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Econometric models used to optimise multiservicing of machine-tools

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The application of multi-service implies the performance of thorough analyses on technical, organisational and economic aspects. First, an optimal use of machine processing capacities and an efficient use of worker's work time must be ensured. To this purpose, the ratio between the time of operation of machines, without supervision and the manual time required for the preparation and supervision of the processing operation must be determined. Depending on this ratio, a decision on the implementation of the multi-service system shall be taken.

Key words: Multiservicing, Markov chain, input, production function, Takacs-Runnenburg model, mechanisation coefficient, loading coefficient.

INTRODUCTION

The multi-service system has a wide range of application in various processing processes, with determinist or fortuitous characteristics. Possibilities of application under efficient conditions are determined by the complexity and features of technological processes and by the constraint factors of the worker(s).

In machines with a regular cycle, multi-serving may be organised more easily, as work parameters, the ratio between the worker and operated machines, and also the constraints the workers are subject to may be established exactly (Chase et al., 2004).

In the case of transforming (processing) places where service and supervision have different durations, with fortuitous times, the issue of optimisation implies more complex studies and calculations.

LITERATURE REVIEW

From a statistical point of view, it is clearly seen that the operation of several machines is a stochastic process where state changes related by probability laws succeed at accidental or determinist intervals.

As it is known, the theory of "Markov chains" considers that the result of any attempt depends "directly" only on the result of the previous attempt (Li, 1995).

The question arises whether this theory may also be applied in the case of operation of several machines or multi-positional workstations (a very frequent name in specialised literature). If a system has n states, we mark with πi (i = 1, 2,…, m) the probability that this system is in the "i" state. This probability is named state probability.

Modrak and Pandian (2010), who drew up the Takacs model, try to show how the issue regarding the general distribution of service times may be solved, at least theoretically.

With a view to solving the issue, the distribution function of the machine operation time is considered:

$P_0(t) = e^{-\lambda \times t}$. The service time complies with a

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general distribution $P(t)=B(t)$, with the average value given by the formula $m=\int_0^\infty dB(t)$, and the value of the deviation $\sigma^2=\int_0^\infty (t-m)^2 \times dB(t)$, considered as finite.

In his Runnenberg (Modrak and Pandian, 2010), the creator of the Runnenberg model, approaches the issue in other terms. Thus, unlike Markov and Takacs, who start from the service of work stations according to the principle of first come, first served, the Runnenburg model considers the supervision and service of workstations according to the principle of the operator's continuous movement. This service method is especially found in textile industry, but also in other industries where the work cycle is performed in a technological line, with quite a large extension. In most cases, in car constructing industry, situations are found when means of processing have a regular cycle (Pearl, 1988).

In order to optimise multi-service in the case of processing on tools, in the multi-serving systems, several methods which may be used in the determination of work parameters between the man and the machine have been studied and discussed, for purposes of providing a higher efficiency.

The method of the mechanisation coefficient is used for solving concrete cases, charts and tables.

ANALYSIS OF MODELS USED

Model based on applying the "Markov chain" theory

Considering the conclusions drawn from the Markov chain theory, the probability that the system should pass from the i state to the j state is Pij, named transition probability. In the case of serving multiple machines, the state of the "worker-machines" system is given by the number of machines operating, idle. If the system has "n" machines, a system may undergo n+1 states, from all machines work to all machines are idle, the probability that the system is in state x is the "state probability" πr(x = 0, 1, 2, …, n). The transition probabilities of the system may be defined as follows:

Px, x – the probability that the state x (x machines operate) is maintained within a given time interval;
Px, x+1 – the probability that a machine should be started in that same interval;
Px, x-1 – the probability that a machine should be stopped in that same interval.

The following additional requirements must be met, so as to analyse the system: the time interval must be chosen so that a single start or stop may appear; the probability law of the process imposes a poissonian nature thereof; the serving mechanism should obey an exponential distribution law (Santana, 2003). If all these requirements are met, the process of serving several machines is a Markov process.

The use of the Markov chain in the case of multi-serving workstations implies:

(1) The analysis of the "input" process (machine stops).

Measurements performed in multi-positional workstations, especially by Palm, proved that the phenomenon of machine stopping may be reproduced quite exactly through an exponential distribution.

One of the n machines is taken and its behaviour is studied in the time interval (0, t):

$$\frac{P_0(t)}{t}, \quad \frac{dT}{dt}. \tag{1}$$

We use P0(t) to mark the probability that the machine should not stop in the interval (0, t). If λ is the frequency of stops (the number of stops in a given time unit), the probability that a stop should appear in the added time interval dt should be λdt. The probability that no stops appear in the interval dt is 1 – λdt. The time interval dt is chosen so that more than one stop or start of the car may not arise. We also suppose that events are independent of the entire interval (0, t); and hence, the probability of stopping the machine in the dt interval is no longer influenced by the length of the (0, t) interval. However, the possibility that no stops of the machine appear in the interval (0, t + dt) is:

$$P_0(t+dt)=P_0(t)(1-\lambda dt) \tag{2}$$

By integration, the solution of the differential equation will be:

$$P0(t)=e^{-\lambda t} \tag{3}$$

The opposed probability, that is, that one or several stops should appear in the interval (0, t) will be:

$$P0(t)=1-e^{-\lambda t} \tag{4}$$

Extending the rationale for x machines, the probability that x machines should not stop in the interval (0, t) will be:

$$P0(t)=e^{-\lambda x t} \tag{5}$$

The frequency of stops, λ, may be interpreted as the inverse value of the average time spent by a machine from one stop to another. This results from the integration of formula (3), depending on the time:

$$T_{fx}=\int_0^\infty t \times \frac{dP_0}{dt} \times dt = \frac{1}{\lambda} \tag{6}$$

For the average time of x machines, one may write:

$$T_{fx}=\frac{1}{\lambda x} \tag{7}$$

(2) The analysis of the serving mechanism

Due to the complicated structure of the work time (te), the exponential distribution of service times is harder to deduce. In order to simplify calculations and, especially, for reducing the work time required for calculating the distribution governing this time, most researchers admit an exponential distribution for the service time as well.

