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Joint determination of lot-size and shipment policy for a vendor-buyer system with rework and an improving delivery plan

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This paper studies a vendor-buyer integrated system with rework and an improving delivery policy for lowering both vendor and buyer's stock holding costs. The objective is to derive the optimal production lot size and number of deliveries that minimizes total costs for the proposed vendor-buyer integrated system. This study extends the work of Chiu et al. (2011) by incorporating an improving n+1 shipment policy into their model, with the purpose of reducing both vendor and buyer's stock holding costs. Under such an enhancing policy, one extra upfront delivery of finished items is distributed to buyer for satisfying product demand during supplier's production and rework times. Then, fixed quantity (n) installments of finished items are delivered to customer at the end of rework. Mathematical modeling along with Hessian matrix equations is employed to derive and prove convexity of the long-run cost function. Closed-form solution in terms of lot size and number of deliveries is obtained. A numerical example is provided to show its practical usage and demonstrate significant reduction in stock holding cost.

Key words: Production lot size, vendor-buyer system, multi-deliveries, rework.

INTRODUCTION

Chiu et al. (2011) studied an optimal replenishment and shipment decisions in an integrated finite production rate model with scrap and rework. Their work can be considered as an extension of the conventional economic production quantity (EPQ) model (Taft, 1918) with special focuses on both production quality assurance and end items delivery issues. Classic EPQ model assumes that all items produced are of perfect quality. However, in real-life manufacturing firms, owing to process deterioration or various other factors, generation of defective items is inevitable. Many studies have been carried out to address the imperfect quality issue in production systems (Barlow and Proschan, 1965; Rosenblatt and Lee, 1986; Wee, 1993; Salameh and Jaber, 2000; Nahmias, 2009; Baten and Kamil, 2009; Grasman, 2009; Jha and Shanker, 2009; Roy et al., 2009; Cheng and Ting, 2010; Lodree et al., 2010; Ma et al., 2010; Saha et al., 2010; Mehdi et al.,

2010), Panda and Maiti, 2009; Liu et al., 2009. The defective products sometimes can be reworked to reduce total production costs (Hutchings, 1976; Chiu et al., 2009a; Cárdenas-Barrón, 2009; Wazed et al., 2009; El Saadany and Jaber, 2010; Chiu et al., 2010a, b, c, d; Taleizadeh et al., 2010; Wahab and Jaber, 2010; Chiu, 2010; Wazed et al., 2010a, b). For example, production processes in plastic injection molding, or in printed circuit board (PCB) assembly, sometimes employs rework as an acceptable process to increase level of quality. Chern and Yang (1999) considered a threshold control policy for an imperfect production system with only a work center handling both regular and rework jobs. The imperfect production system generates defect jobs by factors other than machine failures. A threshold control policy sets the guideline for a work center to switch between regular and rework jobs. They assumed the outcome of each completed regular job is an independent Bernoulli trial with three possibilities: good, rework, or scrap. Once the work center accumulates more than a threshold of rework jobs, it finishes the last batch of regular jobs and switches

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to rework jobs. The objective of their research was to find a threshold ω and a lot size s that maximize the average long-term profit. Chiu et al. (2010a) examined a finite production rate model with scrap, rework and stochastic machine breakdown. Stochastic breakdown rate and random defective rate along with the reworking of nonconforming items were assumed in their study. The objective was to derive the optimal production run time that minimize the long run average production cost.

