A supply chain model for deteriorating items with time discounting under trade credit and quantity discounts

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A supply chain model for deteriorating items with price-dependent demand is developed under inflation, trade credit, and quantity discounts. We apply the discounted cash flow (DCF) approach for analysis of the retail price and replenishment problems over a finite planning horizon. In this paper, a mathematical model is derived under two different circumstances, that is, case I: the credit period is less than or equal to the cycle time for settling the account, and case II: the credit period is greater than the cycle time for settling the account. In addition, an optimal solution procedure is developed to find the optimal number of replenishment, cycle time, retail price and order quantity such that the total present value of profits is maximized. Finally, we provide several numerical examples to illustrate the results.

Key words: Inventory, deterioration, delay in payment, quantity discounts, inflation.

INTRODUCTION

During the past few years, many researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic items and fashion goods. The analysis of deteriorating inventory problems began with Ghare and Schrader (1963), who developed a simple economic order quantity (EOQ) model with a constant rate of deterioration. Since then, many related articles could be found in Aggarwal and Jaggi (1995), Hariga (1995), Sarker et al. (1997), Jaggi et al. (2006), Hou (2006) and Dye et al. (2007). As a result, this paper considers the items which have the exponential distribution for the time to deterioration. On the other hand, the traditional EOQ model assumes that retailer must pay for the items as soon as the items are received. However, in real-life situations, the supplier allows a fixed credit period to settle the account for stimulating retailer's demand. The retailer can sell the goods to accumulate revenue and earn interest before the end of trade credit period. But if the payment is delayed beyond that period, a higher interest will be charged. Such a convenience is likely to motivate customer to order more quantities because paying later indirectly reduces the purchase cost. Recently, Chung and Huang (2007) developed a retailer’s replenishment model to reflect the real-life situations by assumed that the goods are perishable and the storage capacity is limited under trade credit financing. Tsao (2010) considered the condition of advance sales discount and two-echelon trade credits in a one supplier-one retailer supply chain. Other related articles can be found in Goyal (1985), Aggarwal and Jaggi (1995), Khouja and Meherz (1996), Jamal et al. (1997), Chu et al. (1998), Chen and Chuang (1999), Sarker et al. (2000), Chang et al. (2003), Chung and Huang (2006), and their references.

The positive effects of credit period on the product demand can be integrated into the EOQ model through the consideration of retailing situations where the demand rate is a function of the retail price. The availability of the credit period from the supplier enables the retailer to choose the retail price from a wider range of option. Since the retailer’s lot size is affected by the demand rate of the product, the problems of determining the retail price and lot-size are interdependent and must be solved simultaneously (we will call the RPLS problem). Kunreuther and Richard (1971), Ladany and Sternlieb (1974), and Shah and Jha (1991), dealt with the PRLS problem when the demand is a decreasing function of
retail price. Abad (1988) considered the PRLS problem assuming that the supplier offers all-unit quantity discounts and the demand for the product is a decreasing function of price. Wee (1995) studied the PRLS problem for inventory with a constant rate of deterioration. Hwang and Shinn (1997) dealt with the PRLS problem of determining the retail’s optimal price and lot size simultaneously when the supplier permits delay in payment for an order of a product whose demand rate is represented by a constant price elasticity function.

All of the models mentioned above, the inflation and the time value of money were disregarded. It has happened mostly because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. However, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money last several years owing to financial crises or mark-up on international crude oil and so forth. As a result, while determining the optimal retail price and ordering policies, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. Recently, Chang et al. (2010) used a discounted cash-flow approach to establish an inventory model for deteriorating items with trade credit based on the order quantity. Other related articles can be found in Ray and Chaudhuri (1997), Liao et al. (2000), Wee and Law (2001), Balkhi and Benkherouf (2004), Chung and Liao (2006), Hsieh et al. (2008) and their references.

