Qualitative model of multi-criteria analysis for the need of selecting a new product

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The work gives a survey of a qualitative model of multicriteria analysis by means of which alternatives are ranked in full array. The model derives its basic idea and philosophy from Analytic Hierarchy Process (AHP) and Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE) methods, so that it could be said to represent a combination of these two methods by applying appropriate elements of multiple attribute utility theory. This fact alone already understands certain novelties and originality. Besides this, the model includes some of its characteristic elements which do not appear in these methods. The essential change regarding the existing method is contained in the modification of Saaty’s Scale for comparing attributes in pairs. This change also caused changes in other novelties in the model. The proposed scale for comparison of attributes is separately analysed in the work. This scale enables a far greater number of combinations for assigning assessments to superordinate and subordinate attributes, as well as more accurate comparison which corresponds much more to the real conditions. In the conclusion, some of the basic features and advantages of the model are explained.

Key words: Model, criteria, alternatives, assessments, comparison.

INTRODUCTION

Quantitative model (QM) represents an original model of multicriteria analyses by means of which alternatives are ranked in full array. QM draws its basic idea and philosophy from Analytic Hierarchy Process (AHP) (Saaty, 1980) and Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE) (Brans et al., 1984) methods, so that it could be said that QM represents a combination of these methods, alongside the use of appropriate elements of multiple attribute utility theory (Oberstone, 1990). Current form of QM is obtained after considering numerous procedures and models, for example: (Enea and Piazza, 2004; Hwang and Yoon, 1981; Laininen and Hamalainen, 2003; Larichev et al., 2002; Leskinen, 2000; Nochin, 1997; Podinovski, 2002). References (Farinwata et al., 2000; Hoppner et al., 1999; Pedrycz, Gomide, 1998; Royo and Verdegay, 2000; Triantaphyllou, 2000; Yager, 1995), which deal with the area of multicriteria decision-making and fuzzy sets, were also helpful.

QM has been developed for the need of selecting a new product. When comparing alternatives for new products in pairs, there arose the need to slightly emphasise the importance of one alternative in respect to the other. That is the reason why a comparison scale, which is considerably more accurate in comparison with Saaty’s scale was developed first. In order to use the formed scale as objectively as possible, it was necessary to introduce backup changes which would support the proposed scale. Alongside this, some of the basic ideas of the PROMETHEE method and multiple attribute utility theory were being used. All this resulted in creating the QM.
**METHODOLOGY**

The procedure of ranking alternatives to QM consists of six basic steps, some of which develop in several sub-steps. Basic QM steps are: 1. Defining the problem situation (initial conditions of decision making), 2. Determining relative weights of criteria, 3. Assigning of quantitative assessments to all actions for each of the criteria, 4. Selection of the type of criteria function for every single criterion, 5. Evaluation of alternatives for each criterion separately, and 6. Synthesis of the problem situation and final ranking of alternatives. After the completion of all the six steps, alternatives are ranked in full array.

**Defining initial decision making conditions**

Initial decision making conditions are given by means of a group of identified possible alternatives, among which it is necessary to determine the most acceptable one and the group of criteria on the basis of which this selection is performed. The group of alternatives is given by the expression: $\{A_1, A_2, ..., A_n\}$, where: $n$ – the number of alternatives. The group of criteria is given by the expression: $C = \{C_1, C_2, ..., C_p\}$, where: $p$ – the number of criteria. This step is of great importance and its proper performance is the condition for making the correct decision.

**Determining relative weights of the criteria**

**Assigning qualitative values to the criteria**

Taking into consideration the fact that the criteria cannot be expressed quantitatively, it is necessary to give them certain qualitative assessments which will later serve as the basis for comparison of the criteria in a pair. The criteria are given qualitative assessments ranked in five levels: 1 - Very Weak; 2 - Weak; 3 - Average; 4 - Strong; and 5 - Very Strong.

**Quantification of qualitative assessments**

Quantitative assessments are given the corresponding quantitative assessments according to the established scale of intervals (Table 1). In this way, quantification of qualitative assessments is performed.

The difference of the interval scale with relation to the AHP method is reflected in the last row of Table 1, which represents a possible variation of the basic quantitative assessment. This variation is given in the form of an interval which, in a way, represents decision maker’s rough estimate. Variation of the basic assessments is not mandatory, and it is applied only in those cases in which the decision maker is able to carry out a more accurate assessment.

The importance of assessment variation is particularly noticeable when giving quantitative assessments to the criteria which were previously assessed with the same qualitative assessments. Variation then facilitates more accurate observation of the differences in the strength of such criteria. In addition to this, variation of assessments can particularly be important if the decision maker is more frequently faced with one and the same (or similar) decision problem, but in slightly changed conditions. In such situations, it is possible to perform a considerably more objective variation.

**Comparison of criteria in pairs**

Quantitative assessments which are assigned to all the relevant criteria (with or without variation), serve as the starting point for determining relative weights of the criteria. Comparison and evaluation of the criteria is performed in pairs, according to the method which evolves in three stages. The stages are as follows:

1. **Determining the differences in assessments of all pairs of criteria and ASC and AWC values**

   Corresponding assessment is given to the stronger and weaker criterion in each pair for the determined difference of quantitative assessments of every pair of criteria. For a start, it is assumed that each pair of criteria has a collective assessment 100, in which process the stronger criterion in the pair has the assessment $50 + X$, and the weaker criterion in the pair has the assessment $50 - X$ (where $X \in \{0, 1, 2, ..., 50\}$). In this way, for example, possible combinations could be 60:40, 70:30, 95:5, and so on.

