African Journal of Business Management Vol. 7(7), pp. 526-535, 21 February, 2013
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM11.099

Full Length Research Paper

An economic design of the VSSI $\bar{X}$ control charts for the means of positively Skewed distributions

Kun-Lin Hsieh[1]* and Yan-Kwang Chen[2]

[1]Department of Information Management, National Taitung University, 684, Sec.1, Chung-Hua Rd., Taitung, Taiwn R.O.C.
[2]Department of Logistics Engineering and Management, National Taichung Institute of Technology
129 Sanmin Road, Sec. 3, Taichung, Taiwan R.O.C.

Accepted 21 May, 2012

Variable sample size and sampling interval (VSSI) $\bar{X}$ control charts have been shown superior to standard Shewhart (SS) $\bar{X}$ control charts for detection of a small or moderate shift in process mean. However, they might utilize more resources by a more frequent sampling rate and a large sample size to improve its performance. Recently, some economic models were used to express the long-run cost per hour of operating the VSSI $\bar{X}$ control charts and gain insight into the way to design the charts. The usual assumption for the models is the normality of the underlying data or measurements. However, this assumption may not be true in practice. In this paper, an economic design of the VSSI $\bar{X}$ control charts for skewed non-normal data is conducted by using the Markov chain approach and genetic algorithms. Two types of VSSI $\bar{X}$ charts are considered and compared with the SS $\bar{X}$ charts over several numerical examples: the symmetric control limits and asymmetric control limits. Moreover, effects of non-normality on the performance of the VSSI $\bar{X}$ charts with respect to the costs of operating the charts are studied. It is shown that the reduction in cost can be achieved by using the VSSI $\bar{X}$ charts instead of the SS $\bar{X}$ charts. However, an increase on the skewness coefficient results in a decrease on the cost savings. In addition, the asymmetric control limits is a better choice with respect to the costs and the false alarm rate.

Key words: Skewed non-normal, control chart, genetic algorithms, economic design.

INTRODUCTION

The control chart technique can be considered as a graphical expression of statistical hypothesis testing in the industrial process control for detecting a process change. The standard Shewhart (SS) control chart used to detect process mean shifts is one of the most popular control chart techniques. Since it takes samples of fixed size with a fixed time interval between samples, it is also called the fixed sampling rate (FSR) control chart. The advantage of the SS control chart is its simplicity, but its efficiency (in terms of the speed with which process shifts are detected) is poor when the process mean shift is small or moderate. In recent years, it has been found that performance of the SS control chart can be improved by varying the rate of sampling as a function of the data from the process. Several ways are used for variable sampling rate. One way is to vary the sampling interval, in which a short sampling interval is used when there is an indication of process change and a long sampling interval is used when there is no indication of process change. The resulting control chart is called the variable sampling interval (VSI) control chart (Reynolds et al., 1988; Reynolds and Arnold, 1989; Runger and Pignatiello, 1991; Baxley, 1996; Reynolds, 1996). Another way to vary the sampling rate is to vary the sample size, in which a large sample size is employed when there is an indication of process change, and a small sample size is employed when there is no indication of process change. The resulting control chart is called the VSS chart (Prabhu et al., 1993; Costa, 1994). The VSI and VSS schemes can be used together for improving the performance of the SS $\bar{X}$ control chart,

*Corresponding author. E-mail: kunlinnttu@gmail.com.

and the resulting control chart is called the VSSI control chart (Rendtel, 1990; Prabhu et al., 1994; Costa, 1997).

The use of control chart requires the user to select several parameters. For the SS $\bar{X}$ control chart, the sample size, control limits and the sampling interval should be predetermined. Since the work by Duncan (1956), the economic design for the SS $\bar{X}$ control charts has received much attention (Montgomery, 1980; Vance, 1983; Woodall, 1986; Pignatiello and Tsai, 1988). The usual approach to the economic design is to develop a cost model for a particular type of industrial process, and then derive the optimal parameters by minimizing the long-run expected cost per hour. For the VSI, VSS or VSSI $\bar{X}$ control chart, there are additional parameters of multiple sampling interval lengths and/or sample sizes and warning limits to choose. As compared with the SS $\bar{X}$ control chart, little work has been done on the economic design of the variable sampling rate $\bar{X}$ control chart. Park and Reynolds (1994) studied the economic design of the VSS $\bar{X}$ control chart. In 1998, Bai and Lee considered the economic design of the VSI $\bar{X}$ control chart. Moreover, Park and Reynolds (1999) developed the economic design of the VSSI $\bar{X}$ control chart. The models they adopted are similar to that of Duncan (1956).

Traditionally, when the issue on designing control chart is discussed, one usually assumes the measurements in each sample (or say population) are normally distributed; therefore, the sample mean $\bar{X}$ is also normally distributed. However, the assumption may not be tenable in practice. For example, the distributions of measurements from chemical processes, semiconductor processes, or cutting tool wear process are often skewed (Chang and Bai, 2001). Surely, if the sample size is large enough, the statistic $\bar{X}$ will be distributed normally according to the central limit theorem. However, this is often expensive. The non-normal behavior of measurements may imply that the traditional design approach is improper for the operation of control charts. Facing the impropriety, Rahim (1985) presented an economic model of the SS $\bar{X}$ charts under non-normality assumption. Likewise, Chou et al. (2000) used the Burr distribution to represent various non-normal distributions and construct an economic-statistical model for the SS $\bar{X}$ control charts. Recently, Chen (2004) presented an economic design of the VSI $\bar{X}$ charts for non-normal measurements. This economic model is an extension of the works by Bai and Lee (1998) and Chou et al. (2000). However, the economic designs of VSS and VSSI $\bar{X}$ charts by taking the effect of non-normality into consideration have not appeared in the literature.

