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How the information precision and the information frequency observed affect the stock market equilibrium

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The traditional model in the competitive stock market assumes that the observational frequency of information is uniform, and concludes that the stock market equilibrium price which aggregates market information provides a sufficient statistic reflecting all the private information in the market. However, we are the first to assume that the observational frequency of an information is not uniform. Actually, information is heterogeneous among market participants, and there is an information asymmetry among investors. The main purpose of this study is to explore the relationship among information precision, the observational frequency of information and the stock market equilibrium. The study analyze the determination of the price system in a competitive stock market, where there are I sources of information $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$ which are respectively observed f_1, f_2, \dots, f_I times by risk-averse traders. Each informed investor uses information observed to form an estimate for the expected value of the firm's true value, \tilde{v} , and to make decisions to buy the shares to maximize his own expected utility, and hence, determine the stock market equilibrium. Our main findings are as follows: First, we found that the competitive equilibrium price is equal to the rational expectations equilibrium price. Only when the observational frequencies of each piece of market information are equal, will the fully-informed equilibrium become a special case of competitive equilibrium. Second, we found that the market equilibrium price aggregates all the market information, contingent on each observational frequency and its precision. The market equilibrium condition and the expected utility depend not only on the realized information, but also on the observational frequency and the precision of information. The market equilibrium price will fully reflect the precision and the observational frequency of information about the future value of asset. The stock price response to an unexpected change of information is positively related to the observational frequency and the precision of that information. We found that, the heterogeneity of belief about the true value of the risky asset among investors will lead to different regimes of market equilibrium. Third, when the observational frequency of each piece of market information is uniform, Grossman's model (1976) is mathematically equivalent to a special case of our model, and the market equilibrium price could act as a sufficient statistic for all the private information about the intrinsic value of the risky asset. However, the observational frequencies of market information with asymmetry are usually not uniform such that traders still have an incentive to collect costly information. Finally, further research could investigate how accurate the market equilibrium price is as a sufficient statistic for all the market information.

Key words: Information precision, observational frequency of information, stock market equilibrium, information value.

INTRODUCTION

The role of prices in aggregating and revealing information is crucial in the allocation of resources in a competitive economy. The most frequently asked

question in the stock market is as follows: How informationally efficient is the market equilibrium price?, or "Is the market equilibrium price a sufficient statistic for

market information?"

That the market price can transmit information observed by investors was initially modeled by Lintner (1969). He analyzed an economy in which beliefs are exogenous. This leads to a characterization of equilibrium prices as the weighted average of these beliefs (Rubinstein, 1975; Verrecchia, 1980)

Grossman (1976) analyzed an economy in which each trader observes one piece of information about the return of risky assets, and he claimed that the rational expectations equilibrium price reveals all the market information to all traders, and it is a sufficient statistic of that information. A major limitation of this result is that when traders take prices as given, they have no economic incentive to acquire any information, even when the market is free of noise (Diamond and Verrecchia, 1981). Under this situation, private information is a redundancy to investors. Grossman (1981) showed that an equilibrium exists in which prices completely aggregate and reveal the private information of agents in the economy when markets are complete.

Scheinkman and Weiss (1986), Huffman (1987), Dumas (1989), and Campbell and Kyle (1993) considered competitive models, in which investors with homogeneous information trade since they have different preferences and constraints. Kyle (1985), and Admati and Pfleiderer (1988) considered noncompetitive models of stock trading, in which some investors have superior information about the stock value, and thus trade strategically to maximize profits.

Despite its dominance in economic models, the rational expectations model has limitations since it assumes that each informed trader observes only one piece of information. (Grossman, 1976, 1978, 1981; Grossman and Stiglitz, 1980; Hellwig, 1980; Bray, 1981; Paul, 1993; Baigent, 2003). Here, we model traders with multiple sources of information, introduced the concept of the "observational frequency of information" to a competitive stock market and assumed that the observational frequency of information is divergent among market traders.