   A linear dependence, which can be represented with a graph (Figure 1), is established between the differences in assessments given to the criteria and the height of the assessment of the stronger criterion in a pair. On the ordinate of the graph (Figure 1), there is the scale of the assessment of the stronger criterion (ASC), which ranges within the interval between 50 and 100.

   The difference of assessments (DA) is entered in the interval of $[0, 1]$ (all the possible DAs are in this interval). Therefore, the value of ASC linearly and proportionally depends on DA for the observed pair of criteria. ASC is calculated by linear interpolation, that is, through the expression:

   $$ASC = 50 + |DA| \cdot 50 \quad (1)$$

   If the ASC value for some DA on the graph wants to be read, the assessment of the weaker criterion (AWC) in that pair is obtained by the expression:

   $$AWC = 100 - ASC \quad (2)$$

   The dependence of ASC on DA which is here applied as linear, can also be applied differently, depending on whether people want to have a greater or lesser stratification of the criteria according to their importance.

2. **Determining sums of assessments of all pairs of criteria and VSC and VWC values**

   The next idea is that each pair of criteria has a corresponding, common assessment, which should be distributed to the members of the pair. For example, if assessments of one pair are 0.3 and 0.2, and the other 0.9 and 0.8, DA in both cases is 0.1, so that the same

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**Table 1. Scale of intervals for quantification of qualitative assessments.**

<table>
<thead>
<tr>
<th>Qualitative assessment</th>
<th>Very weak</th>
<th>Weak</th>
<th>Average</th>
<th>Strong</th>
<th>Very strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative assessment</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Variation of Assessment</td>
<td>0 – 0.2</td>
<td>0.2 – 0.4</td>
<td>0.4 – 0.6</td>
<td>0.6 – 0.8</td>
<td>0.8 – 1</td>
</tr>
</tbody>
</table>
assessments of strong and weak criteria would be formed for both pairs. On the whole, however, the second pair is considerably "stronger", so that it is illogical that these two pairs have the same aggregate assessment 100. The real aggregate assessment (RAA) of a pair of criteria is obtained by a linear function (Figure 2). On Figure 2, the sum of assessments (SA) of a pair of criteria, which can range within the interval from 0 to 2, is shown on the abscissa. To each sum of assessment (SA) corresponds a real value of the sum of assessments (RAA) for final evaluation of a pair of criteria. The RAA value moves within the interval from 0 to 100, and is read on the ordinate of the graph in Figure 2, that is, through the expression:

$$RAA = \frac{100 \cdot SA}{2} = 50 \cdot SA$$  \hspace{1cm} (3)

The final value of the weaker criterion in the pair (VWC) is determined according to the following expression:

$$VWC = RAA - VSC.$$  \hspace{1cm} (6)

If formula (1) is applied in expression (4) instead of ASC, the study get the expression through which it can directly calculate the VSC value:

$$VSC = \frac{RAA}{2} \cdot (1 + DA).$$  \hspace{1cm} (7)

The first two stages of comparing the criteria in pairs can be represented with a table (Table 2). Designation $C_i$ represents the i-criterion, at which $i = 1, 2, 3, ..., p$. 

![Figure 1. Establishing ASC depending on DA.](image1)

![Figure 2. Establishing RAA depending on SA.](image2)
Table 2. First two stages of the method of comparison of the criteria in pairs.

<table>
<thead>
<tr>
<th>Quantitative assessment of the criterion A(C)</th>
<th>Difference of assessments of pairs DAij</th>
<th>Initial assessments of the criteria in a pair ASCij AWCij</th>
<th>Sum of assessments of a pair of criteria SAij RAAij</th>
<th>Real aggregate assessment of a pair of criteria VSCij VWCij</th>
</tr>
</thead>
<tbody>
<tr>
<td>QA(C1)</td>
<td>DA1,2 ASC1,2 AWC1,2 SA1,2 RAA1,2 VSC1,2 VWC1,2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QA(C2)</td>
<td>DA1,3 ASC1,3 AWC1,3 SA1,3 RAA1,3 VSC1,3 VWC1,3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QA(Cp-1)</td>
<td>DA1,p ASC1,p AWC1,p SA1,p RAA1,p VSC1,p VWC1,p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>QA(Cp)</td>
<td>DA1,p-1 ASC1,p-1 AWC1,p-1 SA1,p-1 RAA1,p-1 VSC1,p-1 VWC1,p-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Calculation of relative weights of the criteria.

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>Cp</th>
<th>Cp-1</th>
<th>Cp-1</th>
<th>Σ(VXCij)</th>
<th>wij</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSC1,2 (VWC1,2)</td>
<td>...</td>
<td>VSC1,p-1 (VWC1,p-1)</td>
<td>VSC1,p (VWC1,p)</td>
<td>Σ(VXC1,i)</td>
<td>wij</td>
<td></td>
</tr>
<tr>
<td>VWC1,2 (VSC1,2)</td>
<td>...</td>
<td>VSC2,p-1 (VWC2,p-1)</td>
<td>VSC2,p (VWC2,p)</td>
<td>Σ(VXC2,i)</td>
<td>wij</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>VWC1,p-1 (VSC1,p-1)</td>
<td>...</td>
<td>VSCp-1,p (VWCp-1,p)</td>
<td>VSCp,p (VWCp,p)</td>
<td>Σ(VXCp-1,i)</td>
<td>wp</td>
<td></td>
</tr>
<tr>
<td>VSC1,p-1 (VWC1,p-1)</td>
<td>...</td>
<td>VSCp-1,p (VWCp-1,p)</td>
<td>VSCp,p (VWCp,p)</td>
<td>Σ(VXCp,i)</td>
<td>wp</td>
<td></td>
</tr>
</tbody>
</table>

Values given in brackets mean that each member in a pair can be stronger or weaker depending on whether it has a higher or lower previous quantitative assessment.