The main objective of this paper is to present an economic design of the VSSI $\bar{X}$ chart for positively skewed process data. The rest of this paper is structured as followed; the VSSI $\bar{X}$ chart for process data with normal and gamma distribution, which stands for a wide-ranging skewed distribution; the cost function associated with the long-run cost per hour of operating the VSSI $\bar{X}$ control chart is developed by means of the Markov chain approach; a searching method based on genetic algorithms (GAs) is introduced for finding the optimal design parameters so that the cost function is minimized; numerical illustrations and comparisons are made; concluding remarks.

BACKGROUND

In this area, we briefly explain the operation of the VSSI $\bar{X}$ control chart and the gamma distribution, which is used to represent a generally skewed distribution.

Review of the VSSI $\bar{X}$ chart

Prabhu et al. (1994) and Costa (1997) separately proposed the $\bar{X}$ control chart with variable sample size and variable sampling interval. To simplify the implementation of the VSSI $\bar{X}$ chart, the sample size and sampling interval length are considered to vary between two values.

Assume that the distribution of the measurements from the process is normal with mean μ and standard deviation σ, and the objective is to detect the shifts in μ from the target value μ_0. In the SS $\bar{X}$ control chart, a random sample of size n_0 is taken every h_0 hour, and the sample mean is plotted on the control chart with the control limits $\mu_0 \pm k\sigma_{\bar{X}}$. The search for assignable cause is carried out when sample mean falls outside the control limit.

In the VSSI $\bar{X}$ control chart, random samples of variable size are taken at the intervals of variable length. Let (n_1, h_1) be a pair of minimum sample size and longest sampling interval, and (n_2, h_2) be a pair of maximum sample size and shortest sampling interval. These pairs are chosen such that $n_1 < n_0 < n_2$ and $h_2 < h_0 < h_1$. The decision to switch between the two pairs of the parameters is made through the warning limits $\mu_0 \pm w\sigma_{\bar{X}}$ and control limits $\mu_0 \pm k\sigma_{\bar{X}}$. For the sake of simplicity on presentation, the sample points plotted on the control chart will be standardized sample means, that is $(\bar{X} - \mu_0)/\sigma_{\bar{X}}$. In the case, the warning limits and control limits will be $\pm w$ and $\pm k$ respectively. If the sample point falls into the central region of $(-w, w)$, then the pair (n_1, h_1) is used to relax the control. If the sample mean falls into the warning region of $(-k, -w]$ or $[w, k)$, then the pair (n_2, h_2) is used to tightening the control. Otherwise, a signal is produced to indicate the process

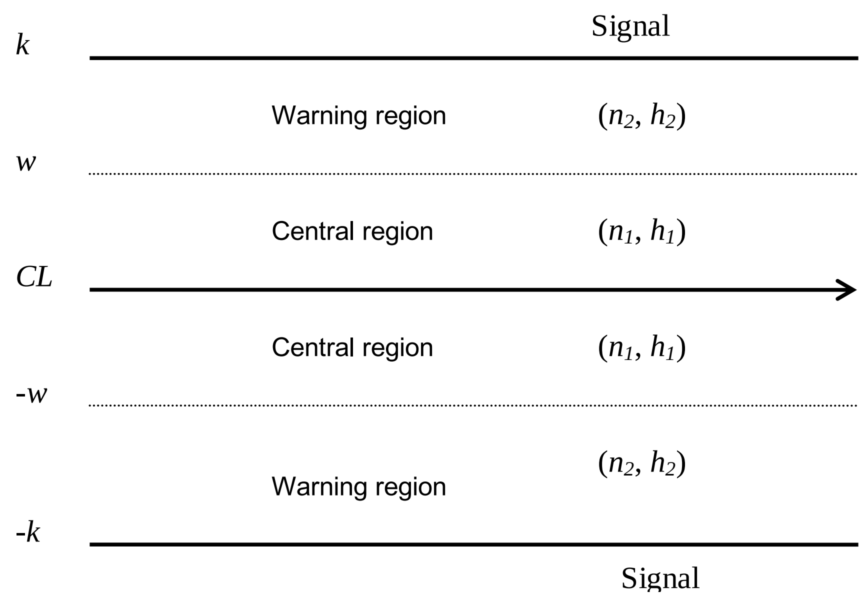

Figure 1. The VSSI $\overline{X}$ control chart.

being out-of-control (Figure. 1).

Since no sample is taken at the start of the process, the first sample size and sampling interval cannot be determined by above switching rule. Prabhu et al. (1994) and Costa (1997) explained that the first sample size and sampling interval can be taken at random. It has a probability of p_0 of being (n_1, h_1) and a probability of $(1-p_0)$ being (n_2, h_2), where p_0 can be simply defined as the conditional probability of a sample mean falling into the central region, given that it did not fall outside two control limits. As an alternative opinion, it may be preferable in practice to use the tightening control--the large sample size and the shortest sampling interval, because it gives additional protection against problems that arise during start-up (Bai and Lee, 1998) (Figure 1).

VSSI $\overline{X}$ chart for process data with gamma distribution

In the economic model, the distribution of the underlying measurements (or data) from the process is assumed to follow the gamma distribution which is a positively skewed distribution.

Let $X_1, X_2, \cdots, X_{n(i)}$ be the measurements of the i^{th} sample. These measurements are independently from the identical gamma distribution, $\text{Gam}\,(\alpha,\gamma)$, with the probability density function,

$$f(x) = \frac{\alpha^{\gamma}}{\Gamma(\gamma)} x^{\gamma-1} e^{-\alpha x}, \qquad x > 0 \qquad (1)$$

By means of its moment generating function, it is easily found that the sample mean $\overline{X}_i$ also follows a gamma distribution, $\text{Gam}\,(n(i)\alpha, n(i)\gamma)$. In monitoring the skewed sample means like the afore-mentioned, a control chart with asymmetric control limits may be preferred to the traditional control chart with symmetric control limits (Tagaras, 1989; Yourstone and Zimmer, 1992). Thus, the warning and control limits of the VSSI $\overline{X}$ control chart can be replaced in the asymmetric condition by $-w'$, $+w$, $-k'$, and k, respectively.

