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Full Length Research Paper

Cost-benefit analysis in selected air trips using a non parametric method

Vitor de Souza Lima, João Carlos C. B. Soares de Mello and Lidia Angulo Meza*

Post-Graduation Production Engineering, Universidade Federal Fluminense, Av. Dos Trabalhadores 420, Vila Santa Cecilia, Volta Redonda, Rio de Janeiro, Brazil.

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The Brazilian airlines industry went through great changes since its deregulation at the beginning of this century. The appearance of Gol Linhas Aéreas Inteligentes, the first low-cost, low-fare, mass Internet ticket vendor airline in Latin America made passenger competition fiercer. The country's airline crisis, with its delayed and cancelled flights, the duopoly created by Gol's purchase of Varig, and the global financial crisis wrought havoc in the market. This study searches to identify these circumstances and within Data Envelopment Analysis (DEA) concepts, which are the fundamental cost/benefit more efficient flights from the passenger viewpoint. This study also shows why these flights are efficient within the sample.

Key words: Data envelopment analysis, air transportation, cost-benefit analysis, Brazilian airlines.

INTRODUCTION

Air transport is responsible for a large part of domestic (in countries of great size like Brazil) and international travel (Fonseca et al., 2010). Brazilian air transport is today one of the main integrators of the national economy. In a country of continental dimensions it is a fundamental means of transportation for both goods and people. In the last twenty years, it has gone in Brazil through opposing moments. In the early nineties, the commercial airways market went through several transformations leading to productivity increases and greater choice for the passengers. On the other hand, questions on the safety of Brazilian skies that caused a wave of delays and cancellations, Gol's purchase of Varig leading to the sector's duopoly and lastly, the global financial crisis resulted in a sector in permanent deep transformations. Against this background, the passenger became the decisive role player for the market's success. Their decision on which flight to prefer became a choice of multiple facets such as price, quality of service, departure and arrival timetables, intermediate stops and connections among others.

In this work, we have tried to identify within a sample of flights in the high demand routes of the domestic market the most efficient from the clients' viewpoint. This was done from a flight cost/benefit analysis. Classing these flights as efficient within the sample was also a goal of this research. Finally, this paper tries to extend the use of a mathematical tool commonly used to measure the efficiency of productive units. Thus, it may become useful for future analysis of the airlines market behaviour whether by clients, companies, or regulators. They can then avoid eventual distortions between fight costs and benefits. To reach these goals the DEA (Charnes et al., 1978) methodology was used. This is a comparative methodology of efficiencies between production units. As this is a very benevolent measurement, evaluating each unit for whatever it does best, it was necessary to introduce a DEA discrimination increase technique. Among

*Corresponding author. E-mail: lidia_a_meza@pq.cnpq.br. Tel: +55-24-21073510.

Abbreviations: DEA, Data envelopment analysis; **CONAC,** Conferences on Commercial Aviation; **DAC,** Civil Air Department; **ANAC,** National Civil Aviation Agency; **DMUs,** decision-making units; **CCR,** Charnes, Cooper and Rhodes; **CRS,** constant returns to scale; **LPP,** linear programming problem; **BCC**, Banker, Charnese Cooper; **CRS,** constant returns to scale; **VRS,** variable returns to scale.

those available, the Inverted Frontier was chosen because of its simplicity and objectivity.

COMMERCIAL AVIATION IN BRAZIL

The first attempt to establish a network of commercial air routes in the country took place in 1924 with the expansion of international air routes. Those were operated by pioneer airlines, established in Europe and the USA during the twenties. However, only after the end of WWII did Brazilian aviation really take off. American aircraft, war leftovers, were bought at low prices and easy financial terms. This led to the appearance of several airlines between 10945 and 1952, when Brazil reached the point of having 34 airlines, the majority of which with a precarious management and financial structure. As a result of an exaggerated initial offer and unbalanced finances, a wave of fusions and bankruptcies took place as early as 1950. However, the number of cities served was never as high as in those years of the fifties having reached 300. What made the gradual reduction of the number of cities served from the sixties onwards was the increase and structure of the highway network. The fierce competition among airlines and their initial economic fragility brought a serious crisis to the airline sector. Short haul route were the most affected.