In real-life, vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly adapted instead of continuous issuing policy (as was assumed by classic EPQ). Schwarz (1973) considered a one-warehouse N-retailer inventory system with the objective of determining optimal stocking policy that minimizes average system cost. Schwarz (1973) derived some necessary properties for the optimal policy as well as the optimal solutions. Heuristic solutions were also provided for the general problem and tested against analytical lower bounds. Studies have since been carried out to address various aspects of supply chains optimization (Goyal, 1977; Schwarz et al., 1985; Hahm and Yano, 1992; Sarker and Parija, 1994; Hill, 1995; Viswanathan, 1998; Buscher and Lindner, 2005; Sarmah et al., 2006; Kim et al., 2008; Mahnam et al., 2009; Chiu et al., 2009b; Sarker and Diponegoro, 2009; Chiu et al., 2011). Selected articles are surveyed as follows. Hahm and Yano (1992) determined the frequencies of production and delivery of a single component with the objective of minimizing the long-run average cost per unit time. Their cost includes production setup costs, inventory holding costs at both the supplier and the customer and transportation costs. For their proposed model, it was proved that the ratio between the production interval and delivery interval must be an integer in an optimal solution. They used these results to characterize situations in which it is optimal to have synchronized production and delivery and discussed the ramifications of these conditions on strategies for setup cost and setup time reductions. Sarmah et al. (2006) considered that coordination between two different business entities is an important way to gain competitive advantage as it lowers supply chain cost, so they reviewed literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. An effort was also made to identify critical issues and scope of future research. Chiu et al. (2009b) incorporated a multi-delivery policy and quality assurance into an imperfect economic production quantity (EPQ) model with scrap and rework. They assumed the reworking of repairable defective items in each production run and the finished items can only be delivered to customers if the whole lot is quality assured in the end of rework. The expected integrated cost function per unit time was derived. A closed-form optimal batch size solution to the problem was obtained. This paper extends the integrated vendor-buyer production-shipment

problem (Chiu et al., 2011) and proposes an $n+1$ delivery policy with the purpose of reducing both vendor and buyer's stock holding costs. The joint effects of the $n+1$ multi-delivery policy and partial rework on the optimal replenishment lot size and shipment policy for such an integrated system are examined.

METHODS

Modelling and formulations

For the purpose of lowering both vendor and buyer's stock holding costs, this study proposed an $n+1$ delivery policy in lieu of n delivery policy in Chiu et al. (2011). Recall such a specific integrated vendor buyer production-shipment problem: it is a typical EPQ model with the quality assurance in production process and multi-delivery policy. Quality assurance issue is in regard that the process may produce an x portion of random nonconforming items at a production rate d . All defective items are considered to be repairable and they are reworked and repaired at a rate P_1 within the same cycle when regular production ends. While under the proposed $n+1$ delivery policy, the first installment of finished products is delivered to customer for satisfying demand during uptime t_1 and rework time t_2 (Figures 1 and 2). Then, in the end of rework when the whole lot is quality assured, fixed quantity n installments of the rest of finished items are delivered to customer at a fixed interval of time during the production downtime t_3 . Figure 1 illustrates vendor's on-hand inventory level of perfect quality items in the proposed $n+1$ delivery model (in blue). It also depicts the vendor's expected reduction in stock holding costs (in a lighter shade of blue) when comparing with model in Chiu et al. (2011) (in black). Figure 2 depicts buyer's stock level in the proposed model (in blue) and the expected reduction in buyer's holding costs (in a lighter shade of blue) when comparing with model in Chiu et al. (2011) (in black). From Figures 1 and 2, the objective of the present study is clearly displayed, that is, to reduce the stock holding costs for both vendor and buyer in such an integrated system. The following are more details in assumption. Consider that the constant production rate P is larger than sum of demand rate λ and production rate d . Thus, $(P-d-\lambda) > 0$, where $d = Px$. Cost parameters include the setup cost K per production, unit manufacturing cost C , unit holding cost h , unit rework cost C_R , holding cost h_1 for each reworked item, fixed delivery cost K_1 , delivery cost C_T per item shipped and the following notation:

$H =$ the level of on-hand inventory for satisfying product demand during vendor's uptime t_1 and rework time t_2 ,
 $H_1 =$ maximum level of on-hand inventory in units when regular production ends,
 $H_2 =$ the maximum level of on-hand inventory in units when rework process finishes,
 $Q =$ production lot size per cycle,
 $n =$ number of fixed quantity installments of the remaining finished items to be delivered to customer during t_3 ,
 $l(t) =$ the level of on-hand inventory of perfect quality items at time t ,
 $l_c(t) =$ the level of buyer's on-hand inventory at time t ,
 $TC(Q, n+1) =$ total production-inventory-delivery costs per cycle for the proposed model,
 $E[TCU(Q, n+1)] =$ the long-run average costs per unit time for the proposed model,

Total production-inventory-delivery costs per cycle $TC(Q)$ consists of the following:

1) The variable manufacturing costs and setup cost per cycle as given in Equation (1):

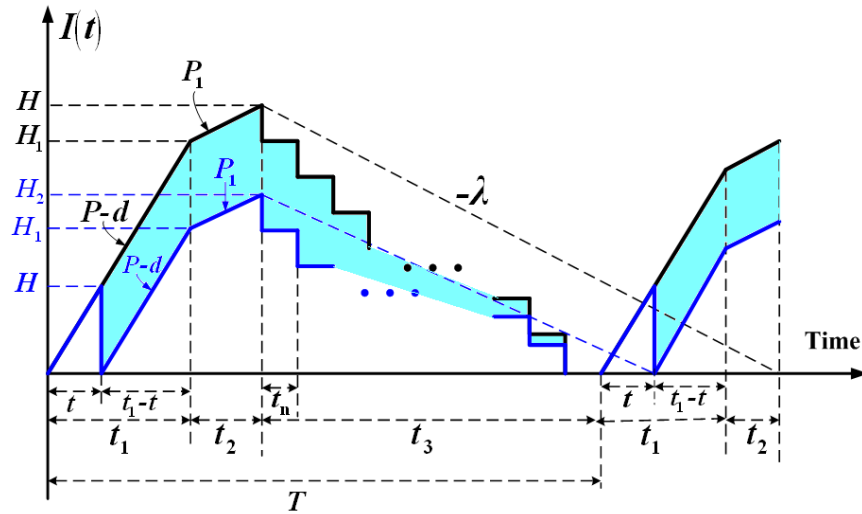


Figure 1. Vendor's inventory level of perfect items in the proposed model (in blue) and the expected reduction in holding costs (in a lighter shade of blue) when comparing with model in Chiu et al. (2011) (in black).

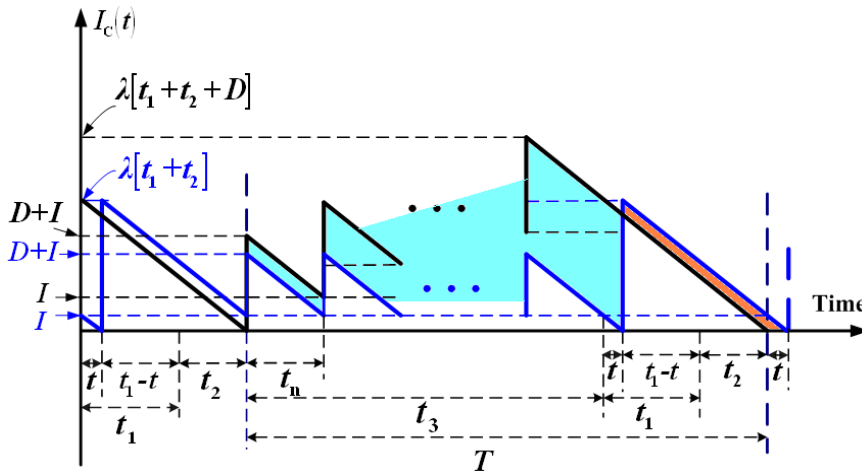


Figure 2. Buyer's inventory level in the proposed model (in blue) and the expected reduction in buyer's holding costs (in a lighter shade of blue) when comparing with model in Chiu et al. (2011) (in black) (T = cycle length, t = the production time needed for producing enough perfect items for satisfying customer's demand during t_1 and t_2 , t_1 = the production uptime, t_2 = rework time, t_3 = production downtime, time to deliver the remaining quality assured finished products, t_n = a fixed interval of time between each installment of products delivered during t_3).

