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Full Length Research Paper

The optimal inventory policies for the economic order quantity (EOQ) model under conditions of two levels of trade credit and two warehouses in a supply chain system

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An inventory problem involves a lot of factors influencing inventory decisions. To understand it, the traditional EOQ model plays rather important role on inventory analysis. Although the traditional EOQ models are still widely used in industry, practitioners frequently question validities of assumptions of these models such that their use encounters challenges and difficulties. So, this paper tries to present a new inventory model by considering two levels of trade credit and two warehouses together to relax the basic assumptions of the traditional EOQ model to improve the environment of the use of it. Keeping in mind cost-minimization strategy, three easy-to-use theorems are developed to characterize the optimal solutions. Finally, the sensitivity analysis is executed to investigate the effects of the various parameters on ordering policies and the annual total relevant costs of the inventory system.

Key words: Economic order quantity (EOQ), permissible delay in payment, trade credit, two warehouses, supply chain system.

INTRODUCTION

The classical economic order quantity (EOQ) model makes the following assumptions:

(1) The retailer's capitals are unrestricted and the stocks ordered are fully paid upon receipt. However, from the viewpoint of practice, this may not be true. In recent years, marketing researchers and practitioners have recognized the phenomenon that the supplier offers a permissible delay to the retailer if the outstanding amount is paid within the permitted fixed settlement period, known as trade credit period. During the trade credit period, the retailer can accumulate revenues by selling items and earning interests.

Stokes (2005) indicates that trade credit represents one of the most flexible sources of short-term financing available to firms principally because it arises spontaneously with the firm's purchases. The decision to offer trade credit and the determination of the firm's terms of sale are important managerial considerations.

In a competitive market, the supplier offers different delay periods to encourage the retailer to order more quantities. Many articles related to the trade credit can be found in Sana and Chaudhuri (2008) and references of the review paper by Chang et al. (2008).

(2) The inventories are stored by a single warehouse with unlimited capacity. As we all know, the capacity of any warehouse is limited. In practice, there usually exist various factors that induce the decision-maker of the inventory system to order more items than can be held in his/her own warehouse. Therefore, for the decision maker, it is very practical to determine whether or not to rent other warehouses.

Many article related to the inventory policy under limited storage capacity can be found in Pakkala and

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Achary (1992), Berkherouf (1997), Goswami and Chaudhuri (1997), Bhunia and Maiti (1998), Chung and Huang (2004, 2006), Zhou and Yang (2005), Yang (2006), Hsieh et al. (2008), Rong et al. (2008), Chung et al. (2009), Lee and Hsu (2009) and their references.

Furthermore, Goyal (1985) established a single-item inventory model under permissible delay in payments. The trade credit policy discussed in Goyal (1985) assumed that the supplier would offer the trade credit but the retailer would not offer the trade credit to his/her customers. That is one level of trade credit.

Recently, Teng (2009) and Huang (2003) explore inventory models under two levels of trade credit policy when the supplier offers the retailer a permissible delay of *M* periods, and the retailer also provides its customers a permissible delay of *N* periods, respectively. That is two levels of trade credit. On the other hand, in practice, there are two payment methods to be adopted to explore the inventory problem under the trade credit. They can be described as follows:

Payment method 1: The retailer pays off all units sold and keeps the profits for other uses.
Payment method 2: The retailer pays off the amount owed to the supplier whenever the retailer has money obtained from sales.

Huang (2006) explored the inventory model with two levels of trade credit and limited storage capacity under payment method 1. Chung and Huang (2007) further generalized Huang (2006) to allow by considering the deteriorating items. Teng et al. (2007) studied the inventory model under two levels of trade credit from the viewpoint of Huang (2003) and payment method 2.

Recently, Teng et al. (2009) presented the different viewpoint of two levels of trade credit from that of Huang (2006) to explore the inventory model under limited storage capacity and payment method.

Incorporating concepts of Teng et al. (2007, 2009), we try to consider two levels of trade credit from the viewpoint of Teng (2009) and two warehouses together under payment method 2. Many related articles can be found in Taleizadeh et al. (2010a, b, 2011, 2012a, b, c). Finally, the sensitivity analysis is executed to study the effects of various parameters on ordering policies and annual total costs of the inventory system.

MODEL FORMULATION

The following notation and assumptions will be adopted in the whole paper.

Notation

D: the annual demand rate.
A: the ordering cost per order.
c: purchasing price per unit.
p: selling price per unit.
h: unit stock holding cost for the item in own warehouse (OW) per year excluding interest charges.
k: unit stock holding cost for the item in the rented warehouse (RW) per year excluding interest charges.
I_e: the annual interest earned on investment per $.
I_k: the annual interest charged per $ in stocks by the supplier.
M: the retailer's trade credit period offered by the supplier in years.
N: the customer's trade credit period offered by the retailer in years.
T: the replenishment cycle time in years, which is a decision variable.
W: the storage capacity of OW.
t_w: the rented warehouse time in years

$$=\begin{cases} \frac{DT-W}{D} & \text{if } DT>W \\ 0 & \text{if } DT\le W \end{cases}.$$

TRC(T): the annual total relevant cost, which is a function of T.
T:* the optimal cycle time of *TRC(T).*

Assumptions

1. Demand rate is known and constant.
2. Shortages are not allowed.
3. Time horizon is infinite.
4. Replenishments are instantaneous.
5. $k \ge h$ and $M \ge N$.
6. If the ordering quantity is larger than retailer's OW storage capacity *W*, the retailer will rent warehouse to store these exceeding items and the RW storage capacity is unlimited. When the demand occurs, it first replenished those exceeding items from the RW which stores these exceeding items.
7. The retailer deposits the sales revenue into an interest bearing account. If $M \ge T + N$ (that is, the permissible delay period is longer than the time at which the retailer receives the last payment from its customers), then the retailer receives all revenues and pays off the entire purchase cost at the end of the permissible delay *M*. Otherwise (if $M \le T + N$), the retailer pays the sum of all units sold by $M - N$ and all interest earned from *N* to *M* to the supplier. Furthermore, the retailer starts paying for the interest charges on the items sold after $M - N$ with rate I_k.

The model

The total annual relevant cost consists of the following elements. Three situations are taken into account:

I. $\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c} > M-N \ge \frac{W}{D}$

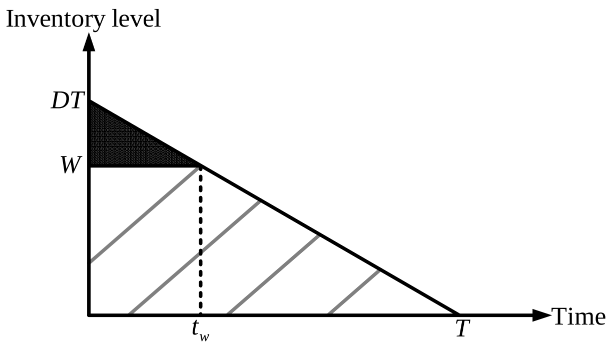

Figure 1. The inventory level and the stock-holding cost when $\frac{W}{D} < T$.