This study develops a deterministic inventory model in supply chain systems with price-dependent demand when a delay in payments and quantity discount are permissible. The effects of the inflation, deterioration, and delay in payment are discussed. Mathematical model is also derived under two different circumstances, that is, case I: The credit period is less than or equal to the cycle time for settling the account. Case II: The credit period is greater than the cycle time for settling the account. Moreover, the expressions for system’s total present value of profits under above two cases are derived. In addition, a computational technique is proposed to obtain the optimal the number of replenishment, cycle time, retail price, and order quantity. Those results of this study can be used to make inventory decisions by companies or retailers which are affected by inflation and the time value of money. Finally, numerical examples are used to illustrate the results obtained in this paper.

**NOTATION AND ASSUMPTIONS**

The following notation is used throughout the paper.

1) \( H \): length of planning horizon
2) \( T \): replenishment cycle time
3) \( n \): number of replenishment during the planning horizon; \( n=H/T \)
4) \( Q \): order quantity, units/cycle
5) \( A \): ordering cost at time zero, \$/order
6) \( v(q) \): per unit material cost as a function of \( q \), where \( q \) is \( i \)-th price break quantity, \( i \) is the number of price break, that is,

\[
v_i(q_i) = \begin{cases} v_1 & 0 = m_1 < q_1 \leq m_2 \\ v_2 & m_2 < q_2 \leq m_3 \\ \vdots \\ v_k & m_{k-1} < q_k \\
\end{cases}
\]

where \( v_i \) is the basic unit cost and \( v_1 > v_2 > \ldots > v_k \); \( m_1, m_2, \ldots, m_k \) are price breaks quantity.
7) \( D(s) \): demand rate for the product, a function of \( s \)
8) \( h \): inventory holding cost per unit per unit time excluding interest charges
9) \( \theta \): deterioration rate, a constant fraction of the on-hand inventory
10) \( r \): discount rate represent the time value of money
11) \( f \): inflation rate
12) \( I_E \): the net discount rate of inflation, \( R = r - f \)
13) \( I_C \): the interest charged per dollar in stocks per unit time by the supplier, \( I_c \geq I_C \)
14) \( M \): the permissible delay in settling account (that is, the trade credit period).

In addition, the following assumptions are made:

1) Demand rate is a decreasing linear function of retail price, that is, \( D(s) = a-bs \) where \( a, b > 0 \) and \( s < a/b \).
2) The replenishment rate is instantaneous, the order quantity and the replenishment cycle time are the same for each period.
3) The system operations for a prescribed period of a planning horizon.
4) Shortages are not allowed
5) The constant rate of deterioration is known and only applied to on-hand inventory.
6) During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. When \( T > M \), the account is settled at \( T = M \) and we start paying for the interest charges on the items in stock. When \( T \leq M \), the account is settled at \( T = M \) and we do not need to pay any interest charge.
6) Product transactions are followed by instantaneous cash flow.

**MATHEMATICAL FORMULATION**

The total time horizon \( H \) has been divided into \( n \) equal parts of length \( T \) so that \( T = H/n \). Hence, the reorder times
over the planning horizon $H$ are $T_j = jT$ $(j = 0, 1, 2 \ldots n-1)$. Let $I(t)$ be the inventory level during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration. So, the variation of $I(t)$ with respect to $t$ is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -D(s) - \theta I(t), \quad 0 \leq t \leq T$$

(2)

with the boundary condition $I(T) = 0$. The solution of (1) can be represented by

$$I(t) = \frac{D(s)}{\theta} \left[ \frac{e^{\theta T}}{\theta} - e^{-\theta t} \right] - \frac{1}{\theta} \int_0^t e^{\theta T - \theta s} \, ds, \quad 0 \leq t \leq T$$

(3)

Consequently, initial inventory after replenishment becomes

$$I(0) = q = \frac{D(s)}{\theta} \left[ 1 - e^{-\theta T/n} \right]$$

(4)

Since there are $n$ replenishments in the entire horizon $H$, the present value of the total replenishment costs is given by

$$C_R = A \sum_{j=0}^{n-1} e^{-jRT}$$

$$= A \left( 1 - e^{-RH/n} \right) \frac{1 - e^{-RT}}{1 - e^{-1/n}}$$

(5)

and the present value of total purchasing costs is given by

$$C_p = v(q) \sum_{j=0}^{n-1} I(0) e^{-jRT}$$

$$= \frac{v(q)}{\theta} \left( e^{\theta T/n} - 1 \right) \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(6)