$$CQ + K \tag{1}$$

2) The quality assurance costs include variable repairing costs and holding costs for reworked items:

$$C_R [xQ] + C_T Q + h_1 \cdot \frac{dt_1}{2} \cdot (t_2) \tag{2}$$

3) The fixed and variable delivery costs per cycle as shown thus:

$$(n+1)K_1 + C_T Q \tag{3}$$

4) Total inventory holding costs for vendor for all end items produced in t_1 , t_2 , and t_3 (Chiu et al., 2011) for computations of the last term in Equation (4):

$$h \left[\frac{H}{2}(t) + \frac{H_1}{2}(t_1 - t) + \frac{dt_1}{2}(t_1) + \frac{H_1 + H_2}{2}(t_2) + \left(\frac{n-1}{2n} \right) H_2 t_3 \right] \tag{4}$$

5) Total stock holding costs for buyer are as follows (Chiu et al., 2011):

$$h_2 \left[\frac{H(t_1+t_2)}{2} + n \left(\frac{D+2I}{2} \right) t_n \right] \tag{5}$$

Therefore, total production-inventory-delivery costs per cycle $TC(Q,n)$ is:

$$\begin{aligned} TC(Q,n+1) = & CQ + K + C_r[xQ] + h_1 \cdot \frac{dt_1}{2} \cdot (t_2) + (n+1)K_1 + C_T Q \\ & + h \left[\frac{H}{2}(t) + \frac{H_1}{2}(t_1-t) + \frac{dt_1}{2}(t_1) + \frac{H_1+H_2}{2}(t_2) + \left(\frac{n-1}{2n} \right) H_2 t_3 \right] \\ & + h_2 \left[\frac{H(t_1+t_2)}{2} + n \left(\frac{D+2I}{2} \right) t_n \right] \end{aligned} \tag{6}$$

Because defective rate x is assumed to be a random variable with a known probability density function, one could use the expected

values of x in the related cost analysis. With further derivation, the long-run average production-inventory-delivery cost per unit time $E[TCU(Q,n+1)]$ is as follows:

$$\begin{aligned} E[TCU(Q,n+1)] = & \lambda \left\{ C + \frac{[(n+1)K_1 + K]}{Q} + [C_r E(x)] + C_T \right\} \\ & + \frac{hQ}{2} \left\{ \frac{2\lambda^3 E_0}{P^3} + \frac{4\lambda^3 E_1}{P^2 P_1} + \frac{2\lambda^3 E_2}{P P_1^2} - \frac{\lambda E_3}{P_1} + 1 - \frac{\lambda}{P} \right\} \\ & + \frac{(h_2-h)Q}{2n} \left[1 - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{P P_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2}{P_1^2} \cdot E_3 \right] \\ & + h_2 Q \left[\frac{\lambda^2}{P^2} E_0 - \frac{\lambda^3}{P^3} E_0 - \frac{2\lambda^3}{P^2 P_1} \cdot E_1 + \frac{\lambda^3}{P P_1^2} \cdot E_2 + \frac{\lambda^2}{P P_1} E_1 \right] \\ & + \frac{(h_2-h)Q}{2} \left[\frac{\lambda^2}{P^2} + \frac{2\lambda^2 E(x)}{P P_1} + \frac{\lambda^2}{P_1^2} \cdot E_3 \right] + \frac{h_1 Q \lambda}{2 P_1} \cdot E_3 \end{aligned} \tag{7}$$

where

$$E_0 = E\left(\frac{1}{1-x}\right); E_1 = E\left(\frac{x}{1-x}\right); E_2 = E\left(\frac{x^2}{1-x}\right); E_3 = [E(x)]^2 \tag{8}$$

Hessian matrix equations (Rardin, 1998) are employed in this study to verify the convexity of $E[TCU(Q,n)]$ as follows:

$$[Q \ n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(Q,n+1)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n+1)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q,n+1)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q,n+1)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} > 0 \tag{9}$$

Convexity of $E[TCU(Q,n)]$

To derive the optimal production-shipment policy for the proposed model, one must first prove that $E[TCU(Q,n)]$ is a convex function.