Designation VXC at the same time represents both VSC and VWC, and is introduced to facilitate labelling.

2. Determining relative weights of the criteria

The values from the last two columns of Table 2 (VSC and VWC) represent the final values of the assessments of the criteria in a pair. These values are entered in a separate table in the way that is similar to AHP method. In a pair, the VSC value is given to the criterion that has a higher previous quantitative assessment, and VWC is given to the criterion which has a lower previous quantitative assessment. It is represented in Table 3 for a general case.

The same criteria are not compared between themselves. That is why the fields across the diagonal on Table 3 are not filled. The values in the next to the last column, Σ(VXC1,i), are obtained by adding up assessments in rows. The sum total of values which make up the Σ(VXC1,i) column is marked as ΣΣ(VXC1,i). With the scope of partial checking of the results, the ΣΣ(VXC1,i) value can be calculated through the formula:

\[ ΣΣ(VXC1,i) = ΣΣRAAij = [ΣFA(Ci)] \cdot (p - 1) \cdot 50 \]  (8)

Relative weights of some wij criteria are read in the last column of Table 3, and are obtained by dividing the values from column Σ(VXC1,i) with the value ΣΣ(VXC1,i):

\[ wij = \frac{Σ(VXC1,i)}{ΣΣ(VXC1,i)} \]  (9)

Assigning of quantitative assessments to all actions for each of the criteria

This is the third step in which assigning of quantitative assessments to all the attributes in the model, that is, to all the attributes which describe alternatives for each of the criteria. In this way, the previous assessments (attributes), both qualitative and quantitative, are reduced to the same order of values, namely, in the interval [0, 1]. Such unification facilitates work in the next step of QM. In this step of QM, procedures of assigning quantitative assessments to qualitative attributes and quantitative attributes are clearly limited.

Assigning the quantitative assessments to the qualitative attributes

Similarly to giving qualitative assessments to the criteria, there are five levels of qualitative assessments. Therefore, there are the
following five attributes which describe the alternatives according to the qualitative criteria: Very Bad, Bad, Medium, Good or Very Good. Upon assigning qualitative marks to the alternatives, it is necessary to translate all these assessments into the quantitative ones by means of the Interval Scale (Table 4). With some criteria, there may be a request for minimisation. The quantitative assessments are also given according to qualitative assessments, but in reverse order (last two rows in Table 4). Regarding the variation of the recommended assessments, the same remarks apply as with assessing the criteria.

Assigning the quantitative assessments to the quantitative attributes

Contrary to the previous case, and according to the quantitative criteria, alternatives can be assessed by quantitative attributes (numbers). However, these numbers can be given in most varied orders of value. Because of that, it is appropriate to reduction of assessments which are given by alternatives both according to qualitative and quantitative criteria, to the same order of values to the interval [0, 1].

The reduction of assessments in to the interval [0, 1] is performed by graphs (Figure 3). On these graphs, the quantitative assessment is entered on the abscissa, and its corresponding assessment from the interval [0, 1] according to the function defined by the decision maker is read from the ordinate. There are two basic types of these graphs, depending on the expressed request for maximisation (Figure 3a, non-decreasing function) or minimisation (Figure 3b, non-increasing function).

Selection of the type of criterion function to each criterion

In this (fourth) step, selection of the type of criterion function is performed for each criterion in the model, which in decision maker’s opinion best describes the observed criterion. The types of criterion functions are given by means of the graph to whose ordinate are imported values of the assessment of the stronger alternative (ASA) in the range from 50 to 100, and to the abscissa are imported the differences in assessments in the range [0, 1]. Criterion functions must be linear. The fact is that for a determined DA, ASA is read from the ordinate. Further procedure will be explained in the fifth step of the QM. All types of criterion functions (which are considered to be of importance in practice) can be generally represented by means of the graph of general criterion function (Figure 4).

The following conditions (general) are valid for the general type of criterion function:

1. \( l_1 \leq l_2 \leq l_3 \leq l_4 \leq 1 \)
2. \( h_1 \leq h_2 \leq h_{21} \leq h_3 \leq h_4 \leq 100 \)

ASA values per interval of absolute RA values are the following:

\[
0 \leq |DA| < l_1 \Rightarrow ASA = 50
\]

\[
|DA| = l_1 \Rightarrow ASA = h_1
\]

\[
l_1 < |DA| < l_2 \Rightarrow ASA = h_1 + (h_2 - h_1) \left( \frac{|DA| - l_1}{l_2 - l_1} \right) \]

\[
l_2 \leq |DA| < l_3 \Rightarrow ASA = h_{21}
\]

\[
|DA| = l_3 \Rightarrow ASA = h_3
\]

\[
l_3 < |DA| < l_4 \Rightarrow ASA = h_3 + (h_4 - h_3) \left( \frac{|DA| - l_3}{l_4 - l_3} \right) \]

\[
l_4 \leq |DA| \leq 1 \Rightarrow ASA = 100
\]

where: \( \alpha \) (like \( \alpha_1 \)) is the exponent on which it depends whether in certain interval function graph will be linear (\( \alpha = 1 \)), concave (\( \alpha < 1 \)) or convex (\( \alpha > 1 \)).

General type of criterion function could also be represented in other ways. It is assumed that such form of general criterion function provides sufficient opportunities for selecting the type of criterion function in concrete circumstances. In order to facilitate orientation among these possibilities some special (far simpler) cases of general type of criterion function can be defined.

