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Full Length Research Paper

Simulation of risk based on ending activeness of the project plan

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This work presents the results of the theoretical-experimental researches of the quantification for the superponed flow time of the two local-autonomous flows in the net of the basic Clark's equations. The computer solution to this basic variant of the general flow model through the net is performed by the methods of the numerical simulation (Monte Carlo), supplemented by frames' method. The numerical experiment is realized by the program tool Mathcad Professional.

Key word: Mathematical model, simulation, project plan.

INTRODUCTION

The host model, for which the Clark's equation of the equivalent (resulting) activity, consists of the oriented graph, where two activities go parallel, they have the common beginning and run up to a terminal "event". In that sense activities can be local - autonomous, until they are totally realized. The results of Clark's equations in this work are compared to the results of the Monte Carlo - frame numerical simulation. Both methods, analytic and numerical ones, are characteristic for studying different phenomena and processes based upon network models of activity flows, resources, energies and likes. Those problems, as we know from the experience of Clark (1961), are mostly of stochastic character, and solving them in an analytic way is often unfeasible without certain approximation. In this way, stimulated by Clark (1961), Clemen (1996), Slyke (1963), Dodin (1984), Fishman (1986, 1999), Haga and O'Keefe (2001), Keefer and Bodily (1983), Keefer and Verdini (1993), Littlefield and Randolph (1987), Lock (2007) and Vose (1996), a network model of flows activity is defined and the contribution to algorithms development is given for solving the general model of critical flows based on row-parallel graph structures. In this analysis, it is supposed that there is a normal distribution of single activity endings with the average (mean) values characteristics and appropriate time deviations of their realization.

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THE FLOWS WITH CRITICAL ACTIVITIES

The unambiguous solution for critical activity flow, and with it, the resulting flow time using the expecting times of the elementary flows-activities, presents one of the most troublesome effects of the network planning application based on stochastic methods. The stochastic (but also deterministic) activities networks formed, for example, the arrow diagram method (ADM) basis structure can in some planning cases be very complex. There are the examples of the network diagrams, for example, in machine-building industry, where the number of activities amounts to several thousands, with several hundreds of identified critical and subcritical flows (paths). From the analysis standpoint of the critical flows, as seen in the study of Clemen (1996), the particular problem encountered is the variant with parallel critical flows, which do not contain one common (unique) activity. The only one common thing with those activities is: the initial and final event and the approximately same or different values realized from the critical and subcritical flows. The final event will be realized if all the critical flows that "run up into it" are realized. In that case, we can rightly put the question: how great is the certainty (as well probability distribution) that the resulting flow time will be completed within the planned period of time Tp, taking into consideration that such activity graph can comprise one, two ar limitless number of critical flows parallel, ordinal ar combined type. For giving a correct answer to this question, it is necessary to define exactly the algorithm for the impact quantification, primarily, critical and subcritical flows and forming the resulting - superponed flow time of flows.

The aim of the paper

The basic aim of this paper is the impact quantification of critical and subcritical flows on forming the resulting, that is, superponed flow time. With its solution, a fundamental base is being created here for defining the function of probability distribution, as well relatively noticing those flows by frames methods application.

The defining of the basic time parameters for the autonomous critical flows

According to the researches by Clark (1961) and Van Slyke (1963), the intervals superponing of the critical and subcritical flow times and their deviations and (their) reducing into an equivalent flow can be deduced by:

• Analytic methods: with Clark's equations for the parallel flows solving, on the basic of the central limit theorem, for ordinal flows solving and
• By numerical method: - Monte Carlo - frame simulation for the parallel - ordinary flows. To illustrate the application of the above-mentioned fundamental algorithms, we shall take the ADM - network with two parallel flows π_1 and π_2 (Figure 1).
• Based on modeling Fuzzy.