Applying Hessian matrix equations one obtains:

$$\begin{aligned} \frac{\partial E[TCU(Q,n+1)]}{\partial Q} = & -\frac{\lambda[(n+1)K_1 + K]}{Q^2} + \frac{h}{2} \left\{ \frac{2\lambda^3 E_0}{P^3} + \frac{4\lambda^3 E_1}{P^2 P_1} + \frac{2\lambda^3 E_2}{P P_1^2} - \frac{\lambda(1-\theta)^2 E_3}{P_1} + 1 - \frac{\lambda}{P} \right\} \\ & + \frac{(h_2-h)}{2n} \left[1 - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{P P_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2}{P_1^2} \cdot E_3 \right] + h_2 \left[\frac{\lambda^2}{P^2} E_0 - \frac{\lambda^3}{P^3} E_0 - \frac{2\lambda^3}{P^2 P_1} \cdot E_1 + \frac{\lambda^3}{P P_1^2} \cdot E_2 + \frac{\lambda^2}{P P_1} E_1 \right] \\ & + \frac{(h_2-h)}{2} \left[\frac{\lambda^2}{P^2} + \frac{2\lambda^2 E(x)}{P P_1} + \frac{\lambda^2}{P_1^2} \cdot E_3 \right] + \frac{h_1 Q \lambda}{2 P_1} \cdot E_3 \end{aligned} \tag{10}$$

$$\frac{\partial^2 E[TCU(Q, n+1)]}{\partial Q^2} = \frac{2\lambda[(n+1)K_1 + K]}{Q^3} \quad (11)$$

$$\frac{\partial E[TCU(Q, n+1)]}{\partial n} = \frac{K_1 \lambda \cdot E_3}{Q} - \frac{Q(h_2 - h)}{2n^2} \left[\frac{1}{E_3} - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2 [E(x)]^2}{P_1^2} \right] \quad (12)$$

$$\frac{\partial^2 E[TCU(Q, n+1)]}{\partial n^2} = -\frac{Q(h - h_2)}{n^3} \left[\frac{1}{E_3} - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2 [E(x)]^2}{P_1^2} \right] \quad (13)$$

$$\frac{\partial E[TCU(Q, n+1)]}{\partial Q \partial n} = -\frac{K_1 \lambda \cdot E_3}{Q^2} - \left(\frac{h_2 - h}{2n^2} \right) \left[\frac{1}{E_3} - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2 [E(x)]^2}{P_1^2} \right] \quad (14)$$

Substituting Equations 10 to 14 in Equation 9, one has:

$$[Q \ n] \cdot \begin{pmatrix} \frac{\partial^2 E[TCU(Q, n+1)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q, n+1)]}{\partial Q \partial n} \\ \frac{\partial^2 E[TCU(Q, n+1)]}{\partial Q \partial n} & \frac{\partial^2 E[TCU(Q, n+1)]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\lambda(K_1 + K)}{Q} > 0 \quad (15)$$

Equation 15 is resulting positive, because $K, K_1, \lambda,$ and Q are all positive. Hence, $E[TCU(Q, n+1)]$ is a strictly convex function for all Q and n different from zero.

shipments n^* , one can differentiate $E[TCU(Q, n+1)]$ with respect to Q and with respect to n , and solve the linear system of Equations 10 and 12 by setting these partial derivatives equal to zero. With further derivations one obtains Q^* and n^* respectively as follows:

RESULTS

Derivation of the optimal production-shipment policy

Derive the optimal lot size Q^* and number of

$$Q^* = \sqrt{\frac{2\lambda[(n+1)K_1 + K]}{h \left\{ \frac{2\lambda^3 E_0}{P^3} + \frac{4\lambda^3 E_1}{P^2 P_1} + \frac{2\lambda^3 E_2}{PP_1^2} - \frac{\lambda(1-\theta)E_3}{P_1} + 1 - \frac{\lambda}{P} \right\} + (h_2 - h) \left[\frac{\lambda^2}{P^2} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2 E_3}{P_1^2} \right] + \frac{h_1 \lambda E_3}{P_1} + \frac{(h_2 - h)}{n} \left[1 - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2 E_3}{P_1^2} \right] + 2h_2 \lambda^2 \left[\frac{E_0}{P^2} - \frac{\lambda E_0}{P^3} - \frac{2\lambda E_1}{P^2 P_1} + \frac{\lambda E_2}{PP_1^2} + \frac{E_1}{PP_1} \right]} \quad (16)$$

$$n^* = \sqrt{\frac{(K_1 + K)(h_2 - h) \left[1 - \frac{2\lambda}{P} - \frac{2\lambda E(x)}{P_1} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2}{P^2} + \frac{\lambda^2 E_3}{P_1^2} \right]}{K_1 \left\{ h \left[-\frac{\lambda E_3}{P_1} + 1 - \frac{\lambda}{P} \right] + h_2 \left[\frac{2\lambda^2 E_0}{P^2} + \frac{2\lambda^2 E_1}{PP_1} \right] + \frac{h_1 \lambda E_3}{P_1} + (h_2 - h) \left[\frac{-2\lambda^3 E_0}{P^3} - \frac{4\lambda^3 E_1}{P^2 P_1} + \frac{2\lambda^3 E_2}{PP_1^2} + \frac{\lambda^2}{P^2} + \frac{2\lambda^2 E(x)}{PP_1} + \frac{\lambda^2 E_3}{P_1^2} \right] \right\}}} \quad (17)$$

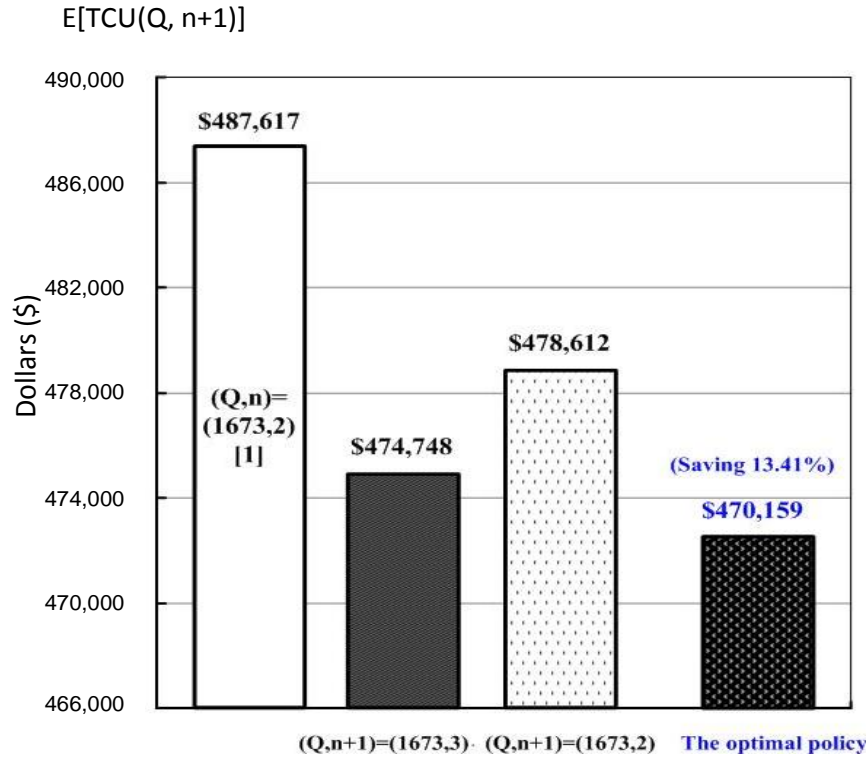


Figure 3. Comparisons of cost reductions among different scenarios in numerical example.