The present value of the holding costs during the first replenishment cycle is

$$h_1 = h \int_0^T h(t) e^{-RT} \, dt$$

$$= \frac{hD(s)}{\theta} \left( \frac{(e^{\theta T/n} - e^{-RH/n})}{(\theta + R)} + \frac{(e^{-RH/n} - 1)}{R} \right)$$

(7)

hence, the present value of the total holding costs over the time horizon $H$ is given by

$$C_h = \sum_{j=0}^{n-1} h e^{-jRT}$$

$$= \frac{hD(s)}{\theta} \left( \frac{(e^{\theta T/n} - e^{-RH/n})}{(\theta + R)} + \frac{(e^{-RH/n} - 1)}{R} \right) \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(8)

During the time period $(0, T)$, the replenished inventory is being consumed due to demand and deterioration. Under instantaneous cash transactions, the present value of sales revenue during the first cycle is

$$E_1 = s \int_0^T D(s)e^{-RT} \, dt = sD(s) \left( 1 - \frac{e^{-RH/n}}{R} \right)$$

(9)

hence, the present value of total sales revenue over the time horizon $H$ is

$$E_H = \sum_{j=0}^{n-1} E_1 e^{-jRT}$$

$$= \left[ sD(s) \left( 1 - \frac{e^{-RH/n}}{R} \right) \right] \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(10)

Since the inventory model considers the effect of delay in payment, there are two distinct types of cases in inventory system.

Case I: $M \leq T$

In this case, the replenishment cycle time $T$ is longer than or equal to $M$. Therefore, the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned. The present value of the interest payable during the first replenishment cycle is

$$I_{c1} = v(q) \int_0^T I(t) e^{-RT} \, dt$$

$$= \frac{v(q)}{\theta} \int_0^T D(s) \left( e^{\theta T/n} - 1 \right) e^{-RT} \, dt$$

$$= \frac{v(q)}{\theta} \int_0^T D(s) \left( \frac{e^{\theta T/n - \theta s} - e^{-RH/n}}{R} \right) + \frac{v(q)}{R} \left( e^{-RH/n} - e^{-RT/n} \right)$$

(11)

hence, the present value of the total interest payable over the time horizon $H$ is

$$I_{cH} = \sum_{j=0}^{n-1} I_{c1} e^{-jRT}$$

$$= \left( \frac{v(q)}{\theta} \int_0^T D(s) \left( e^{\theta T/n - \theta s} - e^{-RH/n} \right) + \frac{v(q)}{R} \left( e^{-RH/n} - e^{-RT/n} \right) \right) \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(12)

Next, the present value of the interest earned during the first replenishment cycle is

$$I_{e1} = \frac{v(q)}{R} \int_0^T D(s) e^{-RT} \, dt$$

$$= \frac{v(q)}{R} \int_0^T D(s) \left( 1 - \frac{e^{-RH/n}}{n} - H e^{-RH/n} \right)$$

(13)
hence, the present value of the total interest earned over the time horizon $H$ is

$$I_{e1}^H = \sum_{j=0}^{n-1} I_j e^{-jRT} = \left[ \frac{v(q_j) I_j D(s)}{R} \left( 1 - e^{-RH/n} \right) \right] \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(14)

The total present value of the costs over the time horizon $H$ is

$$T_{C_1}(s,n) = C_a + C_r + C_n + I_{n-1}^H$$

$$= \left[ A + \frac{v(q_j) D(s)}{\theta} \left( e^{R} - 1 \right) + \frac{hD(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) + \frac{(e^{R/n} - 1)}{R} \right]$$

$$+ \left[ \frac{v(q_j) I_j D(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) + \frac{v(q_j) I_j D(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) - \frac{v(q_j) I_j D(s)}{\theta} \left( 1 - e^{-RH/n} \right) \right] \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(15)