To solve the crisis, the Government and Airlines had three national meetings called National Conferences on Commercial Aviation (CONAC). The first CONAC took place in 1961, the others in 1963 and 1968. These conferences led to two measures that fully transformed Brazilian commercial aviation: incentives to airlines fusions and the introduction of the “controlled competition” model. Brazilian air transport regulations were made flexible in 1991 only during the Fifth CONAC. A new policy was established that, together with the Air Ministry legislation enacted the regulations. That legislation replaced the DAC (Civil Air Department, now ANAC National Civil Aviation Agency) strict price regulation. Airlines can now offer differentiated fares that allow more competition. In 2000 the differentiation between regional and national airlines disappeared and now all designated as domestic regular airlines. At the beginning of the XXIst Century a new series of alterations in the competitive conditions of the sector changed completely the way to think about commercial aviation in Brazil.

In 2001 the total liberalization of regular air fares took place being just monitored by DAC. In that very year Gol entered the market, being the first low cost, low fare airline in Brazil (Evangelho et al., 2005). This model had already been fully tested by airlines such as Southwest Airlines (USA) and Ryanair (Ireland). The year of 2006 saw what the specialized press in Brazil called the Air Crisis or Air Blackout that peaked with Gol’s Boeing 737 crash on September 29th 2006. Problems caused by flight delays and cancellations became constant in the life of passengers of Brazilian air transport system that had a precious reputation of excellent security, quality and punctuality. On top of this, a second accident, now with a TAM Airbus in July 2007 alerted to lack of investment in infrastructure at the country’s main airports. At the end of 2007 another important event was the purchase of Varig by Gol. This caused a narrowing of the market, which became a visible duopoly. New companies have gained space in the last few years and flight offer has increased. If, on the one hand, it is obvious the passenger benefits from the increase in competition, on the other the choice of flights becomes harder because there are more alternatives to consider. This study offers a varied vision involving a cost/benefit analysis between price, cost and distance. The DEA method was adopted and a brief summary of how it woof how it works will be presented in the next section.

DATA ENVELOPMENT ANALYSIS (DEA)

DEA is a mathematical technique, the objective of which is to analyse the decision-making units (DMUs) performance (Biondi et al., 2004). This methodology was presented in 1978 by Charnes, Cooper and Rhodes (Charnes et al., 1978) to compute and to define the comparative efficiency between productive units, DMU whenever financial considerations are either irrelevant or undesired. Farrel (1957) had already presented a technique to determine productive frontiers and relative productive efficiency indicators. Contrary to parametric approaches that optimize a regression from observations, DEA optimizes each individual observation to compute an efficiency frontier. The Pareto efficient units, that is, those that cannot improve some of its performances without worsening the others, determine the efficiency frontier. DEA’s objective is to compare a given number of units with similar tasks and that use the same inputs to produce the same outputs. The differences between these DMUs are the quantities of used resources and generated products.

The similitude in the use of inputs and generation of outputs classes these units in either efficient or non-efficient. Relative measures for efficiency are thus obtained. There are two DEA classic models:

CHARNES, COOPER AND RHODES (CCR)

Charnes, Cooper and Rhodes (CCR) is also known as CRS (Constant Returns to Scale). It Works with constant returns to scale, that is, any variation in inputs leads to a proportional variation in outputs. Its mathematical formulation is based on each DMU k, k = 1, ..., n, being a production unit that uses r inputs x_{ik}, i =1, …, r, to produce outputs y_{jk}, j =1, …, m. This model maximizes the quotient between the linear combinations of outputs and inputs with the restriction that this quotient can never be higher than 1. Using some mathematical artifices, this model can be transformed into a Linear Programming Problem (LPP) shown in (1). Eff_0 is DMU 0 efficiency being studied; x_{i0} and y_{j0} are DMU 0's inputs and outputs; v_i and u_j are the weights calculated by models for inputs and outputs.

$$
\begin{aligned}
& Max\ \ Eff_0 = \sum_{j=1}^{s} u_j y_{j0} \\
& subject\ to \\
& \sum_{i=1}^{r} v_i x_{i0} = 1 \\
& \sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \le 0 \quad , \quad k = 1,...,n \\
& u_j, v_i \ge 0 \quad \forall i, j
\end{aligned}
\qquad (1)
$$

BANKER, CHARNESE COOPER (BCC)