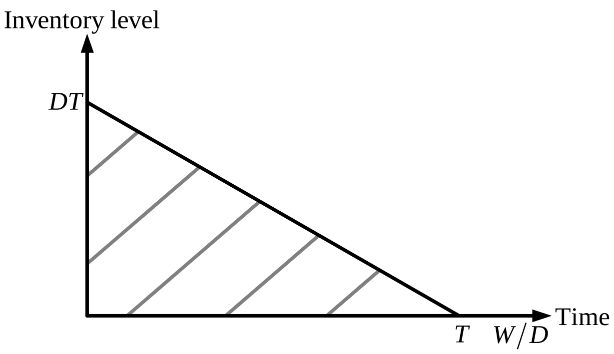

Figure 2. The inventory level and the stock-holding cost when $T \le \frac{W}{D}$.

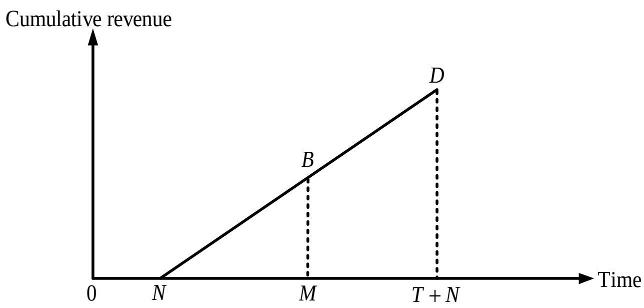

Figure 3. The total interest earned when $M \le T + N$.

II. $\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c} \ge \frac{W}{D} > M - N$

III. $\frac{W}{D} > \frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c} > M - N$

Case I: Suppose $\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c} > M - N \ge \frac{W}{D}$.

(1) Annual ordering cost $= \frac{A}{T}$.

(2) According to assumption (6), annual stock-holding cost (excluding interest charged) can be obtained as follows:

(i) $\frac{W}{D} \le T$, as shown in Figure 1. In this case, the order quantity is larger than retailer's OW storage capacity. Thus the retailer needs to rent the warehouse to store the exceeding items. Hence, the annual stock holding cost = annual stock holding cost of rented warehouse + annual stock holding cost of the storage capacity *W*

$$= \frac{kt_w(DT-W)}{2T} + \frac{h\left[Wt_w + \frac{W(T-t_w)}{2}\right]}{T} = \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT}$$

(ii) $T < \frac{W}{D}$, as shown in Figure 2. In this case, the order quantity is less than retailer's storage capacity. Thus the retailer will not need to rent warehouse to store any items. Hence

Annual stock-holding cost $= \frac{DTh}{2}$.

(3) According to assumption (7), interest earned per year can be obtained as
follows:

(i) $M - N \le T$. In this case, the retailer stares selling products at time 0, but getting the money at time *N*. Consequently, the retailer accumulates revenue in an account that earns I_e per dollar per year starting from *N* through *M*. Therefore, the interest earned per cycle is I_e multiplied by the area of the triangle *NMB* as shown in Figure 3. Hence;

The interest earned per year $= \frac{pI_eD(M-N)^2}{2T}$ (1)

(ii) $0 < T < M - N$. In this case, Figure 4 reveals that the annual interest earned $= \frac{pI_eDT^2}{2T} + \frac{pI_eDT(M-T-N)}{T}$

$$= pI_eD(M-N) - \frac{pI_eDT}{2}. \tag{2}$$

(4) According to assumption (7), cost of interest charged for the items kept in stock per year can be obtained as follows:

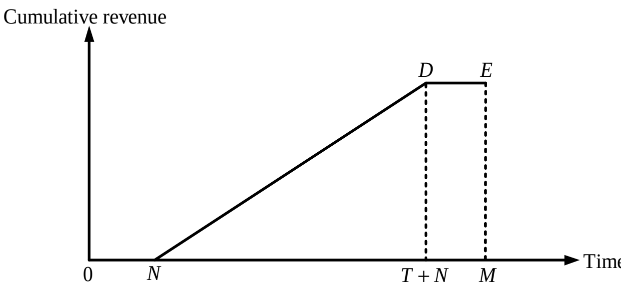

Figure 4. The total interest earned when $T+N<M$.

(i) $M-N\le T$. In this case, the retailer has $pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}$ in the account at time *M*. Since, the retailer buys *DT* units at time 0, the retailer owes the supplier *cDT* at time *M*. From the difference between the purchasing cost and the money in the account, we have the following two cases:

Case A: $pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}<cDT$. If the money in the account $pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}$ at time *M* is less than the purchasing cost *cDT*, then the retailer needs to finance the difference $L=cDT-\left[pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}\right]$ (at interest I_k) at time *M*, and pays the supplier in full in order to get the permissible delay. Thereafter, the retailer gradually reduces the amount of financed loan from constant sales and revenue received. Hence, we obtain the interest charged per year as:

$$\frac{I_kL\left[\dfrac{L}{pD}\right]}{2T}=\frac{I_k\left\{cDT-\left[pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}\right]\right\}^2}{2pDT}\quad \text{if } T>\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}, \tag{3}$$

Case B: $pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}\ge cDT$. If the money in the account $pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}$ at time *M* is greater than or equal to the purchasing cost *cDT*, then there is no interest charged.

(ii) $0<T<M-N$. In this case, the annual interest charged is 0. The annual total relevant cost for the retailer can be expressed as:

$TRC(T)$ = ordering cost + stock-holding cost + interest charged – interest earned.

Based on the foregoing arguments, the annual total relevant cost, $TRC(T)$, is given by:

$$TRC(T)=\begin{cases} TRC_1(T) \ \text{if } 0<T\le \dfrac{W}{D} & \text{a}\\ TRC_2(T) \ \text{if } \dfrac{W}{D}\le T\le M-N & \text{b}\\ TRC_3(T) \ \text{if } M-N\le T\le\left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c & \text{c}\\ TRC_4(T) \ \text{if } \left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c<T & \text{d}\end{cases} \tag{4}$$

Where;

$$TRC_1(T)=\frac{A}{T}+\frac{DTh}{2}+\frac{pI_eDT}{2}-pI_eD(M-N), \tag{5}$$

$$TRC_2(T)=\frac{A}{T}+\frac{(DT-W)^2}{2DT}k+\frac{W(2DT-W)}{2DT}h+\frac{pI_eDT}{2}-pI_eD(M-N), \tag{6}$$

$$TRC_3(T)=\frac{A}{T}+\frac{(DT-W)^2}{2DT}k+\frac{W(2DT-W)}{2DT}h-\frac{pI_eD(M-N)^2}{2T}, \tag{7}$$

$$\begin{aligned}TRC_4(T)=&\frac{A}{T}+\frac{(DT-W)^2}{2DT}k+\frac{W(2DT-W)}{2DT}h\\ &+\frac{I_k\left\{cDT-\left[pD(M-N)+\dfrac{pI_eD(M-N)^2}{2}\right]\right\}^2}{2pDT}-\frac{pI_eD(M-N)^2}{2T}.\end{aligned} \tag{8}$$

Since,

$$TRC_1\left(\frac{W}{D}\right)=TRC_2\left(\frac{W}{D}\right), \tag{9}$$

$$TRC_2(M-N)=TRC_3(M-N), \tag{10}$$

$$TRC_3\left(\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}\right)=TRC_4\left(\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}\right). \tag{11}$$

Equations 4a to d imply that *TRC(T)* is continuous on *T > 0*. The techniques of arguments of *TRC(T)* in Case (I) can be used to develop *TRC(T)* of Cases (II) and (III).