Besides, the present value of total sales revenue over the time horizon $H$ is

$$E_H = \sum_{j=0}^{n-1} E_j e^{-jRT}$$

$$= \left[ sD(s) \left( 1 - \frac{e^{-RH/n}}{R} \right) \right] \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(16)

Thus, the total present value of profits over the time horizon $H$ can be expressed as

$$NP_1(s,n) = E_H - T_{C_1}(s,n)$$

$$= \left[ sD(s) \left( 1 - \frac{e^{-RH/n}}{R} \right) - A \left[ \frac{v(q_j) D(s)}{\theta} \left( e^{R} - 1 \right) \right] - \frac{hD(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) + \frac{(e^{R/n} - 1)}{R} \right]$$

$$+ \left[ \frac{v(q_j) I_j D(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) + \frac{v(q_j) I_j D(s)}{\theta} \left( e^{R/n} - e^{-RH/n} \right) - \frac{v(q_j) I_j D(s)}{\theta} \left( 1 - e^{-RH/n} \right) \right] \frac{1 - e^{-RH}}{1 - e^{-RH/n}}$$

(17)

The total present value of profits $NP_1(s,n)$ is a function of two variables $s$ and $n$ where $s$ is a continuous variable and $n$ is a discrete variable. For a given $n$, we get

$$g_1(s) = dNP_1(s,n)/ds$$

$$= \left\{ (a - 2bs) \left[ \frac{1 - e^{-R/tn}}{R} \right] + \frac{v(q_j)b}{\theta} \right\} \left( e^{R/tn} - 1 \right)$$

$$+ \left\{ h\left( e^{R/tn} - e^{-R/tn} \right) + \frac{v(q_j) I_j}{\theta} \left( e^{R/tn} - e^{-R/tn} \right) \right\} \left( R + \frac{v(q_j) I_j}{\theta} \left( e^{R/tn} - e^{-R/tn} \right) \right)$$

$$+ \left\{ \frac{v(q_j) I_j}{\theta} \left( 1 - \frac{e^{-R/tn}}{n} \right) - \frac{v(q_j) I_j}{R} \left( 1 - e^{-R/tn} \right) \right\} \frac{1 - e^{-R/tn}}{1 - e^{-R/tn/n}}$$

(18)

And

$$g_1(s) = \frac{d^2NP_1(s,n)}{ds^2} = -\frac{2h(1-e^{-R/tn})}{R} \left( \frac{1 - e^{-R}}{1 - e^{-R/tn}} \right) < 0$$

(19)

For a given $n$, the necessary condition for $NP_1(s,n)$ to be maximized is

$$g_1(s) = dNP_1(s,n)/ds = 0$$

(20)

We have $\lim_{s \to 0} g_1(s) = a \left( \frac{e^{-R}}{R} \right) > 0$ and $\lim_{s \to \infty} g_1(s) = -\infty < 0$. Therefore, there exists at least one sign change of $g_1(s)$ from positive to negative. Moreover, from Eq. (19) we show $g_1(s)$ is a decreasing function of $s$ and the equation $g_1(s)=0$ has a unique positive solution and so does $dNP_1(s,n)/ds = 0$. Consequently, we show that $NP_1(s, n)$ is concave with respect to $s$ and $dNP_1(s,n)/ds$ is a decreasing function for a given $n$.

Therefore, for a given positive integer $n$, the optimal retail price $s$ maximizing $NP_1(s,n)$ always exists and is unique. From the decreasing property of $g_1(s)$, the optimal solution $s$ can be obtained from Equation (20) using the Newton-Raphson method.

Case II: $M > T$

The interest charged during the time period $(0, T)$ is equal to zero when $M > T$ because the supplier can be paid in full at the permissible delay, $M$. The interest earned in the first cycle is the interest earned during the time period $(0, T)$ plus the interest earned from the cash invested during the time period $(T, M)$ after the inventory is exhausted at time $T$, and it is given by

$$I_{12} = \nu(q_j) I_j \left[ \int_0^T D(s)e^{-R/T} dt + (M - T)e^{-R/T} \int_0^T D(s) dt \right]$$

$$= \left[ \frac{v(q_j) I_j D(s)}{R} \left( 1 - e^{-R/tn} \right) + \frac{v(q_j) I_j D(s)}{R} \left( e^{-R/tn} - e^{-R/tn} \right) \right]$$

