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Optimal production policy with investment on imperfect production processes

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The imperfect production processes always result in imperfect products and decrease the profit of the business. Improving the production processes by increasing the investment cost will decrease the percentage of defective items. The trade-off between the investment cost and the marginal improvement on products is a key problem. In this study, we develop an EPQ model of deteriorating items with investment on imperfect production processes. An algorithm is developed to derive a replenishment policy such that the expected unit time profit is maximized. Numerical examples are provided to illustrate the theory.

Key words: Economic production quantity, imperfect quality, deterioration, investment.

INTRODUCTION

The imperfect production processes always lead to imperfect products and decrease in the profit of the business. Most studies assumed items to possess perfect quality. Rosenblatt and Lee (1986) were the early researchers who considered defective items and imperfect quality production processes. Salameh and Jaber (2000) developed an inventory model considering imperfect items using the EPQ/EOQ formulae. Increasing the investment cost to improve the production processes will decrease the percentage of defective items. The trade-off between the investment cost and the marginal improvement on products is a key problem.

Improving the firm's business by increasing the investment cost is critical to the managers. Investment options typically involve three parameters: 1) the initial and accumulated costs, 2) the flexibility in timing the investment, and 3) the uncertainty regarding the future rewards (Heikkinen and Pietola, 2009). Nishihara and Fukushima (2008) evaluated the start-up's loss to be a result of incomplete information on the firm's behavior. Kulkarni (2008) considered a multi-product environment where production lot-sizing and investing for quality improvement in several production processes were desired.

Lin (2009) investigated in the continuous review model with backorder price discount and variable lead time to effectively increase investment and to reduce the joint expected annual total cost. Heikkinen and Pietola (2009) studied optimal investment and the dynamic cost of income uncertainty, and applied a stochastic programming approach. Hsu et al. (2010) developed a deteriorating inventory policy when the retailer invested on the preservation technology to reduce the rate of product deterioration. Hou and Lin (2011) determined the optimal capital investment in setup cost reduction and optimal lot sizing policies for an economic order quantity model with random yields. Other researchers such as Moon (1994), Hong and Hayya (1995), Banerjee et al. (1996), Gurnani et al. (2007), Wang et al. (2007), Mathur and Shah (2008), Uc-kun et al. (2008), and Kort et. al. (2010) considered investment constraint issues. Nonetheless, researches of increasing the investment cost on the production process have received little attention.

In this study, we develop an EPQ model of deteriorating items with investment on imperfect production processes to decrease the percentage of defective items. The screening process and demand proceed simultaneously. The renewal theory is used in the modeling. An algorithm is developed to derive a replenishment policy and the investment cost such that the expected unit time profit is maximized.

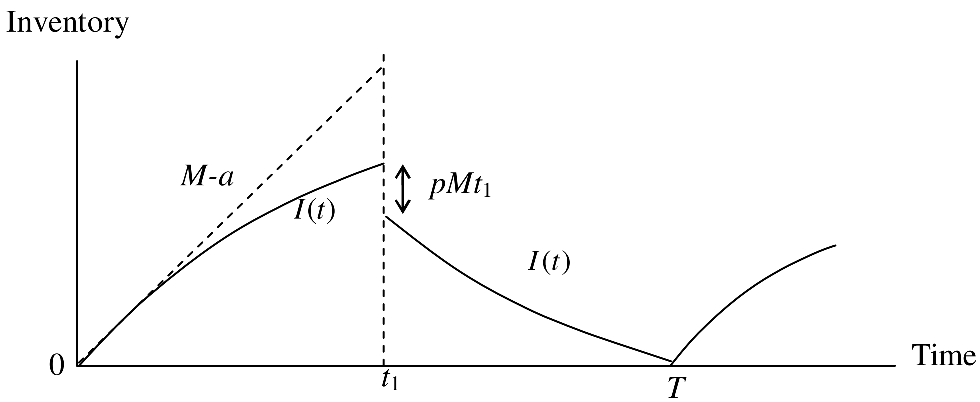

Figure 1. Inventory system of deteriorating items when the screening rate is higher than the production rate.

ASSUMPTIONS AND NOTATION

The mathematical models presented in this study have the following assumptions:

1) The customer's demand $D(t)=a$ during the production run time, while $D(t)=b$ after the production.
2) The production rate, M, is known and constant with $M>a>b>0$.
3) The lead-time is known and constant.
4) The screening process and demand proceeds simultaneously.
5) The defective items exist in each production. The defective percentage, p, has a uniform distribution.
6. No shortages are allowed.
7. A single product is considered.
8. The deterioration rate of the on-hand-stock is small.

The following notations are used: T - the production cycle length; t_1 - the production run time per cycle, decision variable; θ - constant deterioration rate of the on-hand-stock, $\theta>0$; M - the production rate, $M > D(t)$; x- the screening rate, $x > D(t)$; t_x - the screening time per cycle; c - the production cost per unit; K - the setup cost per production; p - the defective percentage in per production, which is a random variable; s - the selling price of good quality items per unit; v- the selling price of defective items per unit, $v < c$; r - the investment cost on production processes, decision variable; d - the screening cost per unit; h - inventory holding cost per unit; TR - the total revenue per cycle; which is the sum of total sales of good quality and imperfect quality items; TC - the total cost per cycle; TPU - the net profit per unit time; $ETPU$ - the expected value of TPU.

Analysis of the model

The deteriorating items having a constant deterioration rate θ and demand $D(t)$ are considered in this study. We assume an imperfect production process having a constant production rate M, a production cost of c per unit and a setup cost of K per production. Each lot produced contains some percentage of defectives, p, with uniformly distribution over $[\alpha, \beta(r)]$, where $\beta(r)$ is a decreasing function as the investment cost of r. It means that more investment on production process will generate less defective items.