The authors of this paper propose four special cases (Table 5). In addition to the graphic presentation, special conditions (relative to the general case) can be read in the same table when forming the given case, then the parameters which the decision maker should define, and finally, the ASA values in certain DA intervals. The selected special cases can in an appropriate way represent most of the situations in practice. That is why there is no reason why the number of special cases should be increased, as it would bring the decision maker to additional, probably unnecessary dilemmas regarding the selection of the type of the criterion function.

Evaluation of alternatives in pairs for each criterion

The procedure is similar to determining relative weights of the criteria, but here the assessment of the superordinate alternative in the pair (ASA) is read (calculated) from the selected graph for the observed criterion. Quantitative assessments which in the third step were given to alternatives according to each criterion serve as starting point for evaluation of alternatives in the pairs. Therefore, comparing and evaluating of alternatives is performed in pairs for each criterion separately according to the procedure which develops in three stages.

### Table 4. Interval Scale for translating qualitative assessments into quantitative.

<table>
<thead>
<tr>
<th>Qualitative Assessment</th>
<th>Very bad</th>
<th>Bad</th>
<th>Medium</th>
<th>Good</th>
<th>Very good</th>
<th>Type of criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative Assessment</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
<td>max</td>
</tr>
<tr>
<td>Variation in quantitative Assessment</td>
<td>0 - 0.2</td>
<td>0.2 - 0.4</td>
<td>0.4 - 0.6</td>
<td>0.6 - 0.8</td>
<td>0.8 - 1</td>
<td>min</td>
</tr>
<tr>
<td>Quantitative Assessment</td>
<td>0.9</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>min</td>
</tr>
<tr>
<td>Variation in quantitative Assessment</td>
<td>1 - 0.8</td>
<td>0.8 - 0.6</td>
<td>0.6 - 0.4</td>
<td>0.4 - 0.2</td>
<td>0.2 - 0</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3. The reduction of quantitative assessments into the interval [0, 1].

Figure 4. General type of criterion function.

Table 5. Special cases of the general type of criterion function.

<table>
<thead>
<tr>
<th>Type of special case of criterion function</th>
<th>Case I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Special conditions</td>
<td>1. $h_1 = h_2$</td>
</tr>
<tr>
<td></td>
<td>2. $h_{21} = h_3 = h_4 = 100$</td>
</tr>
</tbody>
</table>
Table 5. Contd.

Parameters for determining $l_1, h_1, l_2$

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameters for determining</th>
<th>ASA values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case II</td>
<td>$0 \leq</td>
<td>DA</td>
</tr>
<tr>
<td></td>
<td>$l_2 \leq</td>
<td>DA</td>
</tr>
</tbody>
</table>

Case II

Special Conditions

1. $h_1 = 50$
2. $h_2 = h_{21} = h_3 = h_4 = 100$

Graphic representation

Parameters for determining $l_1, l_2, \alpha$

ASA values

$$l_1 < |DA| < l_2 \Rightarrow ASA = 50 + (100 - 50) \left(\frac{|DA| - l_1}{l_2 - l_1}\right)^\alpha$$

$$l_2 \leq |DA| \leq 1 \Rightarrow ASA = 100$$

Case III

Special Conditions

1. $h_{21} = h_3 = h_4 = 100$

Graphic representation

Parameters for determining $l_1, h_1, l_2, h_2, \alpha$
Table 5. Contd.

ASA values

\[ 0 \leq |DA| < l_1 \Rightarrow ASA = 50 \]
\[ |DA| = l_1 \Rightarrow ASA = h_1 \]
\[ l_1 < |DA| < l_2 \Rightarrow ASA = h_1 + (h_2 - h_1) \left( \frac{|DA| - l_1}{l_2 - l_1} \right)^\alpha \]
\[ l_2 \leq |DA| \leq 1 \Rightarrow ASA = 100 \]

Case IV

Special Conditions
1. \( l_1 = 0 \)
2. \( h_1 = 50 \)
3. \( h_2 = h_3 = h_5 \)
4. \( l_2 = l_3 \)
5. \( h_4 = 100 \)

Graphic representation

Parameters for determining

\[ |DA| = 0 \Rightarrow ASA = 50 \]
\[ 0 < |DA| < l_2 \Rightarrow ASA = 50 + (h_2 - 50) \left( \frac{|DA|}{l_2} \right)^\alpha \]
\[ |DA| = l_2 \Rightarrow ASA = h_2 \]
\[ l_2 < |DA| < l_4 \Rightarrow ASA = h_2 + (100 - h_2) \left( \frac{|DA| - l_2}{l_4 - l_2} \right)^\alpha \]
\[ l_4 \leq |DA| \leq 1 \Rightarrow ASA = 100 \]

a) Determining differences of assessments of all pairs of alternatives and values of ASA and AWA

For a certain difference in quantitative assessments of each pair of alternatives, a corresponding assessment is assigned to the stronger and weaker alternative in the pair. The initial value of the stronger alternative assessment in the pair (ASA) is read from the ordinate of the selected criterion function, depending on the difference of the assessment of that pair, all for the criterion observed. As with the criteria each pair of alternatives has the total assessment 100, at which the stronger alternative in the pair has the assessment of \( 50 + X \), and the weaker one \( 50 - X \) (where \( X \in 0,1,2,...,50 \)). Thus, for example, possible combinations would be 60:40, 70:30, 95:5 etc.

When the ASA value is read from the graph (or, if necessary, is calculated according to the appropriate expression), the assessment of the weaker alternative (AWA) is immediately calculated by following the formula:

\[ AWA = 100 - ASA. \]  \hspace{1cm} (11)

b) Determining sums of assessments of all the pairs of alternatives and VSA and VWA values

Sum total of assessments of each of the pairs of alternatives is to be calculated. To each sum of assessments (SFA) corresponds one real aggregate assessment (RAA) for the final evaluation of the pair of alternatives. The RAA value ranges in the interval from 0 to 100, and it is read on the ordinate of Figure 2, namely, it is calculated according to the formula (3) as it is the case with the criteria.