The Superpone time and the flow variant

In the algorithm structuring for the analytical solving of this critical flows variant, the paper starts from Clark's authentic equations. With these equations, the flows parameters are being solved as follows: the superpone flow time P_{12} and its variants $\sigma^2(P_{12})$. For basic oriented graph with two parallel flows, from the initial (*r*) to the terminal (*k*) event (Figure 2), the flow time values P_{12}, are:

• The superponed flow time:

$$\overline{P_{1,2}} = \overline{P_1} \cdot \Phi(\xi_{1,2}) + \overline{P_2} \cdot \Phi(-\xi_{1,2}) + \lambda_{1,2} \cdot \Psi(\overline{\xi_{1,2}}) \quad (1)$$

Where: $\Phi(\xi) = (2\pi)^{-1/2} \cdot \int_{-\infty}^{\xi} \exp(-z^2/2)dz$ - Laplace integral, $\Psi(\xi) = (2\pi)^{-1/2} \cdot \exp(-\xi^2/2)$ - the density function of the centered normal distribution and $\lambda_{1,2} = \sqrt{\sigma^2(P_1) + \sigma^2(P_2)}$, that is $\xi_{1,2} = \frac{1}{\lambda_{1,2}} \cdot (\overline{P}_1 - \overline{P}_2)$ - the parameter of Clark's functions. In addition to it, the predicted or mean values of time intervals are usually taken as:

$$\overline{P_1} = \mu_1 \text{ i } \overline{P_2} = \mu_2 \quad (2)$$

so according to Equations 1 and 2, it follows that:

• The mean superponed flow time is:

$$\mu_{1,2} = \mu_1 \cdot \Phi(\xi_{1,2}) + \mu_2 \cdot \Phi(-\xi_{1,2}) + \lambda_{1,2} \cdot \Psi(\xi_{1,2}) . \quad (3)$$

• The superponed dispersion is presented by the second Clark's equation:

$$\sigma_{1,2}^2 = (\mu_1^2 + \sigma_1^2) \cdot \Phi(\bar{\xi}_{1,2}) + (\mu_2^2 + \sigma_2^2) \cdot \Phi(-\bar{\xi}_{1,2}) + (\mu_1 + \mu_2) \cdot \lambda_{1,2} \cdot \varphi(\bar{\xi}_{1,2}) - \mu_{1,2}^2 \quad (4)$$

With this equation, we can describe characteristics of an equivalent flow instead of the previous two flows (Figure 2).

The growth of the superponed flow time in relation to the critical flow

On the basic of the new superponed function of the time distribution P_{12}, with the characteristics $N \sim [\mu_{12}, \sigma_{12}]]$, one can quantify the time growth $P_{1,2}$ in relation to the single time P_1 or P_2, depending on which of them has the critical feature. For the elementary network with the autonomous flows π_1 and π_2, the growth or "the superponed extract", after a more straightforward derivation results to:

$$\Delta\mu_{1,2} = \lambda_{1,2} \cdot \varphi(\xi_{1,2}) + (\mu_2 - \mu_1) \cdot \Phi(-\xi_{1,2}) \quad (5)$$

Meanwhile, in the case of the reversed choice, it follows:

$$\Delta\mu_{2,1} = \lambda_{2,1} \cdot \varphi(\xi_{2,1}) + (\mu_1 - \mu_2) \cdot \Phi(-\xi_{2,1}) \quad (6)$$

In addition, these values are by their nature (essence) always negative, that is: $\Delta\mu_{1,2} \geq 0$ and $\Delta\mu_{2,1} \geq 0$.