In order to examine the effect of non-normality on the economic design of the VSSI $\overline{X}$ control chart, The nine cases of $\gamma =$0.5, 1, 2, 3, 4, 5, 10, 20, and 30 while holding $\alpha = 1$ are considered. Figure 2 presents partial cases along with a normal distribution with the same mean and variance. These values of α and γ correspond to those used by Schilling and Nelson (1976).

DEVELOPMENT OF COST FUNCTION

In the economical model, a process is assumed to start with an in-control state ($\mu = \mu_0$) but after a random time of in-control operation it will be disturbed by a single assignable cause that causes a fixed shift in the process mean ($\mu = \mu_1$). After the shift, the process remains out-of-control until the assignable cause is eliminated (if possible). The inter-arrival time of the assignable cause disturbing the process is assumed following an exponential distribution with a mean of 1/λ hours.

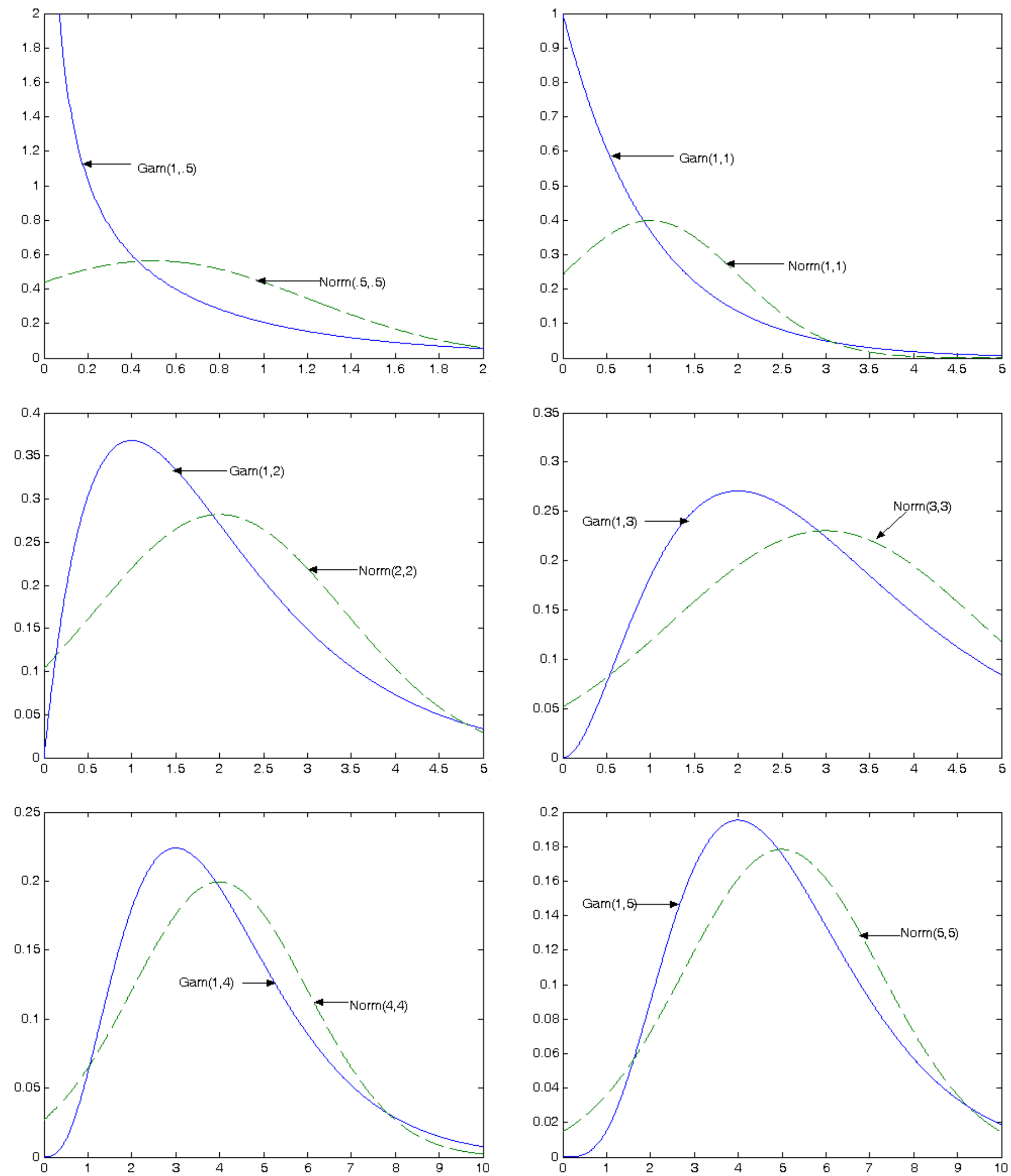

Figure 2. Various gamma and the normal distributions with the same mean and variance.

sample is taken at each sampling time to compute the sample mean. If the sample point falls within the control limit, its position on the chart will be used to decide the next sample size and the next sampling interval. Otherwise, the process is stopped and a search starts to find the assignable cause and adjust the process. The economic design of the VSSI $\bar{X}$ control charts is undertaken by specifying a cost function, and searching the optimal design parameters for minimizing the cost function over a production cycle. The production cycle length is defined as the average time from the start (or restart) of production until the assignable cause is identified and eliminated. Once the expected cycle length is determined, the cost over the production cycle can be converted to an index—long run expected cost per hour (Ross, 1970). The optimal values of the design parameters based on the cost function can be determined by certain optimization techniques such as the grid search, nonlinear programming, or genetic algorithms.