Using μ to mark the frequency of servings in a given time unit for the stopped station, the possibility that this station should be operated in the dt interval is μdt. The probability that serving operations should not end within the interval 0, t + dt is:

$$P_0(t+dt)=P_0(t)(1-\mu dt) \quad (8)$$

Solving this differential equation, an exponential distribution of operation times is obtained:

$$P_0(t)=e^{-\varpi t} \quad (9)$$

It is mentioned that the parameters μ and λ depend on time. The operation frequency μ may be interpreted as an inverse value of the operation time. The following value is obtained for the average operation time:

$$T_{en}=\int_0^\infty \frac{dP_0}{dt}\times dt=\frac{1}{\mu} \quad (10)$$

(3) A detailed analysis of the issue

Admitting that both the input process and the serving mechanism are subject to exponential distributions, therefore, the process of operating several machines is a Markov process.
The Markov process, as established – through the current knowledge of the system, its future behaviour may be determined, without needing past information. This happens because, in the exponential determination of machine stops and operation times, it was admitted that stochastic events in the (dt) interval are independent of the past (interval 0, t).
Establishing an operation method in a temporary order, that is: "first come, first served", one obtains the three following states for the x (x = 1 - n) machines to travel within the given interval.

1. State x – 1 – a machine is started in the dt interval.
2. State x + 1 – a machine is stopped in the dt interval.
3. State x – no machine is started and no machine is stopped in the dt interval.

Based on statistical independence, we obtain:

$$\pi_x(t)=\pi_{x-1}(t)\times P_{x-1,x}+x+\pi_{x+1}(t)\times P_{x+1,x}+\pi_x(t)\times P_{x,x} \quad (11)$$
x = 1, 2, 3, …, n – 1.

The extreme states for: x = n, x = 0, all machines work, respectively are idle, we obtain:

for x = n
$$\pi_x(dt)=\pi_{(x-1)}(t)\times P_{x-1,x}+\pi_x(t)\times P_{x,x} \quad (12)$$

and for x = 0
$$\pi_x(dt)=\pi_{(x+1)}(t)\times P_{x-1,x}+\pi_x(t)\times P_{x,x} \quad (13)$$

Considering the exponential distribution of stops and operations, the transition probabilities for formula (11) are:

$P_{x-1,x}=\mu dt$ - a machine is operated;
$Px+1,x=x$ - an x machine of the operating ones is stopped;
$Px,x=1-(x\lambda dt+\mu dt)=1-(x\lambda+\mu)dt$ - no machine is operating and no machine is stopped;

- For the formula (12) (x = n):
$P_{x-1,x}=\mu dt$ - the only idle machine starts;
$Px,x=1-\lambda dt$ - all machines operate, none is idle.

- For the formula (13) (x = 0):
$P_{x-1,x}=\mu dt$ - the only operating machine is stopped;
$Px,x=1-\lambda dt$ - all machines are idle.

The state probabilities obtained according to formulae (11), (12) and (13) depend on time as long as transition probabilities do not depend on time. The theory of Markov chains proves that state probabilities πx(t) become equal when t tends to be infinite. Therefore:

$$\lim_{t\to\infty}\pi_x(t)=\pi_x$$

While $\pi_x(dt)\to\pi_x$, the process of convergence to the stationary state is developed quickly.
If, at the beginning of the exchange, the worker starts all the machines, the system begins $\pi_{x=0}(0)=1$; all the other state possibilities are zero $\pi_x=0$.

Based on the mentioned theorem, for an infinitely long time, probabilities πx do not change. It has been proved in practice that the stationary state occurs after quite a short time. In this situation, formulae (11), (12) and (13) become, for x = 1, 2, 3, …, n – 1:

$$\pi_x=\pi_{x-1}\times\lambda+\pi_{x+1}\times\lambda+\pi_x\times[1-(x\times\lambda+\mu)]; \quad (14)$$
for x = n

$$\pi_x=\pi_{x-1}\times\lambda+\pi_x\times(1-\mu) \quad (15)$$
for x = 0

$$\pi_x=\pi_{x+1}\times\lambda+\pi_x\times(1-\mu) \quad (16)$$

New equations with new unknown elements are obtained, πx (x = 0, 1, …, n). As one of the formulae is reducible, the condition that the sum of state probabilities should be equal to 1 may be used instead:

$$\sum_{x=0}^{n}\pi_x=1 \quad (17)$$

A possible solution for (17) system of equations is:

$$\pi_x=\frac{\left(\frac{\mu}{\lambda}\right)^x}{x\times\sum_{x=0}^{n}\frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}, \text{ x = 0, 1, 2, …, n.} \quad (18)$$

This formula establishes the distribution of state probabilities, whose quality aspect is presented in Figure 1. The fundamental issue of a waiting phenomenon being solved, namely the determination of the probability law of units in the system, a transition can be made to establish the optimal conditions for

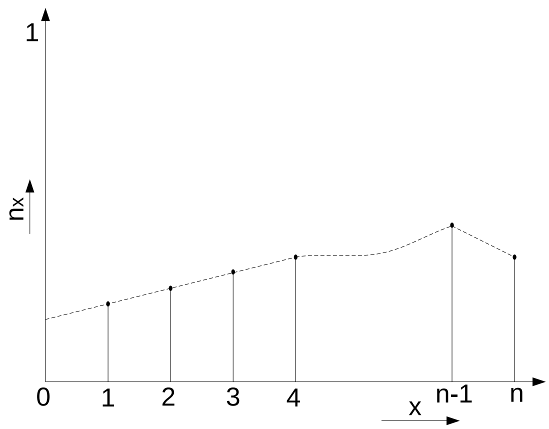

Figure 1. The distribution of the state probability.