It is noted that the optimal number of shipments n^* only takes on integer value, while Equation 17 results likely in a real number. One can use two adjacent integers from the result of Equation 17, plugging into Equation 16 to obtain their corresponding Qs. Then substituting each pair of (Q,n) in $E[TCU(Q,n+1)]$ (that is, Equation 7) to compare and pick whichever (Q,n) that has the minimal cost as our optimal production-shipment policy.

DISCUSSION

To ease the comparison for readers between the proposed model and Chiu et al. (2011) model, we adopt their numerical example and the values for the corresponding system parameters are as follows:

- $\lambda = 3400$ units per year,
- $P = 60,000$ units per year,
- $x =$ random defective rate which follows a uniform distribution over interval $[0, 0.3]$,
- $P_1 = 2,200$ units per year,
- $C = \$100$ per item,
- $K = \$20,000$ per production run,
- $H = \$20$ per item per year,
- $h_1 = \$40$ per item reworked per unit time (year),
- $h_2 = \$80$ per item kept at the customer’s end per unit time

- $C_R = \$60$, repaired cost for each item reworked,
- $K_1 = \$4,350$ per shipment, a fixed cost,
- $C_T = \$0.1$ per item delivered.

Three scenarios are considered here with the purpose of comparing our research results to what was obtained in Chiu et al. (2011).

Scenario 1

Add one extra upfront delivery to Chiu et al. (2011) model and use the same replenishment lot size in Chiu et al. (2011), so we have $(Q,n+1)=(1673,3)$. Applying Equation 7, $E[TCU(1673,3)] = \$ 474,748$. It is noted that even with one extra delivery, our proposed policy results a cost reduction of \$ 12,869 in comparison with what is in Chiu et al. (2011) or 9.55% savings of total other related costs (that is, $E[TCU(Q,n+1)]-(AC)$).

Scenario 2

Let total number of deliveries remain 2 (Chiu et al., 2011) (that is, trigger first delivery during regular production time as proposed by our model) and use the same lot size, so one has $(Q,n+1)=(1673,2)$. Applying Equation 7,

$E[TCU(1673,2)] = \$ 478,612$. This scenario results in a cost saving of \$ 9,005 in comparison with what is in Chiu et al. (2011) or 6.50% savings of total other related costs.

Scenario 3

Use the proposed model. Let us use Equations 16 and 17 to derive the optimal production-shipment policy. We have $n^* = 2.44$, since n^* only take on integer value one can use two adjacent integers, plugging them in Equation 16, and then into Equation 7. The resulting costs are $E[TCU(2265,2)] = \$470,159$ and $E[TCU(2562,3)] = \$470,200$. Therefore, the optimal production-shipment policy is $(Q, n+1) = (2265, 2)$ and the long-run average cost is \$ 470,159. Total cost for the proposed model results in a reduction of \$ 17,458 or 13.41% savings of other related costs (Figure 3).

Conclusions

Chiu et al. (2011) studied a vendor-buyer integrated system with quality assurance issue and multi-delivery policy. They adopted an n multi-delivery plan starts in the end of rework process when the entire lot is quality assured. This paper extends it and proposes an enhancing product delivery policy with the purpose of lowering both vendor and buyer's stock holding costs. Under the proposed enhancing policy, one extra upfront delivery of finished items is distributed to buyer for satisfying product demand during supplier's production and rework times. Then fixed quantity n installments of finished items are delivered to buyer at the end of rework. By using mathematical modeling along with Hessian matrix equations, this study derives optimal production lot size as well as optimal number of deliveries that minimizes total costs for the proposed vendor-buyer integrated system with an enhancing product shipping policy. A numerical example along with a few scenarios is provided to demonstrate the significant savings from the proposed model.

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