BCC (Banker, Charnese Cooper) also known as VRS (Variable Returns to Scale), tackles production efficiencies by changing scale, disregarding proportionality between inputs and outputs. The problem fractional formulation is presented in (2), after having been previously rendered linear (Banker et al., 1984). In (2) Eff_0 is DMU0's efficiency; x_{ik} is input i of DMU_k, y_{jk} is output j of DMU k; v_i is the weigh allocated to input i, v_i is the weigh allocated to output j; u_* is a scale factor.

$$
\begin{aligned}
& Max\ Eff_0 = \sum_{j=1}^{s} u_j y_{j0} - u_* \\
& subject\ to \\
& \sum_{i=1}^{r} v_i x_{i0} = 1 \\
& -\sum_{i=1}^{r} v_i x_{ik} + \sum_{j=1}^{s} u_j y_{jk} - u_* \le 0,\ \forall k \\
& u_j \ge 0, v_i \ge 0,\ \forall i, j \\
& u_* \in \Re
\end{aligned}
\qquad (2)
$$

Besides identifying efficient DMUs, DEA models measure and locate the inefficiency and estimate a linear production function by parts, which becomes the for the inefficient DMUs benchmark. This benchmark is the projection of the inefficient DMUs on the efficiency frontier. The way this projection is obtained defines the model orientation: to inputs when minimizing them and keeping outputs constants is the objective or to outputs when it is desired that outputs are maximized to constant inputs. As DEA is a very benevolent measurement that evaluates each unit for whatever it does best, a discrimination enhancing technique in DEA was required (Adler et al., 2002; Angulo-Meza and Lins, 2002; Podinovski and Thanassoulis, 2007; Soares de Mello et al., 2008). One of the techniques to distinguish between large numbers of efficient DMUs is the concept of Inverted Frontier introduced by Yamada et al. (1994), Lins et al. (2005). This method consists of inverting inputs for outputs and vice-versa. This is an objective method as no further information from the decision-makers is required. Inverted Frontier has at least two interpretations: the first is that as inputs are replaced by outputs, the new frontier is defined by the worse managed DMUs.

It could be called then the inefficient frontier. The second is that those DMUs have the best practices from the opposite viewpoint. In other words, the original frontier can be seen as the seller's point of view, while the Inverted Frontier is the buyer's (Lins et al., 2005). A DMU evaluation index, which will be named "combined efficiency", can be defined to involve the standard efficiency as much as the inverted one. For a DMU to be evaluated as good it must not only perform well at what it excels but it must also not perform badly in its worst criterion. This is achieved calculating the arithmetic average between the standard frontier efficiency and the inverted frontier inefficiency as per equation (3) (Angulo-Meza et al., 2005a; Angulo-Meza et al., 2005b; Soares de Mello et al., 2008).

$$
\text{Combined Efficiency} = \frac{\text{Standard Efficiency} + (1 - \text{Inverted Efficiency})}{2} \qquad (3)
$$

Normalised Combined Efficiency (Combined*) is the combined efficiency divided by the highest value of all the combined efficiency values and is represented by Equation (4).

$$
\text{Combined Efficiency*} = \frac{\text{Combined Efficiency}}{\text{Max (Combined Efficiency)}} \qquad (4)
$$

This method defines a ranking of DMUs, decreasing the number of ties and taking no account of subjective opinions. In the air transportation field, DEA has been used mainly for airport evaluation (Barros and Dieke, 2007; Lam et al., 2009; Pacheco and Fernandes, 2003; Pacheco et al., 2006; Wing Chow and Fung, 2009; Yang and Lu, 2006).

METHODOLOGY

Case study selected route fares cost/benefit analysis

The formulation and analysis of a DEA study require: (1) definition of DMUs; (2) selecting evaluation variables (both inputs and outputs); and (3) choosing the model to be used.

DMUs definition

In a DEA model, DMUs must have the same usage of entries and exits, intensity being the only variation; must be homogeneous, that is, do the same tasks with the same resources; must operate in same market conditions; and must be autonomous in decision making. Taking into account these premises and the objectives of this study the DMU' of this study are the selected flights from the airlines data collected. Thus, a DMU here is a putative flight with the following different characteristics: origin, destination, airline, date and hour and advance of ticket purchase. Take, then, the example of a DMU in this study: the Gol airline flight from Congonhas (Sâo Paulo) airport to Brasília on the 4th of May 2009 at 20.05, the fare having been simulated to be bought 29 days in advance.

