the ratio θ when the study model compared to Kim and Ha's model for the increase in $E[p]$.

Using Equations (20) - (23) the study results of this work are different from Kim and Ha's results. To demonstrate the behavior of the relation between N^* and $N^*_{K\&H}$, the example shown earlier is repeated for different values for $E[p]$ ($0.02 \le E[p] \le 0.25$) in which the values for Ψ in Equation (20) are plotted against those for $E[p]$ as shown in Figure 2. The values for θ in Equation (21) are calculated and scatted against those for $E[p]$, as shown in Figure 3. The values for Φ in Equation (22), in which they demonstrated the behavior of the relation between $Q^{\min}$ and $Q^{\min}_{K\&H}$, are computed and plotted for the same range of $E[p]$, as shown in Figure 4. The values for Ω in Equation (23) are calculated and drawn for the same $E[p]$ range, which is shown in Figure 5. From Figure 2, the number of shipments is still the same value until the expected defective rate becomes greater than 12.25%. This indicates that the defect rate has slight sensitivity to the number of shipment and if the defective rate is greater than the critical value, the number of shipments drops to another level. Figure 3 shows the order quantity increases as the defective rate increases in the same shipments. Note that if the defective rate increases above the critical value, the θ ratio moves sharply down due to the fewer number of shipments. As shown in Figure 4, the minimum delivery lot size is larger than that in Kim and Ha's model under the same shipments. This implies that to achieve the cost savings the supplier must deliver more items to satisfy the customer needs if defects occur. Furthermore, in Figure 5, the increase in the total expected cost per unit time from using the study model is intuitively at a high $E[p]$ value. The defect rate is especially critical for the supplier in this cooperative system. It can be concluded that the supplier may reduce the

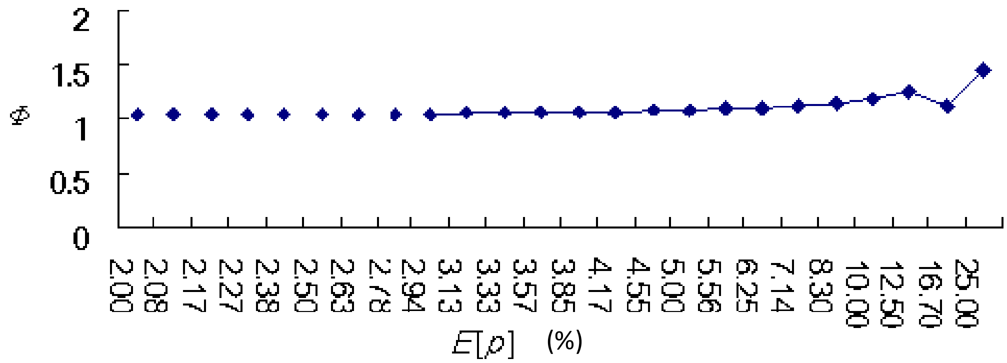

Figure 4. The behaviour of the ratio of Ø when the study model compared to Kim and Ha's model for the increase in $E[p]$.

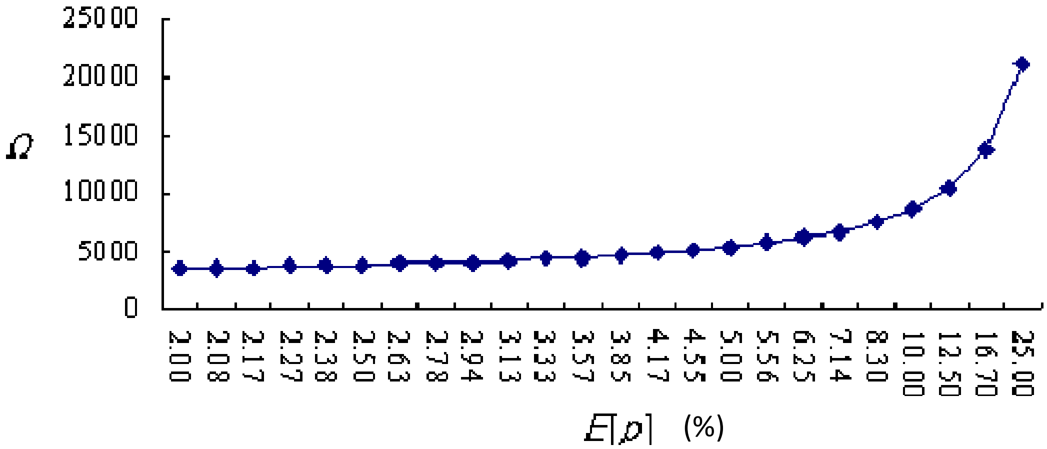

Figure 5. The behaviour of the ratio of Ω when the study model compared to Kim and Ha's model for the increase in $E[p]$.

number of shipments and increase the delivery size for each trip to avoid extra cost if defective items with disposal cost occur.

Conclusions

This paper developed an inventory model, in which a 100% inspection process is performed on the received lot, for items with defects and disposal cost in a Just-in-time (JIT) environment. Specially, this paper investigated the effectives of a JIT lot-splitting shipment strategy on the integrated buyer-supplier total relevant cost by examining the optimal lot size, number of delivers and shipment size under defective items with disposal cost. An algorithm free of using convexity was developed to determine the optimal solution. Compared with the results provided by Kim and Ha (2003), the fewer order lot size and greater cost were obtained in our model with the same number of shipments. This result may also lead to the supplier increasing the delivery size for each trip. The numerical results show that (1) when the expected defect rate increases, the order lot size increases under the same number of shipments. (2) As the expected defect rate increases below the critical value, the number of shipments remains at the same level. Alternatively, when the expected defect rate increases above the critical value, the number of shipments drops down to another level. (3) The minimum order quantity increases when the expected defect rate increases, except for the critical value. Simultaneously, a sharp reduction for the ratio of order lot size occurs at the critical point. (4) The total expected cost increases when the expected defect rate increases. Note that if all items are perfect (that is, there are no need for screening and no salvage value), the study model reduces to Kim and Ha's (2003) model.

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