The study captures two types of heterogeneity among investors. Firstly, we considered that the observational frequency of information is not uniform and there is an asymmetry of information among investors. This differs from the noisy rational expectations models, which commonly study the information asymmetry via informed and uninformed trading. The study focused on the relationship among the observational frequency of information, the market equilibrium price, the trading quantity and the value of information. Secondly, we assumed that the market information is heterogeneous in

precision.

This study assumed that there are I sources of information, that is, $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$ with different precision, which is respectively observed f_1, f_2, \dots, f_I times by N risk-averse traders, where $0 \leq f_i \leq N$, $i = 1, 2, \dots, I$. If $f_i = 0$, it implies that no investor has ever observed information \tilde{x}_i . Imagine there are three informed investors A, B and C in the economy. A observes information \tilde{x}_1 , B observes information \tilde{x}_1 and \tilde{x}_2 , and C observes information \tilde{x}_2 and \tilde{x}_3 . Then the observational frequencies of information, \tilde{x}_1, \tilde{x}_2 and \tilde{x}_3 are $f_1 = 2$ (observed by A and B), $f_2 = 2$ (observed by B and C), and $f_3 = 1$ (observed by C only).

Informed investors utilize this information to form an estimate for the expected value of the firm's true value, \tilde{v} , and then decide how to invest to maximize their own utility. This generates trading and price movement in the stock market. We show that the stock equilibrium price aggregates all the market information based on the precision of each piece of information and its observational frequency. The stock equilibrium price, trading quantity and the expected utility are functions of both the observational frequency and the precision of information. We explore the conditions under which the stock equilibrium price informativeness could be a sufficient statistic for the market information.

This study subsequently presents the basic stock market to be analyzed. In particular, the concept of the observational frequency of information was considered. This was followed by a derivation of the Walrasian competitive market equilibrium, the rational expectations equilibrium and the fully-informed economy equilibrium of the model, after which the study discussed how the stock equilibrium price, the expected utility and the trading quantity were influenced by the investor's risk coefficient, and the precision and observational frequency of information. Furthermore, a set of necessary and sufficient conditions was established for the stock price to be a sufficient statistic for the information about the true value of the risky asset. Finally, conclusions and further suggestions were also provided.

THE ANALYTICAL MODEL

Information Structure Assumptions

Assume the true value of a risky asset, denoted by $\tilde{v} \sim N(0, \Lambda_v^{-1})$, is standardized and distributed normally with zero mean and variance $\Lambda_v^{-1} \equiv \text{Var}(\tilde{v})$. Following Titman and Trueman (1986), assumption that the prior distribution of \tilde{v} was diffused.

In addition to the information of observed security price per share \tilde{p} , assume there are I kinds of information, denoted by $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$, which are available in the market. Each kind of information is related to the realization of the true value of that risky asset and is an unbiased estimate of that true value. Specifically,

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$$\tilde{x}_i = \tilde{v} + \tilde{\mathcal{E}}_i, \quad i=1,2,\dots,I \quad (1)$$

There is a noise term $\tilde{\mathcal{E}}_i \sim N(0, \Lambda_i^{-1})$, which deters traders from learning the true value of \tilde{v} . The increasing value of $\Lambda_i^{-1} \equiv \text{Var}(\tilde{\mathcal{E}}_i)$ can be seen as the decreasing precision of information \tilde{x}_i . Assume that noise terms $\tilde{\mathcal{E}}_1, \tilde{\mathcal{E}}_2, \dots, \tilde{\mathcal{E}}_I$ are jointly normally distributed and their covariances are zero, that is, $\text{Cov}(\tilde{\mathcal{E}}_i, \tilde{\mathcal{E}}_j) = 0 \quad \forall i \leq I, j \leq I, i \neq j$.

Each investor searches for information set $\tilde{\theta}$ which is a subset of the market information, that is, $\tilde{\theta} \subseteq \{\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I, \tilde{p}\}$. Let $C_{\tilde{\theta}}$ represent the pecuniary cost of searching information set $\tilde{\theta}$, the precision of which is $\Lambda_{\tilde{\theta}}$.