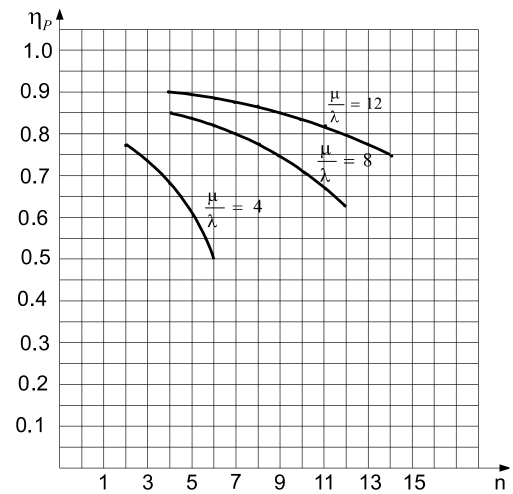

Figure 2. The variation in the active production capacity.

the multi-positional work station.

(4) The determination of the production function and costs

The machine production index

The machine production index is defined as the ratio between the basic time (tMb-) and the total time ™. In case of serving several machines regarded as a waiting system, the production index is given by the ratio between the average number of operating machines and the number of machines in the system.
Considering the total time (tM) of the n machines in the case of multi-serving, the machine time of a station may be determined:

$$t'_M = \frac{t_M}{n} \tag{19}$$

If the average number of operated stations is l, the basic time of the system may be obtained:

$$t_{Mb} = t'_M \times l \, .$$

The index of the active production capacity of the system at a certain moment will be:

$$\eta_P = \frac{t_{Mb}}{t_M} = \frac{t'_M}{t_M \times n} = \frac{1}{n} \tag{20}$$

The average number of operating machines results from the arithmetic average of the distributions of state probabilities:

$$l = \sum_{x=0}^{n} x \times \pi_x \tag{21}$$

By replacement, one gets, for the index of the active production capacity:

$$\eta_P = \frac{1}{n} = \frac{\sum_{x=0}^{n} x \times \pi_x}{n} \tag{22}$$

or by replacements and transformations:

$$\eta_P = \frac{\sum_{x=0}^{n-1} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}{\sum_{x=0}^{n} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}} \tag{23}$$

According to the formula, one may notice that the index of the active production capacity, of productivity at a certain time, depends on the number of stations n and the ratio $\frac{\mu}{\lambda}$. This index shows somehow the efficiency of the multi-serving system.

The graphical representation of the formula (23) for various values of the ratio $\frac{\mu}{\lambda}$ shows us an image of the variation in the index of the active production capacity, in accordance with the number of the machines in the system. η_P decreases with the growth and, as the waiting time of the machine increases (Figure 2). The reduction in the productivity index increases in intensity with the growth of the frequency of stops (λ) in comparison with the frequency of servings μ.

The production function

The production function is defined as the connection between the production factors entering the manufacture process and the results of this process. As main production factors for the undertaken study, the followings are considered: the operator (worker) and the machine, being compared quantitatively through times (te) and (tt). As production factors are limited, the quantitative participation of a factor solely depends on the amount of achieved production.
The dependence of the production amount has been shown through the function:

$$q = f \times \left(\frac{t_f}{t_e}\right)$$

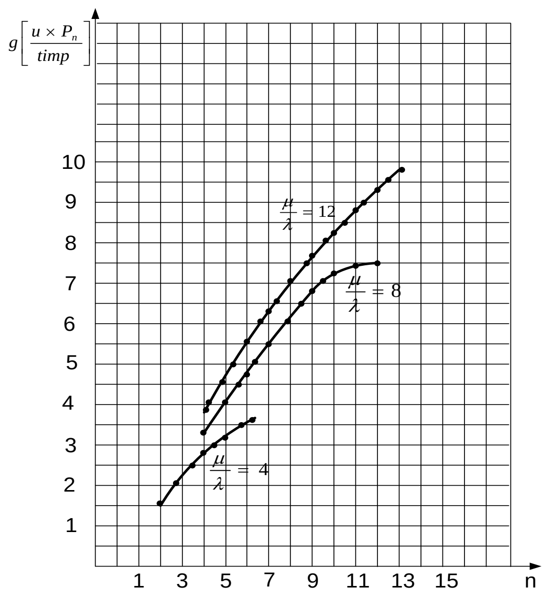

Figure 3. The variation of production in a time unit, according to the ratio $\frac{\mu}{\lambda}$.

The quantity produced in the time unit of a multi-positional workstation may be determined according to the formula:

$$q = \frac{1}{t_{Mb}} \times \eta_P \times n, \tag{24}$$

where: $\frac{1}{t_{Mb}}$ - represents the amount produced in the time unit of a machine with uninterrupted operation; ηP - the index of the machine production capacity; n - the number of operated machines.
Using the formula (23) for ηP, one obtains:

$$q = \frac{1}{t_{Mb}} \times \frac{\mu}{\lambda} \times \frac{\sum_{x=0}^{n-1} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}{\sum_{x=0}^{n} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}} \tag{25}$$

The formula indicates that the time unit (q) contains a function of the number of stations n and by λ and μ a function of production factors – man-machine. Actually, $\frac{1}{\lambda} = T_f$ is the average travel time of a machine, and $\frac{1}{\lambda} = T_a$ represents the waiting time. Figure 3 presents graphically the formula for the various values of the parameter $\frac{\mu}{\lambda}$ depending on n.

Limit costs in the case of serving several machines

For the calculation of costs within serving several machines, two cost categories may be considered: staff costs and technical manufacture costs. Some of these costs depend on time, and others depend on the amounts of achieved products. Costs depending on the amount of products are constant in relation to the product unit and, hence, shall have no influence on the optimisation of production factors.

The optimal number of operated machines may be established considering the variation of time-dependent costs (calculated as standards or hourly quotes):

$$C = C_M \times t_M + C_m \times T_{en}, \tag{26}$$

where: C - represents total costs in the case of serving several machines; C_M - costs for the machine; Cm - staff costs.