(21)
hence, the present value of the total interest earned over the time horizon $H$ is

$$I^H_{e_2} = \sum_{j=0}^{n-1} \int e^{-jRT}$$

$$= \left[ \frac{v_i(q_j)D(s)}{R} \left(1 - e^{-RH/n} \right) \right] + \left[ \frac{v_i(q_j)D(sj)}{R} \left(1 - e^{-RH/n} \right) \right] + \left[ \frac{v_i(q_j)D(sjH)}{R} \left(1 - e^{-RH/n} \right) \right] \frac{1 - e^{-R}}{1 - e^{-RH/n}}$$

Since the replenishment cost, purchasing cost, and inventory holding cost over the time horizon $H$ are the same as Case I, the total present value of the costs, $TC_2(s, n)$, is given by

$$TC_2(s, n) = C_r + C_p + C_h - I^H_{e_2}$$

Therefore, the total present value of profits over the time horizon $H$ can be expressed as:

$$NP_2(s, n) = E_{HI} - TC_2(s, n)$$

$$= \left[ sD(s) \left(1 - e^{-RH/n} \right) \right] - A - \frac{v_i(q_j)D(s)}{\theta} (e^{\theta HI/n} - 1)$$

$$- \frac{hD(s)}{\theta} \left[ \frac{1}{\theta + R} (e^{\theta HI/n} - e^{-RH/n}) + \frac{1}{R} (e^{-RH/n} - 1) \right]$$

$$+ \frac{v_i(q_j)D(s)}{R^2} (1 - e^{-RH/n}) - \frac{v_i(q_j)D(s)H}{R} e^{-RH/n}$$

$$+ \left[ \frac{MH}{n} - \frac{H}{n} \right] \frac{v_i(q_j)D(s)e^{-RH/n}}{R} \left(1 - \frac{1 - e^{-R}}{1 - e^{-RH/n}} \right)$$

So we get

$$g_2(s) = dNP_2(s, n) / ds$$

$$= \left[ a - 2bs \right] \left(1 - e^{-RH/n} \right) + \frac{v_i(q_j)b}{\theta} (e^{\theta HI/n} - 1)$$

$$+ \frac{hb}{\theta} \left[ \frac{1}{\theta + R} (e^{\theta HI/n} - e^{-RH/n}) + \frac{1}{R} (e^{-RH/n} - 1) \right]$$

$$- \frac{v_i(q_j)b}{R^2} (1 - e^{-RH/n}) + \frac{v_i(q_j)bH}{R} e^{-RH/n}$$

$$+ \left[ \frac{MH}{n} - \frac{H}{n} \right] \frac{v_i(q_j)b e^{-RH/n}}{R} \left(1 - \frac{1 - e^{-R}}{1 - e^{-RH/n}} \right)$$

and

$$g_2'(s) = \frac{d^2NP_2(s, n)}{ds^2} = - \frac{2b(1 - e^{-RH/n})}{R} \left(1 - \frac{1 - e^{-R}}{1 - e^{-RH/n}} \right) < 0$$

Similarly, for a given $n$, the necessary condition for $NP_2(s, n)$ in Equation (24) to be maximized is $g_2(s) = dNP_2(s, n)/ds = 0$.

We note that Equation (26) is the same as Equation (19) in Case I. Consequently, the $NP_2(s, n)$ is concave with respect to $s$. In addition, $g_2(s)$ is a decreasing function with $\lim_{s \to 0^+} g_2(s) > 0$ and $\lim_{s \to \infty} g_2(s) = -\infty < 0$. Similar to the Case I, for a given positive integer $n$, the optimal $s^*$ always exists and is unique.