The selling price of good quality item is s per unit. The items with imperfect quality assumed a 100% screening at a constant rate of x. The screening process is needed for quality control (Britney, 1972). Poor quality items are kept in stock and sold prior to the next production at a discounted price of v. No shortages are allowed. Two case scenarios are considered: (i) when the screening rate, x, is higher than the production rate, M (analysis and discussion of this case had been presented in the First Asian Conference on Intelligent Information and Database Systems 2009), and (ii) when the screening rate is lower than the production rate (analysis and discussion of this case had been presented in the 5th International Congress on Logistics and SCM Systems 2009). The optimum operating inventory strategy is obtained by trading off the total revenues, the production cost, the investment cost, the inventory holding cost and the item screening cost so that the total revenue per unit time will be a maximum.

When the screening rate, x, is higher than the production rate, M

The behavior of the inventory level is illustrated in Figure 1. To avoid shortages, it is assumed that the inventory at t_1 is positive during production time, that is:

$$I(t_1) - pMt_1 \geq 0 \quad (1)$$

Let $I(t)$ be the inventory level during [0, T]. The differential equation governing the transition of the system during [0, t_1) is:

$$\frac{dI(t)}{dt} = -\theta I(t) + M - a, \quad 0 \le t < t_1. \tag{2}$$

For initial condition $I(0)=0$, solving equations, one has:

$$I(t) = \frac{(M-a)(1-e^{-\theta t})}{\theta}, \quad 0 \le t < t_1. \tag{3}$$

The differential equation during [t_1, T] is:

$$\frac{dI(t)}{dt} = -\theta I(t) - b, \quad t_1 \le t \le T. \tag{4}$$

For initial condition $I(t_1)=\frac{(M-a)(1-e^{-\theta t_1})}{\theta} - pMt_1$, solving the equation, one has:

$$I(t) = \frac{-b}{\theta} + \frac{e^{-\theta(t-t_1)}}{\theta}[b+(M-a)(1-e^{-\theta t_1}) - pMt_1\theta], \quad t_1 \le t \le T. \tag{5}$$

Solving $I(T)=0$, one has:

$$T(t_1,p) = 1/\theta \{\ln[b+(M-a)(1-e^{-\theta t_1}) - pMt_1\theta] - \ln b\} + t_1. \tag{6}$$

When 0< θ <<1, $T(t_1,p)$ can be rewritten as (By L'Hospital Rule):

$$T(t_1,p) \approx (M - pM - a + b)t_1 / b. \tag{7}$$

The random variable p is uniformly distributed over $[\alpha, \beta]$, where $0 \le \alpha < \beta < 1$, α a constant and $\beta = \beta(r)$ is assumed to be a decreasing function when the investing cost is r. Define $TR(t_1, p)$ as the total revenue that is the sum of total sales of good quality and the imperfect quality items. One has:

$$TR(t_1,p) = [at_1 + b(T(t_1,p) - t_1)]s + pMt_1 v$$
$$= Mt_1(s - ps + pv). \tag{8}$$

$TC(r, t_1, p)$ is the sum of setup cost per cycle, investment cost per cycle, production cost per cycle, screening cost per cycle, and holding cost per cycle. One has:

$$TC(r,t_1,p) = K + r + cMt_1 + dMt_1 + h\left\{\int_0^{t_1} I(t)dt + \int_{t_1}^{T(t_1,p)} I(t)dt\right\} \tag{9}$$

Using $e^{-x} \approx 1 - x + x^2/2$, for 0<$x$<<1,

$$\int_0^{t_1} I(t)dt = (e^{-\theta t_1} + \theta t_1 - 1)(M-a)/\theta^2$$
$$\approx \frac{t_1^{\,2}}{2}(M-a), \tag{10}$$

and

$$\int_{t_1}^{T(t_1,p)} I(t)dt = [\; e^{\theta t_1(-M+pM+a)/b}(a - b - M + pMt_1\theta) +$$
$$e^{\theta t_1(-M+pM+a-b)/b}(M-a) +$$
$$(1 - \theta t_1 - e^{-\theta t_1})(M-a) + b \;]/\theta^2$$

$$\approx \frac{t_1^2}{2b}(M^2 - 2Ma + a^2 - 2pM^2 + p^2M^2 + 2pMa). \tag{11}$$

The total profit per unit time of $TPU(r, t_1, p)$ given by dividing the total profit per cycle by the cycle length of T is:

$$TPU(r,t_1,p) = \frac{TR(t_1,p) - TC(r,t_1,p)}{T(t_1,p)} \tag{12}$$

The expected value of $TPU(r, t_1, p)$ is:

$$ETPU(r,t_1) = E[\frac{TR(t_1,p) - TC(r,t_1,p)}{T(t_1,p)}]. \tag{13}$$

Since the process generating the profit is renewal (with renewal points at production epochs), the expected profit per unit time is given by the renewal-reward theorem (Ross, 1996; Theorem 3.6.1) as:

$$ETPU(r,t_1) = \frac{E[TR(t_1,p) - TC(r,t_1,p)]}{E[T(t_1,p)]}.$$

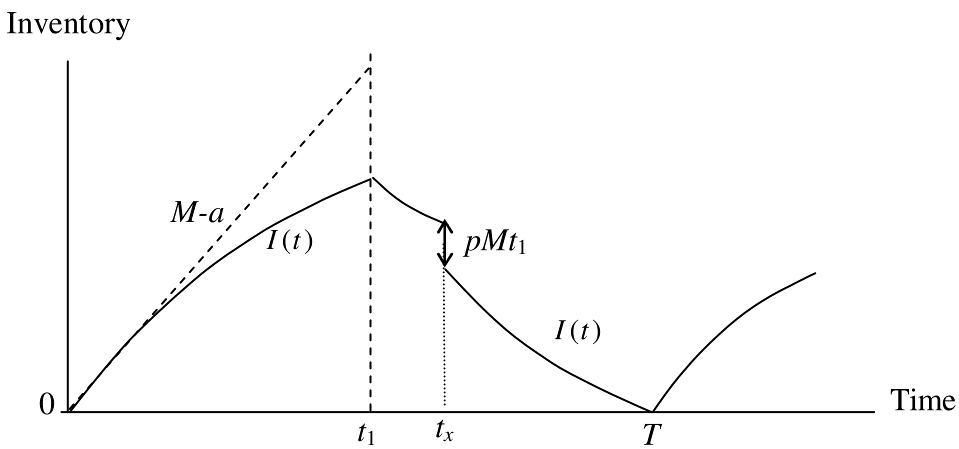

Figure 2. Inventory system of deteriorating items when the screening rate is lower than the production rate.

={ $Mt_1[s - E(p)s + E(p)v] - (K + r + cMt_1 + dMt_1)$

$- \frac{ht_1^2}{2b}[M^2 - 2Ma + a^2 + Mb - ab - 2E(p)M^2 + E(p^2)M^2 + 2E(p)Ma]$ }/

{ $[M - E(p)M - a + b]t_1 / b$ }. (14)

Where $E(p) = \int_{\alpha}^{\beta(r)} pf(p)dp$ is a function of *r*. Our problem can be formulated as:

Max: *ETPU*(r, t_1)

Subject to: $I(t_1)$-$pMt_1 \geq 0$, $r \geq 0$, $t_1 \geq 0$. (15)

The domain constraint is considered for feasibility of the model and the solution procedure is used.

Solution procedure: Under the constraint of $I(t_1)$-$pMt_1 \geq 0$, higher defective percentage of *p* would reduce the domain of *ETPU*(r, t_1). The maximal *p* is considered in order to look for possible solution. For the concave function of *ETPU*(r, t_1), the optimum occurs either at the interior or at the boundary of domain (Apostol, 1977). Therefore, the solution procedure of optimal (r^*, t_1^*) is described as follows:

Step 1: verify the concavity of *ETPU*(r, t_1).
Step 2: given the maximal defective percentage, named, *pm*, find the optimal solution of *ETPU*(r, t_1) by setting $\frac{\partial ETPU}{\partial r} = 0$ and $\frac{\partial ETPU}{\partial t_1} = 0$ when without domain constraint. If the optimal solution satisfies the domain constraint (that is, $I(t_1)$-$pmM\, t_1 \geq 0$, $r \geq 0$, $t_1 \geq 0$, with $pm = \beta(r)$), then go to Step 4. Otherwise, go to Step 3.
Step 3: find the optimal solution of *ETPU*(r, t_1) on the boundary of domain (that is, (that is, $I(t_1)$- $\beta(r)\, M\, t_1 = 0$, $t_1 \geq 0$.).
Step 4: Stop. The solution is obtained.

Due to the complexity of *ETPU*(r, t_1), the concavity with the closed form of r^* and t_1^* is difficult to find. The mathematical software MATHCAD and MAPLE 8 are used in the analysis.

When the screening rate, x, is lower than the production rate, M

The behavior of the inventory level is illustrated in Figure 2. Let $I(t)$ be the inventory level during [0, *T*]. The differential equation governing the transition of the system during [0, t_1) is:

$$\frac{dI(t)}{dt} = -\theta I(t) + M - a, \qquad 0 \leq t < t_1. \tag{16}$$

For initial condition *I*(0)=0, one has:

$$I(t) = \frac{(M-a)(1-e^{-\theta t})}{\theta}, \qquad 0 \leq t < t_1. \tag{17}$$

The differential equation during [t_1, t_x) is:

$$\frac{dI(t)}{dt} = -\theta I(t) - b, \qquad t_1 \leq t < t_x \tag{18}$$

Where,

$$t_x = \frac{Mt_1}{x} \tag{19}$$

For initial condition $I(t_1)=\frac{(M-a)(1-e^{-\theta t_1})}{\theta}$, one has:

$$I(t)=\frac{-b}{\theta}+\frac{e^{-\theta(t-t_1)}}{\theta}[b+(M-a)(1-e^{-\theta t_1})], \quad t_1 \le t < t_x. \tag{20}$$

The differential equation during [t_x, T] is:

$$\frac{dI(t)}{dt}=-\theta I(t)-b, \quad t_x \le t \le T. \tag{21}$$

For initial condition:

$$I(t_x)=\frac{-b}{\theta}+\frac{e^{-\theta(t_x-t_1)}}{\theta}[b+(M-a)(1-e^{-\theta t_1})]-pMt_1, \tag{22}$$

one has:

$$I(t)=\frac{-1}{\theta}[b+(M-a)e^{-\theta t}-(M-a+b)e^{-\theta(t-t_1)}$$
$$+pMt_1\theta e^{-\theta(tx-Mt_1)/x}], \quad t_x \le t \le T. \tag{23}$$

For $I(T)=0$, one has:

$$T(t_1,p)=1/\theta$$
$$\{\ln[(M-a+b)e^{-\theta t_1 M/x}-(M-a)e^{-\theta t_1(M+x)/x}$$
$$-pMt_1\theta e^{-\theta t_1}]-\ln b\}+\theta t_1(M+x)/\theta x$$

