

*Full Length Research Paper*

# **An integrated supply chain design model with random disruptions consideration**

**Mir-Bahador Aryanezhad, Seyed Gholamreza Jalali and Armin Jabbarzadeh\***

Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.

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**This paper investigates a supply chain design problem where distribution centers are subject to random disruptions. As a result of disruptions, one or more of the distribution centers may fail to serve the customers. It is assumed that customers have random demands; thus, each distribution center maintains some amount of safety stocks in order to provide suitable service level for the customers it serves. The proposed model for this study is formulated as a nonlinear integer programming to minimize the expected total cost which includes costs of location, inventory, transportation and lost sales. The model simultaneously determines the location of distribution centers and the allocation of customers to distribution centers. In order to solve the resulted mathematical model, an efficient solution approach based on genetic algorithm is developed. Finally, computational results for several instances of the problem are presented to demonstrate the effectiveness of the proposed algorithm.**

**Key words:** Supply chain management, inventory management, disruptions, location model.

## **INTRODUCTION**

Traditional supply chain design models typically assume that facilities will never fail. However, in the real world cases, facilities are always vulnerable to disruptions of various sorts due to natural disasters, strikes, changes of ownership and other factors (Snyder and Daskin, 2005). There are many evidences that facility disruptions can be costly. For instance, a research by Hendricks and Singhal (2003) examines stock market reactions when firms publicly announce that they are experiencing supply chain disruptions. Results of the study of 519 supply chain problem announcements reveal that stock market reactions reduced shareholder value by 10.28%. Also, delivery failure of two critical parts resulted in over \$2.6 billion loss for the Boeing (Radjou, 2002). Both Hurricane Katrina and Rita caused shutdowns of numerous facilities and consequently significant economic losses (Barrionuevo and Deutsch, 2005). These examples and other events demonstrate crucial need to plan for facility disruptions in designing supply chain systems so that they perform well even after disruptions.

This paper presents an integrated supply chain design model, which considers impacts of the facility disruptions on both the strategic facility location and tactical inventory decisions. Specifically, we study a supply chain system which comprised a single supplier, distribution centers (DCs) and customers. It is assumed that customers have uncorrelated probabilistic demands with normal probability distribution. In this supply chain system, a supplier ships one type of product to customers in order to satisfy their demands. DCs function as the direct intermediary between the supplier and customers for the shipment of the product, that is, DCs combine the orders from different customers and then order from the supplier. Each DC retains safety stocks in order to ensure pre-specific level of service.

A key problem is that DCs are always subject to disruptions. As a result, each DC, at any time, may become unavailable and fail to serve the customers. In order to overcome this problem, we adopt the strategy that each customer is assigned to multiple DCs: a primary DC and a number of backup DCs. A primary DC, assigned to a customer, is responsible for satisfying all the demands of the customer in normal circumstances. As soon as the primary DC becomes unavailable, the first

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\*Corresponding author. E-mail: [arminj@iust.ac.ir](mailto:arminj@iust.ac.ir).

backup DC which is allocated to the customer provides its demands. If the first backup DC is disrupted, then the second backup DC will serve the customer's demands, and so on. If all of the DCs assigned to the customer are disrupted, or when the total cost of satisfying the customer's demands becomes higher than lost sales cost, the customer is not served. In this case, the system incurs lost sales cost.

The model determines the location of DCs and assignment of customers to DCs in order to minimize the expected total cost which includes: (1) the fixed cost to locate DCs, (2) the working inventory cost at the DCs, (3) the safety stock cost at the located DCs, (4) the shipment cost from DCs to customers, and (5) the lost sales cost.

The remainder of this paper is organized as follows. Some relevant models in the literature are discussed, then, the integrated supply chain design model for the problem is proposed. Also, an efficient solution approach for the mathematical model is developed. Finally, the related computational results along with conclusions are provided.

## Literature review

In the recent years, researchers have focused on the integrated models in which location and nonlinear inventory costs are included in the same model. For instance, Erlebacher and Meller (2000) provide a joint location inventory model with complicated nonlinear objective function. They applied a continuous approximation along with some heuristics techniques to solve the model. Shen (2000), Shen et al. (2003) and Daskin et al. (2002) introduce a location model with risk pooling (LMRP) that incorporates inventory decisions into the location model. Shen (2000) and Shen et al. (2003) use column generation, while Daskin et al. (2002) presents Lagrangian relaxation to solve LMRP. Another efficient approach to solve the LMRP is presented by Shu et al. (2005).