Traders could collect many pieces of information about \tilde{v} to make trading decisions. After traders have observed the information set $\tilde{\theta}$, they become informed to infer the true value of firm's shares and decide to trade shares $q_{\tilde{\theta}}$ competitively. It is reasonable to assume that the searching cost of information set $\tilde{\theta}$ is an increasing function of the precision of that information set, that is, $\frac{\partial C_{\tilde{\theta}}}{\partial \Lambda_{\tilde{\theta}}} > 0$.

The investor's objective function

Consider an economy where there are N homogeneous-preference traders with negative exponential utility. Assume that, the total outstanding shares of the risky asset are Q in the market.

Let $\tilde{w}_{\tilde{\theta}}$ be the capital gain of financial wealth of any investor who has observed the information set $\tilde{\theta}$. Let $g(\tilde{v} | \theta)$ be the probability density of \tilde{v} , given $\theta = \tilde{\theta}$.

The objective of any trader who has observed the information set $\tilde{\theta}$ is intended to maximize his own expected utility derived from wealth. Thus, the objective function of any trader can be written:

$$\text{Max.} \quad EU(\tilde{w}_{\tilde{\theta}}) = \int U(\tilde{w}_{\tilde{\theta}}) g(\tilde{v} | \theta) d\tilde{v} \quad (2)$$

$$= EU[(\tilde{v} - \tilde{p})q | \theta]. \quad (3)$$

The determination of market trading function

The market trading of the risky asset depends on the risk attitude, the expected price change and the precision of the information observed. Specifically, we could derive the market trading function of the risky asset depending on the information set $\theta = \tilde{\theta}$:

$$q_{\theta} = \frac{1}{r} \text{var}^{-1}(\tilde{v} | \theta) E[\tilde{v} - \tilde{p} | \theta] \quad (4)$$

Where r is defined as the constant absolute risk aversion, $-\frac{U''(w)}{U'(w)} = r$, which is negatively related to the market trading

quantity q_{θ} , whereas q_{θ} is positively related to both the precision of information set and the expected share's capital gain.

Investors observe the specific information set $\tilde{\theta} = \{\tilde{x}_k\}$ to form a more precise estimate for the expected value of the firm's intrinsic value \tilde{v} .

Recall Equation 1:

$$\tilde{x}_k = \tilde{v} + \tilde{\mathcal{E}}_k$$

Where $\tilde{v} \sim N(0, \Lambda_v^{-1})$ and $\tilde{\mathcal{E}}_k \sim N(0, \Lambda_k^{-1})$ are independent of \tilde{v} .

Given \tilde{x}_k we could prove the conditional expectation and the inverse of the variance of the true value \tilde{v} are equal to \tilde{x}_k and Λ_k , respectively:

$$E(\tilde{v} | \tilde{x}_k) = \tilde{x}_k \quad (5)$$

$$\text{Var}^{-1}(\tilde{v} | \tilde{x}_k) = \Lambda_k \quad (6)$$

They imply that the posterior distribution of \tilde{v} conditional on \tilde{x}_k is normal with mean \tilde{x}_k and variance Λ_k^{-1} , where Λ_k means the conditional precision or accuracy of the specific information observed given by \tilde{x}_k . The utility-maximizing value of market trading, q_k , could be solved as:

$$q_k = \frac{\Lambda_k(\tilde{x}_k - \tilde{p})}{r} \quad (7)$$

Substituting Equation 7 into the trader's objective utility function (Equation 2) gives:

$$EU(\tilde{w}_{\tilde{\theta}} | \tilde{x}_k) = \frac{\Lambda_k(\tilde{x}_k - \tilde{p})^2}{2r} - C_k \quad (8)$$

We assume that each investor could observe various kinds of information and that the market trading quantity of the risky asset of an investor who has observed the information set $\theta = \{\tilde{x}_i, \tilde{x}_j\}$ is just equal to the sum of the market trading quantity of an investor who has observed the information \tilde{x}_i and who has observed the information \tilde{x}_j due to the given budget constraint. Specifically,

$$q_{i,j} = q_i + q_j \quad (9)$$

$$q_{1,2,\dots,I} = q_1 + q_2 + \dots + q_I \quad (10)$$

STOCK MARKET EQUILIBRIUM AND COMPARATIVE ANALYSIS

Competitive equilibrium versus rational expectations equilibrium

The market clearing condition for the risky asset is a function of the observed information set $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$. It implies that different information about the true value of the risky asset leads to different market trading behavior of the risky asset, and also results in different market equilibrium regimes.