Considering that: $t_M = \frac{t_{Mb}}{\eta_P}$ and the formula (26) may be written as:

$$C = C_M \times \frac{t_{Mb}}{\eta_P} + C_m \times \frac{t_{Mb}}{\eta_P \times n} = \frac{t_{Mb}}{\eta_P \times n} \times \left(C_M \times n + C_n\right) \tag{27}$$

or

$$C = \frac{t_{Mb}}{n} \times \frac{C_M \times n + C_m}{\frac{\eta}{\lambda \times \eta_P} \times \frac{\sum_{x=0}^{n-1} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}{\sum_{x=0}^{n} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}}$$

one obtains:

$$C = t_{Mb} \times \frac{\lambda}{\mu} \times \frac{\sum_{x=0}^{n-1} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}}{\sum_{x=0}^{n} \frac{\left(\frac{\mu}{\lambda}\right)^{n-x}}{(n-x)!}} \times \left(C_M \times n + C_m\right) \tag{28}$$

For an easier outline of the variation of costs depending on n, λ, μ, and the partial quotes in the costs, the graphical representation is made for different values of the ratio $\frac{C_m}{C_M}$. It is noticed that for higher values of hourly machine quotes, the optimal number of allocated stations is lower (Figure 4).
This means that, in high machine costs, stop times must be reduced, and the service time must be lowered in high staff costs.

Temporary requests of the worker

Under the conditions admitted for the exponential stochastic

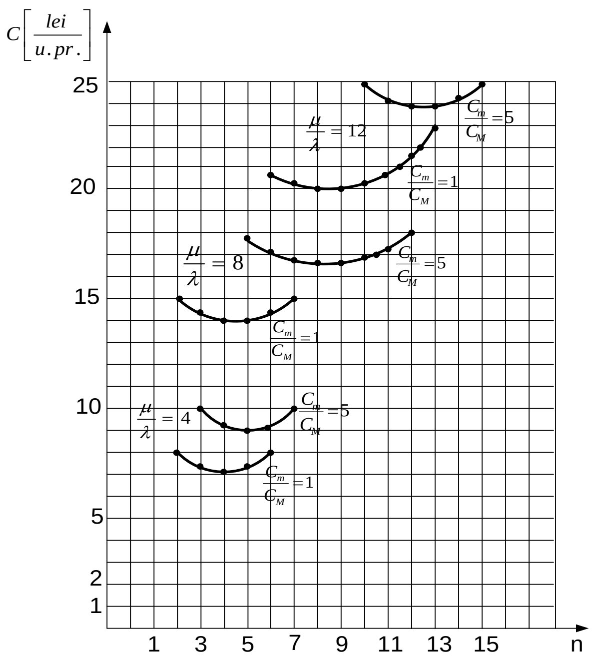

Figure 4. The variation of costs depending on n and $\frac{\mu}{\lambda}$.

distribution with Markov properties, applied to the operation of several machines, the times of the worker and machine times measured at the workplace are average statistical values. According to the analysis of the formula (17), it results that for $\frac{\lambda \times n}{\eta} = 1$, the probability πn (all machines operate) has a very low value. The (temporary) constraint of the worker is complete if the loading coefficient $k = \frac{\lambda \times n}{\mu} = 1$ or the worker's constraint is complete when the average operation time of the operated machines is equal to the average serving time, that is:

$$T_{fn} = T_{en} = \frac{1}{n \times \lambda} = \frac{1}{\mu}$$

where:

$$\frac{n \times \lambda}{\mu} = 1 \qquad (29)$$

For a more rigorous analysis, it must be considered that the worker shall no longer have a waiting "time", if, outside the machine where he/she works, all the other (n-1) operate and their average time of operation is equal to the average service time. These considerations imply, hence, that:

$T_f \times (n-1) = T_{en}$, resulting into:

$$\frac{\lambda \times (n-1)}{\eta} = 1 \qquad (30)$$

Hence, for $\frac{\lambda \times n}{\eta} = 1$, the worker shall not be constrained completely. The time difference may be regarded as a supervision time. The expression of the worker's loading coefficient for the service of a station, using the notions of the stochastic process, shall be:

$$k_m = \frac{\lambda}{\mu} \qquad (31)$$

Takacs-Runnenburg model

In Takacs-Runnenburg model, the service method is considered: first come, first served (Askin et al., 1993).
Admitting a general distribution B(t), probabilities are obtained by formula:

$$\pi_r = \sum(-1) \times \left(\frac{i}{x}\right) \times Bi\,;\ \sum_{x=0}^{n} \pi_x = 1 \qquad (32)$$

where Bi = 1 for i = 0

$$Bi = \frac{n \times C_i - 1}{i} \times \frac{\sum_{k=i-1}^{n-1} \left(\frac{n-1}{k}\right) \times \left(\frac{i}{C_k}\right)}{(1 + n \times m) \times \left(\frac{n-1}{k}\right) \times \left(\frac{i}{C_k}\right)}, \qquad (33)$$

I = 1, 2, …n, and Ci = 1 for i = 0

and

$$C_i = \underset{k=1}{\pi} \times \frac{\varphi \times (k \times \lambda)}{1 - \varphi \times (k \times \lambda)} \qquad (34)$$

i = 1, 2, …, (n-1), and $\varphi \times (k \times \lambda)$ is the Laplace transformation of B(t)

$$\varphi \times (k \times \lambda) = \int_0^{\infty} e^{-k \times \lambda \times t} \times dB(t) \qquad (35)$$

If the exponential distribution of the service time is admitted for B(t), formulae (3 2 to 35) correspond to the Markov relations and more general models may be established for: the average number of operating stations; the productivity index; the production function; the function of expenses and benefits.
Under the conditions of non-exponential distributions, these expressions become very complex, so that, for their solution, electronic computers must be used.
The following restrictions are considered for this model:

machine stops are distributed exponentially in time; the serving time complies with the general distribution P(t) = B(t).