Case II: Suppose $\dfrac{p(M-N)}{c}+\dfrac{pI_e(M-N)^2}{2c}\ge\dfrac{W}{D}>M-N$. Under this case, *TRC(T)* can be expressed as follows:

$$TRC(T)=\begin{cases} TRC_1(T) \ \text{if } 0<T\le M-N & \text{a}\\ TRC_5(T) \ \text{if } M-N\le T\le \dfrac{W}{D} & \text{b}\\ TRC_3(T) \ \text{if } \dfrac{W}{D}\le T\le\left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c & \text{c}\\ TRC_4(T) \ \text{if } \left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c\le T & \text{d}\end{cases} \tag{12}$$

Where;

$$TRC_5(T)=\frac{A}{T}+\frac{DTh}{2}+\frac{pI_eD(M-N)^2}{2T}. \tag{13}$$

Since,

$$TRC_1(M-N)=TRC_5(M-N), \tag{14}$$

and

$$TRC_5\left(\frac{W}{D}\right)=TRC_3\left(\frac{W}{D}\right), \tag{15}$$

Combining equations (14), (15) and (11), then equations 12 (a to d) imply that *TRC(T)* is continuous on *T > 0*.

Case III: Suppose $\frac{W}{D}>\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}>M-N$. Under this case, *TRC(T)* can be expressed as follows:

$$TRC(T)=\begin{cases} TRC_1(T) \text{ if } 0<T\le M-N & \text{a} \\ TRC_5(T) \text{ if } M-N\le T\le\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c & \text{b} \\ TRC_6(T) \text{ if } \left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\le T\le\frac{W}{D} & \text{c} \\ TRC_4(T) \text{ if } \frac{W}{D}\le T & \text{d}\end{cases} \tag{16}$$

Where;

$$TRC_6(T)=\frac{A}{T}+\frac{DTh}{2}+\frac{I_k\left\{cDT-\left[pD(M-N)+\frac{pI_eD(M-N)^2}{2}\right]\right\}^2}{2pDT}+\frac{pI_eD(M-N)^2}{2T}. \tag{17}$$

Since

Since,

$$TRC_5\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)=TRC_6\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right), \tag{18}$$

and

$$TRC_6\left(\frac{W}{D}\right)=TRC_4\left(\frac{W}{D}\right), \tag{19}$$

Combining Equations (18), (19) and (14), then equations 16 (a to d) imply that *TRC(T)* is continuous on *T > 0*. For convenience, all TRC_i *(T) (i = 1 - 6)* and *TRC(T)* are defined on *T > 0*.

THE CONVEXITIES OF $TRC_i(T)$ $(i = 1 - 6)$

Equations (5) to (8), (13) and (17) yield the first derivatives $TRC_i(T)$ and the second derivatives *TRC(T)* with respect to *T* as follows:

$$TRC_1'(T)=-\frac{A}{T^2}+\frac{D(h+pI_e)}{2}, \tag{20}$$

$$TRC_1''(T)=\frac{2A}{T^3}>0, \tag{21}$$

$$TRC_2'(T)=-\frac{A}{T^2}+\frac{D^2T^2-W^2}{2DT}k+\frac{W^2}{2DT^2}h+\frac{pI_eD}{2}, \tag{22}$$

$$TRC_2''(T)=\frac{2A}{T^3}+\frac{W^2}{DT^3}(k-h)>0, \tag{23}$$

$$TRC_3'(T)=-\frac{A}{T^2}+\frac{D^2T^2-W^2}{2DT^2}k+\frac{W^2}{2DT^2}h+\frac{pI_eD(M-N)^2}{2T^2}, \tag{24}$$

$$TRC_3''(T)=\frac{2A}{T^3}+\frac{W^2}{2DT^3}(k-h)-\frac{pI_eD(M-N)^2}{T^3}, \tag{25}$$

$$TRC_4'(T)=-\frac{A}{T^2}+\frac{D^2T^2-W^2}{2DT^2}k+\frac{W^2}{2DT^2}h+I_k\left\{\frac{c^2D}{2p}-\frac{\left[pD(M-N)+\frac{pI_eD(M-N)^2}{2}\right]^2}{2pDT^2}\right\}^2+\frac{pI_eD(M-N)^2}{2T^2}, \tag{26}$$

$$TRC_4''(T)=\frac{2A+\frac{W^2}{D}(k-h)+\frac{I_k}{pD}\left[pD(M-N)+\frac{pI_eD(M-N)^2}{2}\right]^2}{T^3}-\frac{pI_eD(M-N)^2}{T^3}, \tag{27}$$

$$TRC_5'(T)=-\frac{A}{T^2}+\frac{Dh}{2}+\frac{pI_eD(M-N)^2}{2T^2}, \tag{28}$$

$$TRC_5''(T)=\frac{2A}{T^3}+\frac{pI_eD(M-N)^2}{T^3}=\frac{2A+pI_eD(M-N)^2}{T^3}, \tag{29}$$

$$TRC_6'(T)=-\frac{A}{T^2}+\frac{Dh}{2}+I_k\left\{\frac{c^2D}{2p}-\frac{\left[pD(M-N)+\frac{pI_eD(M-N)^2}{2}\right]^2}{2pDT^2}\right\}+\frac{pI_eD(M-N)^2}{2T^2}, \tag{30}$$

and

$$TRC_6''(T)=\frac{2A}{T^3}+I_k\left\{\frac{\left[pD(M-N)+\frac{pI_eD(M-N)^2}{2}\right]^2}{pDT^3}\right\}-\frac{pI_eD(M-N)^2}{T^3}. \tag{31}$$

Let,

$$G_3=2A+\frac{(k-h)W^2}{D}-pI_eD(M-N)^2, \tag{32}$$

$$G_4 = 2A + \frac{(k-h)W^2}{D} + \frac{I_k}{pD}\left[pD(M-N) + \frac{pI_eD(M-N)^2}{2}\right]^2 - pI_eD(M-N)^2, \qquad (33)$$

$$G_5 = 2A - pI_eD(M-N)^2, \text{ and} \qquad (34)$$

$$G_6 = 2A + \frac{I_k}{pD}\left[pD(M-N) + \frac{pI_eD(M-N)^2}{2}\right]^2 - pI_eD(M-N)^2. \qquad (35)$$

Hence, we have the following results:

Lemma 1:

(A) *$TRC_i(T)$* is convex on T > 0 for *i* = 1 and 2.
(B) *$TRC_i(T)$* is convex on T > 0 if $G_i > 0$. Furthermore, $TRC_i(T) > 0$ and *$TRC_i(T)$* is increasing on T > 0 if $G_i \leq 0$ for *i* = 3, 4, 5 and 6.