Shen and Daskin (2005) extend the LMRP to include a customer service element and propose useful techniques for evaluation of cost/service trade-offs. A profit-maximizing supply chain network design model is studied by Shen (2006), where DCs can charge different prices. Ozsen et al. (2008) develop LMRP when each DC has limited capacity. Shen and Qi (2007) study an integrated supply chain design model that contains location, inventory and routing decisions; in fact, they add routing decisions to the LMRP framework. Ozsen et al. (2009) analyzed the effect of multi-sourcing by introducing a capacitated location-inventory model that minimizes the sum of the fixed location costs, the transportation costs and the inventory costs.

Snyder et al. (2007) proposed a stochastic version of LMRP (called SLMRP) that handles uncertainty by describing discrete scenarios. The goal of SLMRP is to

minimize the expected system cost across identified scenarios. However, the authors argue on how to use SLMRP to solve multi-commodity and multi-period problems. Sourirajan et al. (2008) examined a two-stage supply chain with a production facility in which the replenishment lead time at a DC depends on the volume of flow through the DC. They presented a genetic algorithm to solve the model and imply that the proposed algorithm outperforms the Lagrangian relaxation approach. The reader referred to Shen (2007) and Melo et al. (2009) for a thorough review of the integrated supply chain design models.

Another body of literature which is closely related to the present paper is the literature on facility location with disruptions. Snyder and Daskin (2005) examine facility location problems in which facilities may fail with a given probability. Their models minimize the weighted sum of two objectives: the first objective represents the cost of the system when no disruptions occur, while the second objective indicates the expected transportation cost after accounting for disruptions. They assume that facilities have equal probability of failure. Berman et al. (2007) and Lim et al. (2009) developed models that are similar to Snyder and Daskin's (2005) models, which permitted different facilities to have different failure probabilities.

Cui et al. (2010) also relax the uniform failure probability assumption in the Snyder and Daskin's (2005) models using a continuum approximation model. However, they ignore inventory costs in their model. Qi et al. (2010) proposed an integrated location-inventory model, where the supplier and retailers are disrupted randomly. Their model assumes that the demands are deterministic and the lead time for order processing is zero.

Related models are studied by Church and Scaparra (2007) and Scaparra and Church (2008). They formulated problem in places where existed facilities can be protected against disruptions by fortification resources. Since fortification resources are limited in their problem, protecting all the facilities is not possible. They formulate models that determine what facilities must be protected so that the impact of interdiction on the remaining system operation is minimized. Snyder et al. (2006) present a tutorial reviewing a broad range of models for designing supply chains resilient to disruptions. Snyder and Daskin (2007) investigate models for the design of reliable facility location systems under a variety of risk measurements.

The present paper differs from the earlier literature on facility location models with disruptions. First, the proposed model in this study does not ignore nonlinear inventory costs. Moreover, demands for customers are considered probabilistic instead of deterministic. This research, also, is different from the earlier literature on the joint location-inventory models. Namely, in the paper, we dismiss the common restrictive assumption in LMRP that the variance and mean of the daily demand are equal for each customer. Besides, there is no assumption

that the demands of all customers must be satisfied necessarily. Finally, unlike the literature on joint location-inventory models, facility disruptions are considered in the presented model for making it more realistic.

### Model formulation

Here, a model for the problem is formulated. The objective is to minimize the expected total cost including: (1) the fixed cost to locate DCs, (2) the working inventory cost at the located DCs (containing order costs, shipment costs from supplier to DCs and holding costs), (3) the safety stock cost at the located DCs, (4) the shipment cost from located DCs to customers, and (5) the lost sale cost of not serving customers. The following notations will be used throughout the paper:

- $I$  : set of customers indexed by  $i$  ;
- $J$  : set of candidate DC locations indexed by  $j$  ;
- $\mu_i$  : mean of daily demand at customer  $i$  ;
- $\sigma_i^2$  : variance of daily demand at customer  $i$  ;
- $f_j$  : fixed cost of locating a DC at  $j$  ;
- $F_j$  : fixed cost of placing an order at  $j$  ;
- $g_j$  : fixed cost per shipment from the plant to DC  $j$  ;
- $a_j$  : per-unit shipment cost from the plant to DC  $j$  ;
- $h$  : inventory holding cost per unit of product per year;
- $d_{ij}$  : per-unit cost to ship from DC  $j$  to customer  $i$  ;
- $\alpha$  : desired percentage of customers orders that are satisfied;
- $\beta$  : weight factor associated with the shipment cost;
- $\theta$  : weight factor associated with the inventory cost;
- $z_\alpha$  : standard normal deviate, such that  $P(z \leq z_\alpha) = \alpha$  ;
- $L$  : lead time from supplier to DCs, in days;
- $x$  : number of days in a year
- $u_i$  : penalty cost of not serving the customer  $i$ , per unit of demand (it can be interpreted as lost sales cost or the cost of serving customer  $i$  by purchasing product from a competitor);
- $q$  : probability of disruptions for each DC;
- $p$  : number of DCs that must be located;

Without loss of generality, it is assumed that sets  $I$  and  $J$  are the same (Daskin et al., 2002).

### Shipment cost

Transporting the product from each DC  $j$  to each customer  $i$  that has linear shipment cost. Let  $D_{ij}$  denote the expected (annual) demand of customer  $i$  which is assigned to DC  $j$ . Then, the shipment cost from DC  $j$  to the customers can be obtained by:

$$\beta \sum_{i \in I} d_{ij} D_{ij} \quad (1)$$

### Lost sales cost

In order to model the lost sales cost, dummy DC with index  $u$  is

added to the set  $J$  ; it is assumed that the dummy DC  $u$  never faces disruptions. Assigning the customer  $i$  to this dummy DC, represents not serving the customer  $i$ . Regarding DC  $u$ , we assume that it has the shipment cost  $d_{iu} = u_i$  to customer  $i \in I$  and there is no other cost.

### Working inventory cost

Here, details of the inventory policy used in the DCs are given. Each DC orders from the supplier using an approximation to the  $(Q, r)$  model with Type I service (Nahimas, 1997). This approximation consists of two steps. The first step determines the quantity of order at each DC using economic order quantity model (EOQ). In the next step, the reorder point at each DC is determined in the way that the probability of a stockout does not exceed the specified value. The reason why we utilize this two-step approach is that it provides a good approximation to the optimal order quantity and order point values (Zheng, 1992; Axsater, 1996).

Following the two-step approach, first, the order quantity is determined. Let  $D_j$  denote the expected total (annual) demand that is assigned to the DC  $j$  (it is obvious that  $D_j = \sum_{i \in I} D_{ij}$ ) and  $n$  be the unknown number of orders per year. Then, the expected shipment size per shipment from supplier to DC  $j$  is equal to  $\frac{D_j}{n}$ . Furthermore, the annual working inventory cost at DC  $j$  (including order, shipment from supplier to DC  $j$  and holding cost) is obtained by:

$$Fn + \beta(g_j + \frac{a_j D_j}{n})n + \frac{\theta h D_j}{(2n)} \quad (2)$$

The first term of equation (2) is the fixed cost of placing  $n$  orders, while the second term indicates the cost of shipping  $n$  orders of size  $\frac{D_j}{n}$ . The last term represents the cost of holding average  $\frac{D_j}{(2n)}$  units of inventory per year. To determine the optimal number of orders per year, we take the derivative of (2) in respect to  $n$  and set the derivative to zero:

$$F + \beta g_j - \frac{\theta h D_j}{(2n^2)} = 0 \quad (3)$$

Solving Equation (3) for  $n$ , we obtain  $n = \sqrt{\frac{\theta h D_j}{2(F + \beta g_j)}}$ .