Figure 3 depicts the production cycle, which is divided into four time intervals of in-control period, out-of-control period, searching period due to false alarm, and the time

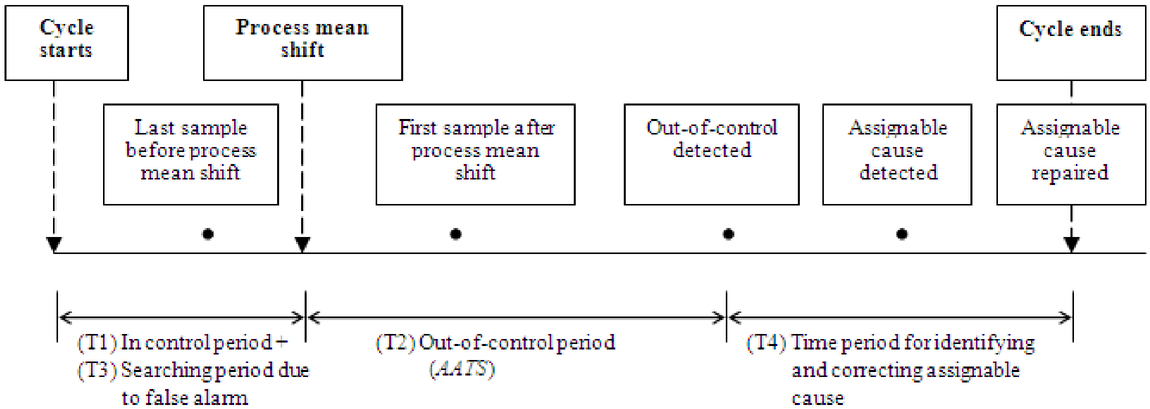

Figure 3. Production cycle considered in the cost model.

period for identifying and correcting the assignable cause. Individuals are now illustrated before they are grouped together.

(T1) The expected length of in-control period is $1/\lambda$ and (T2) The expected length of out-of-control represents the average time needed for the control chart to produce a signal after the process mean shift. This average time is called the adjusted average time to signal (*AATS*), which is the most widely used statistical measure for comparing the efficiencies of different variable sampling rate control charts. The memoryless property of the exponential distribution allows the computation of *AATS* using the Markov chain approach. Costa (2001) and Lin and Chou (2005) used this approach to calculate *AATS* of VSSI $\bar{X}$ chart. Here we basically follow the process developed by Costa. The fundamental concepts of the Markov chain approach can be found in Cinlar (1975).

Let M be the average time from the start of the cycle to the time the chart first signals after the process shift. Then,

$$AATS = M - 1/\lambda \,. \qquad (2)$$

At each sampling moment during the period *M*, one of the four transient states is reached according to the status of the process (in or out-of-control) and the position of sample mean (warning region or central region):

State 1: Sample point falls in the central region and the process is in-control at sampling moment;
State 2: Sample point falls in the warning region and the process is in-control at sampling moment;
State 3: Sample point falls in the central region and the process is out-of-control at sampling moment;
State 4: Sample point falls in the warning region and the process is out-of-control at sampling moment;
The control chart produces a signal if the sample point goes beyond the control limits. At this time, if State 1 or 2 are arrived, the signal is a false alarm; if State 3 or 4 are arrived, the signal is a true alarm. We define the absorbing state-- State 5, which arrives if a true alarm is signaled. The transition probability matrix is given by

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 \\ p_{21} & p_{22} & p_{23} & p_{24} & 0 \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & p_{43} & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (3)$$

where p_{lm} denotes the transition probability that l is the prior state and m is the current state. Thus, we have

$$\begin{aligned} p_{11} &= \Pr\{\text{sample point} \in (-w', w) \mid \text{sample point} \in (-k', k)\} \Pr\{U > h_1\} \\ &= \frac{F_{n_1\alpha, n_1\gamma}(\mu_0 + w\sigma_{\bar{X}}) - F_{n_1\alpha, n_1\gamma}(\mu_0 - w'\sigma_{\bar{X}})}{F_{n_1\alpha, n_1\gamma}(\mu_0 + k\sigma_{\bar{X}}) - F_{n_1\alpha, n_1\gamma}(\mu_0 - k'\sigma_{\bar{X}})} e^{-\lambda h_1} \end{aligned} \qquad (4)$$

where U represents the remaining time until the process shift, from the $(i-1)^{\text{th}}$ sampling moment, that is exponentially distributed with parameter λ; $F_{n_1\alpha, n_1\gamma}(\cdot)$ is the distribution function of gamma function, $\text{Gam}\,(n_1\alpha, n_1\gamma)$. If we denote $a_1 = F_{n_1\alpha, n_1\gamma}(\mu_0 + k\sigma_{\bar{X}})$, $b_1 = F_{n_1\alpha, n_1\gamma}(\mu_0 + w\sigma_{\bar{X}})$, $c_1 = F_{n_1\alpha, n_1\gamma}(\mu_0 - w'\sigma_{\bar{X}})$, and $d_1 = F_{n_1\alpha, n_1\gamma}(\mu_0 - k'\sigma_{\bar{X}})$ to facilitate the presentation, then p_{11} can be rewritten as

$$p_{11} = \frac{b_1 - c_1}{a_1 - d_1} e^{-\lambda h_1} \,.$$

Similarly, we have

$$p_{12} = \frac{(a_1 - b_1) + (c_1 - d_1)}{a_1 - d_1} e^{-\lambda h_2} \quad p_{13} = \frac{b_1 - c_1}{a_1 - d_1}(1 - e^{-\lambda h_1}) \,,$$

$$p_{14}=\frac{(a_1-b_1)+(c_1-d_1)}{a_1-d_1}(1-e^{-\lambda h_2}),$$

$$p_{21}=\frac{b_2-c_2}{a_2-d_2}e^{-\lambda h_1},\ p_{22}=\frac{(a_2-b_2)+(c_2-d_2)}{a_2-d_2}e^{-\lambda h_2}$$

$$p_{23}=\frac{b_2-c_2}{a_2-d_2}(1-e^{-\lambda h_1}),\quad p_{24}=\frac{(a_2-b_2)+(c_2-d_2)}{a_2-d_2}(1-e^{-\lambda h_2}),$$