Variables definition

In this work's model, the efficiency of a flight was evaluated from the passenger's point of view, taking as evaluation variables three that are basic on a journey: distance between departure and arrival locations, cost and travel time.

Distance: Distance chosen as model output because, essentially the objective of a traveller is to go from the departure point to destination. So, displacement is the simplest and more objective result, or benefit, of a journey. The greater the distance, the greater the consumer's benefit.

Journey time: Journey time defined as one of the inputs, or costs, of air trip because during the flight the passenger cannot use his/her time in a different way. Therefore, the passenger's objective is to minimise that resource. It can also be classed as an undesirable output because it is a result of the trip since travelling is time consuming; and it is undesirable because one always wishes to decrease it. For other details, see (Gomes and Lins, 2008).

Fare (ticket and taxes)

Fare (Ticket and Taxes): defined as another input, it is the clearest and most effective cost of a trip. It is the resource the consumer desires to minimize, that is, to spend the minimum to travel the required distance. It should be noted that a model is coherent only if each output/input relationship makes sense on its own (Gomes et al., 2009). In this model, there is a cause effect relationship between distance and flight time: average flight speed. Maximizing that ratio, that is, travelling at the highest possible speed is in the passenger's interest. Flight distance/fare is another relationship that is important for the passenger to maximize. For a better understanding of this ratio, it can also be analysed from the input/output viewpoint, that is, the ratio between the fare and distance, or cost of mile travelled. In this case, the passenger's interest is to minimize the ratio.

Model and orientation selection

Model BCC will be chosen, as there is an obvious lack of proportionality between inputs and outputs. The model CCR requires that proportionality. Distance being an output and flight time an output the ratio between the two is the mean speed. As in the short distance flights the low speed manoeuvres (taking off, approach, landing and taxiing) are an important share of travelling time and thus, mean speed is lower. Proportionality does not ob-tain. Insofar as model orientation is concerned, that is, how DMUs will reach the efficiency frontier, input orientation was chosen. In this orientation, efficiency is reached through an equiproportional reduction of inputs. It should be noted that output orientation would be senseless because the output is the destination.

Research characteristics

This research was carried out with the intent of collecting a domestic flight data sample. It should be representative enough to feed good parameters to subsequent analyses and mainly, to be a departure point for the use of proposed approach for domestic commercial aviation market behaviour studies. Initially, flights were defined. The option fell on high demand domestic flights that were to be served by more than one airline and several daily flights. Chosen cities were: Rio de Janeiro, São Paulo, Belo Horizonte and Brasília, differentiating between airports of the same city as Rio de Janeiro (Galeão and Santos Dumont), São Paulo (Congonhas and Guarulhos) and Belo Horizonte (Pampulha and Confins). 9 routes were selected: Confins (CNF), Brasília (BSB); Pampulha (PLU), Brasília (BSB); Galeão (GIG), Congonhas (CGH); Galeão (GIG), Guarulhos (GRU); Santos Dumont (SDU), Congonhas (CGH); Congonhas (CGH), Brasília (BSB); Guarulhos (GRU), Brasília (BSB); Congonhas (CGH), Confins (CNF); Guarulhos (GRU), Confins (CNF).

Methodology of data collection: some principles established in the works of Huse and Oliveira (2009) were used together with some singularities of this study.

Thus, the data research was directly carried out from the airlines Internet sites, a purchase simulation having been made following the procedures here are as follow:

(1) Collecting all data on the same day, on the same period, to avoid price oscillations that eventually may occur during the day.
(2) Choosing the cheapest available fare.
(3) Compulsory choice of non-connecting flights but allowing flights with stops.
(4) The final price includes the ticket price and all taxes.
(5) Sampling of at least 1 flight on each daily period, if it exists, to try obtaining price differences according to the period of the day:

a) Early morning: 0:00 h to 5:59 h – preferably around 03:00 h;
b) Morning: 6:00 h to 11:59 h – preferably around 09:00 h;
c) Afternoon: 12:00 h to 17:59 h – preferably around 15:00 h;
d) Night: 18:00 h to 23:59 h – preferably around 21:00 h.