The Invariant ability testing of the flow model

The invariant ability ought to prove that the invented values remained unchanged and unambiguously fixed while changing the flows rank order in the reckoning process. It is well-known that, already with only two flows with two parameters each for every flow, we can have nine relations. In other words, analyzing the next possible relations between the expected times and the deviations of single flows, we have:

$$\mu_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \mu_2 \quad \text{and} \quad \sigma_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sigma_2 \quad (7)$$

It can be concluded that nine different combinations can be totally formed. Accepting that:

$$\lambda_{1,2} = \lambda_{2,1},\ \xi_{1,2} = -\xi_{2,1},\ \text{and } \Phi(-\xi_{1,2}) = \Phi(\xi_{2,1}) \quad (8)$$

The paper gets the invariant relations of the basic tested values that are connected to the superponed flow, that is:

$$\mu_{1,2} = \mu_{2,1};\ \Delta\mu_{1,2} = \Delta\mu_{2,1} \text{ and } \sigma^2(P_{1,2}) = \sigma^2(P_{2,1}) \quad (9)$$

It can be concluded that any of the two flows that will be observed as critical or subcritical is irrelevant. This characteristic of the model's invariant ability (Letić, 1996) is very essential and in addition to the

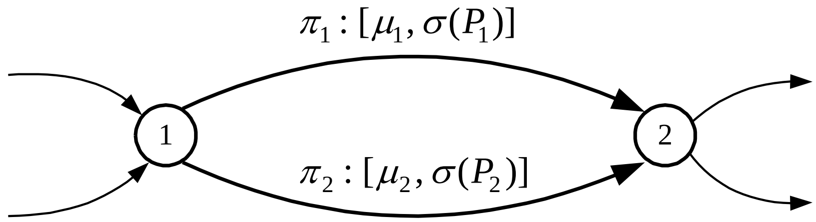

Figure 1. The flow network with two local autonomous activities flows.

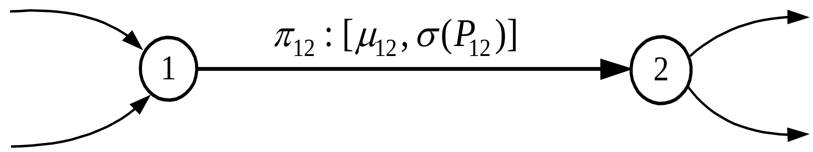

Figure 2. The equivalent - superponed activity flow.

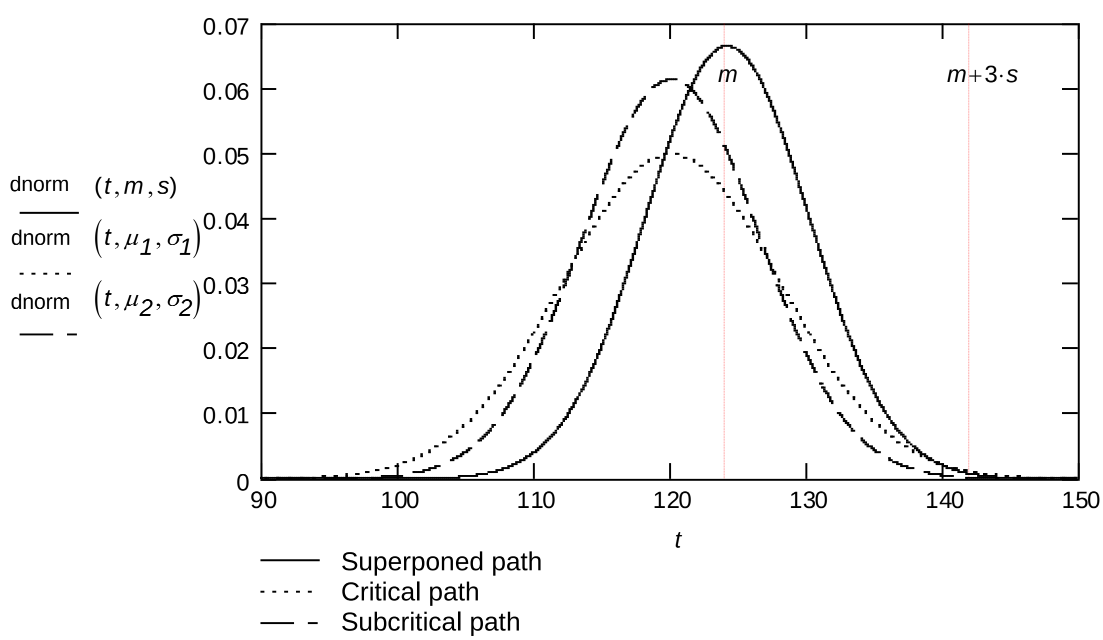

Figure 3. The distributions of probability for critical, subcritical and superponed flow time.

analytical verification, there can also be a numerical performed model: Fishman (1986), Haga and O'keefe (2001), with the Monte Carlo simulation.