By continuous movement, the worker visits all the n workstations. The time of movement from one station to another is constant and has the form $C = \sum C_i$.

If the πx probability is admitted, that is x machines are idle, we may obtain:

$$\pi_x = k \times \prod_{j=0}^{x-1} \left(e^{-\lambda \times C} \times j^{-1} - 1 \right), \tag{36}$$

with:

$$k = \frac{1}{\sum_{x=0}^{n} \left(\frac{n}{x} \right) \times \prod_{j=0}^{x-1} \left(e^{-\lambda \times C} \times j^{-1} - 1 \right)}, \tag{37}$$

and:

$$J = \int_0^{\infty} e^{-\lambda \times t} \times dB(t) \tag{38}$$

When an exponential distribution of the serving time is considered, for B(t):

$$J = \frac{\mu}{\mu + \lambda} \tag{39}$$

The other formulae for the average number of stations, the productivity index, the production function, the function of costs and earnings are approximately the same as in the case of the Markov model.

The model of the “mechanisation coefficient”

The issue in the model of the “mechanisation coefficient” is dealt with by considering certain characteristic notions of the means of processing, regarding the mechanisation coefficient, the loading coefficient (Courtois et al., 2000).
The application of this method implies:

- The determination of the mechanisation coefficient (φ)

The mechanisation coefficient is defined as a ratio between the automatic operation times of the tool and the manual time, being influenced by the type and productivity of the means of production. An increase in the productivity of the tool may be obtained through the reduction of the manual time and through a better use of the machine capacity; the size of the part batch. By increasing the size of the batch, the mechanisation coefficient increases, because the preparation-completion time for each product unit decreases. The size of the addition is more important in processing by cutting; the capacity to process the material is influential in other procedures.

The mechanisation coefficient must fulfil the following conditions: determination must be made in a unitary way for all units, in order to give the possibility of comparison; the degree of development of mechanisation and automation must be expressed in real terms; it must reflect the dynamics of the manufacture process; determination shall be made easily. If work is solely manual in a certain processing, the mechanisation coefficient is 0. With the increase in technicality, it tends to be infinite.

The increase in the mechanisation coefficient, respectively the decrease in the manual time may be achieved by using machines with a high degree of automation and various devices (quick-detaching devices, arresters, feeders etc). With the increase in the coefficient of mechanisation, the premises required for the

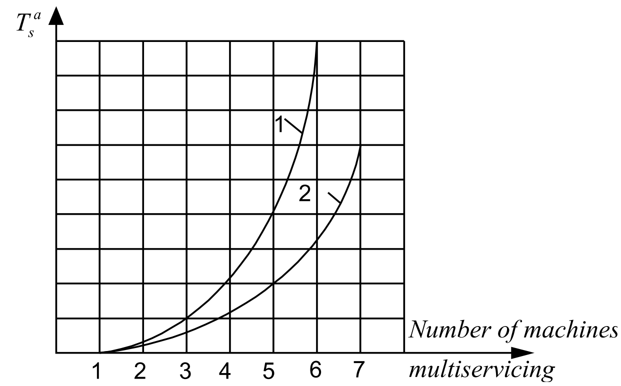

Figure 5. Examples of variation of the superposition time $\left(T_s^a\right)$ in the case of individual (curve 1) and collective service (curve 2).

operation of several machines are created. The mechanisation coefficient is an indicator which may show the trends and state of the mechanisation and automation of production processes within a company or productive sector. The mechanisation coefficient is determined, as it has been shown, by relating the automatic operation time of the machine (tf) to the worker time (te). The machine time includes the basic time (tMb) and the auxiliary time (tMa):

$$t_f = t_{Mb} + t_{Ma} \tag{40}$$

The worker’s time includes the service time (td), the preparation-completion time (tpi) or tpi/n for products and the technical and organisational time at the workstation (tto):

$$t_e = t_d + t_{pi} + t_{t0} \tag{41}$$

In the case of serving several machines, the mechanisation coefficient is calculated as:

$$\varphi = \frac{\sum t_{fi}}{\sum \left(t_e + t_0 \right) + T_{fi}^a} \tag{42}$$

where t_0 represents the time for rest and physiological requirements; T_f^a - the superposition time determining stops of the machines, the technological parameters of the processing process.
The superposition time decreases with the increase of the mechanisation coefficient, because the operator shall be less and less busy with effective intervention and busier with supervision.

Practice has shown us that the superposition time decreases if, instead of singular multi-positional service, the serving of several machines in a group (collective serving) is undertaken, which must be considered in the organisation of the service of several machines.
Figure 5 presents an example of variation of the superposition time, given individual (1) or collective (2) multi-serving.

The time reduction may be explained through the fact that, in case of collective serving, the operation of a group machine stopping at a given moment may be done by one of the operators which are idle at that moment (Lee and Lee, 2005).

Of course, the team must include trained operators, allowing this

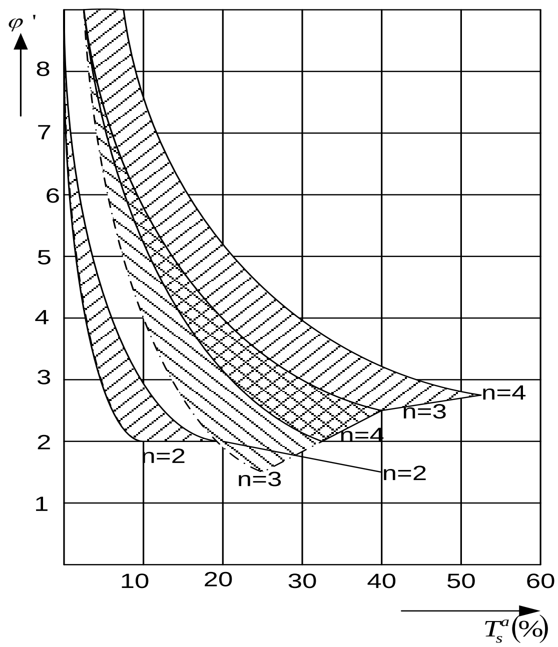

Figure 6. Examples of variation of the superposition time $\left(T_s^a\right)$ depending on the provisional loading degree (φ').