We showed that the equilibrium price reflects both the observational frequency and the precision of the market information. We found that the competitive equilibrium price \tilde{p}^c is equal to the rational expectations equilibrium price \tilde{p}^e , and that \tilde{p}^c (or \tilde{p}^e) is equal to the fully-informed economy equilibrium price p^f if the observational frequency of the market information is homogeneous.

Without considering the effect of the observational frequency of information, Grossman (1976) proved that the market equilibrium price reflects a simple average of the market information and verified that, the market equilibrium price is a sufficient statistic for all the market information. We showed that Grossman's result occurs only when the value of information, $f_i \Lambda_i$, is uniform, that is, $f_1 \Lambda_1 = f_2 \Lambda_2 = \dots = f_I \Lambda_I$.

We showed that the rational expectations equilibrium price \tilde{p}^e is equivalent to the competitive equilibrium price \tilde{p}^c which reflects all the market information $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$, their corresponding observational frequencies f_i 's and their precision Λ_i 's. We also found that \tilde{p}^e or \tilde{p}^c is negatively related to the magnitude of risk aversion and the total number of outstanding shares.

$$\tilde{p}^e = \tilde{p}^c = \tilde{x} - \frac{rQ}{\sum_{i=1}^I f_i \Lambda_i}, \quad \tilde{x} \equiv \frac{\sum_{i=1}^I f_i \Lambda_i \tilde{x}_i}{\sum_{i=1}^I f_i \Lambda_i} \tag{11}$$

The properties of the equilibrium price in Equation 11 are characterized as below:

$$\tilde{p}^e = \tilde{p}^c = \tilde{v} + \tilde{\varepsilon} - \frac{rQ}{\sum_{i=1}^I f_i \Lambda_i}, \quad \tilde{\varepsilon} \equiv \frac{\sum_{i=1}^I f_i \Lambda_i \tilde{\varepsilon}_i}{\sum_{i=1}^I f_i \Lambda_i} \tag{12}$$

$$E(\tilde{p}^e) = E(\tilde{p}^c) = E[\tilde{v}] - \frac{rQ}{\sum_{i=1}^I f_i \Lambda_i} \tag{13}$$

$$Var(\tilde{p}^e) = Var(\tilde{p}^c) = Var(\tilde{x}) = Var(\tilde{v}) + Var(\tilde{\varepsilon}),$$

$$Var(\tilde{\varepsilon}) = \frac{\sum_{i=1}^I f_i^2 \Lambda_i}{(\sum_{i=1}^I f_i \Lambda_i)^2}; \tag{14}$$

$$\frac{\partial \tilde{p}^e}{\partial f_k} = \frac{\partial \tilde{p}^c}{\partial f_k} = \frac{\Lambda_k \tilde{x}_k}{\sum_{i=1}^I f_i \Lambda_i} > 0 \tag{15}$$

Equation 11 shows that the market competitive equilibrium price is reflecting a weighted average of all the market information with weights of the corresponding observational frequency and the precision of information set, that is, $(f_1 \Lambda_1, f_2 \Lambda_2, \dots, f_I \Lambda_I)$. This implies that the informationally efficient price system could reflect diverse information $(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I)$.

Equation 13 implies that the expected market equilibrium price is smaller than the expected intrinsic value \tilde{v} . If the outstanding shares of the risky asset Q or risk aversion measure r is equal to zero, or the aggregate precision of information raises to become relatively large, then the expected market equilibrium price would approach the intrinsic value of the risky asset. Only under this situation, the market competitive equilibrium price or the rational expectations equilibrium price could act as an unbiased estimator of the intrinsic value of the risky asset.