The final value of the stronger alternative in the pair (VSA) is determined according to the formula:

\[ VSA = \frac{ASA}{100} \cdot RAA. \]  \hspace{1cm} (12)
Table 6. The first two stages of the procedure of comparison of the alternatives in pairs.

<table>
<thead>
<tr>
<th>Quantitative assessment of alternatives FA(A_i)</th>
<th>Difference of assessment of alternatives DA_{i,j}</th>
<th>Initial assessments of alternatives in pairs ASA_{i,j}, AWA_{i,j}</th>
<th>Sum of assessments of a pair of alternatives SA_{i,j}</th>
<th>Real aggregate assessment of a pair of alternatives RAA_{i,j}</th>
<th>Final value of assessments of alternatives in Pairs VSA_{i,j}, VWA_{i,j}</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA_{1,2}</td>
<td>ASA_{1,2}, AWA_{1,2}</td>
<td>SA_{1,2}</td>
<td>RAA_{1,2}</td>
<td>VSA_{1,2}, VWA_{1,2}</td>
<td></td>
</tr>
<tr>
<td>DA_{1,3}</td>
<td>ASA_{1,3}, AWA_{1,3}</td>
<td>SA_{1,3}</td>
<td>RAA_{1,3}</td>
<td>VSA_{1,3}, VWA_{1,3}</td>
<td></td>
</tr>
<tr>
<td>DA_{1,n}</td>
<td>ASA_{1,n}, AWA_{1,n}</td>
<td>SA_{1,n}</td>
<td>RAA_{1,n}</td>
<td>VSA_{1,n}, VWA_{1,n}</td>
<td></td>
</tr>
<tr>
<td>DA_{2,3}</td>
<td>ASA_{2,3}, AWA_{2,3}</td>
<td>SA_{2,3}</td>
<td>RAA_{2,3}</td>
<td>VSA_{2,3}, VWA_{2,3}</td>
<td></td>
</tr>
<tr>
<td>DA_{2,4}</td>
<td>ASA_{2,4}, AWA_{2,4}</td>
<td>SA_{2,4}</td>
<td>RAA_{2,4}</td>
<td>VSA_{2,4}, VWA_{2,4}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>DA_{2,n}</td>
<td>ASA_{2,n}, AWA_{2,n}</td>
<td>SA_{2,n}</td>
<td>RAA_{2,n}</td>
<td>VSA_{2,n}, VWA_{2,n}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>DA_{n-1,1}</td>
<td>ASA_{n-1,1}, AWA_{n-1,1}</td>
<td>SA_{n-1,1}</td>
<td>RAA_{n-1,1}</td>
<td>VSA_{n-1,1}, VWA_{n-1,1}</td>
<td></td>
</tr>
<tr>
<td>DA_{n,1}</td>
<td>ASA_{n,1}, AWA_{n,1}</td>
<td>SA_{n,1}</td>
<td>RAA_{n,1}</td>
<td>VSA_{n,1}, VWA_{n,1}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>DA_{n,2}</td>
<td>ASA_{n,2}, AWA_{n,2}</td>
<td>SA_{n,2}</td>
<td>RAA_{n,2}</td>
<td>VSA_{n,2}, VWA_{n,2}</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>DA_{n,n}</td>
<td>ASA_{n,n}, AWA_{n,n}</td>
<td>SA_{n,n}</td>
<td>RAA_{n,n}</td>
<td>VSA_{n,n}, VWA_{n,n}</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Calculating the relative participation of alternatives according to some criterion.

<table>
<thead>
<tr>
<th>A_1</th>
<th>A_2</th>
<th>...</th>
<th>A_{n-1}</th>
<th>A_n</th>
<th>\sum(VXA_{i,j})</th>
<th>u_{i,j}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>VSA_{1,2}</td>
<td>...</td>
<td>VSA_{1,n-1}</td>
<td>VSA_{1,n}</td>
<td>\sum(VXA_{1,j})</td>
<td>u_{1,j}</td>
</tr>
<tr>
<td>A_2</td>
<td>VWA_{1,2}</td>
<td>...</td>
<td>VWA_{2,n-1}</td>
<td>VWA_{2,n}</td>
<td>\sum(VXA_{2,j})</td>
<td>u_{2,j}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>A_{n-1}</td>
<td>VWA_{n-1,1}</td>
<td>...</td>
<td>VWA_{n-1,n}</td>
<td>VWA_{n-1,n}</td>
<td>\sum(VXA_{n-1,j})</td>
<td>u_{n-1,j}</td>
</tr>
<tr>
<td>A_n</td>
<td>VWA_{n,1}</td>
<td>...</td>
<td>VWA_{n,n}</td>
<td>VWA_{n,n}</td>
<td>\sum(VXA_{n,j})</td>
<td>u_{n,j}</td>
</tr>
</tbody>
</table>

Values given in brackets designate that each alternative in the pair can be superordinate or subordinate depending on whether it has a higher or lower previous quantitative assessment.

Designation VXA at the same time represents both VSA and VWA, and it is introduced in order to facilitate designation.

and the final value of the subordinate alternative in the pair (VWA) is calculated according to the formula:

\[ VWA = RAA - VSA. \]  \hspace{1cm} (13)

The first two stages of the procedure of alternative comparison in the pairs can be represented by Table 6.

c) Determining the relative participation of alternatives according to the criterion observed

Values from the last two columns of Table 6 (VSA and VWA) represent the final assessments of alternatives in pairs. These values are entered in a separate table in a way which is similar to AHP method. In a pair, VSA value is assigned to the alternative which has a higher previous quantitative assessment, and VWA value is assigned to the value which has a lower previous quantitative assessment. In a general case, this is represented in Table 7.