$$p_{33}=b_1^{'}-c_1^{'},\quad p_{34}=(a_1^{'}-b_1^{'})+(c_1^{'}-d_1^{'}),\quad p_{35}=1-a_1^{'}+d_1^{'},$$

$$p_{43}=b_2^{'}-c_2^{'},\ p_{44}=(a_2^{'}-b_2^{'})+(c_2^{'}-d_2^{'}),$$

$$p_{45}=1-a_2^{'}+d_2^{'},$$

where

$$a_2=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0+k\sigma_{\bar{X}}),$$
$$b_2=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0+w\sigma_{\bar{X}}),$$
$$c_2=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0-w'\sigma_{\bar{X}}),$$
$$d_2=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0-k'\sigma_{\bar{X}}),$$
$$a_1^{'}=\mathrm{F}_{n_1\alpha,n_1\gamma}(\mu_0+k\sigma_{\bar{X}}-\delta\sigma),$$
$$b_1^{'}=\mathrm{F}_{n_1\alpha,n_1\gamma}(\mu_0+w\sigma_{\bar{X}}-\delta\sigma),$$
$$c_1^{'}=\mathrm{F}_{n_1\alpha,n_1\gamma}(\mu_0-w'\sigma_{\bar{X}}-\delta\sigma),$$
$$d_1^{'}=\mathrm{F}_{n_1\alpha,n_1\gamma}(\mu_0-k'\sigma_{\bar{X}}-\delta\sigma),$$
$$a_2^{'}=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0+k\sigma_{\bar{X}}-\delta\sigma),$$
$$b_2^{'}=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0+w\sigma_{\bar{X}}-\delta\sigma),$$
$$c_2^{'}=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0-w'\sigma_{\bar{X}}-\delta\sigma),$$
$$d_2^{'}=\mathrm{F}_{n_2\alpha,n_2\gamma}(\mu_0-k'\sigma_{\bar{X}}-\delta\sigma).$$

According to the elementary properties of Markov Chains (Cinlar, 1975), $r'(I-Q)^{-1}$ provides the average number of transitions in each transient state before the true alarm signals, where $r^{'}=(p_{11},p_{12},p_{13},p_{14})$ is the vector of starting probability; I is the identity matrix of order 4; and Q is the transition matrix where the elements associated with the absorbing state have been deleted. The product of the average number of transitions in the transient state and the corresponding sampling interval, determines the period M,

$$M=r'(I-Q)^{-1}t, \tag{5}$$

where $t^{'}=(h_1,h_2,h_1,h_2)$ is the vector of the next sampling intervals corresponding to the four transient states. (T3) Let t_0 be the average amount of time wasted searching for the assignable cause when the process is in-control, and $E(FA)$ be the expected number of false alarms per cycle given by

$$E(FA)=r'(I-Q)^{-1}f \tag{6}$$

where $f^{'}=(\alpha_1,\alpha_2,0,0)$ gives the probability of producing false alarms in each transient state. Then, the expected length of searching period due to false alarm is given by $t_0E(FA)$.

(T4) The time to identify and correct the assignable cause following an action signal is a constant t_1.

As a result, the expected length of a production cycle can be aggregately represented as

$$E(T)=M+t_0E(FA)+t_1 \tag{7}$$

If one defines V_0 as the hourly profit earned when the process is operating in control state; V_1 as the hourly profit earned when the process is operating in out-of-control state; C_0 as the average search cost if the given signal is false; C_1 as the average cost to discover the assignable cause and adjust the process to in-control state; and s as the cost for each inspected measurement, then the expected net profit from a production cycle is given by

$$E(C)=V_0(1/\lambda)+V_1(M-1/\lambda)-C_0E(FA)-C_1-sE(N) \tag{8}$$

where $E(N)$ is the average number of observations (measurements) to signal (ANOS) during production cycle, and it is given by

$$E(N)=r'(I-Q)^{-1}\eta \tag{9}$$

where $\eta^{'}=(n_1,n_2,n_1,n_2)$ is the vector of the sample sizes corresponding to the four transient states taken for next sampling.

Finally, the expected loss per hour $E(L)$ is given by

$$E(L)=V_0-E(C)/E(T) \tag{10}$$

SEARCHING METHOD

The cost function (10) is a function of the process parameters (t_0, t_1, λ, δ, α, γ) the cost parameters (s, C_0, C_1, V_0, V_1), and the design parameters (n_1, n_2, h_1, h_2, w,

$w^{'}$, k, $k^{'}$). The economic design of a control chart for a particular application is to derive the design parameters that minimize the cost function, provided the process and cost parameters are given. In doing so, we apply the genetic algorithms (GAs) since the minimization problem has a non-linear objective function with the following constraints: n_1 and $n_2 \in \boldsymbol{Z}^+$ (positive integers), $0 \le n_1 \le n_2$, $h_1 \ge h_2 \ge 0$, $0 \le w \le k$, and $0 \le w^{'} \le k^{'}$ that result in mixed continuous-discrete decision variables and a discontinuous and non-convex solution space. If typical non-linear programming techniques are used to solve this optimization problem, they may be inefficient and time-consuming.

GAs are global search and optimization techniques motivated by the process of natural selection in a biological system (Davis, 1991; Goldberg, 1989). They have been used successfully in the optimization field of the parameters of quality control charts (Aparisi and Garicía-Díaz, 2003; He et al., 2002; He and Grigoryan, 2002; Chen, 2004). GAs are different from other search procedures in the following ways (Karr and Gentry, 1993): (1) GAs consider many points in the search space simultaneously, rather than a single point; (2) GAs work directly with strings of characters representing the parameter set, not the parameters themselves; and (3) GAs use probabilistic rules to guide their search, not deterministic rules. Since GAs considers many points in the search space simultaneously, there is a less chance of converging to local optima. Furthermore, in a conventional search, based on a decision rule, a single point is considered, and that is unreliable in multimodal space.