(6) Selecting three different days for the flights including simulation of ticket purchase for the next day, 2 weeks later and 4 weeks later to check price behaviour in respect to advance ticket purchasing.
(7) Selecting dates that would not correspond to abnormal demand on account of previously known factors such as holidays and similar.

As already mentioned, Huse and Oliveira (2009) used some of these parameters in their study on airline fare discounts. Regarding point (6), we singled out two types of passengers who purchase tickets in advance: those who travel for leisure and those who travel on business. The former can plan and purchase their ticket in advance; the latter are subject to their occupational constraints, often not amenable to planning. The demand created by business traveller tends thus to be less price elastic. Soares de Mello et al. (2005) have also studied different types of passengers. Three types of ticket purchase have been our work: (i) spot purchase (1 day advance at most); (ii) casual purchase (little advance, two weeks maximum); and planned purchase (minimum 4 weeks).

Table 1. Total flights analysed by route and airline.

Airline	(CFN)-(BSB)	(CGH)-(BSB)	(CGH)-(CFN)	(GIG)-(CGH)	(GIG)-(GRU)	(GRU)-(BSB)	(GRU)-(CFN)	(PLU)-(BSB)	(SDU)-(CGH)	Total
GOL	9	8	9	11	9	8	9	0	11	74
OCEANAIR	2	3	1	0	2	5	0	0	9	22
PASSAREDO	0	0	0	0	0	2	0	0	9	11
TAM	9	9	9	8	9	9	9	0	0	62
TRIP	0	0	0	0	0	0	0	2	0	2
TOTAL	20	20	19	19	20	24	16	2	29	171

Collecting data

The data were collected on the 5th April 2007 between 8:00 am e 10:00 am by three researchers. It should be noted that the fares were computed as the final price paid by the passenger: fare plus airport tax. Google Earth measured the distance, and time was given by the difference between flight arrival and departure times, both shown on airlines sites where ticket purchasing was simulated. Data for 171 were collected for the routes and airlines as shown in Table 1.

RESULTS

As mentioned previously, calculations and analyses were carried out using the BCC input oriented model. We use the SIAD software (Angulo-Meza et al., 2005a). This analysis showed that 21 flights were efficient as shown in Table 2. This table includes such a large number of efficient flights that renders impossible the differentiation of efficient from inefficient flights. The Inverted Frontier method was used to decrease the number of ties. Then a division in tertiles was carried out so as to group together DMUs to obtain a conclusion. For a better analysis, the results are also shown by: airline, advance purchase, route, average speed and period of the day. Table 3, results were grouped by airline, that is, results show for instance that 38,0% of all studied TAM flights had a low efficiency and TAM flights represented 45% of all analysed flights. It should be noted that the airlines with high efficiency better percentage flights are Ocean Air and Gol. The point to be enhanced to differentiate the two airlines is hat Gol has no more than 21. 6% of its flights on the worst tertile, while Ocean Air has 45. 5% of its flights in the very same position. This shows that despite Ocean Air being the company with the largest number of flights on the most efficient tertile, it also concentrates a large number of its flights on the most inefficient tertile.

This makes that choosing this company, from the data and selected routes point of view is a risk option. As many efficient flights can be found as inefficient ones. On the contrary, selecting Gol that concentrates more than 78% of its flights on the 2nd and 3rd tertiles is more likely to result in a good choice and the risk to pick up an inefficient flight is thus less probable. Lastly, TRIP airline on route Pampulha – Brasília, and Passaredo, on route Guarulhos – Brasília, placed themselves in the least efficient tertile since their average speeds are low. Average speed, the quotient of distance by flight duration, had an adverse influence on their efficiency analyses. In Table 4 we analyse the show the distribution of combined efficiencies according to the delay between purchase and flight in the combined efficiency. Results show that purchases named in this study as casual (fortnight in advance) and planned (29 days in advance) have placed more than 45% of their totals on the most efficient tertile. Spot purchases (1 day in advance) placed more than half of its flights on the most inefficient tertile. This result confirms the assumption that higher fares for last minute purchases lead to the inefficiency of the journey. It also important to note that casual purchase is slightly more efficient than planned purchases for the sample chosen. This is an indication that the difference in the number of days for advance purchasing proposed in this study was not relevant to change the fares. Table 5 shows the results per route.