Equation 14 shows that the market competitive equilibrium price of the risky asset, relative to its intrinsic value, tends to exhibit excess volatility.

Equation 15 shows that the response of the market competitive equilibrium price to the observational frequency of information f_k is positive, and it is increasingly related to the product value of the precision and the actual observation of that information, that is, $\Lambda_k \tilde{x}_k$. Recent empirical research also verifies a positive association between the information disclosure level and the share price of equity (Botosan, 1997), but we emphasize the specific information value of its precision.

Equation 16 indicates that raising both the frequency and the precision of information will increase the aggregate precision of information set, $\sum_{i=1}^I f_i \Lambda_i$, and the

investor will become better informed, however, in the meantime the specific information such as financial reports \tilde{x}_k will convey a relatively smaller value of the total information set such that the price value-added effect of the specific frequency f_k of that information declines although, the investor becomes overall better informed.

Empirically, we could derive the elasticity of security price to the specific information \tilde{x}_k as η_k , which is

positively related to r, Q, f_k, Λ_k and \tilde{x}_k^2 .

$$\eta_k = \frac{\partial \tilde{p}}{\partial \tilde{x}_k} \frac{\tilde{x}_k}{\tilde{p}} = \frac{f_k \Lambda_k \tilde{x}_k^2}{\sum f_i \Lambda_i \tilde{x}_i - rQ} \quad (16)$$

Fully-informed equilibrium

The fully-informed equilibrium price is that each trader observes the whole information set and then makes his trading decision for the risky asset (Grossman, 1981).

The fully-informed market equilibrium price denoted by \tilde{p}^f aggregates all the market information associated with their precision. \tilde{p}^f is just a special case of the competitive or rational expectations equilibrium price when the observational frequency of information is homogeneous, and \tilde{P}^f is determined by:

$$\tilde{p}^c = \tilde{p}^e = \tilde{p}^f = \frac{[\sum_{i=1}^I \Lambda_i \tilde{x}_i] - rQ}{\sum_{i=1}^I \Lambda_i} \quad (17)$$

Moreover, we could show that the precision of the intrinsic value \tilde{v} conditional on the full information set $(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I)$ is not less than the precision of \tilde{v} conditional on the observed price \tilde{p} as follows:

$$Var^{-1}[\tilde{v} | \tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I] - Var^{-1}[\tilde{v} | \tilde{p}] = \sum_{i=1}^I \sum_{j=1}^I (f_i - f_j)^2 \Lambda_i \Lambda_j \geq 0. \quad (18)$$

The observed price can act as a sufficient statistic for a full information set only when each observational frequency of information is homogeneous, that is, $f_1 = f_2 = \dots = f_I$. If they are not homogeneous, then $Var[\tilde{v} | \tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I] < Var[\tilde{v} | \tilde{p}]$, implying that the observed price can not act as an efficient estimator of the intrinsic value of the risky asset, so that traders still have an incentive to collect and to search for other more valuable information to improve their estimate and bring it closer to the intrinsic value of the risky asset.

Comparative analysis

Inserting the competitive equilibrium price, Equation 11, into Equations 7 and 8, the optimal trading quantity and the optimal expected utility corresponding to the specific information observed \tilde{x}_k was obtained:

$$q_k^* = \frac{\Lambda_k (\tilde{x}_k - p^*)}{r} = \frac{\Lambda_k \frac{\sum_{i=1}^I f_i \Lambda_i (\tilde{x}_k - \tilde{x}_i) + rQ}{\sum_{i=1}^I f_i \Lambda_i}}{r} \quad (19)$$

$$EU_k^* = \frac{\Lambda_k}{2r} \left[\frac{\sum_{i=1}^I f_i \Lambda_i (\tilde{x}_k - \tilde{x}_i) + rQ}{\sum_{i=1}^I f_i \Lambda_i} \right]^2 - C'(\Lambda_k) \quad (20)$$

Equations 19 and 20, imply that the optimal trading volume for the risky asset and the optimal expected utility gained by an investor who has observed the specific information \tilde{x}_k reflect the aggregate differences between the specific information observed \tilde{x}_k and the other information observed \tilde{x}_i .