The primary distinguishing features of GAs include encoding, fitness function, selection mechanism, crossover mechanism, mutation mechanism, and culling mechanism. The algorithm for GAs can be formulated based on the following steps:

1) Randomly generate an initial solution set (population) of N individuals and evaluate each solution (individual) by a fitness function. Usually an individual is represented as a numerical string.
2) If the termination condition is not met, repeatedly do {Select parents from population for crossover.
Generate offspring; mutate some of the numbers; merge mutants and offspring into population;
cull some members of the population.}
3) Stop and return the best fitted solution.

When applying GAs to the minimization problem, a decimal encoding of individuals is adopted so that each individual in the form of decimal string represents a possible solution for (n_1, n_2, h_1, h_2, w, $w^{'}$, k, $k^{'}$). The fitness value of each individual is evaluated by its cost value. Based on the “elitist” strategy of the aforementioned algorithm, that is, the survival of the fittest, the evolution of a population of N individuals has been pursued. The termination condition is achieved when the number of generations is large enough or a satisfactory fitness value is obtained.

NUMERICAL COMPARISONS

In this area, the SS $\overline{X}$ charts and the VSSI $\overline{X}$ charts with symmetric or asymmetric control limits are compared with respect to their operating cost. In the SS $\overline{X}$ charts, $E(L)$ can be obtained by letting $n_1 = n_2$, $h_1 = h_2$, $w = w^{'} = 0$, and $k = k^{'}$, which implies that $p_{11} = p_{21} = 0$ and $p_{13} = p_{23} = p_{33} = p_{43}$ =0.

The process and cost parameters used for the comparisons borrows directly from Costa (2001). Table 1 gives the values of s, C_0, C_1, V_0, V_1, t_0, t_1, λ, and δ for each example. A genetic optimization package (EVOLVER 4.0.2) is coded to minimize the cost function $E(L)$ of the SS and VSSI $\overline{X}$ control charts. The following settings of control parameters for the package manipulation have been used: population size N=50; crossover probability =0.5; mutation rate =0.25; the number of generation =200,000. Since the minimum time-period between samples has to take into consideration to generate the required sample size, it requires $h_2 \ge 0.01$ in the following numerical comparisons.

Table 2 shows the optimal design parameters and the values of $E(L)$ for the SS $\overline{X}$ charts and VSSI $\overline{X}$ charts with symmetric or asymmetric control limits. The distribution of the underlying process data is assumed to follow a gamma distribution, $Gam(1,2)$, in these examples. The percent reduction (%) in $E(L)$ defined by

$$\frac{E(L)_{SS} - E(L)_{VSSI}}{E(L)_{SS}} \times 100$$

is also given in the table. Several findings from Table 2 are spelled out as follows.

1) The values of $E(L)$ for the VSSI $\overline{X}$ charts are consistently smaller than that for the corresponding SS $\overline{X}$ charts. In addition, asymmetric control limits utilized on the VSSI $\overline{X}$ charts has a larger percent reduction in $E(L)$ than symmetric control limits.

2) The VSSI $\overline{X}$ charts seem to require frequent sampling with smaller sample size and larger control limits than the SS $\overline{X}$ charts, which results in its provision of better protection against false alarm, $E(FA)$.

3) Percent reduction is small when δ, t_1, or λ is large 4) The lower control limit coincides with the lower warning limit when the asymmetric control limits are taken into

Table 1. The process and cost parameters borrowed from Costa (2001).

EX.	s	C_0	C_1	V_0	V_1	t_0	t_1	λ	δ
1	5	500	500	500	0	5	1	0.01	1
2	10	500	500	500	0	5	1	0.01	1
3	5	250	500	500	0	5	1	0.01	1
4	5	500	500	250	0	5	1	0.01	1
5	5	500	500	500	0	2.5	1	0.01	1
6	5	500	500	500	0	5	1	0.01	1.5
7	5	500	50	500	0	5	1	0.01	1
8	5	500	500	500	0	5	10	0.01	1
9	5	500	500	500	0	5	1	0.01	0.75
10	5	500	500	500	0	5	1	0.01	0.5
11	5	500	500	500	0	5	1	0.05	1

Table 2. The optimal designs of the SS and VSSI $\bar{X}$ charts with symmetric/asymmetric control limits for process data of Gamma distribution Gam(1,2).

EX.	**SS**					**VSSI**												
						Symmetric						**Asymmetric**						
	n	**h**	**k**	**E(FA)**	**E(L)**	**n1 / n2**	**h1 / h2**	**w / k**	**E(FA)**	**E(L)**	**%**	**n1 / n2**	**h1 / h2**	**w' / k'**	**w / k**	**E(FA)**	**E(L)**	**%**
1	17	6.07	2.82	0.1	43.50	7/13	4.12/0.01	1.43/3.74	0.03	35.31	19	7/14	4.30/0.09	3.74/3.74	1.19/3.68	0.04	34.37	21
2	14	7.88	2.53	0.16	54.24	7/12	5.87/0.01	1.41/3.40	0.05	45.4	16	7/12	6.10/0.09	3.74/3.74	1.14/3.34	0.05	43.64	20
3	16	5.87	2.77	0.12	43.24	7/14	4.08/0.01	1.47/3.74	0.03	35.23	19	7/12	4.29/0.01	3.74/3.74	1.13/3.72	0.03	33.56	22
4	14	7.93	2.57	0.14	29.81	7/13	5.87/0.01	1.45/3.50	0.04	25.19	15	7/11	6.10/0.01	3.74/3.74	1.13/3.43	0.04	24.04	19
5	15	5.72	2.61	0.18	42.03	7/13	4.07/0.01	1.44/3.61	0.05	34.88	17	7/11	4.27/0.01	3.74/3.74	1.09/3.54	0.05	33.20	21
6	10	4.70	3.26	0.06	34.51	7/8	4.07/0.01	2.03/3.74	0.03	29.93	13	7/9	3.95/0.01	3.74/3.74	1.98/4.26	0.01	29.50	15
7	17	6.04	2.83	0.1	39.22	8/15	4.30/0.01	1.55/3.81	0.03	31.14	21	9/14	4.66/0.01	4.24/4.24	1.35/3.75	0.02	30.38	23
8	16	6.13	2.77	0.11	79.46	7/13	4.28/0.01	1.43/3.74	0.03	72.38	9	7/13	4.44/0.01	3.74/3.74	1.17/3.76	0.03	70.91	11
9	26	7.47	2.6	0.13	52.12	11/22	5.10/0.01	1.40/3.44	0.04	43.31	17	11/18	5.43/0.01	4.69/4.69	1.00/3.36	0.05	40.57	22
10	45	9.85	2.28	0.22	68.01	23/41	7.39/0.01	1.35/2.94	0.07	58.96	13	21/35	7.50/0.02	6.48/6.48	0.93/2.89	0.07	54.06	21
11	16	2.86	2.73	0.05	114.74	8/14	2.12/0.01	1.53/3.47	0.02	102.09	11	8/13	2.18/0.01	4.00/4.00	1.26/3.44	0.02	99.59	13