When 0<θ<<1, $T(t_1,p)$ can be rewritten as (By L'Hospital Rule):

$T(t_1,p)\approx(M-pM-a+b)t_1/b$.(the same as Equation(7)) (24)

The total revenue per cycle (the sum of total sales of good quality and the imperfect quality items) is:

$$TR(t_1,p)=[at_1+b(T(t_1,p)-t_1)]s+pMt_1v$$
$$=Mt_1(s-ps+pv). \tag{25}$$

The total cost per cycle (the sum of setup cost, investment cost, production cost, screening cost, and holding cost) is:

$$TC(r,t_1,p)=K+r+cMt_1+dMt_1$$
$$+h\left\{\int_0^{t_1}I(t)dt+\int_{t_1}^{t_x}I(t)dt+\int_{t_x}^{T(t_1,p)}I(t)dt\right\}. \tag{26}$$

with $e^{-x}\approx 1-x+x^2/2$, when 0<x<<1, (that is, when 0<θ<<1):

$$\int_0^{t_1}I(t)dt \approx \frac{t_1^{\;2}}{2}(M-a), \tag{27}$$

$$\int_{t_1}^{t_x}I(t)dt \approx \frac{t_1^2}{2x^2}$$
$$(-2xaM-bM^2+2xbM-x^2b-2x^2M+2xM^2+2x^2a), \tag{28}$$

and

$$\int_{t_x}^{T(t_1,p)}I(t)dt \approx (\frac{t_1^2}{2x^2b})$$
$$(b^2M^2+2xaMb+x^2a^2-2xM^2b+x^2p^2M^2+2xbM^2p+x^2M^2-2xb^2M$$
$$+2x^2apM-2x^2aM-2x^2M^2p-2x^2ab-2x^2bpM+2x^2Mb+x^2b^2). \tag{29}$$

With simplification by software Maple 8, one has:

$$\int_0^{t_1}I(t)dt+\int_{t_1}^{t_x}I(t)dt+\int_{t_x}^{T(t_1,p)}I(t)dt$$
$$=\frac{t_1^2}{2xb}(xbM-xab-2xaM+2xapM-2xM^2p+2bM^2p+xa^2+xM^2$$
$$-2xbpM+xp^2M^2). \tag{30}$$

The total profit per unit time is:

$$TPU(r,t_1,p)=\frac{TR(t_1,p)-TC(r,t_1,p)}{T(t_1,p)} \tag{31}$$

The expected value of $TPU(r, t_1, p)$ is:

$$ETPU(r,t_1)=E[\frac{TR(t_1,p)-TC(r,t_1,p)}{T(t_1,p)}]. \tag{32}$$

$$=\frac{E[TR(t_1,p)-TC(r,t_1,p)]}{E[T(t_1,p)]}$$
$$=\{Mt_1[s-E(p)s+E(p)v]-(K+r+cMt_1+dMt_1)$$
$$-\frac{ht_1^2}{2xb}$$
$$[xbM-xab-2xaM+2xaE(p)M-2xM^2E(p)+2bM^2E(p)+xa^2+xM^2$$
$$-2xbE(p)M+xE(p^2)M^2]\}/\{[M-E(p)M-a+b]t_1/b\} \tag{33}$$

To avoid shortages, it is assumed that the inventory at t_x

is positive during production time, that is:

$$I(t_x) \geq 0. \quad (34)$$

Our problem can be formulated as:

Max: $ETPU(r, t_1)$

Subject to: $I(t_x) \geq 0$, $r \geq 0$, $t_1 \geq 0$. (35)

The domain constraint is considered to ensure the feasibility of the model.

NUMERICAL RESULTS

Example 1: When the screening rate, *x*, is higher than the production rate, *M*

The preceding theory can be illustrated using the numerical example. The parameters are as follows (Salameh and Jaber, 2000):

Demand $D(t)=a$ during the production run time (a=50000 unit/year)
Demand $D(t)=b$ after the production (b=40000 unit/year)
Production rate, M (M=80000 unit/year)
Screening rate, x (x=90000 unit/year)
Ordering cost, K (K=$ 500 /cycle)
Deterioration rate of the on-hand-stock, θ (θ=0.01)
Holding cost, h (h=$ 5 /unit/year)
Screening cost, d (d=$ 0.5 /unit)
Purchase cost, c (c=$ 35/unit),
Selling price of good quality items, s (s=$ 50/unit)
The salvage value of defective item, v (v =$ 5/unit).

The percentage defective random variable, p, can take any value in the range [α, $\beta(r)$] with α=0, and $\beta(r) = \frac{\beta_2}{1+\beta_1 r} = \frac{0.1}{1+0.01r}$, where r is the investment cost on production processes. That is, uniformly distributed over [0, $\frac{0.1}{1+0.01r}$]. Then $E(p)=\frac{0.1}{2+0.02r}$, $E(p^2)=\frac{0.01}{3(1+0.01r)^2}$. The graphical representations showing the concave function ETPU are given in Figures 3 and 4. Figure 3 shows the graph of ETPU. Figure 4 shows the graph of Hessian function# of *ETPU*, which means the values of Hessian function of *ETPU* are all positive. With the given data, the optimal decision is obtained by using software MATHCAD. Solving $\frac{\partial ETPU}{\partial r} = 0$ and $\frac{\partial ETPU}{\partial t_1} = 0$ simultaneously, the solution is r^*=$ 1010 and t_1^*=0.108