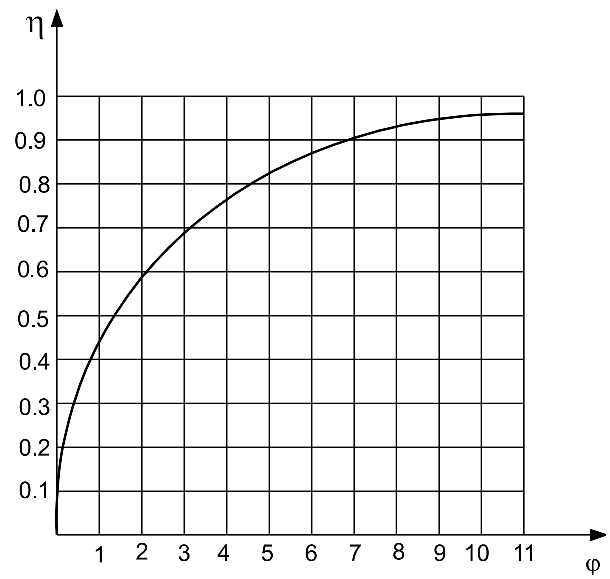

Figure 7. The variation curve of the function $\eta = f(\varphi)$.

intervention, that is, multi-qualified operators, in order to be able to use this method.

The variation of the superposition time depending on the provisional mechanisation degree and the number of operated machines is presented in Figure 6.

This variation may be determined through the method of calculation of the probability or even through experimental methods. Slight differences are obtained between the two methods, which increase with the increase in the number of machines, respectively due to the stop time.

Measurements and tables were made by Rosemberg and Prochorov (Giard, 2010), for obtaining the value of the stop time. The figures show the idle times, depending on the provisional mechanisation coefficient φ', and the operated machines and number of workers operating the machines. The provisional mechanisation coefficient complies with the value obtained in the formula:

$$\varphi' = \frac{\sum t_{fi}}{\sum t_{ei}} \tag{43}$$

According to this formula, it may be seen that the calculation only includes the actual time used by the worker for operating the machine, but, practically, the superposition time must also be considered. For these reasons, this coefficient is admitted in the first phase, then it is adjusted by introducing the superposition time as well;

- *Determining the loading coefficient (η)*

The loading coefficient of the machine, η, is defined as a ratio between the automatic operation time tf and the total time:

$$\eta = \frac{\sum t_{fi}}{\sum t_{fi} + \sum t_{ei} + T_a^s} \tag{44}$$

and expresses the actual load of the machine. Since $\varphi = \frac{\sum t_{fi}}{\sum t_{ei} + T_a^s}$, it is noticed that η may be expressed as:

$$\eta = \frac{\varphi}{\varphi + 1} \tag{45}$$

The variation of η depending on φ is presented in Figure 7.

According to Figure 7, the increase of the mechanisation coefficient results in an increase of the loading coefficient towards 1.

The loading coefficient may also be used for determining the time standards for the machines in the multi-serving system through the formula:

$$t_{fn} = \sum F_r \times \eta\,, \tag{46}$$

where: F_r – represents the actual time of the tool.

The increase may be determined by considering the values before and after the implementation of mechanisation:

$$\Delta\varphi = \varphi_1 - \varphi_2\,, \tag{47}$$

$$\Delta\eta = \eta_1 - \eta_2\,, \tag{48}$$

The efficiency of mechanisation actions may be shown if the variation of the return coefficient is represented according to the degree of mechanisation:

$$\lambda = f(\varphi)\,, \tag{49}$$

If, for a certain plan period, various tasks (processing, assortments) are established, resulting in the determination of usage coefficients for differential tools, an average machine usage coefficient (ηaverage) may be determined for multi-serving conditions:

$$\eta_{mediu} = \frac{\sum_{i=1}^{n} \eta_i \times t_i}{t_{per}}, \quad (50)$$

where: ηi - represents the loading coefficients concerned;
ti - the time period the loading degree refers to.

According to the variation shape of the curve $\lambda = f(\varphi)$ a quick increase in the usage coefficient in the area of the small values of the mechanisation and automation coefficient may be noticed. It may be retained, hence, as a conclusion, that, for the low values of the mechanisation coefficient of tools, the measures for increasing this coefficient will have a strong favourable effect on the loading degree; hence a more efficient use of machines.

IMPLEMENTATIONS AND RESULT OF USING THE MODELS

Using any model from those presented results in determining the influence of production costs on the number of multiserviced machine-tools (Giard, 2010).

The knowledge of the average loading degree leads to a better presentation of the way tools are used, to which extent the minimisation of expenses and the use of fixed funds are considered, for ensuring a superior production and efficiency in the production activity.

For extending the application of the multi-serving system, other measures creating a favourable attitude should be considered. The method of determination of the workers' remuneration is an important factor for the application of the system.

A quick calculation formula may be established for remuneration by agreement. If the basic remuneration is marked R_B, the tariff remuneration is marked R_T, the time standard for the machine, t_{fn}.

The remuneration may be calculated as:

$$R_B = \frac{R_T \times T}{t_{fn}}, \quad (51)$$

where T – represents the time period
However, since the machine time may be expressed as:

$$t_{fn} = \sum T \times \eta = n \times T \times \eta, \quad (52)$$

the basic remuneration will be,

$$R_B = \frac{1}{n \times \eta} \times R_T, \quad (53)$$

where $\frac{1}{n \times \eta}$ - represents a factor of remuneration by agreement.