Although we have shown that $NP_1(s, n)$ and $NP_2(s, n)$ have a unique optimal solution, these optimal solutions do not have closed-form expressions. Hence, a search procedure is required to obtain the optimal solution for Case I or Case II. In addition, we can obtain only the approximate expressions for them whenever we take approximations on $\theta$ and $R$. But this approach may drive the solution of the model away from the optimal solution. However, the concavity of $NP_1(s, n)$ or $NP_2(s, n)$ for a given $n$ and the decreasing property of $dNP_1(s, n)/ds$ or $dNP_2(s, n)/ds$ imply that if we use the Newton-Raphson method to solve the Equation (20) or (27), the algorithm will converge to the unique root $s$ of Equation (20) or (27), that is, for a given $n$, the optimal solution $s^*$ can be obtained (without taking approximations on $\theta$ and $R$). Hence, the computational technique is summarized in the following procedure.

**OPTIMIZATION PROCEDURE**

Deducing the Equations (1), (17) and (23), the total present value of profit $NP_j(s, n, v_i(q_j))$, for case $j = 1, 2$ with three price breaks for given $q_i$, $i = 1, 2, 3$ are depicted in Figure 1 and has the following relationship:

For case I

$$NP_1(s, n, v_i(q_j)) > NP_2(s, n, v_i(q_j)) > NP_3(s, n, v_i(q_j))$$

For case II

$$NP_2(s, n, v_i(q_j)) > NP_1(s, n, v_i(q_j)) > NP_3(s, n, v_i(q_j))$$

Since $v_1 > v_2 > v_3$, it follows that the maximum profit point, $q^*$, on the curve $NP_j(s, n, v_i(q_j))$ for $j = 1, 2$ has the following relationship:

$$q_1^* < q_2^* < q_3^*$$
The following computational technique (e.g., Hariga, 1995; Wee and Law, 2001) is used to derive the optimal $s$, $n$, $T$, $q$ and $NP$ values:

Step 1: Start by choosing a discrete variable $n$, where $n$ is any integer number equal or greater than 1.

Step 2: For different integer $n$ values and discounted price $v(q)$ ($i = 1,2,\ldots,N$).

(i) If $T = H / n \geq M$, derive $s$ from (20) and let $s_1 = s$. Substitute $(s_1, n)$ into Equation (4) to derive $q$. If $m_1 < q$ then $v(q)$ is feasible unit cost otherwise $v(q)$ is infeasible. Compute the total profits, $NP_1(s_1, n, v(q))$, by substituting $s_1$, $n$, and $v(q)$ into Equation (17). Let

$$NP_1(s_1, n) = \max \{\mathcal{M}_1(s_1, n, v(q)), NP_{1a1}(s_1, n, v(m_{1a1})), \ldots, NP_{1n}(s_1, n, v(m_{1n}))\}$$

(ii) If $T = H / n \leq M$, derive $s$ from (27) and let $s_2 = s$. Substitute $(s_2, n)$ into Equation (4) to derive $q$. If $m_1 < q$ then $v(q)$ is feasible unit cost otherwise $v(q)$ is infeasible. Compute the total profits, $NP_2(s_2, n, v(q))$, by substituting $s_2$, $n$, and $v(q)$ into Equation (23). Let

$$NP_2(s_2, n) = \max \{\mathcal{M}_2(s_2, n, v(q)), NP_{2a1}(s_2, n, v(m_{2a1})), \ldots, NP_{2n}(s_2, n, v(m_{2n}))\}$$

Step 3: Repeat Step 1 and 2 for all possible $n$ values with $T = H / n \geq M$ until the maximum $NP_1(s_1, n)$ is found from (17), and let $s_1 = s_1$, $n_1 = n$. For all possible $n$ values with $T = H / n \leq M$ until the maximum $NP_2(s_2, n)$ is found from (23), and let $s_2 = s_2$, $n_2 = n$. The $(s_1, n_1)$, $(s_2, n_2)$, $NP_1(s_1, n_1)$, and $NP_2(s_2, n_2)$ values constitute the optimal solution and the satisfy the following conditions:

$$\Delta NP_1(s_1^*, n_1^*) < 0 < \Delta NP_1(s_1^*, n_1^* - 1) \quad (31)$$

$$\Delta NP_2(s_2^*, n_2^*) < 0 < \Delta NP_2(s_2^*, n_2^* - 1) \quad (32)$$

where $\Delta NP_1(s_1^*, n_1^*) = NP_1(s_1^*, n_1^* + 1) - NP_1(s_1^*, n_1^*)$, and $\Delta NP_2(s_2^*, n_2^*) = NP_2(s_2^*, n_2^* + 1) - NP_2(s_2^*, n_2^*)$.