Plugging this into (2), an annual working inventory cost can be calculated as follows:

$$\sqrt{2\theta h D_j (F + \beta g_j)} + \beta a_j D_j \quad (4)$$

### Safety stock cost

Each DC retains a certain amount of safety stocks to deal with

possible stockouts during replenishment lead time. Assuming that lead time demand at the DC  $j$  is normally distributed with expected variance of  $V_j^2$ , the needed safety stock to guarantee that the

stockouts occur with a probability of  $\alpha$  or less, which is  $Z_\alpha \sqrt{V_j^2}$ . Maintaining this amount of safety stocks incurs the holding cost at DC  $j$  which is calculated by:

$$\theta h z_\alpha \sqrt{V_j^2} \tag{5}$$

**Integrated model**

To determine the locations of the DCs and customers-DCs assignment, two sets of decision variables are defined:

•  $X_j = 1,$

if  $j$  is selected as a DC location, and 0, if otherwise for each  $j \in J$  ;

•  $Y_{ijr} = 1,$

if customer  $i$  at level  $r$  is assigned to a DC located at  $j$ , and 0 if otherwise for  $i \in I, j \in J, \text{ and } r = 0, 1, \dots, P-1$ . To clarify variables  $Y$ , note that each customer  $i \in I$  is assigned to multiple DCs at multiple levels. In fact, customer  $i$  is assigned to its primary DC at level 0, its first backup DC at level 1, its second backup DC at level 2 and its  $r^{\text{th}}$  backup DC at level  $r$ . Recall that in a normal circumstance, the customer is served by its primary DC. However, when the primary DC is disrupted, the customer is served by its first back up DC. If the first back up DC fails, the customer's demand is provided by its second back up DC and so on. Noteworthy, if customer  $i$  is assigned to dummy DC  $u$  at level  $r$ , there is no need to assign it to any other DC at upper level  $s$  (where  $s > r$ ). The reason is that the dummy DC  $u$  never fails and does not require any backup DC.

Now  $D_{ij}$  and  $D_j$  can be written in terms of decision variables:

$$D_{ij} = \sum_{r=0}^{P-1} (1-q)q^r \chi \mu_i Y_{ijr} \tag{6}$$

$$D_j = \sum_{i \in I} D_{ij} = \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r \chi \mu_i Y_{ijr} \tag{7}$$

Also, considering the fact that customer demands are uncorrelated, the expected variance of lead time demand at the DC  $j$ ,  $V_j^2$ , can be obtained based on the decision variables:

$$V_j^2 = L \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r \sigma_i^2 Y_{ijr} \tag{8}$$

To explain (6) and (8), note that each customer  $i \in I$  is served by its  $r^{\text{th}}$  back up DC (name it  $j$ ) if distribution center  $j$  does not face disruptions (this occurs with probability  $1-q$ ) and if all the  $r$  distribution centers, which were assigned to the customer  $i$  at lower

levels (levels 0, 1, 2...  $r-1$ ) are not disrupted (this occurs with probability  $q^r$ ). Now the model can be formulated as follows:

$$\begin{aligned} \text{Min} & \left( \sum_{j \in J} f_j X_j \right) + \left( \beta \sum_{j \in J} \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r d_{ij} \chi \mu_i Y_{ijr} \right) \\ & + \left( \sum_{j \in J} \sqrt{2\theta h (F_j + \beta g_j)} \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r \chi \mu_i Y_{ijr} \right) \\ & + \left( \beta \sum_{j \in J} a_j \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r \chi \mu_i Y_{ijr} \right) \\ & + \left( \sum_{j \in J} \theta h z_\alpha \sqrt{L \sum_{i \in I} \sum_{r=0}^{P-1} (1-q)q^r \sigma_i^2 Y_{ijr}} \right) \end{aligned} \tag{9}$$

subject to:

$$\sum_{j \in J} Y_{ijr} + \sum_{s=0}^{r-1} Y_{ius} = 1 \quad \forall i \in I, \forall r = 0, 1, 2, \dots, P-1 \tag{10}$$

$$Y_{ijr} \leq X_j \quad \forall i \in I, \forall j \in J, \forall r = 0, 1, 2, \dots, P-1 \tag{11}$$

$$X_j \leq Y_{ju0} \quad \forall j \in J \tag{12}$$

$$X_u = 1 \tag{13}$$

$$\sum_{j \in J} X_j = P + 1 \tag{14}$$

$$\sum_{r=0}^{P-1} Y_{ijr} \leq 1 \quad \forall i \in I, \forall j \in J \tag{15}$$

$$X_j \in \{0,1\} \quad \forall j \in J \tag{16}$$

$$Y_{ijr} \in \{0,1\} \quad \forall i \in I, \forall j \in J, \forall r = 0, 1, 2, \dots, P-1 \tag{17}$$

The objective function (9) is composed of four components separated by parentheses. The first component represents the fixed cost of locating DCs, while the second part indicates the expected shipment cost from the DCs to customers. Recall that we added dummy DC  $u$  to the set  $J$  in order to take into account lost sales cost in the model. Considering Equations (4) and (7), it is easy to find that the third component represents the working inventory cost. Finally, the fourth part indicates safety stock cost and can be obtained by considering Equations (5) and (8).