For a certain effective period, t_{ef}, the following formula will be used:

$$R_A = R_B \times t_{ef} \quad (54)$$

Monthly costs are a starting basis for the determination of the costs of serving several machines, which are then related to the achieved production (Schatteles, 1992).

These costs are: machine amortisation; costs for the maintenance and repair of machines; amortisation for buildings and installations; maintenance and repair costs for buildings and installations; tool costs; energy consumption costs; material costs; other general expenses; labour remuneration costs.

Their sum represents total costs by month, C_t. Production cost by assortments and piece is given as:

$$P_{cost} = \frac{C_f (t_f + t_e) n_s}{\sum [(t_f + t_e) n_s] \cdot n_s}, \quad (55)$$

where: ns - represents the number of products in an assortment.

For the economic planning of various types of processing, a company should draw up sheets by assortments, calculating the provisional mechanisation coefficient, φ', which may be deduced easily. As items with the same φ' or a close φ' result in a constant loading coefficient of machines η, these sheets should be grouped according to the same value of φ'.

For determining the optimal number of machines operated in terms of minimal costs, one has to consider the fact that, by increasing the number of machines allocated for operation, pursuant to the increase in the number of parts produced by a worker, salary costs by product unit decrease. On the other hand, by increasing the number of allocated machines, machine stop times increase, fixed costs related to the machine, by product unit, shall increase.

For this opposite trend, a minimal level of costs appears for a certain number of machines, showing the number of machines which may be operated.
The decrease of costs in labour remuneration by product and by time unit will be:

$$\Delta C_e = \frac{100}{100 + T_{sa}} \left(C_e - \frac{C_e}{n} \right), \quad (56)$$

where: C_e - represents the hourly quote of remuneration;
n - the number of operated machines.

The formula $C_e - \frac{C_e}{n}$ expresses remuneration savings in the case of operation of n machines, in comparison to the operation of one machine.
The increase in machine costs by product and by time unit will be:

$$\Delta C_f = \frac{100}{100 - T_s^a} \times C_f - C_f, \qquad (57)$$

where: Cf – represents the hourly quote for a machine. The factor $\frac{100}{100 - T_s^a}$ considers the increase of costs, due to the stop time T_s^a.
The total savings of the cost will be:

$$E_c = \Delta C_e - \Delta C_f = \left[\frac{100}{100 + T_s^a} \times \left(C_e - \frac{C_e}{n}\right)\right] - \left[\frac{100}{100 - T_s^a} \times C_f - C_f\right], \qquad (58a)$$

or

$$E_c = \Delta C_e - \Delta C_f = \frac{100}{100 + T_s^a} \times \left(\frac{n-1}{n}\right) - \left(\frac{T_s^a}{100 - T_s^a}\right) \times Q_c, \qquad (58b)$$

where:

$$Q_c = \frac{C_f}{C_e}$$

The maximal reduction indicates the optimal number of operated machines.

The introduction of the multi-positional work method imposes the performance of analyses regarding the production process and the structure of costs.

As it has been shown, the conditions for the implementation of the method refer to the study of the production capacity, labour forces and, especially, the expenses determining the cost of achieved production.
With the operation of several machines, the superposition time T_s^a increases with the increase of the number of machines, resulting in a reduction in the tool production capacity; on the other hand, increasing the energy consumption and operating expenses. The existence of other factors which may influence the application of the method negatively results in the necessity to attentively analyse the determination of the optimal number of machines operated, with the achievement of minimal costs.

One of the possibilities to fulfil this wish consists of activities which may be performed through the operation of several machines, considering the superposition time.

The formula allowing for the expression of savings, as it has been shown before (58b), is the following:

$$E_c = \frac{100}{100 + T_s^a} \times \left(\frac{n-1}{n}\right) - \frac{T_s^a}{100 - T_c} \times Q_c, \qquad (59)$$

According to the analysis of the formula, the ratio Cf/Ce appears in the calculation of the economic efficiency. Practically, this ratio is not constant. It has different values for different companies. Moreover, it is modified, through the continual process of equipment with tools, through the degree of qualification and improvement in the workers' remuneration.

Of course, with highly technical machines, hourly expenses for machines increase and, normally, the mechanisation degree increases as well.
Hourly costs for groups of tools are also determined by the number of tools.

It is, hence, seen that the Qc coefficient depends on several factors, which, in turn, vary with time. Therefore, in practical applications, the factors influencing this value must be monitored, in order to make a decision on the optimal number of tools which may be operated.
The graphical representation of the dependence between the degree of mechanisation, the number of operated machines and the Qc coefficient, in terms of being operated by a single worker, as presented in Figure 8, shows that, with the increase of the mechanisation degree, the number of machines which may be operated also increases, as they have the same Qc coefficient.

If a constant value is considered for the mechanisation degree, it is noticed that, through the increase of Qc, the number of machines which may be economically operated decreases. It must also be noticed that, for a certain value, although Qc increases, the optimal numbers of machines which may be operated no longer increases. This may be explained by the fact that the influence of the superposition time is higher and higher.
As for costs by product unit, they vary with the number of machines, as seen in Figure 9.

Because, generally, the number of parts in the batch to be processed has a strong influence on the cost by product unit, the optimal batch of parts should also be determined in the case of operation of several machines.
Hourly costs by product unit, in the case of operation of a single machine, in the processing of a batch N of parts, may be expressed as:

$$C_0 = (C_f + C_e) \times \left(\frac{t_{pi}}{N} + t_f\right) \times \frac{1}{60}, \qquad (60)$$

and, for the operation of n machines:

$$C_0 = (C_{fn}^n + C_e) \times \left(\frac{t_{pi}}{N} + K_{fm} \times t_f\right) \times \frac{1}{60}, \qquad (61)$$