THE APPLICATION OF THE SIMULATION MODELS

The use of the Monte Carlo method for solving Clark's flow model

Because the elementary activities of the flow time have the normal distribution with the parameters $N \sim [\mu_j, \sigma_j]$, ($j = 1,2$) as convenient method, for the modeling of the observed random changeable value, the method of inverse functions is accepted. Figure 3 present the results of the numeric simulation of the $n = 5x10^5$ replications for chosen values $N \sim [\mu_1 = 120, \sigma_1 = 8]$ and $N \sim [\mu_1 = 110, \sigma_1 = 6.5]$. Here are also obtained:

- theoretic values: $N \sim [\mu_{12} = 120.908747, \sigma_{12} = 7.1046795]$,
- simulation values: $N \sim [m_{12} \approx 120.897371, s_{12} \approx 7.109556]$.

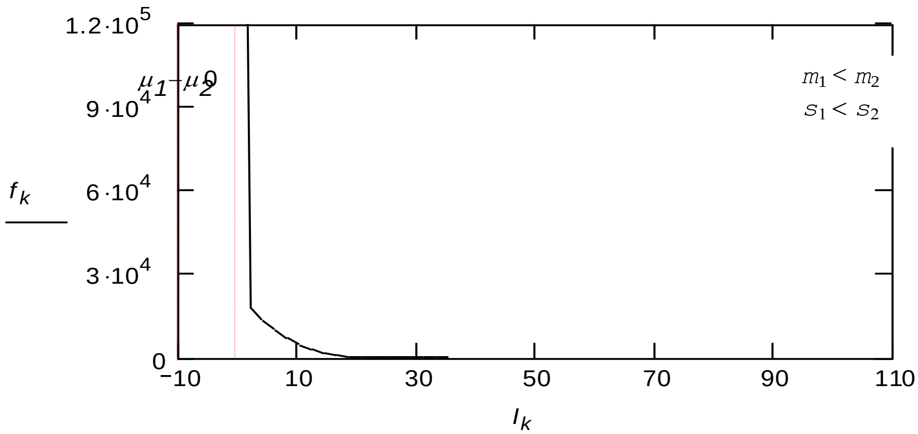

Figure 4. The frame for the values: $[\mu_1 = 120,\ \sigma_1 = 8]$ *and* $[\mu_2 = 130,\ \sigma_2 = 6.5]$.

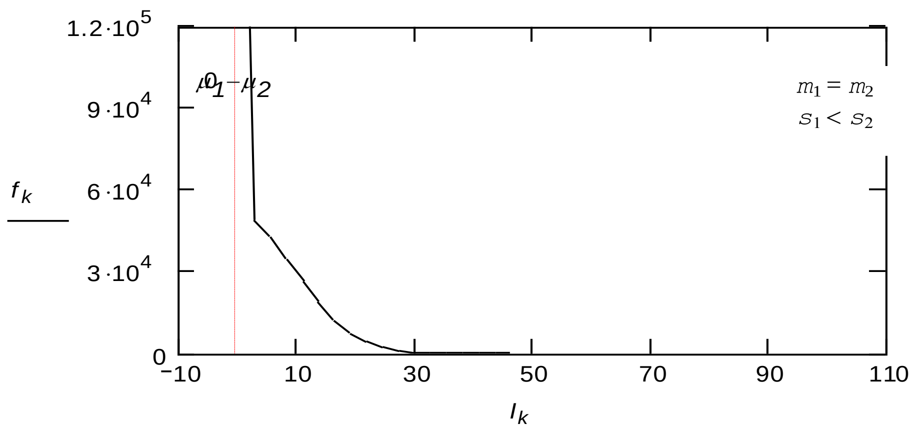

Figure 5. The frame for the values: $[\mu_1 = 120,\ \sigma_1 = 8]$ *and* $[\mu_2 = 120,\ \sigma_2 = 6.5]$.