The same alternatives are not mutually compared as it would make no sense. That is why the diagonal fields on Table 7 are blank. The values in the second-last column \( \sum(VXA_{i,j}) \) are obtained by adding up the assessments by rows. The sum total of values which make the column \( \sum(VXA_{i,j}) \) is marked as \( \sum(VXA_{i,j}) \). The relative participation of an alternative is according to that criterion obtained by dividing the values from column \( \sum(VXA_{i,j}) \) with the value \( \sum(VXA_{i,j}) \):

\[ u_{i,j} = \frac{\sum(VXA_{i,j})}{\sum(VXA_{i,j})} \]  \hspace{1cm} (14)

Problem synthesis and final ranking of alternatives

Now the ranks of alternatives are set, taking into account all the criteria together. This procedure is simple and it is the same as with the AHP method. A table is created for that purpose, which in a general case looks like Table 8. After having performed such a synthesis of a problem situation, alternatives are ranked in full array, which can be read from the last column of Table 8.
According to Saaty's nine point scale, assessments within the sum of assessments which are given according to Saaty's scale also correspond certain RAA values, and within them certain VSC and VWC values (or, observed for the VSA and VWA alternatives). This analogy is shown in Table 9, which is made in the following way:

1. The first column consists of the assessment according to Saaty's scale.
2. The second column consists of the values which reflect the share in terms of percentage of stronger and weaker assessments within the sum of assessments which are given to a pair of attributes according to Saaty's scale. For example, the following proportion can be established for a pair which has the assessment of 7 and 1/7:

\[
7.14286 : 100 = 7 : x \implies x = \frac{7 \cdot 100}{7.1428} = 98.
\]

This means that the value 98 would correspond to the ASC value (or ASA value), and value 2 to the AWC value (or AWA value).

3. The third column consists of the RAA values adapted to Saaty's scale. With regard to the fact that the maximum RAA value on Saaty's scale is 9.1111, this value is taken to correspond to the maximum RAA according to the QM, and that is 100. Analogous to this, the other values within the third column are established. If we observe the pair that had the assessment of 7 and 1/7 again, the following ratio is established:

\[
9.1111 : 100 = 7.14286 : x \implies x = \frac{7.14286 \cdot 100}{9.1111} = 78.397244.
\]

4. In the fourth column is shown how the RAA values which correspond to Saaty's scale are distributed to the values which are assigned to the stronger and weaker criterion. This method corresponds here to the proposed method for determining the final assessments of the attributes, and they are VSC and VWC (for criteria) or VSA and VWA (for alternatives). For comparing criteria in pairs, the final value of the stronger criterion in the pair is established by following expression (4). So, for example, for a pair of criteria which has the assessment 7 and 1/7, the VSC is:

\[
\sum u_{ij} \cdot w_j = 7.14286 : 100 = 7 : x
\]

**RESULTS**

Introduction of changes in the procedure of comparison of attributes in pairs represents a significant change in regard to the AHP method where Saaty's scale of nine points is used for the same purpose. This change represents the basic idea of QM and because of that it merits special consideration. Between evaluating attributes in pairs according to Saaty's nine point scale and evaluating attributes according to the QM, there can be established the following analogy: to the assessments which are given according to Saaty's scale also correspond certain RAA values, and within them certain VSC and VWC values (or, observed for the VSA and VWA alternatives). This analogy is shown in Table 9, which is made in the following way:

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Criteria $C_j$ with weights $w_j$</th>
<th>$\sum u_{ij} \cdot w_j$</th>
<th>RANK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>$C_1w_1$ $C_2w_2$ $\ldots$ $C_mw_p$</td>
<td>$\sum u_{ij} \cdot w_j$</td>
<td></td>
</tr>
<tr>
<td>$A_1$</td>
<td>$u_{1,1}$ $u_{1,2}$ $\ldots$ $u_{1,p}$</td>
<td>$\sum u_{1j} \cdot w_j$</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>$u_{2,1}$ $u_{2,2}$ $\ldots$ $u_{2,p}$</td>
<td>$\sum u_{2j} \cdot w_j$</td>
<td></td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$</td>
<td>$\ldots$</td>
<td></td>
</tr>
<tr>
<td>$A_n$</td>
<td>$u_{n,1}$ $u_{n,2}$ $\ldots$ $u_{n,p}$</td>
<td>$\sum u_{nj} \cdot w_j$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1.0000 1.0000 1.0000 1.0000 1.0000</td>
<td>1.0000</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 9. Analogy between Saati's scale and the proposed scale.**