Letting

$$TRC_i'(T) = 0 \quad (i = 1-6) \qquad (36)$$

Solving equation (36), we obtain

$$T_1^* = \sqrt{\frac{2A}{D(h+pI_e)}} \qquad (37)$$

$$T_2^* = \sqrt{\frac{2A + \frac{W^2}{D}(k-h)}{D(k+pI_e)}} \qquad (38)$$

$$T_3^* = \sqrt{\frac{2A + (k-h)\frac{W^2}{D} - pI_eD(M-N)^2}{Dk}} \quad \text{if } G_3 > 0, \qquad (39)$$

$$T_4^* = \sqrt{\frac{2A + (k-h)\frac{W^2}{D} + \frac{I_k}{pD}\left[pD(M-N) + \frac{pI_eD(M-N)^2}{2}\right]^2 - pI_eD(M-N)^2}{D(k + \frac{c^2I_k}{p})}} \quad \text{if } G_4 > 0, \qquad (40)$$

$$T_5^* = \sqrt{\frac{2A - pI_eD(M-N)^2}{Dh}} \quad \text{if } G_5 > 0, \text{ and} \qquad (41)$$

$$T_6^* = \sqrt{\frac{2A + \frac{I_k}{pD}\left[pD(M-N) + \frac{pI_eD(M-N)^2}{2}\right]^2 - pI_eD(M-N)^2}{D\left(h + c^2\frac{I_k}{p}\right)}} \quad \text{if } G_6 > 0, \qquad (42)$$

as the respective solutions of equation (36). Also if T_i^* exists, then,

$$TRC_i'(T)\begin{cases} < 0 & \text{if } 0 < T < T_i^* \quad \text{a} \\ = 0 & \text{if } T = T_i^* \quad \text{b} \\ > 0 & \text{if } T > T_i^* \quad \text{c} \end{cases}$$

d (43)

Equations 43 (a to c) imply that $TRC(T)$ is decreasing on $(0, T_i^*]$ and increasing on $[T_i^*, \infty)$ for $i = 1-6$.

PROPERTIES OF Δ_{ij}

There are three cases to be discussed.

Case I: Suppose that $\dfrac{p(M-N) + \dfrac{pI_eD(M-N)^2}{2}}{c} > M - N \geq \dfrac{W}{D}$, Equations (20), (22), (24) and (26) yield:

$$TRC_1'\left(\frac{W}{D}\right) = TRC_2'\left(\frac{W}{D}\right) = \frac{\Delta_{12}}{2\left(\frac{W}{D}\right)^2}, \qquad (44)$$

$$TRC_2'(M-N) = TRC_3'(M-N) = \frac{\Delta_{23}}{2(M-N)^2}, \qquad (45)$$

$$TRC_3'\left(\left[p(M-N) + \frac{pI_eD(M-N)^2}{2}\right]\Big/c\right) = TRC_4'\left(\left[p(M-N) + \frac{pI_eD(M-N)^2}{2}\right]\Big/c\right)$$
$$= \frac{\Delta_{34}}{2\left(\left[p(M-N) + \frac{pI_eD(M-N)^2}{2}\right]\Big/c\right)^2}, \qquad (46)$$

Where;

$$\Delta_{12} = -2A + D(h + pI_e)\left(\frac{W}{D}\right)^2, \qquad (47)$$

$$\Delta_{23} = -2A + D(k + pI_e)(M-N)^2 + \frac{W^2}{D}(h-k) \qquad (48)$$

and

$$\Delta_{34} = -2A + D\left\{\left[\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c}\right]^2 k + pI_e(M-N)^2\right\} + \frac{W^2}{D}(h-k). \qquad (49)$$

Equations (47) to (49) imply

$$\Delta_{34} > \Delta_{23} > \Delta_{12}. \qquad (50)$$

Case II: Suppose that $\dfrac{p(M-N)+\dfrac{pI_eD(M-N)^2}{2}}{c} \geq \dfrac{W}{D} > M-N$, Equations (20), (28) and (24) yield:

$$TRC_1'(M-N) = TRC_5'(M-N) = \frac{\Delta_{15}}{2(M-N)^2}, \tag{51}$$

$$TRC_5'\left(\frac{W}{D}\right) = TRC_3'\left(\frac{W}{D}\right) = \frac{\Delta_{53}}{2\left(\dfrac{W}{D}\right)^2}, \tag{52}$$

Where;

$$\Delta_{15} = -2A + D(h+pI_e)(M-N)^2,\ and \tag{53}$$

$$\Delta_{53} = -2A + \frac{W^2}{D}h + pI_eD(M-N)^2. \tag{54}$$

Equations (53), (54) and (49) imply

$$\Delta_{34} > \Delta_{53} > \Delta_{15}. \tag{55}$$

Case III: Suppose that $\dfrac{W}{D} > \dfrac{p(M-N)+\dfrac{pI_eD(M-N)^2}{2}}{c} \geq M-N$, Equations (28), (30) and (26) yield:

$$TRC_5'\left(\frac{p(M-N)}{c}+\frac{pI_eD(M-N)^2}{2c}\right) = TRC_6'\left(\frac{p(M-N)}{c}+\frac{pI_eD(M-N)^2}{2c}\right) = \frac{\Delta_{56}}{2\left(\dfrac{p(M-N)}{c}+\dfrac{pI_eD(M-N)^2}{2c}\right)^2}, \tag{56}$$

$$TRC_6'\left(\frac{W}{D}\right) = TRC_4'\left(\frac{W}{D}\right) = \frac{\Delta_{64}}{2\left(\dfrac{W}{D}\right)^2}, \tag{57}$$

Where;

$$\Delta_{56} = -2A + pI_e(M-N)^2 + Dh\left[\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}\right]^2,\ and \tag{58}$$

$$\Delta_{64} = -2A + pI_e(M-N)^2 + D\left(h+\frac{c^2I_k}{p}\right)\left(\frac{W}{D}\right)^2 - \frac{I_k}{pD}\left[pD(M-N)+\frac{pI_e(M-N)^2}{2}\right]^2. \tag{59}$$

Equations (53), (58) and (60) imply

$$\Delta_{64} > \Delta_{56} > \Delta_{15}. \tag{60}$$

Lemma 2:

(A) If $\Delta_{23} \leq 0$, then

(a1) $G_3 > 0$,

(a2) T_3^* exists,

(a3) $TRC_3(T)$ is convex on $T > 0$.