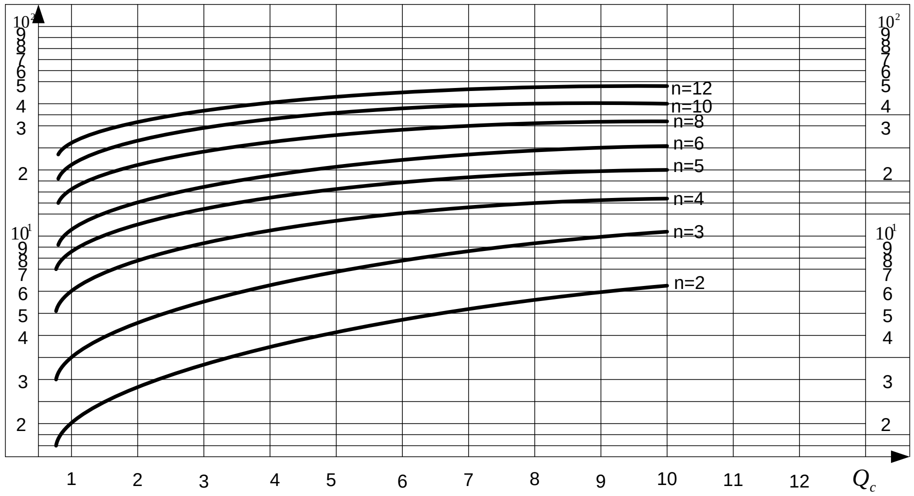

Figure 8. The dependence between the degree of mechanisation, the number of operated machines and the qc coefficient.

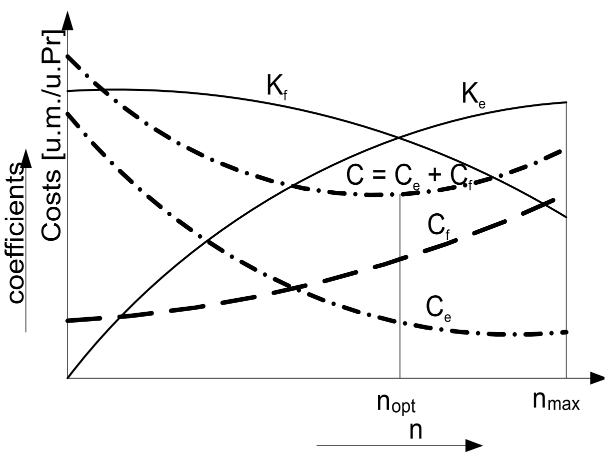

Figure 9. The variation of expenses by product unit, depending on the number of service stations.

where: K_{fm} - represents a coefficient in the case of serving several machines ($K_{fm} < 1$).

For the determination of the number of parts in a batch, considering the two values of expenses, the limit condition is stated, that is, $C_e = C_0$, resulting in the number of parts in a batch for which expenses are identical and, hence, no savings shall be obtained over this number of parts in a batch:

$$N_{lim} = \frac{t_{pi} \times [C_f \times (n-1) - C_e + C_0]}{t_f \times [C_i \times (1 - n \times K_f) + C_e - K_f \times C_0]} = K_e \times \frac{t_{pi}}{t_f} \quad (62)$$

The Kf coefficient has values determined by the dimension and complexity of processed parts and it actually represents the correction required for the cases where additional time is needed for the adjustment of the machine and for securing parts for processing. Because of the experimental results of various investigations, it has been found that, in order to accomplish an efficient operation, in the case of processing part batches, the batch size established according to formula (63) must be calculated:

$$N_L = \frac{1}{a} \times N_{lim} \quad (63)$$

The specialised literature provides values of 0.04-0.08 for a, meaning that the size of the optimal batch, in technical terms, should be of approximately 20-25 times higher than the number of limit parts obtained in the calculation.
The conditions for the application of the method must be established in the introduction of the multi-positional labour method, considering the production capacity, labour forces and, especially, the expenses determining the production costs.

It is generally considered that the amount of products increases with the number of means of production. As it has been seen before, with the operation of several machines, the superposition time T_s^a increases with the increase of the number of machines, resulting in a reduction in the tool production capacity; at the same time, energy is consumed inefficiently and the amortisation quota is higher.

These contrary influences prove the necessity to establish an optimal number of tools to be grouped for multi-service.

Stating the condition that a minimal cost of the processing process should be accomplished, the optimal number of machines which may be operated can be determined.

Regarding the method used within the studies performed at car constructing companies, with examples being presented in this paper, the followings were established. Although the elements used in calculations are based on a high amount of observations, they cannot satisfy the solution of technical and economic issues, under all aspects, given their method of determination. Even the determination of the provisional degree of mechanisation, which, as it has been seen, represents a determinant factor, does not rigorously explain the value of the ratio between automatic operation or without necessity of supervision or direct intervention of the worker during the processing process.

It must be mentioned that, by using the provisional degree of mechanisation, this does not present the actual situation in many cases. In order to obtain more realistic values, a great deal of instantaneous observation is required, especially under the conditions of a higher mechanisation degree and for a group of machines with different degrees of mechanisation. On the other hand, the dynamic and strong increase in the degree of technicality of tools and the increase in workers' qualification result in a difficult and, hence, improper application of the method of calculation for the number of tools operated in terms of maximal efficiency.

CONCLUSION AND LIMITATIONS

The results of the study, both in the cases presented in the paper and in other cases, even if they cannot be fully applied in practice, have also proven, even though only informatively, a range of failures and difficulties regarding the way of preparation, application and development of the multi-service process. Among these we can state the following: the absence of attentive studies regarding production organisation, the location of machines, the succession of operations, the assurance of machine supervision possibilities etc. In many cases, failures were found regarding the supply of parts to be processed, the equipment with proper tools, devices and checkers.

Likewise, failures were found regarding the global development of the process, the faulty elaboration of the technological documentation and, especially, of the indication of cutting systems, the correct standardisation of the works to be executed (Dima, 2013).

Given these observations, one must use those parameters which could objectively express the real advantages obtained in pursuant of the application of this method, which has proven its efficiency in practice. Among these, the degree of loading of tools and workers may result in an appreciation of the advantages of multi-service.

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