Meanwhile, as this algorithm is computer fixed, the main point of the problem is now within the purview of simulation, where one session of the simulation of $n = 5x10^5$ replications was done by testing only one chosen variant, when it is $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$ of the possible nine ones.

The application of the Monte Carlo - frame methods in solving the Clark's flow model

The purview extension of the Monte Carlo method is possible to be done by frames using Equation 7, with computer frames by means of changing the fixed parameter, through the vector value μ_1. In the set algorithm, Monte Carlo is formed the convenience for understanding and the visualization of a broader class of appearances, than it was in previous stereotype-satic view on the process and the simulation results. In that sense the supposition (Equation 9) can be sum up to three variants in one simulation session:

$$\mu_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \mu_2 \text{ and } \sigma_1 < \sigma_2 . \quad (10)$$

The frame number depends on the problem complexity, that is, the studied process. One should not neglect the esthetic moment of the frames presentation, as a result, the integrated Monte Carlo method-frame has also very educative role. Here are frames which are totally, spontaneously connected for the simulation process, and so broadened Monte Carlo simulation for "a new dimension", what can be partially presented by a series of selected frames (Figures 3, 4, 5, 6, 7, 8 and 9).

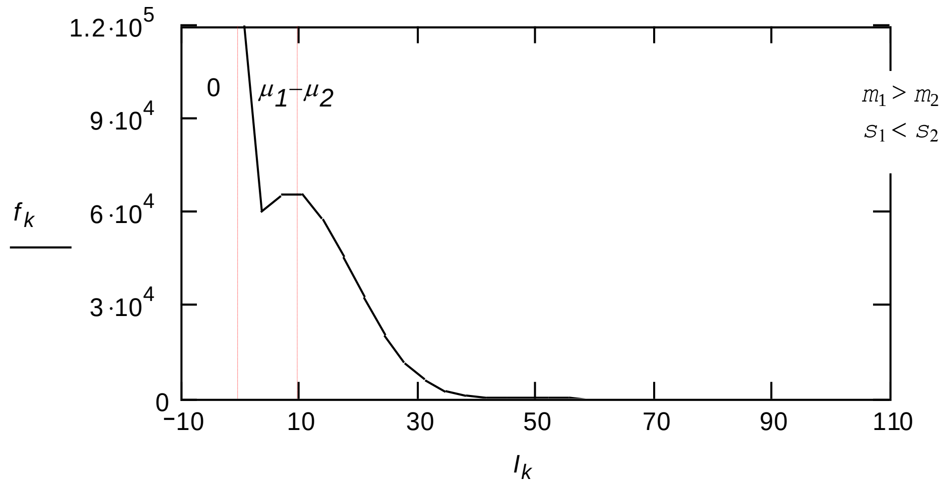

Figure 6. The frame for the values: $[\mu_1 = 120, \ \sigma_1 = 8]$ *and* $[\mu_2 = 110, \ \sigma_2 = 6.5]$.