(B) If $\Delta_{34} \leq 0$, then

(b1) $G_4 > 0$,

(b2) T_4^* exists,

(b3) $TRC_4(T)$ is convex on $T > 0$.

(C) If $\Delta_{15} \leq 0$, then

(c1) $G_5 > 0$,

(c2) T_5^* exists,

(c3) $TRC_5(T)$ is convex on $T > 0$.

(D) If $\Delta_{53} \leq 0$, then

(d1) $G_3 > 0$,

(d2) T_3^* exists,

(d3) $TRC_3(T)$ is convex on $T > 0$.

(E) If $\Delta_{56} \leq 0$, then

(e1) $G_6 > 0$,

(e2) T_6^* exists,

(e3) $TRC_6(T)$ is convex on $T > 0$.

(F) If $\Delta_{64} \leq 0$, then

(f1) $G_4 > 0$,

(f2) T_4^* exists,

(f3) $TRC_4(T)$ is convex on $T > 0$.

Proof: (A) If $\Delta_{23} \leq 0$, then;

$$2A \geq D(k+pI_e)(M-N)^2 + \frac{W^2}{D}(h-k). \tag{61}$$

Both Equations (32) and (61) imply

$$G_3 \geq Dk(M-N)^2 > 0. \tag{62}$$

So, Equations (25) and (39) demonstrate that (a1) to (a3) hold.

(B) If $\Delta_{34} \le 0$, then;

$$2A \ge D\left\{\left[\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c}\right]^2 k + pI_e(M-N)^2\right\} + \frac{W^2}{D}(h-k) \quad (63)$$

So, Equations (33) and (63) imply

$$G_3 \ge D\left[\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c}\right]^2 k > 0 . \quad (64)$$

Equations (27) and (40) demonstrate that (b1) to (b3) hold.

(C) If $\Delta_{15} \le 0$, then;

$$2A \ge D(h + pI_e)(M-N)^2 . \quad (65)$$

Equations (34) and (65) imply

$$G_3 \ge Dh(M-N)^2 > 0 . \quad (66)$$

So, Equations (29) and (41) demonstrate that (c1) to (c3) hold.

(D) If $\Delta_{53} \le 0$, then;

$$2A \ge \frac{W^2}{D}h + pI_e D(M-N)^2 . \quad (67)$$

Equations (32) and (67) imply

$$G_3 \ge \frac{W^2}{D}k > 0 . \quad (68)$$

So, Equations (25) and (39) demonstrate that (d1) to (d3) hold.

(E) If $\Delta_{56} \le 0$, then;

$$2A \ge pI_e D(M-N)^2 + Dh\left[\frac{p(M-N)}{c} + \frac{pI_e(M-N)^2}{2c}\right]^2 . \quad (69)$$

Equations (35) and (69) imply

$$G_6 \ge \frac{I_k}{pD}\left[pD(M-N)^2 + \frac{pI_e D(M-N)^2}{2}\right]^2 > 0 \quad (70)$$

So, Equations (31) and (42) demonstrate that (e1) to (e3) hold.

(F) If $\Delta_{64} \le 0$, then;

$$2A \ge pI_e D(M-N)^2 + D\left(h + \frac{c^2 I_k}{p}\right)\left(\frac{W}{D}\right)^2 - \frac{I_k}{pD}\left[pD(M-N) + \frac{pI_e D(M-N)^2}{2}\right]^2 . \quad (71)$$

Equations (33) and (71) imply

$$G_4 \ge D\left(h + \frac{c^2 I_k}{p}\right)\left(\frac{W}{D}\right)^2 + \frac{(k-h)}{D}W^2 > 0 . \quad (72)$$

So, Equations (27) and (40) demonstrate that (f1) to (f3) hold. Incorporating the foregoing arguments, we have completed the proof of Lemma 2.

THEOREMS FOR THE OPTIMAL CYCLE TIME T^* OF $TRC(T)$

Theorem 1: Suppose $\frac{W}{D} \le M - N$. Hence,

(A) If $\Delta_{12} \ge 0$, then $T^* = T_1^*$.

(B) If $\Delta_{12} \le 0 < \Delta_{23}$, then $T^* = T_2^*$.

(C) If $\Delta_{23} \le 0 < \Delta_{34}$, then $T^* = T_3^*$.

(D) If $\Delta_{34} \le 0$, then $T^* = T_4^*$.

Proof: (A) If $\Delta_{12} > 0$, then $\Delta_{34} > \Delta_{23} > \Delta_{12} > 0$. We have

$$TRC_1'\left(\frac{W}{D}\right) = TRC_2'\left(\frac{W}{D}\right) \ge 0 ,$$

$$TRC_2'(M-N) = TRC_3'(M-N) > 0$$ and

$$TRC_3'\left(\left[p(M-N) + \frac{pI_e(M-N)^2}{2}\right]\Big/c\right) = TRC_4'\left(\left[p(M-N) + \frac{pI_e(M-N)^2}{2}\right]\Big/c\right) > 0.$$

Then, Lemma 1(A, B) and Equations 43 (a to c) imply:

(a1) $TRC_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $\left[T_1^*, \frac{W}{D}\right]$.

(a2) $TRC_2(T)$ is increasing on $\left[\frac{W}{D}, M-N\right]$.

(a3) $TRC_3(T)$ is increasing on $\left[M-N, \left[p(M-N) + \frac{pI_e(M-N)^2}{2}\right]\Big/c\right]$.

(a4) $TRC_5(T)$ is increasing on $\left[[p(M-N)+\frac{pI_e(M-N)^2}{2}]/c,\ \infty\right)$.

Since $TRC(T)$ is continuous on $T>0$, combining (a1)-(a4), we conclude that $TRC(T)$ is decreasing on $(0,T_1^*]$ and increasing on $[T_1^*,\infty)$.

Consequently, $T^*=T_1^*$.

(B) If $\Delta_{12}\le 0<\Delta_{23}$, then $\Delta_{34}>\Delta_{23}>0>\Delta_{12}$. We have $TRC_1'\left(\frac{W}{D}\right)=TRC_2'\left(\frac{W}{D}\right)\le 0$,

$TRC_2'(M-N)=TRC_3'(M-N)>0$ and

$TRC_3'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)=TRC_4'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)>0.$

Then, Lemma 1(A, B) and equations 43(a-c) imply:

(b1) $TRC_1(T)$ is decreasing on $\left(0,\frac{W}{D}\right]$.

(b2) $TRC_2(T)$ is decreasing on $\left[\frac{W}{D},T_2^*\right]$, and increasing on $[T_2^*,M-N]$.

(b3) $TRC_4(T)$ is increasing on $\left[M-N,\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right]$.

(b4) $TRC_5(T)$ is increasing on $\left[\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c,\infty\right)$.