Step 4: Select the optimal retail price $s$ and number of replenishment $n$ such that

$$NP(s^*, n^*) = \max \begin{cases} NP_1(s_1^*, n_1^*) & \text{if } H / n_1^* \geq M \\ NP_2(s_2^*, n_2^*) & \text{if } H / n_2^* \leq M \end{cases}$$

Hence, optimal cycle time, $T = H / n^*$.

**NUMERICAL EXAMPLES**

Example 1: This example is devised here to illustrate the effect of the general model developed in this paper with the following data:

Unit material cost is
the replenishment cost, $A$, is $25/order, the holding cost per year excluding interest charges, $h$, is $1.5/unit/year, the constant rate of deterioration, $\theta$, is 0.15, the net discount rate of inflation, $R$, is $0.1/\$\$/year, the interest charged per $ \$ in stocks per year by the supplier, $I_c$, is $0.18/\$\$/year, the interest earned per $ \$ per year, $\ell_c$, is $0.16/\$\$/year, the demand rate per year, $D(s) = 1000-4s$ unit/year and the planning horizon, $T$, is 5 year. The permissible delay in settling account (that is, the trade credit period), $M$, is 60 days = 60/360 years (assume 360 days of operation per year).

Using the solution procedure, we have the computational results shown in Table 1. We find the Case II is optimal option in credit policy. From the case, the maximum present value of profits is found when the number of replenishment, $n$, is 37. With 37 replenishments, the optimal cycle time $T$ is 0.135 year. The optimal retail price, $s$, is $30.057, the optimal order quantity, $Q$, is 120.10 units, and the optimal total present value of profits is $68301.09.

Example 2: In this example, by considering the same parameters used in Example 1 except $R = 0.05, 0.10, 0.15$. The computed results are shown in Table 2. Table 2 shows that total present value of profits and order quantity will increase as $R$ decreases. That is, the change in $R$ will lead to the adverse changes in $Q$ and $P$.

Example 3: To examine the effects of change in the value of $M$ (let $M = 30/360, 60/360, 90/360$ year) on the optimal solutions, we also use the same example as in Example

---

**Table 1.** Numerical results.

<table>
<thead>
<tr>
<th>$n^*$</th>
<th>$v(q)$</th>
<th>$s^*$</th>
<th>$T'$</th>
<th>$Q^*$</th>
<th>$NP(s^<em>, n^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T ≥ M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>12</td>
<td>30.129</td>
<td>0.192</td>
<td>170.81</td>
<td>64489.77&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>27</td>
<td>12</td>
<td>30.123</td>
<td>0.185</td>
<td>164.4</td>
<td>64491.28&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>28</td>
<td>12</td>
<td>30.118</td>
<td>0.179</td>
<td>158.46</td>
<td>64490.45&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>45</td>
<td>12</td>
<td>31.02</td>
<td>0.111</td>
<td>98.14</td>
<td>64525.17&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>46</td>
<td>12</td>
<td>31.016</td>
<td>0.109</td>
<td>95.99</td>
<td>64520.16&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>47</td>
<td>12</td>
<td>31.012</td>
<td>0.106</td>
<td>93.93</td>
<td>64514.53&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>26</td>
<td>11</td>
<td>30.624</td>
<td>0.192</td>
<td>171.21</td>
<td>66382.10&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>27</td>
<td>11</td>
<td>30.619</td>
<td>0.185</td>
<td>164.78</td>
<td>66382.78&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>28</td>
<td>11</td>
<td>30.614</td>
<td>0.179</td>
<td>158.82</td>
<td>66381.25&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>37</td>
<td>11</td>
<td>30.558</td>
<td>0.135</td>
<td>119.83</td>
<td>66414.93&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>38</td>
<td>11</td>
<td>30.553</td>
<td>0.132</td>
<td>116.65</td>
<td>66415.57&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>39</td>
<td>11</td>
<td>30.547</td>
<td>0.128</td>
<td>113.63</td>
<td>66415.17&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>30.125</td>
<td>0.2</td>
<td>178.57</td>
<td>68280.24&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>T ≥ M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>10</td>
<td>30.12</td>
<td>0.192</td>
<td>171.6</td>
<td>68282.44&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>27</td>
<td>10</td>
<td>30.114</td>
<td>0.185</td>
<td>165.16</td>
<td>68282.29&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>36</td>
<td>10</td>
<td>30.063</td>
<td>0.139</td>
<td>123.47</td>
<td>68300.70&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>37</td>
<td>10</td>
<td>30.057</td>
<td>0.135</td>
<td>120.1</td>
<td>68301.09&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>38</td>
<td>10</td>
<td>30.052</td>
<td>0.132</td>
<td>116.91</td>
<td>68300.43&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> global optimal solution; <sup>b</sup> local optimal solution; <sup>c</sup> feasible solution; <sup>d</sup> infeasible solution.