The table shows that for three of the four longest routes, the vast majority of their flights belong in the tertile most efficient tertile: (1) Congonhas - Brasília; (2) Guarulhos - Brasília (DF); (3) Pampulha (MG) - Brasília (DF) e (4) Confins (MG) - Brasília (DF), the exception is Pampulha (MG) - Brasília (DF), operated only by TRIP among the sample of airlines chosen. These results confirm the first conclusions of this work. BCC model shows that average speed has a major importance deciding on the model's efficiency. In longer routes the aircraft fly longer at cruising speed and so the average speed is higher as the reduced speed effects caused by take offs, approaches, landings and taxiing are diluted over the longer flight. TRIP operated flights are an exception to this conclusion as turbo prop aircraft ATR 42 and ATR 72 fly them. These planes have lower average speeds than the other aircraft in long distance routes. Besides, the two shorter routes and lower average speeds, Galeão (RJ) - Congonhas (SP) and Galeão (RJ) - Guarulhos (SP), have no flights on the efficient tertile. This reinforces the conclusion that the average speed is the by far the most important factor in this analysis. Table 6 shows the DMUs combined efficiencies according to the route Average Speed. It is easy to see that the average speed is a dominant factor on the efficiency composition for the model as more than 70% of higher

Table 2. Efficient DMUs using the BCC model.

Flight date	Advance purchase (days)	Flight	Number of stops	Total price (R$) input 1	Period of the day	Time flight (Minutes) input 2	Distance (Km) output	Average speed (Km/h)
April/06	1	DMU 19 (TAM: SDU-CGH)	0	544.92	Afternoon	54	366	592.00
April/20	15	DMU 22 (TAM: SDU-CGH)	0	354.92	Afternoon	54	366	582.48
May/04	29	DMU 25 (TAM: SDU-CGH)	0	354.92	Afternoon	54	366	582.48
April/20	15	DMU 94 (OCEAN AIR: GIG-GRU)	0	130.62	Early morning	70	334	493.07
May/04	29	DMU 95 (OCEAN AIR: GIG-GRU)	0	130.62	Early morning	70	334	493.07
April/20	15	DMU 99 (OCEAN AIR: SDU-CGH)	0	164.42	Morning	65	366	525.82
April/20	15	DMU 100 (OCEAN AIR: SDU-CGH)	0	214.42	Afternoon	55	366	554.82
May/04	29	DMU 103 (OCEAN AIR: SDU-CGH)	0	164.42	Afternoon	65	366	525.82
April/06	1	DMU 115 (GOL: CNF-BSB)	0	449.62	Afternoon	60	592	592.00
April/20	15	DMU 118 (GOL: CGH-BSB)	0	338.62	Night	95	873	551.37
April/20	15	DMU 120 (GOL: GRU-BSB)	0	198.62	Night	100	854	512.40
April/20	15	DMU 123 (GOL: CNF-BSB)	0	258.62	Night	70	592	552.71
May/04	29	DMU 128 (GOL: GRU-BSB)	0	198.62	Afternoon	100	854	512.40
May/04	29	DMU 132 (GOL: CNF-BSB)	0	258.62	Night	70	592	552.71
April/20	15	DMU 163 (OCEAN AIR: GRU-BSB)	0	192.62	Morning	105	854	488.00
April/20	15	DMU 164 (OCEAN AIR: GRU-BSB)	0	192.62	Afternoon	105	854	488.00
May/04	29	DMU 165 (OCEAN AIR: CGH-BSB)	1	206.62	Morning	190	873	289.29
May/04	29	DMU 166 (OCEAN AIR: CGH-BSB)	0	206.62	Morning	105	873	509.44
May/04	29	DMU 167 (OCEAN AIR: CNF-BSB)	0	157.62	Morning	85	592	491.20
May/04	29	DMU 168 (OCEAN AIR: GRU-BSB)	0	192.62	Morning	105	854	488.00
May/04	29	DMU 169 (OCEAN AIR: GRU-BSB)	0	192.62	Afternoon	105	854	488.00

Table 3. Distribution of combine efficiencies by airline.

Airlines	1st Tertile (low efficiency) (%)	2nd Tertile (average efficiency)(%)	3rd Tertile (high efficiency) (%)	Total (%)
TAM	38.0	35.2	26.8	41.5
OCEAN AIR	45.5	9.1	45.5	12.9
TRIP	100.0	0.0	0.0	1.2
PASSAREDO	100.0	0.0	0.0	1.2
GOL	21.6	40.5	37.8	43.3

average speed flights are found in 3rd tertile and 100% of lowest speedare found in the 1st tertile.