Constraints (10) stipulate that for each customer  $i$  and each level  $r$ , customer  $i$  should be assigned to exactly one DC at level  $r$  unless  $i$  was assigned to DC  $u$  at any lower level  $s$  ( $s < r$ ). In other words, if customer  $i$  at level  $s$  is assigned to dummy DC  $u$ , it is not assigned to any of the DCs at any higher level  $r$  ( $s < r$ ). Note that in case  $r =$

0, we take  $\sum_{s=0}^{r-1} Y_{ius} = 0$ . Constraints (11) state that customers can only be assigned to candidate sites that are selected as DCs. Constraints (12) require that if a DC is located at  $j$ , this DC should serve the customer at  $j$  as a primary DC. Constraint (13) requires

the dummy DC  $u$  to be located. Constraint (14) assures that the number of located DCs should be exactly  $P + 1$  (this means that  $P$  distribution centers must be located in addition to dummy DC  $u$ ). Constraints (15) state that a customer cannot be assigned to a given DC at more than one level. However, constraints (16) and (17) are binary constraints.

### Solution approach

Solving the model (9) in the simplest condition (when  $q = \theta = 0$ ,  $f_j = a_j = 0$  for each  $j \in J$ ,  $\beta = 1$ , and  $u_i$  are

extremely large for each  $i \in I$ ) is identical to solving the  $P$ -median problem which is NP-hard (Garey and Johnson, 1979). This shows that solving the study's model in reasonable time is extremely hard. Therefore, in order to solve this complex nonlinear model, we use an efficient meta-heuristic method based on genetic algorithm (GA) similar to Zhou and Liu (2003).

GA is a stochastic search and heuristic optimization technique based on the mechanism of natural genetics which has been successfully applied to various complex problems. It starts with an initial set of random solution called population. Each solution in the population is called chromosome and each component of chromosome is designated by gene. The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated, using some measures of fitness.

To create the next generation, new chromosomes (called offspring) are formed by crossover or mutation operators. Crossover operator combines two chromosomes from current generation, while mutation operator modifies a chromosome to form offspring. A new generation is created by (a) selecting some of the current chromosomes (called parents) and offspring based on the fitness values, and (b) rejecting others so as to keep the population size constant. However, fitter chromosomes have higher probabilities of being selected. After several generations, the algorithms converge to the best chromosome, which may represent the optimum or suboptimal solution to the problem (Gen and Cheng, 1996).

### Chromosome representation

In the study's GA-based approach, each chromosome is represented as a single dimensional array having two kinds of genes: location and assignment genes. If  $m$  denotes the number of candidate DCs, each arbitrary chromosome  $C$  can be demonstrated by,

$$C = (X_j, Y_{ir}) = (X_1, X_{21}, \dots, X_m, Y_{10}, Y_{11}, \dots, Y_{1(P-1)}, Y_{20}, Y_{21}, \dots, Y_{2(P-1)}, \dots, Y_{m0}, Y_{m1}, \dots, Y_{m(P-1)}).$$

Where  $X_j$  correspond to the location genes and  $Y_{ir}$  to the assignment genes.

Associated with each candidate DC location, there is a location gene in the chromosome. Location genes determine where the DCs are located. More precisely, if  $X_j = 1$ , it means that candidate site  $j$  is selected as a DC location, while if  $X_j = 0$ , candidate location  $j$  is not chosen as a DC site. Note that the gene  $X_{m+1}$  corresponds to

dummy DC  $u$ ; thus, it always takes the value 1.