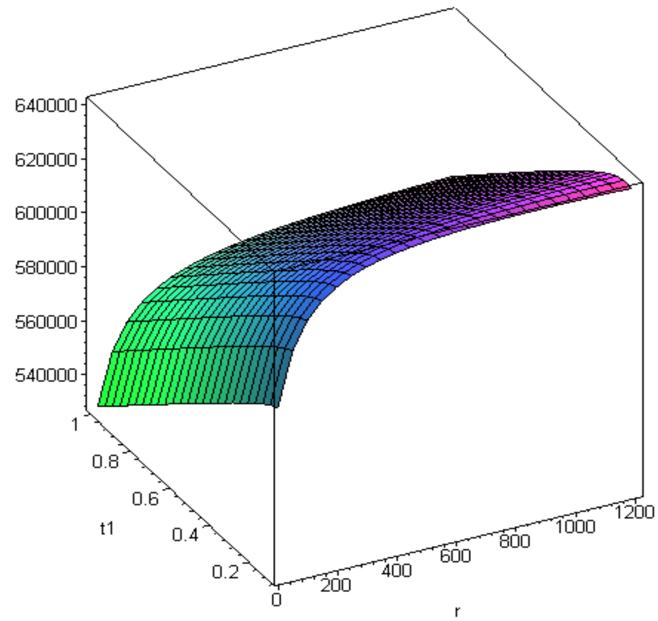

Figure 3. Graph of *ETPU*.

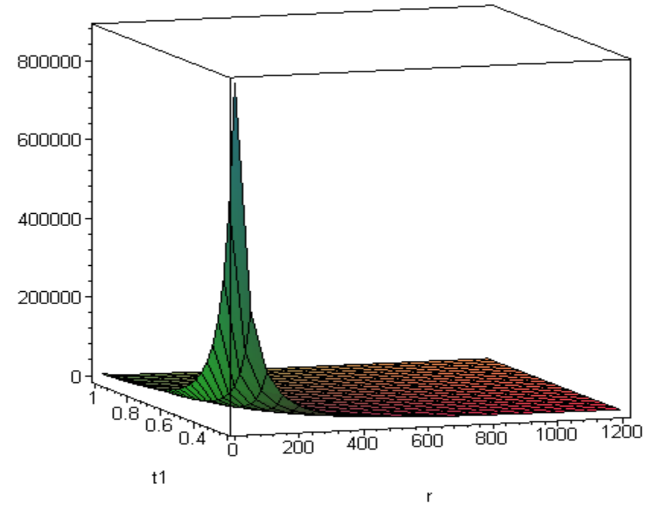

Figure 4. Graph of Hessian function of ETPU.

year, which satisfies the constraint of $I(t_1)-\beta(r)Mt_1 \geq 0$. The EPQ is $Q^*=Mt_1^*$=8626 units, the production cycle length T=0.186 year and the maximum profit per year $ETPU(r^*, t_1^*)$=$640883. When without investment, that is r=0, then t_1^*=0.065 year with $ETPU(0, t_1^*)$=$584645. The profit increase is 9.6%.

Example 2: When the screening rate, *x*, is lower than the production rate, *M*

In this example, a=50000unit/year, b=40000unit/year,

M=80000unit/year, x=70000unit/year, k=\$ 500/cycle, θ=0.01, h=\$ 5/unit/year, d=\$ 0.5/unit, c=\$ 25/unit, s=\$ 50/unit, v =\$20/unit and $\beta(r) = \frac{0.3}{1+0.01r}$. Then

$E(p)=\frac{0.3}{2+0.02r}$, $E(p^2)= \frac{0.03}{(1+0.01r)^2}$.

The graphical representations showing the concave function *ETPU* are given in Figures 5 and 6. Figure 5 shows the graph of *ETPU*. Figure 6 shows the graph of Hessian function of *ETPU*, which means the values of Hessian function of *ETPU* are all positive. Solving $\frac{\partial ETPU}{\partial r} = 0$ and $\frac{\partial ETPU}{\partial t_1} = 0$ simultaneously, the solution is r^*=\$ 366 and t_1^*=0.0836 year, which satisfies the constraint of $I(t_x) \geq 0$. The *EPQ* is $Q^*=Mt_1^*$=6688 units, the production cycle length T=0.141 year and the maximum profit per year $ETPU(r^*, t_1^*)$=\$ 1104650. Whenwithout investment, that is r=0, then t_1^*=0.0684 year with $ETPU(0, t_1^*)$=\$1093368. The profit increase is 1%.

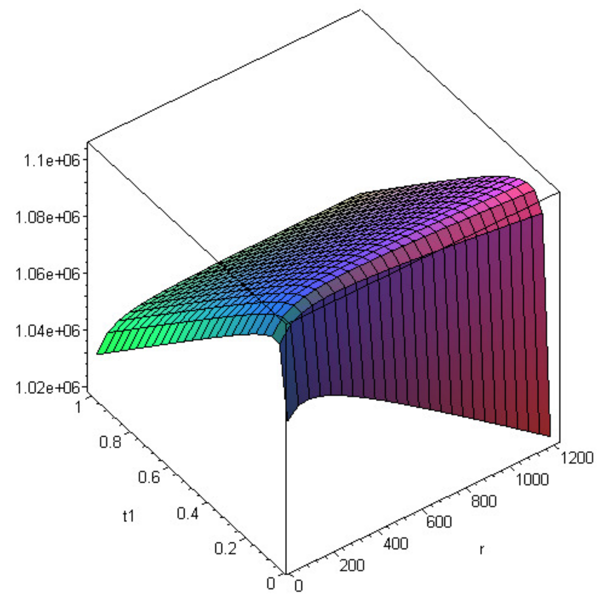

Figure 5. Graph of ETPU.