Since $TRC(T)$ is continuous on $T>0$, combining (b1)-(b4), we conclude that $TRC(T)$ is decreasing on $(0,T_2^*]$ and increasing on $[T_2^*,\infty)$.

Consequently, $T^*=T_2^*$.

(C) If $\Delta_{23}\le 0<\Delta_{34}$, then $\Delta_{34}>0\ge\Delta_{23}>\Delta_{12}$, we have $TRC_1'\left(\frac{W}{D}\right)=TRC_2'\left(\frac{W}{D}\right)<0$,

$TRC_2'(M-N)=TRC_3'(M-N)\le 0$ and

$TRC_3'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)=TRC_4'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)>0.$

Then, Lemmas 1(A,B), 2(A) and equations 43(a-c) imply:

(c1) $TRC_1(T)$ is decreasing on $\left(0,\frac{W}{D}\right]$.

(c2) $TRC_2(T)$ is decreasing on $\left[\frac{W}{D},M-N\right]$.

(c3) $TRC_3(T)$ is decreasing on $[M-N,T_3^*]$ and increasing on $\left[T_3^*,\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right]$.

(c4) $TRC_4(T)$ is increasing on $\left[\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c,\infty\right)$.

Since $TRC(T)$ is continuous on $T>0$, combining (c1)-(c4), we conclude that $TRC(T)$ is decreasing on $(0,T_3^*]$ and increasing on $[T_3^*,\infty)$. Consequently, $T^*=T_3^*$.

(D) If $\Delta_{34}\le 0$, then $0\ge\Delta_{34}>\Delta_{23}>\Delta_{12}$. We have $TRC_1'\left(\frac{W}{D}\right)=TRC_2'\left(\frac{W}{D}\right)<0$,

$TRC_2'(M-N)=TRC_3'(M-N)<0$ and

$TRC_3'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)=TRC_4'\left(\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right)>0.$

Then, Lemmas 1(A, B), 2(B) and equations 43(a-c) imply:

(d1) $TRC_1(T)$ is decreasing on $\left(0,\frac{W}{D}\right]$.

(d2) $TRC_2(T)$ is decreasing on $\left[\frac{W}{D},M-N\right]$.

(d3) $TRC_3(T)$ is decreasing on $\left[M-N,\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c\right]$.

(d4) $TRC_5(T)$ is decreasing on $\left[\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c,T_4^*\right]$ and increasing on $[T_4^*,\infty)$.

Since $TRC(T)$ is continuous on $T>0$, combining (d1)-(d4), we conclude that $TRC(T)$ is decreasing on $(0,T_4^*]$ and increasing on $[T_4^*,\infty)$. Consequently, $T^*=T_4^*$.

Incorporating the previous argument, this completes the proof of Theorem 1.

With Lemmas 1 (A and B), 2 (C, D and F) and Equations

43 (a to c), the techniques of arguments in Theorem 1 can be used to demonstrate the following results:

Theorem 2: Suppose $M-N<\frac{W}{D}\leq\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c$. Hence,

(A) If $\Delta_{15}>0$, then $T^*=T_1^*$.

(B) If $\Delta_{15}\leq 0<\Delta_{53}$, then $T^*=T_5^*$.

(C) If $\Delta_{53}\leq 0<\Delta_{34}$, then $T^*=T_3^*$.

(D) If $\Delta_{34}\leq 0$, then $T^*=T_4^*$.

With Lemmas 1 (A and B), 2 (C, E and F) and Equations 43 (a to c), the techniques of arguments in Theorem 1 can be used to demonstrate the following results:

Theorem 3: Suppose $M-N<\left[p(M-N)+\frac{pI_e(M-N)^2}{2}\right]\Big/c<\frac{W}{D}$. Hence,

(A) If $\Delta_{15}>0$, then $T^*=T_1^*$.

(B) If $\Delta_{15}\leq 0<\Delta_{56}$, then $T^*=T_5^*$.

(C) If $\Delta_{56}\leq 0<\Delta_{64}$, then $T^*=T_6^*$.

(D) If $\Delta_{64}\leq 0$, then $T^*=T_4^*$.

THE SENSITIVITY ANALYSIS

Given $A=$\$45/order, $c=$ \$0.5/unit, $p=$\$1.0/unit, $h=$ \$0.5/unit/year, $I_k=$\$0.06/\$/year, $I_e=$ \$0.03/\$/year, $M=$90 days $=$90/365 years and $N=$30 days $=$ 30/365 years. There are three cases:

Case I. $\left[\frac{W}{D}\leq M-N<\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}\right]$: D = 2000 units and W = 100 units.

Case II. $\left[M-N<\frac{W}{D}\leq\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}\right]$: $D=$ 1300 units/year and $W=$300 units,

Case III. $\left[M-N<\frac{p(M-N)}{c}+\frac{pI_e(M-N)^2}{2c}<\frac{W}{D}\right]$: $D=$1000 units/year and $W=$600 units, to be executed the sensitivity analyses. Tables 1 to 3 can be obtained for all cases. Based on the computational results as shown in Tables 1 to 3, we can get the following phenomena:

(1) About Tables 1 to 3 (1): A higher value of the interest rate earned I_e causes lower values of T^* and $TRC(T^*)$. So, the retailer will order more frequently and obtain more interest to reduce the annual total relevant cost when he/she has the larger value of the interest rate.

(2) About Tables 1 to 3 (2): A higher value of the customer's trade credit period N causes higher values of $TRC(T^*)$. So, if the retailer gives the larger trade credit to his/her customers, it will increase the annual total relevant cost to meet the customer's demand.

(3) About Tables 1 to 3 (3): For Tables 1 to 2, a higher value of the interest rate charged I_k does not impact on the values of T^* and $TRC(T^*)$. However, for Table 3, a higher value of the interest rate charged I_k causes lower values of T^* and higher values of $TRC(T^*)$. So, for Cases (I) and (II), the larger value of interest rate charged I_k does not influence the order quantity and the annual total relevant cost. However, for Case (III), the larger value of the interest rate charged will make the retailer to order less quantity and increase the annual total relevant cost.

(4) About Tables 1 to 3 (4): For Table 1, a higher value of the unit cost c does not impact on the values of T^* and $TRC(T^*)$. However, for Tables 2 to 3, a higher value of c causes lower values of T^* and higher values of $TRC(T^*)$. So, for Case (I), the larger value of the purchasing cost will do not influence the order quantity and the annual total relevant cost of the retailer. However, for Cases (II)-(III), the larger value of the purchasing cost will make the retailer to order more frequently and increase the annual total relevant cost.

(5) About Tables 1 to 3 (5): A higher value of the holding cost for own warehouse h causes lower values of T^* and higher values of $TRC(T^*)$. So, the larger value of holding cost for down warehouse will make the retailer to order less quantity and increase the total annual cost.