The larger the differences are, the more active trading behavior investors will adopt, thus leading to a higher level of the optimal trading volume and the expected utility. Investors trade among themselves because they have different valuations about the intrinsic value of the risky asset.

The behavior of the trading volume is closely linked to the information heterogeneity observed among investors. Policies that eliminate information dispersion will reduce the optimal level of the trading quantity and the expected utility.

Differentiating Equation 20 with respect to Λ_k yields:

$$\frac{\partial EU_k^*}{\partial \Lambda_k} = \frac{1}{2r} \left[\frac{\sum_{i=1}^I f_i \Lambda_i (\tilde{x}_k - \tilde{x}_i) + rQ}{\sum_{i=1}^I f_i \Lambda_i} \right]^2 = \frac{r}{2\Lambda_k^2} (q_k^*)^2 > 0. \quad (21)$$

The result shows that the optimal expected utility derived from the specific information \tilde{x}_k rises as the precision of the corresponding information increases.

Differentiating equation 20 with respect to r yields:

$$\frac{\partial EU_k^*}{\partial r} = \frac{-\Lambda_k}{2r^2} \left[\frac{\sum_{i=1}^I f_i \Lambda_i (\tilde{x}_k - \tilde{x}_i) + rQ}{\sum_{i=1}^I f_i \Lambda_i} \right]^2 = -\frac{(q_k^*)^2}{2\Lambda_i} < 0. \quad (22)$$

Equation 22 implies that the higher the risk aversion of the investor k , the lower the optimal expected utility derived from the specific information \tilde{x}_k .

Differentiating equation 19 with respect to f_k gives:

$$\frac{\partial q_k^*}{\partial f_k} = \frac{\Lambda_k}{r} \left[\frac{-rQ\Lambda_k}{\left(\sum_{i=1}^I f_i\Lambda_i\right)^2} \right] \leq 0. \tag{23}$$

Equation 23 shows that the optimal marginal trading quantity of the observational frequency of the specific information \tilde{x}_k is nonpositive. If the stock market does not exit, the outstanding shares of the initial risky asset Q , the optimal trading quantity q_k^* traded by an investor who has observed the specific information x_k will be a constant.

THE CONDITION OF PRICE AS A SUFFICIENT STATISTIC FOR THE MARKET INFORMATION

Grossman (1976) argued that the market competitive equilibrium price is a simple average over the market information collected by traders and is a sufficient statistic for the market information. However, we find that Grossman (1976) just discussed a special case in our model where the information value denoted by $f_i\Lambda_i$ is uniform, that is,

$$f_1\Lambda_1 = f_2\Lambda_2 = \dots = f_I\Lambda_I \tag{24}$$

Moreover, we prove that the homogeneity of the observational frequency of information is a sufficient condition for the market competitive equilibrium price as a sufficient statistic for the information about the true value of the risky asset. When the observational frequency of information is not homogeneous, the market equilibrium price is not a sufficient statistic for the market information about the true value of the risky asset. Therefore, information asymmetry persists in the market equilibrium.

As all the random variables are normally distributed, the conditional distribution of the equilibrium price could be characterized by the conditional mean and variance. According to Bray (1981), if all the random variables are normal, and if $\frac{E(\tilde{v} | \tilde{x}_k, \tilde{p})}{Var(\tilde{v} | \tilde{x}_k, \tilde{p})} = \frac{E(\tilde{v} | \tilde{p})}{Var(\tilde{v} | \tilde{p})}$, then \tilde{p} could act as a sufficient statistic for the information \tilde{x}_k about the true value of the risky asset \tilde{v} . Bray's condition can also be expressed as follows:

If and only if $\frac{Cov(\tilde{p}, \tilde{x}_k)}{V(\tilde{p})} = 1$ or $[Cov(\tilde{p}, \tilde{x}_k) - V(\tilde{v})]\tilde{p} - [V(\tilde{p}) - V(\tilde{v})]\tilde{x}_k = 0$, the random variable \tilde{p} could act as a sufficient statistic for the information \tilde{x}_k about the true value of the risky asset \tilde{v} . The condition $\frac{Cov(\tilde{p}, \tilde{x}_k)}{V(\tilde{p})} = 1$ is

equivalent to the coefficient on \tilde{p} when we regress \tilde{x}_k on \tilde{p} using OLS.