Sensitivity analysis

In order to further investigate the effect of the optimal profit per year under the investment cost, different parameter values are assumed.

In Table 1 and Figure 7, the parameters, a=50000, b=40000, M=80000, x=90000, k=500, θ=0.01, h=5, d=0.5, c=35, s=50, v=5 andβ_2=0.1 are used as the standard values, but $\beta_1$is a variable. When β_1 increases, the investment cost r and the production run time per cycle t_1, decrease, but the expected profit per unit time, *ETPU* increases. In Table 2 and Figure 8, the parameters, a=50000, b=40000, M=80000, x=90000, k=500, θ=0.01, h=5, d=0.5, c=35, s=50, v=5, and β_1=0.01 are used as the standard values, but β_2 is a variable. When β_2 increases, r and t_1 increase, but *ETPU* decreases. Tables 2 and 3 show significant extent of percentage increase. In Table 3 and Figure 9, the parameters, a=50000, b=40000, M=80000, k=500, θ=0.01, h=5, d=0.5, c=35, s=50, v=5, β_1=0.01, and β_2=0.1 are used as the standard values, but x is a variable. When x increases, t_1 and *ETPU* increase, but r decreases. In Table 4a ($x>M$) and b ($x<M$) to Table 8a and b, the sensitivity analysis of parameters h, d, c, s, v are considered. In Table 4a and b, when h increases, both % increase and *ETPU* decrease. In Table 5a and b, when d increases, % increase increases but *ETPU* decreases. In Table 6a and b, when c increases, % increase increases but *ETPU* decreases. In Table 7a and b, when s increases, % increase decreases but *ETPU* increases. In Table 8.a and 8.b, when v increases, % increase decreases but *ETPU* increases.

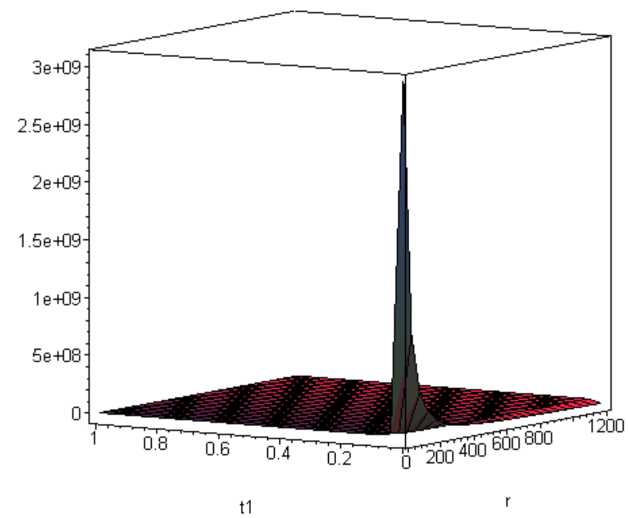

Figure 6. Graph of Hessian function of ETPU.

CONCLUSION

Inspecting the products during production process is a critical work in quality management. Many industries focus on increasing inspection cost to maintain the products quality. In fact, it is more profitable to improve the production process such as the maintenance of machine, training of employee, etc. It requires not only lowering the percentage of defective items but also lowering the inspection cost. This study develops an EPQ deteriorating item model with investment on improving the production process to decrease the percentage of

Table 1. Sensitivity analysis of β_1 .

β_1	*r*	t_1	ETPU	ETPUr	% increase
0.003	1965	0.138	632796	584645	8.2
0.004	1678	0.13	635045	584645	8.6
0.005	1484	0.124	636644	584645	8.9
0.006	1341	0.119	637862	584645	9.1
0.007	1231	0.116	638833	584645	9.3
0.008	1143	0.113	639632	584645	9.4
0.009	1071	0.11	640305	584645	9.5
0.01	1010	0.108	640883	584645	9.6
0.011	958	0.106	641387	584645	9.7
0.012	913	0.104	641832	584645	9.8
0.013	873	0.103	642229	584645	9.8
0.014	838	0.101	642585	584645	9.9
0.015	807	0.1	642908	584645	9.9
0.016	779	0.099	643203	584645	10
0.017	753	0.098	643473	584645	10

a=50000, b=40000, M=80000, k=500, θ=0.01, h=5, d=0.5, c=35, s=50, v =5, β_2 =0.1, x=90000. $ETPUr(r, t_1)$:= $ETPU(0, t_1)$ which means *ETPU* without investment; % increase= (*ETPU* / *ETPUr*)*100%.

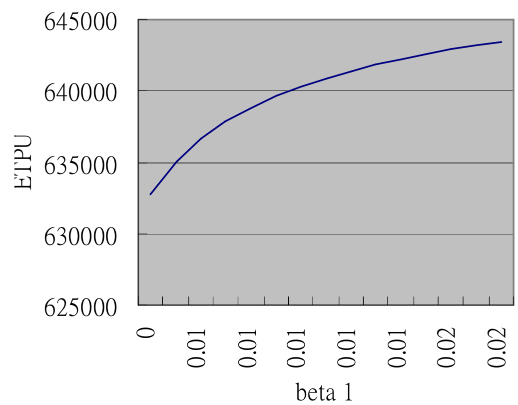

Figure 7. The effect of β_1 on the expected value of *TPU*.

Table 2. Sensitivity analysis of β_2.

β_2	*r*	t_1	ETPU	ETPUr	% increase
0.03	439	0.085	646541	584645	10.6
0.04	538	0.089	645428	584645	10.4
0.05	629	0.093	644471	584645	10.2
0.06	713	0.097	643621	584645	10.1
0.07	793	0.099	642852	584645	10
0.08	868	0.103	642147	584645	9.8
0.09	941	0.105	641494	584645	9.7
0.1	1010	0.108	640883	584645	9.6
0.11	1077	0.11	640309	584645	9.5
0.12	1142	0.112	639766	584645	9.4
0.13	1206	0.115	639250	584645	9.3

Table 2. Contd.