Also, associated with each customer, there are  $P - 1$  assignment genes. Assignment genes determine the assignment of customers to DCs at different levels. Specifically,  $Y_{ir} = j$  represents that customer  $i$  at level  $r$  is assigned to DC  $j$ . If customer  $i$  at level  $r$  is assigned to a dummy DC  $u$ , the corresponding assignment gene takes the value of  $m+1$ ; in other words,  $Y_{ir} = m + 1$ . In this case, the customer is not assigned to any DC at upper levels, that is,  $Y_{is} = 0$  for  $s > r$ .

### Initialized chromosome population

In the study, we initialize *pop-size* chromosomes from the feasible region randomly. In order to achieve feasible chromosomes, constraints (10) - (15) must be considered closely. Based on the model formulation in constraints (10), if customer  $i$  at level  $s$  is allocated to dummy DC  $u$ , it is not assigned to any DC at upper levels than  $s$ .

Thus, if  $Y_{is} = m + 1$ ,

then  $Y_{ir} = 0$  where  $s < r$ .

Constraints (11) state that customers can be only allocated to the located DCs, that is,

if  $Y_{ir} = j$ ,

then  $X_j = 1$ .

According to constraints (12), if a DC is located at  $j$ , customer  $j$  at level 0 is assigned to this DC.

Hence, if  $X_j = 1$ ,

then  $Y_{j0} = j$ .

Based on constraints (13) and (14), excluding dummy DC  $u$ , the number of located distribution centers must be  $P$ .

Therefore, the number of variables  $X_j$  taking the value of 1 is equal to  $P$ . According to constraints (15), if any customer  $i$  at level  $r$  is assigned to a given DC  $j$ , it cannot be assigned to  $i$  at any level of  $s$ , where  $s \neq r$ .

Consequently, if  $Y_{ir} = j$ ,

then  $Y_{is} \neq j$  for  $s \neq r$ .

### Chromosomes fitness

The rank-based evaluation function is defined as the objective function (9) for the chromosomes. In fact, we calculate the objective function (9) for each of the chromosomes. Obviously, the chromosome which results in less value of the objective function (9) has the better rank.

### Crossover operator

Crossover operator generates offspring chromosomes by merging parent chromosomes. In order to determine which of the chromosomes  $C_k$ ,  $k = 1, 2, \dots, pop\text{-size}$  are selected as parents for crossover operation, we repeat the following procedure from  $k = 1$  to  $pop\text{-size}$ , that is, by generating a random number  $r$  from the interval  $[0, 1]$ , the chromosome  $C_k$  will be selected as a parent provided that  $r < PC$ , where the parameter  $PC$  is the probability of crossover. Then randomly, we group the selected parents  $C'_1, C'_2, C'_3, \dots$  to the pairs  $(C'_1, C'_2), (C'_3, C'_4), \dots$ . Without loss of generality, the crossover operator on each pair by  $(C'_1, C'_2)$  will be explained.

Crossover operator assigns each customer  $i$  at level  $r$  in offspring chromosome either to the DC which is allocated to customer  $i$  at level  $r$  in parent chromosome  $C'_1$ , or to the DC which is assigned to customer  $i$  at level  $r$  in parent chromosome  $C'_2$ . This occurs randomly and with a probability of 0.5. However, the resulted offspring may be infeasible. If a customer is allocated to the dummy DC  $u$  at any level, it is not assigned to any DC at upper levels, but if a customer is allocated to an unselected candidate DC site, this infeasibility is removed by locating the DC in that candidate location. If the number of located DCs exceeds  $P + 1$ , the number of selected DCs is reduced to  $P + 1$  by closing some DCs randomly. The customers which are allocated to these closed DCs are allocated randomly to one of the opened DCs. If a customer is assigned to a given DC in several levels, the assignments of customer at upper levels are modified, that is, the customer at upper levels is allocated to other DCs randomly in order to prevent from assigning the customer to a given DC at more than one level.

### Mutation operator

Mutation operator may modify chromosomes  $C_k$ ,  $k = 1, 2 \dots$  and  $pop\text{-size}$  to form offspring chromosomes. In order to determine which of the chromosomes  $C_k$  undergo mutation, there will be a repetition of the following practice from  $k = 1$  to  $pop\text{-size}$ : by generating a random number  $r$  from the interval  $[0, 1]$ , the chromosome  $C_k$  will be selected as a parent provided that  $r < Pm$ , where the parameter  $r < Pm$  is the probability of mutation. Each selected chromosome is modified by one of the two following types of mutation, several times (each type of mutation occurred with a probability of 0.5).