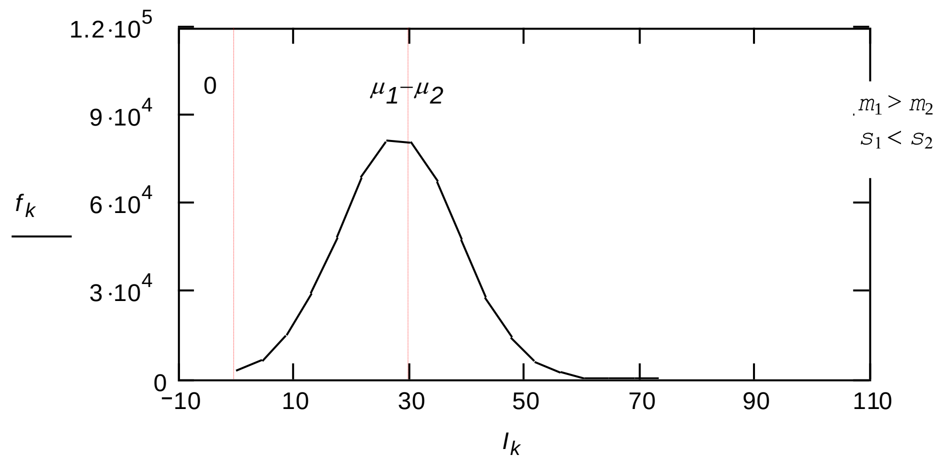

Figure 7. The frame for the values: $[\mu_1 = 120, \ \sigma_1 = 8]$ *and* $[\mu_2 = 90, \ \sigma_2 = 6.5]$.

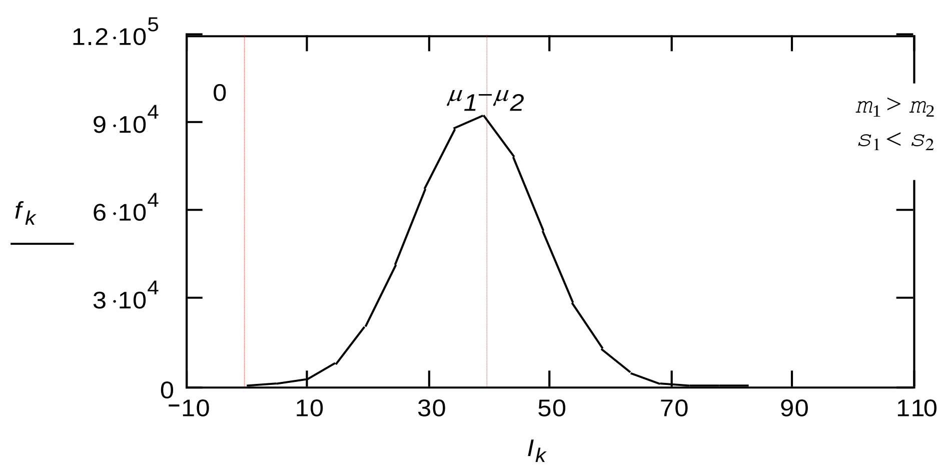

Figure 8. The frame for the values: $[\mu_1 = 120, \ \sigma_1 = 8]$ *and* $[\mu_2 = 80, \ \sigma_2 = 6.5]$.

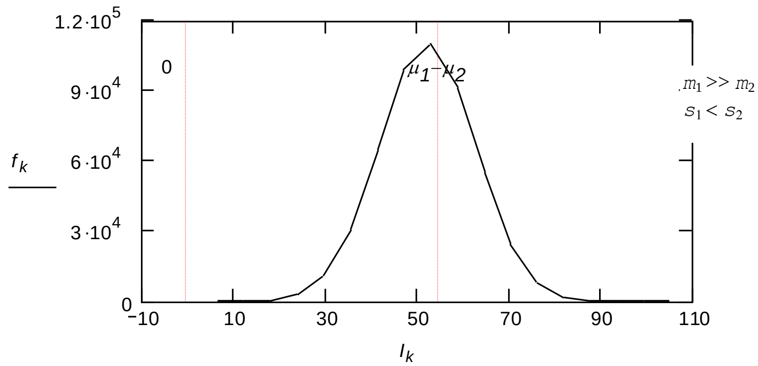

Figure 9. The frame for the values: $[\mu_1 = 120,\ \sigma_1 = 8]$ *and* $[\mu_2 = 65,\ \sigma_2 = 6.5]$.