0.14	1267	0.117	638758	584645	9.3
0.15	1327	0.119	638287	584645	9.2
0.16	1386	0.121	637836	584645	9.1
0.17	1443	0.122	637401	584645	9.0

a=50000, *b*=40000, *M*=80000, *k*=500, *θ*=0.01, *h*=5, *d*=0.5, *c*=35, *s*=50, v =5, β_1 =0.01, *x*=90000. *ETPUr*(*r*, t_1):= *ETPU*(0, t_1) which means *ETPU* without investment; % increase= (*ETPU* / *ETPUr*)*100%.

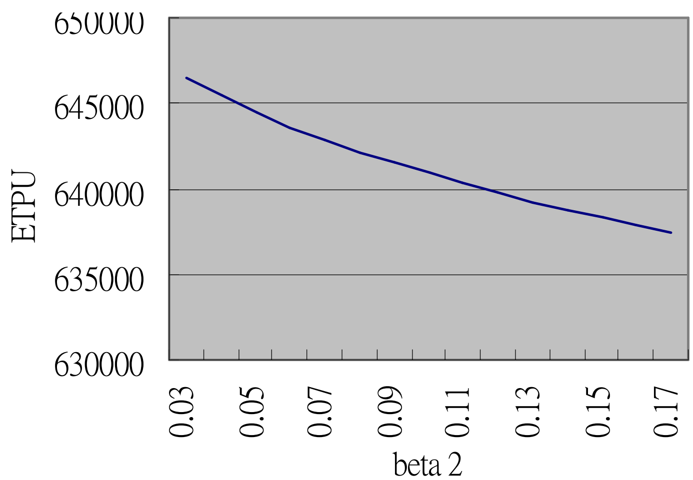

Figure 8. The effect of β_2 on the expected value of *TPU*.

Table 3. Sensitivity analysis of *x*.

x	*r*	t_1	ETPU
52000	1015	0.108	640823
54000	1014	0.108	640830
56000	1014	0.108	640836
58000	1013	0.108	640841
60000	1013	0.108	640846
62000	1013	0.108	640851
64000	1012	0.108	640855
66000	1012	0.108	640860
68000	1012	0.108	640864
70000	1011	0.108	640867
72000	1011	0.108	640871
74000	1011	0.108	640874
76000	1011	0.108	640877
78000	1010	0.108	640880
80000	1010	0.108	640883

a=50000, *b*=40000, *M*=80000, *k*=500, *θ*=0.01, *h*=5, *d*=0.5, *c*=35,*s*=50, v =5, β_1 =0.01, β_2 =0.1

defective items. The tradeoff between the investment cost and the marginal improvement on products is a key problem. Two case scenarios corresponding to the screening rate are considered in this study. Numerical examples are provided to illustrate the theory. Sensitivity analysis shows that higher screening rate leads to higher

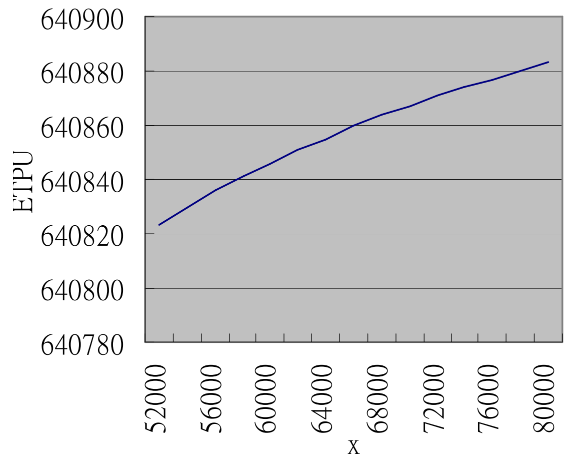

Figure 9. The effect of *x* on the expected value of *TPU*.

Table 4a. Sensitivity analysis of unit holding cost, *h* for *x*>*M*.

h	*r*	t_1	ETPU	ETPUr	% increase
1	1725	0.292	650551	589783	10.3
2	1371	0.19	647096	588061	10
3	1198	0.147	644613	586740	9.9
4	1088	0.124	642603	585627	9.7
5	1010	0.108	640883	584645	9.6
6	950	0.096	639363	583758	9.5
7	903	0.088	637991	582943	9.4
8	862	0.081	636733	582183	9.4
9	829	0.075	635568	581470	9.3

a=50000, *b*=40000, *M*=80000, *k*=500, *d*=0.5, *c*=35, *s*=50, v =5, β_1 =0.01, β_2 =0.1, *x*=90000.

Table 4b. Sensitivity analysis of unit holding cost, *h* for *x*<*M*.

h	*r*	t_1	ETPU	ETPUr	% increase
1	627	0.211	1111820	1098940	1.2
2	498	0.141	1109310	1097070	1.1
3	435	0.112	1107470	1095640	1.1
4	395	0.095	1105960	1094430	1
5	366	0.084	1104650	1093370	1
6	344	0.075	1103480	1092410	1
7	326	0.069	1102420	1091520	1
8	312	0.064	1101450	1090700	1
9	300	0.06	1100530	1089920	1

a=50000, *b*=40000, *M*=80000, *k*=500, *d*=0.5, *c*=25, *s*=50, v =20, β_1 =0.01, β_2 =0.3, *x*=70000.