The first type of mutation generates offspring by modifying the assignment genes of parent chromosome. Namely, in the first type of mutation, two located DCs are selected randomly; let  $s$  and  $t$  denote them. Then, if any customer in parent chromosome is assigned to  $s$ , that customer will be assigned to  $t$  and if any customer is assigned to  $t$ , it will be allocated to  $s$ .

The second type of mutation modifies location genes of parent chromosome to form offspring. Indeed, the second type of mutation randomly selects a location in which no DC is located; let  $t$  denote it. Next, a DC is selected randomly from the located DCs and is named  $s$ . This type of mutation closes DC  $s$  instead of locating a DC at  $t$ . Then, all the customers assigned to DC  $s$ , are allocated to DC  $t$ . Note that this is similar to the crossover process, in that if the resulted offspring does not belong to a feasible region, it is repaired to be a feasible chromosome.

## COMPUTATION RESULTS AND DISCUSSION

Here, the study summarizes the computational experience with the genetic algorithm outlined in the previous section. The algorithm was coded in Visual Basic.Net 2008 and executed on Pentium 5 computer with 1.00 GB RAM and 2.00 GHz CPU.

### Evaluating robustness of the GA

The proposed genetic algorithm was tested on the 49-node, 88-node and 150-node data sets described in Daskin (1995). The 49-node data set indicates the capitals of the lower 48 United States plus Washington, DC; while the 88-node data set represents the 50 largest cities in the 1990 U.S. census along with the 49-node data set, minus duplicates; and the 150-node data set includes the 150 largest cities in the 1990 U.S. census.

For all three data sets, the mean and variance of daily demand were obtained by dividing the population data given in Daskin (1995) by 1000. The per-unit cost to ship from DC  $j$  to customer  $i$  was set to the great-circle distance between  $i$  and  $j$  in Daskin (1995). Fixed costs of locating DCs were gained by dividing the fixed costs in Daskin (1995) by 10 for the 49-node problem and by 100 for 88-node problem. For the 150-node problem, fixed costs of locating DCs were set to 10000. However, we set holding cost to be 1,  $q = 0.05, P = 20, \beta = 0.00001, \theta = 0.001, \chi = 1, z_\alpha = 1.96, L = 1, F_j = 10, g_j = 10, a_j = 5$  for all  $j \in J$  and  $u_i = 1000$  for all  $i \in I$ .

Although  $\chi = 1$  may seem unrealistic, the difference between the daily and yearly parameters can be realized through the weights  $\beta$  and  $\theta$ .

Tables 1, 2 and 3 summarize the results for the computational study on 49-node, 88-node and 150-node problems. The number of generations was set to 400 and was considered as a stopping rule of GA. The column labeled  $pop\text{-size}$  gives the number of initial feasible chromosomes, while  $Pc$  gives the probability of crossover and  $Pm$  gives the probability of mutation.

The column marked CPU indicates the total number of CPU seconds required and 'cost' indicates the objective value of the solution. The last column gives the parameter error which shows the deviations of objective values (costs). This parameter can be obtained by: (objective value - the best objective value) / the best objective value, where the best objective value is the least value in the column marked 'cost'.

It follows from Tables 1 - 3 that the error for the 49-node, 88-node and 150-node problems does not exceed 0.000008, 0.000007 and 0.000002, respectively. These small values of error show that costs differ little from each other when different parameters are selected. We can also see from the three tables, when the parameters are varied, that the CPU time changes slightly.