<table>
<thead>
<tr>
<th>Ratio of assessments in Saati's scale</th>
<th>Participation of Saati's scale assessments in terms of percentage</th>
<th>RAA Values adapted to Saati's scale</th>
<th>Distribution of RAA values to pair members for Saati's scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : 1</td>
<td>50 : 50</td>
<td>2 $\rightarrow$ 21.95122</td>
<td>10.97561 : 10.97561</td>
</tr>
<tr>
<td>2 : 0.5</td>
<td>80 : 20</td>
<td>2.5 $\rightarrow$ 27.43902</td>
<td>21.95122 : 5.4878</td>
</tr>
<tr>
<td>3 : 0.3333</td>
<td>90 : 10</td>
<td>3.3333 $\rightarrow$ 36.58536</td>
<td>32.92683 : 3.658536</td>
</tr>
<tr>
<td>4 : 0.25</td>
<td>94.11765 : 5.88235</td>
<td>4.25 $\rightarrow$ 46.64634</td>
<td>43.90244 : 2.7439</td>
</tr>
<tr>
<td>5 : 0.2</td>
<td>96.15385 : 3.84615</td>
<td>5.2 $\rightarrow$ 57.07317</td>
<td>54.87805 : 2.19512</td>
</tr>
<tr>
<td>6 : 0.16667</td>
<td>97.29729 : 2.70271</td>
<td>6.16667 $\rightarrow$ 67.68293</td>
<td>65.85365 : 1.829276</td>
</tr>
<tr>
<td>7 : 0.14286</td>
<td>98 : 2</td>
<td>7.14286 $\rightarrow$ 78.397244</td>
<td>76.8293 : 1.56794</td>
</tr>
<tr>
<td>8 : 0.125</td>
<td>98.46154 : 1.53846</td>
<td>8.125 $\rightarrow$ 89.17683</td>
<td>87.80488 : 1.37195</td>
</tr>
<tr>
<td>9 : 0.1111</td>
<td>98.78049 : 1.21951</td>
<td>9.1111 $\rightarrow$ 100</td>
<td>98.78049 : 1.21951</td>
</tr>
</tbody>
</table>
Table 10. Comparison of criteria according to Saati’s scale.

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>Σ</th>
<th>w_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>12.5</td>
<td>12.5</td>
<td>0.3058727</td>
</tr>
<tr>
<td>C₂</td>
<td>0.33333</td>
<td>4</td>
<td>3</td>
<td>0.25</td>
<td>7.583333</td>
<td>0.1855628</td>
<td></td>
</tr>
<tr>
<td>C₃</td>
<td>0.2</td>
<td>0.25</td>
<td>0.25</td>
<td>0.16667</td>
<td>0.8667</td>
<td>0.02120718</td>
<td></td>
</tr>
<tr>
<td>C₄</td>
<td>0.25</td>
<td>0.33333</td>
<td>4</td>
<td>0.33333</td>
<td>4.916667</td>
<td>0.1203099</td>
<td></td>
</tr>
<tr>
<td>C₅</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>15</td>
<td>15</td>
<td>0.3670473</td>
</tr>
<tr>
<td>Σ</td>
<td>40.86667</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11. Comparison of criteria according to adapted corresponding Saati’s scale assessments.

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>Σ(VXCᵢ,j)</th>
<th>w_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>32.92683</td>
<td>54.87805</td>
<td>43.90244</td>
<td>5.4878</td>
<td>137.19512</td>
<td>0.3058727</td>
<td></td>
</tr>
<tr>
<td>C₂</td>
<td>3.658536</td>
<td>43.90244</td>
<td>32.92683</td>
<td>2.7439</td>
<td>83.231706</td>
<td>0.1855628</td>
<td></td>
</tr>
<tr>
<td>C₃</td>
<td>2.195122</td>
<td>2.7439</td>
<td>2.7439</td>
<td>1.829276</td>
<td>9.512196</td>
<td>0.02120718</td>
<td></td>
</tr>
<tr>
<td>C₄</td>
<td>2.7439</td>
<td>3.658536</td>
<td>43.90244</td>
<td>3.658536</td>
<td>53.963412</td>
<td>0.203099</td>
<td></td>
</tr>
<tr>
<td>C₅</td>
<td>21.95122</td>
<td>43.90244</td>
<td>65.85365</td>
<td>32.92683</td>
<td>164.63414</td>
<td>0.3670473</td>
<td></td>
</tr>
<tr>
<td>Σ(VXCᵢ,j)</td>
<td>448.536574</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VSC = ASC / 100 \cdot RAA = \frac{98}{100} \cdot 78.397244 = 76.8293, and VWC is, according to expression (6):

VWC = RAA – VSC = 78.397244 – 76.8293 = 1.56794.

An example of comparison of the proposed scale to Saati’s scale

The following test was performed in some cases: for the existing examples of criteria and alternatives ranking according to the AHP method, assessments (given according to Saati’s scale) were replaced with the corresponding assessments from the fourth column in Table 9. When doing this, the following approximation was performed: values along the diagonal line of Table 9 were not taken into account (with AHP method there are 1, 1, 1, ..., along the diagonal).

Comparison of 5 criteria in pairs, which was performed by applying the AHP method (Table 10), was carried out with the adapted corresponding assessments of Saati’s scale (Table 11). Here normalisation was performed following a somewhat simpler method, but it had no impact on the final results: these were the same, even when normalisation was performed following the AHP method.

Results obtained by applying both methods were identical. On the basis of this, and some other cases, the conclusion is drawn that Saati’s scale can be considered a special case of the scale which has been proposed in this paper. This special case consists of only nine RAA values: 21.9, 27.4, 36.6, 46.6, 57.1, 67.7, 78.4, 89.2; and 100. Within each of these nine RAA values, there is only one combination of assessments for the stronger and the weaker criterion in the pair.

The performed analysis also shows that Saati’s scale can be replaced by 9 pairs of assessments from the proposed scale. The question now arises: why Saati’s scale should not be extended to other combinations for evaluation in pairs? That is what has been done in the method set forth here. In other words, in the method proposed here, there are incomparably more possible RAA values, and within each of these RAA values, there are incomparably more combinations for evaluation in pairs.