**Table 2.** Optimal production policy with changing the parameter $R$

<table>
<thead>
<tr>
<th>$R$</th>
<th>$n^*$</th>
<th>$v(q)$</th>
<th>$s^*$</th>
<th>$T'$</th>
<th>$Q^*$</th>
<th>$NP(s^<em>, n^</em>)$</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>25</td>
<td>10</td>
<td>30.1</td>
<td>0.2</td>
<td>178.59</td>
<td>76883</td>
<td>I</td>
</tr>
<tr>
<td>0.1</td>
<td>37</td>
<td>10</td>
<td>30.057</td>
<td>0.135</td>
<td>120.1</td>
<td>68301</td>
<td>II</td>
</tr>
<tr>
<td>0.15</td>
<td>37</td>
<td>10</td>
<td>30.075</td>
<td>0.135</td>
<td>120.09</td>
<td>60999</td>
<td>II</td>
</tr>
</tbody>
</table>

$$v_i = \begin{cases} 
$12.0 & \text{for } 0 < q_1 \leq 100 \\
$11.0 & \text{for } 100 < q_2 \leq 120 \\
$10.0 & \text{for } 120 < q_3 
\end{cases}$$
Table 3. Optimal production policy with changing the parameter $M$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$n'$</th>
<th>$v(q)$</th>
<th>$s'$</th>
<th>$T'$</th>
<th>$Q'$</th>
<th>$NP(s', n')$</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/360</td>
<td>30</td>
<td>10</td>
<td>30.121</td>
<td>0.167</td>
<td>148.43</td>
<td>68199.30</td>
<td>I</td>
</tr>
<tr>
<td>60/360</td>
<td>37</td>
<td>10</td>
<td>30.057</td>
<td>0.135</td>
<td>120.10</td>
<td>68301.09</td>
<td>II</td>
</tr>
<tr>
<td>90/360</td>
<td>37</td>
<td>10</td>
<td>29.991</td>
<td>0.135</td>
<td>120.14</td>
<td>68551.17</td>
<td>II</td>
</tr>
</tbody>
</table>

1. The computational results are shown in Table 3. From Table 3, we see that total present value of profits increases as $M$ increases. That is, the retailer expects the supplier can provide the wide delay payment to earn the more profits.

However, the retailer will replenish greater order quantity as supplier provides a narrow delay payment.

Conclusions

This study develops a deterministic inventory model for deteriorating items over a finite planning horizon when the supplier provides a permissible delay in payments and quantity discounts. The model considers the effects of deterioration, inflation, trade credit and quantity discounts. Based on the DCF approach we permit a proper recognition of the financial implication of the opportunity cost in inventory analysis. In addition, we have presented an optimal solution procedure to find the optimal number of replenishment, retail price, cycle time and order quantity to maximize the total present value of profits. It should be noted that the effect of inflation and time value of money and credit period on total present value of profits is significant. Further research can be done for case with stochastic market demand when the supplier provides other credit terms include credit period and cash discount.

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REFERENCES