Finally, Table 7 shows efficiencies by time of day.

A counter intuitive result obtains from the table: early morning flights that at first sight should be the most efficient have only 10% of its flights in

Table 4. Distribution of combined efficiencies by advance purchases.

Delay between purchase and flight (Day)	1st Tertile (%) (Low efficiency)	2nd Tertile (%) (Average efficiency)	3rd Tertile (%) (High efficiency)	Total (%)
1	51.7	39.7	8.6	33.9
15	21.6	31.4	47.1	29.8
29	25.8	29.0	45.2	36.3

Table 5. Distribution of combined efficiencies by route.

Flight	1st Tertile (%) (Low efficiency)	2nd Tertile (%) (Average efficiency)	3rd Tertile (%) (High efficiency)	Total (%)
CONFINS-BRASÍLIA	15.0	25.0	60.0	11.7
CONGONHAS-BRASÍLIA	15.0	25.0	60.0	11.7
CONGONHAS-CONFINS	26.3	36.8	36.8	11.1
GALEÃO-CONGONHAS	57.9	42.1	0.0	11.1
GALEÃO-GUARULHOS	65.0	35.0	0.0	11.7
GUARULHOS-BRASÍLIA	16.7	33.3	50.0	14.0
GUARULHOS-CONFINS	16.7.	33.3	50.0	10.5
PAMPULHA-BRASÍLIA	100.00	0.0	0.0	1.2
STS. DUMONT-CONGONHAS	44.8	37.9	17.2	17.0

Table 6. Distribution of combined efficiencies by average speed.

Average speed	1st Tertile (%) (Low efficiency)	2nd Tertile (%) (Average efficiency)	3rd Tertile (%) (High efficiency)	Total (%)
[200 – 300)	100.0	0.0	0.0	8.2
[300 – 400)	49.2	46.2	4.6	38.0
[400 – 500)	16.4	31.1	52.5	35.7
[500 – 600)	3.2	25.8	71.0	18.1

Table 7. Distribution of combined efficiencies by period of the day.

Period of the day	1st Tertile (%) (Low efficiency)	2nd Tertile (%) (Average efficiency)	3rd Tertile (%) (High efficiency)	Total (%)
Morning	37.9	27.6	34.5	33.9
Afternoon	26.9	32.7	40.4	30.4
Night	31.4	39.2	29.4	29.8
Early morning	50.0	40.0	10.0	5.8

the most efficient tertile. This goes against common sense be-cause in principle; (1) they could fly at a higher average speed as airport congestion is lower; and (2) they could offer lower fares as this period has a lower demand. These contra-dicting results may be explained as follows: (1) all early morning flights are the Rio – São Paulo; Santos Dumont – Congonhas; Galeão – Congonhas; and Galeão – Guarulhos routes, departures from 04.00 am. This is a high demand departure time as it allows passengers to arrive at destination before the morning rush of typical of those two cities. Thus, no need for discounts nor significant gains of mean speed; (2) the flights to Guarulhos may be feeding flights to a hub to catch other flights. In that case it is not very interesting for the airline to transport passengers who will fly with that company only to that airport.

Conclusions

This study has tried to identify, within a sample of highly demanded domestic routes flights that were the most cost/benefit efficient from the clients' viewpoint. Presenting the main variables that made these flights as efficient within the sample was also a goal of this research. The DEA methodology and, more specifically the concept of Inverted Frontier were used for this. DEA measures the comparative efficiency of productive units. The Inverted Frontier concept is based on the premise that it is not enough for a productive unit to perform well in what it does best, but that it need not to have performance in what it does worst. The obtained results enhance Ocean Air's ambiguous position, Gol's high efficiency and that it is almost indifferent to buy tickets two or three weeks in advance. It should be noted that this analysis was not exhaustive and that new work should be carried out to reach firmer results. Further studies, using for instance the Tobit Regression associated to DEA, known as two stages DEA could be useful for a better understanding of the factors that influence these routes efficiencies. It should be taken in consideration, however, that this technique does not allow a number of analyses, such as where does Ocean Air stand. It has been classed as a risky option as it operates both very efficient and inefficient flights within the sample.

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