consideration for the VSSI $\bar{X}$ chart. We call the VSSI $\bar{X}$ chart with such a feature the half-VSSI $\bar{X}$ chart for convenience. The minimum sample size and longest sampling interval are used for the next sampling if the current sample mean falls into the lower side of the chart. Table 3 continues the first example and shows the optimal design parameters under various gamma distributions (γ =0.5, 1, 2, 3, 4, 5, 10, 20, 30 while holding

Table 3. The optimal designs of the SS and VSSI $\bar{X}$ charts with symmetric/asymmetric control limits for Gamma distributed process data with various parameters.

Process data			SS					VSSI												
								Symmetric						Asymmetric						
Distribution	α3	α4	n	h	k	E(FA)	E(L)	n1 / n2	h1 / h2	w / k	E(FA)	E(L)	%	n1 / n2	h1 / h2	w' / k'	w / k	E(FA)	E(L)	%
Gam(1,.5)	2.83	15.00	18	6.61	2.87	0.13	44.46	20/21	6.60/0.01	2.40/3.16	0.07	42.91	3.5	19/21	6.54/0.03	3.08/3.08	2.30/3.90	0.02	41.33	7.0
Gam(1,1)	2.00	9.00	17	6.26	2.82	0.12	43.90	12/15	5.42/0.01	1.76/3.46	0.05	37.40	14.8	12/16	5.33/0.01	3.46/3.46	1.64/3.82	0.02	36.75	16.3
Gam(1,2)	1.41	6.00	17	6.07	2.82	0.10	43.50	7/13	4.12/0.01	1.43/3.74	0.03	35.31	18.8	7/14	4.30/0.09	3.74/3.74	1.19/3.68	0.04	34.37	21.0
Gam(1,3)	1.15	5.00	17	6.02	2.83	0.09	43.36	7/14	4.02/0.01	1.48/3.75	0.03	35.24	18.7	7/13	4.20/0.01	4.58/4.58	1.17/3.72	0.03	33.57	22.6
Gam(1,4)	1.00	4.50	16	5.77	2.8	0.10	43.27	7/14	4.00/0.01	1.49/3.71	0.02	35.20	18.7	7/13	4.17/0.01	5.28/5.29	1.18/3.68	0.02	33.51	22.6
Gam(1,5)	0.89	4.20	16	5.75	2.8	0.10	43.22	6/13	3.71/0.01	1.41/3.68	0.03	35.11	18.8	7/13	4.14/0.01	5.59/5.69	1.19/3.65	0.02	33.43	22.7
Gam(1,10)	0.63	3.60	16	5.68	2.82	0.09	43.12	6/13	3.65/0.01	1.43/3.60	0.02	34.98	18.9	7/12	4.11/0.01	6.55/8.37	1.15/3.53	0.02	33.29	22.8
Gam(1,20)	0.45	3.30	16	5.65	2.82	0.09	43.08	6/13	3.61/0.01	1.44/3.55	0.02	34.90	19.0	5/11	3.52/0.01	8.04/10.00	1.01/3.52	0.02	32.22	25.2
Gam(1,30)	0.37	3.20	16	5.64	2.83	0.09	43.06	6/13	3.59/0.01	1.45/3.54	0.02	34.87	19.0	4/11	3.16/0.01	6.70/10.52	0.97/3.52	0.02	31.92	25.9
Norm(2,2)	0.00	3.00	15	5.34	2.81	0.09	43.04	6/13	3.55/0.01	1.46/3.46	0.02	34.83	19.1	---	---	---	---	---	---	---

α_3 and α_4 symbolize the skewness and kurtosis of distributions, respectively.

α =1) as well as a contrastive normal distribution, $Norm(2,2)$, which has the same mean and variance as $Gam(1,2)$. The results of Table 3 show that;

1) Decreasing the value of γ will reduce the percent reduction. In other words, skewed non-normal distribution will limit the saving per hour operation in the VSSI $\bar{X}$ chart, especially when the distribution is highly skewed.
2) Asymmetric control limits on the VSSI $\bar{X}$ chart offers more robust protection against false alarm to skewed non-normality in comparison with symmetric one.
3) The half-VSSI $\bar{X}$ chart is well suited to highly skewed process data. Oppositely, the ordinary VSSI $\bar{X}$ chart, where the control limit and warning limit did not coincide, is well suited to near normally distributed process data.

CONCLUDING REMARKS

An economic design of the VSSI control chart for control of those processes whose observations are drawn from positively skewed distributions, has been presented to achieve a reasonable balance between the cost of administering and the cost due to delays in trying to detect changes in the mean. In the modeling procedure, the assumption of the time the process remains in control following an exponential distribution allowing the probability structure on the process the same as the Markov chains. Using the well-known properties of the Markov chains, statistical measures of performance, such as AATS and average number of observations to signal (ANOS) are obtained.

Such measures were put together to arrive an expression of average loss per unit time, which is regarded as a cost function of the process parameters, cost parameters, and design parameters.