Table 5a. Sensitivity analysis of unit screening cost, *d* for *x*>*M*.

d	*r*	t_1	ETPU	ETPUr	% increase
0.1	999	0.107	659264	604039	9.1
0.2	1002	0.108	654669	599191	9.3

Table 5a. Contd.

0.3	1005	0.108	650074	594342	9.4
0.4	1008	0.108	645478	589494	9.5
0.5	1010	0.108	640883	584645	9.6
0.6	1013	0.108	636288	579797	9.7
0.7	1016	0.108	631693	574948	9.9
0.8	1018	0.108	627098	570100	10
0.9	1021	0.108	622503	565251	10.1

a=50000, b=40000, M=80000, k=500, h=5, c=35, s=50, $\mathcal{V}$ =5, β_1 =0.01, β_2 =0.1, x=90000.

Table 5b. Sensitivity analysis of unit screening cost, *d* for *x<M*.

d	*r*	t_1	ETPU	ETPUr	% increase
0.1	307	0.081	1123680	1115440	0.7
0.2	322	0.082	1118920	1109920	0.8
0.3	337	0.082	1114150	1104400	0.9
0.4	352	0.083	1109400	1098890	1
0.5	366	0.084	1104650	1093370	1
0.6	380	0.084	1099910	108785	1.1
0.7	393	0.085	1095170	1082330	1.2
0.8	407	0.085	1090440	1076820	1.3
0.9	421	0.086	1085710	1071300	1.3

a=50000, b=40000, M=80000, k=500, h=5, c=25, s=50, $\mathcal{V}$ =20, β_1=0.01, β_2 =0.3, x=70000.

Table 6a. Sensitivity analysis of unit screening cost, *c* for *x>M*.

c	*r*	t_1	ETPU	ETPUr	% increase
27	778	0.099	1009000	972524	3.7
29	839	0.102	917000	875554	4.7
31	898	0.104	824737	778585	5.9
33	955	0.106	732797	681615	7.5
35	1010	0.108	640883	584645	9.6
37	1064	0.11	548993	487676	12.6
39	1116	0.111	457123	390706	17
41	1167	0.113	365272	293736	24.4
43	1217	0.115	273437	196767	39

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, s=50, $\mathcal{V}$ =5, β_1 =0.01, β_2 =0.1, x=90000.

Table 6b. Sensitivity analysis of unit screening cost, *c* for *x<M*.

c	*r*	t_1	ETPU	ETPUr	% increase
21	0	0.068	1314060	1314060	0
22	0	0.068	1258890	1258890	0
23	0	0.068	1203710	1203710	0
24	203	0.076	1152510	1148540	0.3
25	366	0.084	1104650	1093370	1
26	496	0.089	1057410	1038200	1.9
27	610	0.094	1010460	983020	2.8
28	713	0.098	963690	927850	3.9
29	809	0.101	917050	872680	5.1

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, s=50, $\mathcal{V}$ =20, β_1 =0.01 β_2, =0.3, x=70000.

Table 7a. Sensitivity analysis of unit selling price of good quality items, *s* for *x*>*M*.

s	*r*	t_1	ETPU	ETPUr	% increase
42	1037	0.109	274935	216161	27.2
44	1030	0.109	366422	308282	18.9
46	1024	0.108	457909	400403	14.4
48	1017	0.108	549396	492524	11.5
50	1010	0.108	640883	584645	9.6
52	1003	0.108	732371	676767	8.2
54	997	0.107	823859	768888	7.1
56	990	0.107	915348	861009	6.3
58	983	0.107	1007000	953130	5.6

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, c=35, v=5, β_1=0.01, β_2=0.1, x=90000.

Table 7b. Sensitivity analysis of unit selling price of good quality items,*s* for *x*<*M*.

s	*r*	t_1	ETPU	ETPUr	% increase
42	496	0.089	737410	718200	2.7
44	466	0.088	829180	818990	2.1
46	434	0.086	920980	905780	1.7
48	401	0.085	1012800	999570	1.3
50	366	0.084	1104650	1093370	1
52	330	0.082	1196530	1187160	0.8
54	291	0.08	128846	128095	0.6
56	249	0.078	1380440	1374750	0.4
58	204	0.076	1472510	145854	0.3

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, c=25, v=20, β_1=0.01, β_2=0.3, x=70000.

Table 8a. Sensitivity analysis of unit selling price of defective items, v for *x*>*M*.

v	*r*	t_1	ETPU	ETPUr	% increase
1	1103	0.111	640089	574948	11.3
2	1080	0.11	640282	577373	10.9
3	1057	0.109	640478	579797	10.5
4	1034	0.109	640679	582221	10
5	1010	0.108	640883	584645	9.6
6	986	0.107	641093	587070	9.2
7	962	0.106	641306	589494	8.8
8	938	0.105	641525	591918	8.4
9	913	0.104	641750	594342	8

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, c=35, s=50, β_1=0.01, β_2=0.1, x=90000.

Table 8b. Sensitivity analysis of unit selling price of defective items, v for *x*<*M*.

v	*r*	t_1	ETPU	ETPUr	% increase
16	762	0.1	1100360	106026	3.8
17	675	0.096	1101210	1068540	3.1
18	583	0.093	1102180	1076820	2.4
19	481	0.088	1103290	1085090	1.7

Table 8b. Contd.

20	366	0.084	1104650	1093370	1
21	227	0.077	1106460	1101640	0.4
22	15	0.069	1109950	1109920	0
23	0	0.068	1118200	1118200	0
24	0	0.068	112647	112647	0

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, c=25, s=50 β_1=0.01, β_2 **=**0.3, x=70000.

profit. Also, enhancing production process by increasing the investment cost will improve the business significantly.

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