(6) About Tables 1-3 (6): For Tables 1 to 2, a higher value of the storage capacity W causes higher values of T^* and lower values of $TRC(T^*)$. However, for Table 3, the value of W does not impact on the values of T^* and

Table 1. The sensitivity analysis for Case 1.

S/N	Parameter values	T*	TRC(T*)	S/N	Parameter values	T*	TRC(T*)
	I_e				k		
1	0.01	$T_3^* = 0.2144$	378.8579	7	0.8	$T_3^* = 0.2370$	349.2175
	0.02	$T_3^* = 0.2138$	377.5959		1.0	$T_3^* = 0.2132$	376.3301
	0.03	$T_3^* = 0.2132$	376.3301		1.5	$T_3^* = 0.1764$	429.2788
	0.04	$T_3^* = 0.2125$	375.0606		3.0	$T_2^* = 0.1301$	528.2677
	0.05	$T_3^* = 0.2119$	373.7872		5.0	$T_2^* = 0.1057$	603.9744
	N				D		
2	10	$T_2^* = 0.2119$	373.3696	8	1000	$T_3^* = 0.3069$	256.9028
	20	$T_3^* = 0.2125$	374.9546		1500	$T_3^* = 0.2478$	321.7203
	30	$T_3^* = 0.2132$	376.3301		2000	$T_3^* = 0.2132$	376.3301
	40	$T_3^* = 0.2137$	377.4906		2500	$T_3^* = 0.1897$	424.2714
	50	$T_3^* = 0.2142$	378.4377		3000	$T_3^* = 0.1725$	467.4013
	I_k				M		
3	0.02	$T_3^* = 0.2132$	376.3301	9	60	$T_4^* = 0.2139$	379.3456
	0.04	$T_3^* = 0.2132$	376.3301		90	$T_3^* = 0.2132$	376.3301
	0.06	$T_3^* = 0.2132$	376.3301		120	$T_2^* = 0.2119$	371.7258
	0.08	$T_3^* = 0.2132$	376.3301		150	$T_2^* = 0.2119$	366.7943
	0.10	$T_3^* = 0.2132$	376.3301		180	$T_2^* = 0.2119$	361.8628
	c				A		
4	0.3	$T_3^* = 0.2132$	376.3301	10	25	$T_2^* = 0.1596$	268.9987
	0.4	$T_3^* = 0.2132$	376.3301		35	$T_3^* = 0.1883$	326.5068
	0.5	$T_3^* = 0.2132$	376.3301		45	$T_3^* = 0.2132$	376.3301
	0.6	$T_3^* = 0.2132$	376.3301		55	$T_3^* = 0.2355$	420.9112
	0.7	$T_3^* = 0.2132$	376.3301		65	$T_3^* = 0.2558$	461.6223
	h				p		
5	0.3	$T_3^* = 0.2143$	358.6693	11	0.75	$T_3^* = 0.2136$	377.2798
	0.4	$T_3^* = 0.2138$	367.5013		1.00	$T_3^* = 0.2132$	376.3301
	0.5	$T_3^* = 0.2132$	376.3301		2.00	$T_3^* = 0.2113$	372.5100
	0.6	$T_3^* = 0.2126$	385.1557		5.00	$T_3^* = 0.2054$	360.8367
	0.7	$T_3^* = 0.2120$	393.9780		10.00	$T_3^* = 0.1953$	340.6068
	W						
6	25	$T_3^* = 0.2104$	408.2967				

	50	$T_3^* = 0.2110$	396.9092

Table 1. Contd.

	100	$T_3^* = 0.2132$	376.3301
	150	$T_3^* = 0.2168$	358.5982
	200	$T_3^* = 0.2218$	343.5734

Table 2. The sensitivity analysis for Case 2.

S/N	Parameter values	*T**	*TRC(T*)*	S/N	Parameter values	*T**	*TRC(T*)*
	I_e				k		
	0.01	$T_3^* = 0.3092$	251.9245		0.8	$T_3^* = 0.3248$	247.7928
	0.02	$T_3^* = 0.3087$	251.3560		1.0	$T_3^* = 0.3083$	250.7867
1	0.03	$T_3^* = 0.3083$	250.7867	7	1.5	$T_3^* = 0.2848$	255.3782
	0.04	$T_3^* = 0.3079$	250.2166		3.0	$T_3^* = 0.2592$	260.8857
	0.05	$T_3^* = 0.3074$	249.6456		5.0	$T_3^* = 0.2482$	263.4280
	N				D		
	10	$T_3^* = 0.3073$	249.4552		1000	$T_4^* = 0.3658$	216.3459
	20	$T_3^* = 0.3078$	250.1690		1200	$T_3^* = 0.3247$	239.6571
2	30	$T_3^* = 0.3083$	250.7867	8	1300	$T_3^* = 0.3083$	250.7867
	40	$T_4^* = 0.3082$	251.3450		1500	$T_3^* = 0.2814$	272.1090
	50	$T_4^* = 0.3080$	251.6530		1800	$T_3^* = 0.2512$	302.0768
	I_k				M		
	0.02	$T_3^* = 0.3083$	250.7867		75	$T_4^* = 0.3080$	251.6530
	0.04	$T_3^* = 0.3083$	250.7867		80	$T_4^* = 0.3082$	251.3450
3	0.06	$T_3^* = 0.3083$	250.7867	9	90	$T_3^* = 0.3083$	250.7867
	0.08	$T_3^* = 0.3083$	250.7867		100	$T_3^* = 0.3078$	250.1690
	0.10	$T_3^* = 0.3083$	250.7867		110	$T_3^* = 0.3073$	249.4552
	c				A		
	0.3	$T_3^* = 0.3083$	250.7867		25	$T_3^* = 0.2535$	179.5906
	0.4	$T_3^* = 0.3083$	250.7867		35	$T_3^* = 0.2822$	216.9196
4	0.5	$T_3^* = 0.3083$	250.7867	10	45	$T_3^* = 0.3083$	250.7867
	0.6	$T_4^* = 0.3076$	250.8373		55	$T_4^* = 0.3323$	282.0071
	0.7	$T_4^* = 0.3065$	251.1086		65	$T_4^* = 0.3543$	311.1353
5	h			11	p		
	0.3	$T_3^* = 0.3251$	212.6464		0.75	$T_4^* = 0.3075$	251.3703

	0.4	$T_3^* = 0.3168$	231.8616		1.00	$T_3^* = 0.3083$	250.7867

Table 2. Contd.

	0.5	$T_3^* = 0.3083$	250.7867		2.00	$T_3^* = 0.3070$	249.0739
	0.6	$T_3^* = 0.2995$	269.3970		5.00	$T_3^* = 0.3030$	243.8908
	0.7	$T_3^* = 0.2905$	287.6639		10.00	$T_3^* = 0.2962$	235.0972
	W						
	220	$T_3^* = 0.2876$	263.9385				
	250	$T_3^* = 0.2948$	258.2492				
6	300	$T_3^* = 0.3083$	250.7867				
	350	$T_3^* = 0.3235$	245.5710				
	400	$T_4^* = 0.3401$	242.3040				

Table 3. The sensitivity analysis for Case 3.