The cost function was minimized by genetic algorithms to acquire the optimal design parameters of the chart, given the process parameters and cost parameters. Numerical comparisons between the SS $\bar{X}$ chart and VSSI $\bar{X}$ chart with symmetric or asymmetric control limits have been made over 11 examples and 9 gamma distributions, each of which is positively skew with various degrees.

The result of the numerical comparisons reveals that the VSSI $\bar{X}$ chart with asymmetric control limits can be more efficient than both the SS $\bar{X}$ chart and the VSSI $\bar{X}$ chart with symmetric control limits in terms of average loss per unit time. Another advantage of putting asymmetric control limits to use is that it provides fairly robust protection against false alarm to highly skewed non-normality.

Also, the result of the economic designing for the VSSI $\bar{X}$ chart may be affected by non-normality. The loss savings resulting from the use of the VSSI $\bar{X}$ charts are substantially limited when monitoring highly skewed process data.

REFERENCES

Aparisi F, Garcia-daiz JC (2003). Optimization of EWMA and MEWMA control charts using genetic algorithms. Comput. Oper. Res. 31:1437-1454.

Bai DS, Lee KT (1998). An economic design of variable sampling interval X-bar control charts. Int. J. Prod. Econ., 54: 57-64.

Baxley JR (1996). An application of variable sampling interval control charts. J. Qual. Technol. 27:275-282.

Chang YS, Bai DS (2001). Control charts for positively-skewed populations with weighted standard deviations. Qual. Reliab. Eng. Int. 17:397-406.

Chen YK (2004). Economic design of $\bar{X}$ control charts for non-normal data using variable sampling policy. Int. J. Prod. Econ. 92:61-74.

Chou CY, Chen CH, Liu HR (2000). Economic-statistical design of $\bar{X}$ charts for non-normal data by considering quality loss. J. Appl. Statist. 27(8):939-951.

Cinlar E (1975) Introduction to Stochastic Processes, Prentice Hall, Englewood Cliffs: NJ.

Costa AFB (1994). $\bar{X}$ control chart with variable sample size. J. Qual. Technol. 26:155-163.

Costa AFB (1997). $\bar{X}$ charts with variable sample sizes and sampling intervals. J. Qual. Technol. 29:197-204.

Costa AFB (2001). Economic design of $\bar{X}$ charts with variable parameter: the Markov chain approach. J. Appl. Statist. 28:875-885.

Davis L (1991). Handbook of Genetic Algorithms, Ed. Van Nostrand Reinhold: NY.

Duncan AJ (1956). The economic design of $\bar{X}$ charts used to maintain current control of a process. J. Am. Stat. Assoc. 51:228-242.

Goldberg DE (1989). Genetic Algorithms in Search, Optimization, and Machine Learning Reading, MA: Addison-Wesley.

He D, Grigoryan A (2002). Construction of double sampling S-control charts for agile manufacturing. Qual. Reliab. Eng. Int. 18:343-355.

He D, Grigoryan A, Sigh M (2002). Design of double and triple-sampling $\bar{X}$ control charts using genetic algorithms. Int. J. Prod. Res. 40:1387-1404.

Karr CL, Gentry EJ (1993). Fuzzy control of pH using genetic algorithms. IEEE Trans. Fuzzy Syst. 1(1):46-53.

Lin YC, Chou CY (2005). On the design of variable sample size and sampling intervals $\bar{X}$ charts under non-normality. Int. J. Prod. Econ. 96:249-261.

Montgomery DC (1980). The economic design of control charts: a review and literature survey. J. Qual. Technol. 12:75-87.

Park C, Reynolds JR (1994). Economic design of a variable sample size $\bar{X}$ -charts. Communi. Statist. Simul. Comput. 23:467-483.

Park C, Reynolds JR (1999). Economic design of a variable sample rate $\bar{X}$ charts. J. Qual. Technol. 31(4):427-443.

Pignatiello JJ, Tsai A (1988). Optimal economic design of control chart when cost model parameters are not known. IIE Trans. 20(1):103-110.

Prabhu SS, Runger GC, Keats JB (1993). An adaptive sample size $\bar{X}$ chart. Int. J. Prod. Res. 31:2895-2909.

Prabhu SS, Montgomery DC, Runger GC (1994). A combined adaptive sample size and sampling interval $\bar{X}$ control chart. J. Qual. Technol. 26:164-176.

Rahim MA (1985). Economic model of $\bar{X}$ chart under non-normality and measurement errors. Comput. Oper. Res. 12(3):291-299.

Rendtel U (1990). CUSUM schemes with variable sampling intervals and sample sizes. Statist. Papers 31:103-118.

Reynolds JR, Amin RW, Arnold JC, Nachlas JA (1988). $\bar{X}$ -charts with variable sampling interval. Technometrics 30:181-192.

Reynolds JR, Arnold JC (1989). Optimal one-side Shewhart control chart with variable sampling intervals. Sequent. Anal. 8(1):51-77.

Reynolds JR (1996). Variable sampling interval control charts with sampling at fixed times. IIE Trans. 28:497-510.

Ross SM (1970). Applied Probability Models with Optimization Applications. Holden-Day, San Francisco, CA.

Runger GC, Pignatiello JR (1991). Adaptive sampling for process control. J. Qual. Technol. 23(2):133-155.

Schilling EG, Nelson PR (1976). The effect of non-normality on the control limits of $\bar{X}$ charts. J. Qual. Technol. 8(4):183-188.

Tagaras G (1989) Economic X-bar charts with asymmetric control limits. J. Qual. Technol. 21(3):147-154.

Vance LC (1983). A bibliography of statistical quality control chart techniques, 1970-1980. J. Qual. Technol. 15:59-62.

Woodall WH (1986). Weakness of the economic design of control charts. Technometrics 28(4):408-409.

Yourstone SA, Zimmer WJ (1992). Non-normality and design of control charts for averages, Decis. Sci. 23:1099-1113.