**Table 1.** Comparison solution of 49-node problem.

	Pop-size	Pc	Pm	CPU	Cost	Error
1	25	1	0.33	4	979078.3	0.000007
2	25	0.95	0.3	4	979079.8	0.000008
3	25	0.75	0.1	4	979075.6	0.000004
4	25	0.65	0.1	4	979072.1	0.000001
5	25	0.7	0.1	4	979077.7	0.000006
6	30	0.9	0.3	5	979071.5	0.000000
7	30	0.95	0.25	5	979074.8	0.000003
8	30	0.8	0.15	5	979073.3	0.000002
9	30	0.8	0.25	5	979077.8	0.000006
10	30	0.75	0.25	5	979076.8	0.000005

**Table 2.** Comparison solution of 88-node problem.

	Pop-size	Pc	Pm	CPU	Cost	Error
1	25	1	0.33	2	1174629	0.000002
2	25	0.9	0.3	2	1174631	0.000003
3	25	0.8	0.2	2	1174635	0.000007
4	25	0.75	0.1	2	1174634	0.000006
5	25	0.65	0.05	2	1174630	0.000003
6	30	1	0.33	3	1174627	0.000000
7	30	0.95	0.25	3	1174629	0.000002
8	30	0.8	0.1	3	1174629	0.000002
9	30	0.75	0.05	3	1174630	0.000003
10	30	0.75	0.09	3	1174628	0.000001

**Table 3.** Comparison solution of 150-node problem.

	Pop-size	Pc	Pm	CPU	Cost	Error
1	25	1	0.33	11	2000073	0.000002
2	25	0.95	0.3	11	2000074	0.000002
3	25	0.8	0.2	11	2000069	0.000000
4	25	0.7	0.15	11	2000071	0.000001
5	25	0.95	0.25	11	2000072	0.000001
6	30	0.9	0.3	14	2000072	0.000001
7	30	0.8	0.3	14	2000069	0.000000
8	30	0.85	0.3	14	2000070	0.000000
9	30	0.95	0.3	14	2000069	0.000000
10	30	0.85	0.25	14	2000070	0.000000

slightly. Thus, the study's genetic algorithm is robust to the parameter setting and effective to solve the model.

### Measuring performance of the GA

Here, we compare solutions from the study's genetic

algorithm with LINGO 8.00 optimization software and finally compute the error. For this experiment, five data sets were employed: a 10-node, 15-node, 20-node, 25-node and 30-node data set. These five data sets respectively consist of 10, 15, 20, 25 and 30 nodes with the highest demands from the 49-node data set given in Daskin (1995). For each data set, different values of

**Table 4.** Comparing GA with Lingo 8.00 optimization software solution.

	Nodes	$\beta$	$\theta$	P	GA time*	Lingo time*	GA cost	Lingo cost	Error
1	10	0.01	0.0004	5	1	1	283513.9	283504.6	0.000033
2	10	0.01	0.0004	3	1	2	146222.7	146212.1	0.000072
3	10	0.001	0.00001	5	1	11	283321.8	283320.9	0.000003
4	10	0.001	0.00001	3	1	2	146023.1	146021.5	0.000011
5	15	0.01	0.0004	8	1	7	472317.6	472296.2	0.000045
6	15	0.01	0.0004	6	1	6	334534	334507	0.000081
7	15	0.001	0.00001	8	1	9	472122.6	472120.1	0.000005
8	15	0.001	0.00001	6	1	373	334324.1	334321.1	0.000009
9	20	0.01	0.0004	12	2	25	759517.8	759511.6	0.000008
10	20	0.01	0.0004	10	2	38	614548.9	614542.7	0.000010
11	20	0.001	0.00001	12	2	3974	759322.5	759321.9	0.000001
12	20	0.001	0.00001	10	2	171	614325.4	614324.9	0.000001
13	25	0.01	0.0004	12	2	99	734867.8	734861	0.000009
14	25	0.01	0.0004	10	2	84	592776.1	592766	0.000017
15	25	0.001	0.00001	12	2	2764	734628.4	734626.9	0.000002
16	25	0.001	0.00001	10	2	2072	592529.1	592527	0.000004
17	30	0.01	0.0004	12	2	410	689514.6	689498	0.000024
18	30	0.01	0.0004	10	2	126	556922.9	556905	0.000032
19	30	0.001	0.00001	12	3	3850	689322.7	689321	0.000002
20	30	0.001	0.00001	10	3	320	556723.4	556721	0.000004

\*Time is in second.

This work can be extended in some directions; however, it would be interesting to model the problem when DCs have different or dependent probabilities of disruptions. Also, the model can be expanded to consider the possibility of disruptions for the supplier. Another development for this study could be adding the constraints on the maximum capacity of inventory at DCs or on the maximum demand that can be provided by a supplier.

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