Recall $\tilde{p} = \frac{\sum f_i\Lambda_i\tilde{x}_i}{\sum f_i\Lambda_i}$, $Cov(\tilde{p}, \tilde{x}_k) - V(\tilde{v}) = \frac{f_k}{\sum f_i\Lambda_i}$, and

$V(\tilde{p}) - V(\tilde{v}) = \frac{\sum f_i^2\Lambda_i}{(\sum f_i\Lambda_i)^2}$, substituting them into the above

Bray's condition, we get the informationally efficient condition for $\tilde{x}_k = \frac{f_k \sum f_i\Lambda_i\tilde{x}_i}{\sum f_i^2\Lambda_i}$, $\forall k=1, 2, \dots, I$. It is

obvious that \tilde{p} differs from \tilde{x}_k due to $f_k \neq f_i$. Therefore, it demonstrates that information asymmetry among investors persists in the market equilibrium.

If the observational frequency of each piece of information is homogeneous, that is, $f_1 = f_2 = \dots = f_k$, then the market competitive equilibrium price \tilde{p}^c , the rational expectations equilibrium price \tilde{p}^e and the fully-informed economy equilibrium price \tilde{p}^f are all equal, and it is the price which could act as a sufficient statistic for the market information about the true value of the risky asset \tilde{v} .

MAIN FINDINGS AND FURTHER RESEARCH SUGGESTIONS

When some agents behave irrationally, or when some markets operate inefficiently, opportunities exist for others to make profit. The profit seeking behavior tends to eliminate these opportunities as some investors attempt to earn extra return from exploiting asymmetric information, then the market equilibrium price will be affected and the information will be transmitted across the market.

Comparing to related works such as Grossman (1976, 1978, 1981), Grossman and Stiglitz (1980), Hellwig (1980), Bray (1981), Paul (1993), and Baigent (2003), we analyze a more general model of a competitive stock market equilibrium in which each trader could observe various pieces of information about the true value of risky assets. We are the first to introduce the concept of the observational frequency of information to a competitive stock market and to assume that informed traders could observe not only one piece of information but also several pieces so that the observational frequency of the market information observed by traders is heterogeneous.

Consider a stock market where there are N homogeneous-preference traders with negative exponential utility. Assume that total number of outstanding stock shares are Q in the market where there are I kinds of information $\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_I$ being observed with f_1, f_2, \dots, f_I frequencies respectively by risk-averse

traders. Each informed investor is based on the information observed to form the best estimate for the expected value of the firm's true value, \tilde{v} , and then decides whether or not to invest to maximize his own utility. The stock market equilibrium is then determined. Our main findings are as follows:

Firstly, we showed that the market equilibrium price aggregates all the market information including not only the precision of information but also the observational frequency of information. The price response to an unexpected change of information is positively related to both the precision and the observational frequency of that information.

Secondly, we found that the market competitive equilibrium price is the same as the rational expectations equilibrium price. The fully-informed equilibrium price becomes only a special case of the market competitive equilibrium price under the condition when the observational frequencies of each market information are equalized.

Thirdly, when the observational frequencies of each market information are uniform, the market equilibrium price could act as a sufficient statistic of the true value of the risky asset. However, the observational frequencies of each market information are usually not uniform, this may motivate some investors to collect and to search for costly information to improve their estimate of the true value of the risky asset.

Finally, further research is needed to extend the economy models that assume each investor observes only one piece of information.

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