According to these observations, the method of evaluation of criteria in pairs which is set forth here can, in fact, be accepted as extended Saati’s scale which requires corresponding preliminary calculations. The advantages of the scale defined in this way are the following:

1. A considerably greater number of combinations of criteria comparison.
2. It is possible to compare much more accurately the attribute that corresponds more to the real conditions.
3. The number of pair assessments is always n \cdot (n – 1) / 2, where n – the number of alternatives or criteria which are being compared. (Triantaphyllou, 2000) A conclusion begs to be made that it is easier and more objective to assign n of assessments than n \cdot (n – 1) / 2 pair assessments. In this way a more objective assessment in pairs is achieved. For example: for n = 10, the number of pairs is 45, and for n = 15, the number of pairs is 105.
4. The proposed scale does not provide complete consistency, but assessment in pairs is certainly performed...
with a high degree of orderliness.

A disadvantage of the proposed method is represented by the occurrence of subjectivity during quantification of the importance of the criteria, but as it has already been said, it is better to assign n quantitative assessments than n · (n – 1) / 2 pair assessments.

The proposed scale also shows the same compatibility with the scale proposed by Ma and Zheng (1991). Theirs is a scale which also has 9 points with uniform intervals from 0.1111 to 1, and the remaining nine values are reciprocal to them.

**Conclusions**

1. QM can be said to represent a combination of AHP and PROMETHEE methods, with the use of appropriate elements of multiple attribute utility theory. This fact alone already understands certain novelties and originality. Besides that, QM contains some of its characteristic elements which do not exist in these methods. The new elements are:

   a. Changes in the procedure of assigning the relative weight to the criteria.
   b. Introduction of variation of assessments assigned according to qualitative criteria.
   c. Formation of a different scale for evaluation of the criteria and alternatives in the pair.
   d. Changes in the form of graphs of criteria functions.
   e. Quantification of all the assessments of alternatives according to all the criteria (regardless of whether the criteria are qualitative or quantitative). This is the element of multiple attribute utility theory which is used in QM.
   f. Changes in the scale on the abscissa and ordinate of the graph of criterion function.
   g. From the above-stated result the changes in the procedure of evaluation of the criteria and alternatives in pairs.

2. In general, the most important advantages of QM are:

   a. Assigning of relative weights to the criteria is performed after defined recommendations and estimates.
   b. Quantification of qualitative and quantitative assessments contributes to their unification and realistic evaluation, and differences of quantitative assessments always have the same regular values and same importance in respect to all the criteria. This provides the decision maker with a good view of the criteria space, better orientation and feeling in it. Moreover, a better coordination and comparability of the selected criteria functions is achieved.
   c. By quantifying of the previous quantitative assessment it is achieved that each alternative gets the assessment of how much it is really valuable to the decision maker. By quantification of all the attributes it is avoided to absolutely equally evaluate all identical differences in assessments in the complete range of originally stated quantitative attributes. This is one of the most important advantages of QM with respect to the PROMETHEE method.
   d. A great number of possibilities for defining criterion functions by means of which the non-linearity of the criteria is taken into account.
   e. Changed scale for comparison of attributes in pairs facilitates far more combinations for assessment awarding to superordinate and subordinate attributes. Besides this, this scale provides a more accurate comparison, which is far more appropriate to real conditions.
   f. Although there are incomparably more possibilities for evaluating attributes in pairs, this does not lead the decision maker to the dilemma regarding the selection of the pair assessment. On the contrary, establishing the values which are given to each pair of the attributes is defined by an exact estimate and performed with a high degree of orderliness. This is one of the important advantages of QM in respect to AHP method. Namely, it is easier and more objective to assign n of quantitative assessments than n · (n – 1) / 2 pair assessments.
   g. QM assumption makes it possible to perform, by means of a survey, to research the assessment of the importance of the criteria and forms of criteria function for typical problems of multicriteria selection.

3. Advantages of QM are especially distinct in the following situations:

   a. Decision maker is an expert who can make maximum use of the advantages offered by QM.
   b. There is a knowledge base obtained by the research into the criteria for a given problem of decision making.
   c. All alternatives are equally good, so that it is necessary to assess accurately.
   d. In the criteria group, the criteria according to which alternatives can be assessed only by qualitative assessments prevail.
   e. In the group of criteria there are criteria which simultaneously have a request for both maximisation and minimisation. Quantification considerably helps here.
   f. Same or similar decision problems appear in slightly changed conditions.

4. After having performed the analysis, it can be concluded that QM, in relation to the models of multicriteria analysis which are similar to it, has two basic disadvantages. They represent the source of subjectivity in QM. They are as follows:

   a. Occurrence of subjectivity when assigning quantitative assessments to all the attributes in the model.
   b. Occurrence of subjectivity when selecting and defining the criteria functions.

The cited disadvantages are real. However, they are
caused by alternatives which, on the other hand, bring numerous advantages to QM. These disadvantages can be mitigated by research into the assessment of the importance of the criterion and the forms of criterion functions for typical problems of selection of multicriteria. Subjectivity remains only at the stage of assigning quantitative assessments to the alternatives, which is inevitable.

5. QM has a significant number of advantages in relation to the methods which make its basis (AHP and PROMETHEE methods). There are the two above-cited disadvantages, but on the basis of the analyses performed, it seems that they are not so serious and that the advantages to a great extent prevail and thus justify the application of QM.

6. QM has been tested on some ten examples of selection of new product, and it has shown itself to be successful in these situations. It is assumed that it is also applicable to other problems of multiple criteria selection.

7. The real results of application of QM can be expected only after it has been applied for some time in practice. Thus, managers will gain experience in working with QM. This will enable a more objective research of assessment of the importance of criteria and forms of criteria functions for typical problems of multicriteria selection and formation of corresponding knowledge bases for these cases. Software to support QM is being planned. The importance of software would be particularly stressed in conditions of existence of a greater number of alternatives and criteria.

REFERENCES