S/N	Parameter values	T*	TRC(T*)	S/N	Parameter values	T*	TRC(T*)
	I_e				k		
	0.01	$T_6^* = 0.4174$	211.9768		0.8	$T_6^* = 0.4162$	211.3326
	0.02	$T_6^* = 0.4168$	211.6550		1.0	$T_6^* = 0.4162$	211.3326
1	0.03	$T_6^* = 0.4162$	211.3326	7	1.5	$T_6^* = 0.4162$	211.3326
	0.04	$T_6^* = 0.4155$	211.0099		3.0	$T_6^* = 0.4162$	211.3326
	0.05	$T_6^* = 0.4149$	210.6866		5.0	$T_6^* = 0.4162$	211.3326
	N				D		
	10	$T_5^* = 0.4209$	210.4267		500	$T_6^* = 0.5899$	150.1089
	20	$T_6^* = 0.4155$	210.8678		700	$T_6^* = 0.4981$	177.2406
2	30	$T_6^* = 0.4162$	211.3326	8	1000	$T_6^* = 0.4162$	211.3326
	40	$T_6^* = 0.4167$	211.8545		1300	$T_6^* = 0.3645$	240.5064
	50	$T_6^* = 0.4172$	212.4325		1500	$T_6^* = 0.3390$	258.0777
	I_k				M		
	0.02	$T_6^* = 0.4203$	211.2260		60	$T_6^* = 0.4176$	213.0657
	0.04	$T_6^* = 0.4182$	211.2787		75	$T_6^* = 0.4170$	212.1366
3	0.06	$T_6^* = 0.4162$	211.3326	9	90	$T_6^* = 0.4162$	211.3326
	0.08	$T_6^* = 0.4141$	211.3878		100	$T_6^* = 0.4155$	210.8678
	0.10	$T_6^* = 0.4122$	211.4442		130	$T_5^* = 0.4189$	209.4614
	c				A		
4	0.3	$T_5^* = 0.4223$	211.1745	10	25	$T_5^* = 0.3137$	156.8269

	0.4	$T_6^* = 0.4184$	211.1845		35	$T_6^* = 0.3665$	186.0447

Table 3. Contd.

	0.5	$T_6^* = 0.4162$	211.3326		45	$T_6^* = 0.4162$	211.3326
	0.6	$T_6^* = 0.4105$	212.3580		55	$T_6^* = 0.4605$	233.9595
	0.7	$T_6^* = 0.4032$	214.3215		65	$T_6^* = 0.5009$	254.6219
	h				p		
	0.3	$T_6^* = 0.5321$	164.2019		0.25	$T_6^* = 0.4004$	219.8115
	0.4	$T_6^* = 0.4636$	189.2028		0.50	$T_6^* = 0.4112$	213.9580
5	0.5	$T_6^* = 0.4162$	211.3326	11	1.00	$T_6^* = 0.4162$	211.3326
	0.6	$T_6^* = 0.3808$	231.3994		1.25	$T_6^* = 0.4169$	210.9496
	0.7	$T_6^* = 0.3532$	249.8909		1.50	$T_5^* = 0.4214$	210.6941
	W						
	500	$T_6^* = 0.4162$	211.3326				
	550	$T_6^* = 0.4162$	211.3326				
6	600	$T_6^* = 0.4162$	211.3326				
	650	$T_6^* = 0.4162$	211.3326				
	700	$T_6^* = 0.4162$	211.3326				

$TRC(T^*)$. So, for Cases (I)-(II), the larger storage capacity will make the retailer to order more quantity and reduce the total annual cost. However, for Case (III), the larger storage capacity will do not influence the order quantity and total annual cost of the retailer.

(7) About Tables 1 to 3 (7): For Tables 1 to 2, a higher value of the holding cost for the rented warehouse k causes lower values of T^* and higher values of $TRC(T^*)$.

However, for Table 3, the value of k does not impact on the values of T^* and $TRC(T^*)$. So, for Cases (I)-(II), the larger value of the holding cost for rented warehouse will make the retailer to order more frequently ant increase the total annual cost. However, for Case (III), the larger value of the holding cost for rented warehouse will do not influence the order quantity and total annual cost of the retailer.

(8) About Tables 1 to 3 (8): A higher value of the demand rate D causes lower values of T^* and higher values of $TRC(T^*)$. So, the retailer will order more frequently and increase the total annual cost when he/she has larger demand rate.

(9) About Tables 1 to 3 (9): A higher value of the retailer's trade credit M offered by the supplier causes lower values of $TRC(T^*)$. So, if the supplier gives the larger value M will make the retailer to obtain more interest to reduce the annual total relevant cost.

(10) About Tables 1 to 3 (10): A higher value of the ordering cost A causes higher values of T^* and $TRC(T^*)$.

So, the retailer will order more quantity and increase the total annual cost when he/she has the larger value of the ordering cost.

(11) About Tables 1 to 3 (11): A higher value of the selling price causes lower values of $TRC(T^*)$. So, if the retailer has larger selling price will obtain more interest to reduce the annual total relevant cost.

THE SPECIAL CASE

When the retailer's OW storage capacity W is infinity, only Case III holds. Then, Equations 16 (a to d) will be simplified into the following equations.

$$TRC(T)=\begin{cases} TRC_1(T) & \text{if } 0<T\le M-N, & \text{a} \\ TRC_5(T) & \text{if } M-N\le T\le\left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c, & \text{b} \\ TRC_6(T) & \text{if } \left[p(M-N)+\dfrac{pI_e(M-N)^2}{2}\right]\Big/c\le T. & \text{c} \end{cases} \quad (73)$$

Huang (2003) studied the inventory model under two levels of trade credit for payment method 1. However, Equations 73 (a to c) develop the inventory model under two levels of trade credit for payment method 2.

Conclusion

Notation and assumptions in Teng et al. (2007) present two payment methods described in the introduction in this paper. Teng et al. (2009) explore the inventory model with two levels of trade credit and limited storage capacity under payment method 1. Adopting the different viewpoint from Teng et al. (2009), we discuss the inventory model with two levels of trade credit and limited storage capacity under payment method 2. The arguments of the total annual relevant cost $TRC(T)$ should be divided into three situations (I) to (III) described in the model.

Three theorems are provided to determine the optimal cycle time T^*. When the optimal order quantity $Q^*=DT^*$ is more than the retailer's OW storage capacity W, the retailer should rent a warehouse to store those exceeding items. Otherwise, the retailer does not need to rent a warehouse. Furthermore, when the retailer's OW storage capacity W is infinity, only Case (III) holds. So, this paper will modify the inventory model under two levels of trade credit of Huang (2003) for payment method 2. Finally, the sensitivity analysis is executed to investigate the effects of various parameters on the ordering policies and total annual cost of the inventory system.

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