CONCLUSION

The most important advantage of Monte Carlo simulation method, in solving this flow problem through the network, presents the possibility of function modeling for function distribution of superponed flow time possibility for the essential flow time of the basic network model, presented in Figure 1. Meanwhile, the advancement of Monte Carlo frame simulation method is substantially increased because of the possibility for dynamic flow modeling through network. The frames provide more reliable basic for further acquirement and expanding of knowledge in this field, especially in relation to the relativity of the critical flow activity. Using combine procedures of Monte Carlo and frames, there can be more authentic performed time planning for the critical flows, than what is achieved by standard procedures of network planning and managing, for example, through program evaluation and review technique (PERT). With the classical PERT, the flow time planning was established on the expected values of the elementary flow times and so a considerable mistake was done in the planning, since in principle, the influence of subcritical flows and forming of the total superponing flow time was neglected. Van Slyke (1963) indicated that, in the flow network with ten (sub)critical flows of autonomous type, the resulting flow time increases by 11% from the time we should get by calculating using the PERT method. This “planning mistake” as theoretical result, verified also by simulation for two parallel flows, is $\Delta = 3.313\%$. The result obtained by frames is not unambiguous but it depends on the chosen value pairs $N \sim [\mu_j, \sigma_j]$, ($j = 1{,}2$). In addition to it, we can distinctly see from one frame to another, the domination losing of the critical flow in favor of the subcritical one, if the mean value of this latter is increased as regard to the former one. Of course, it is possible by Monte Carlo frame method to test also the remaining cases, for example, when there is put the deviation vector, instead of the mean value vector as in Equation 10:

$$\mu_1 = \mu_2 \text{ and } \sigma_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sigma_2 \qquad (11)$$

These influences Equation 10 and 11 can be by simulation explicitly perceived with more complex ADM networks (Equation 10). The consequences of the essence of ignoring the obtained results can be very negative especially in the planning and control cased of the complex stochastic flows activities through network.

REFERENCES

Clark CE (1961). The greatest of finite set of random variables. Oper. Res. 12(9): 145-162.
Clemen RT (1996). Making Hard Decisions: An Introduction to Decision Analysis. Duxbury Press.
Dodin B (1984). Determining the (k) most critical paths in PERT networks. Oper. Res. 32(4):859 877.
Fishman SG (1986). A Monte Carlo sampling plan for estimating network reliability. Oper. Res., 34(4):581-594.
Fishman SG (1999). Monte Carlo Concepts, Algorithms and Applications. Springer Series in Operations Research, Springer.
Haga AW, O’keefe T (2001). Crashing PERT networks: A simulation Approach. 4th International conference of the Academy of Business and Administrative Sciences Conference Quebec City, Canada.
Keefer DL, Bodily SE (1983). Three-point approximations for continuous random variables. Manag. Sci., 29(5): 595-609.
Keefer DL, Verdini WA (1993). Better estimation of PERT activity time parameters. Manag. Sci., 39(9): 1086–1091.
Letić D (1996). Edukativni i opšti model kritičnih protoka materijala Precedence Diagramming strukture. (PhD thesis). Tehnički fakultet "Mihajlo Pupin", Zrenjanin.
Letić D, Davidović B, Berković I, Petrov T (2007). Mathcad 13 u matematici i vizuelizaciji. Kompjuter biblioteka, Beograd.
Littlefield TK Jr, Randolph PH (1987). An answer to Sasieni’s question on PERT times. Manag. Sci. 33(10):1357-1359.
Lock D (2007). Project Management (9e ed.). Gower Publishing, Ltd., 2007.
Slyke Van MR (1963). Monte Carlo methods and PERT problem. Oper. Res. 11(5):839-860.
Vose D (1996). Quantitative Risk Analysis: A Guide to Monte Carlo Simulation Modeling. Wiley.