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A new two-stage data envelopment analysis (DEA) model for evaluating the branch performance of banks

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The purpose of this paper is to provide a framework for evaluating the overall performance of bank branches in terms of profitability efficiency and effectiveness by means of a two stage data envelopment analysis (DEA) model. DEA is a linear programming problem approach for evaluating efficiency of decision making units (DMU) that have multiple inputs and outputs. In this paper, we use a two stage DEA model for efficiency evaluation of DMUs. In the two stage model all the outputs from the first stage are the only inputs to the second stage, in addition to the inputs to the first stage and the outputs from the second stage. The outputs from the first stage to the second stage are called intermediate measures. Many papers have regressed non parametric estimate of efficiency in the two stage DEA procedure. All of these studies have several problems in the benchmark or projection unit and present the DMU with relative efficiency. Most of these papers do not present an efficient benchmark unit and cannot evaluate the relative efficiency unit. In this paper we review these studies and then create a two stage model that does not have these problems. Our model presents the efficient benchmark unit and also presents DMU with relative efficiency. In the next step we apply our two stage model in bank branches of a large commercial bank in Iran. This study aggregates profitability efficiency and effectiveness into the overall performance. Some relations between profitability efficiency and effectiveness are very important. Without these relations we cannot obtain superior insight about overall performance. So we assume that profitability efficiency and effectiveness are dependent, and with this assumption we design a two stage DEA model. This study shows the importance of profitability efficiency and effectiveness in the overall performance in bank branches in Iran.

Key words: Banking, process efficiency, organizational effectiveness, data envelopment analysis (DEA), banks, banks performance.

INTRODUCTION

Nowadays, due to the continuous change in economic conditions, evaluating the performance of industrial and economical units has become one of the most important factors in their improvement. The banking sector has experienced profound changes over the past two decades or so. Globalization, deregulation, financial innovation, automation and speed of exchanges have been significant, leaving their impact on the performance of the banking sector (Akhtar, 2010). So to maintain viability and continuous improvement, bank branches must evaluate their performance, efficiency and productivity. The banks are forced to evaluate their branch networks efficiency and effectiveness in order to improve the overall performance of their branches. Effectiveness and efficiency are the central terms used in the overall performance of the organizations (Kumer and Gulati, 2010). Performance can be defined as an appropriate combination of efficiency and effectiveness (Kumer and Gulati, 2010). Drucker (1977) distinguished efficiency and effectiveness. He defined efficiency as “doing things right” and effectiveness as “doing the right things”. So we can say that efficiency and effectiveness are two components of overall performance.

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Industrial and economical units should be evaluated by utilizing scientific methods in order to evaluate and improve the performance and structuring a good position in comparison to other units.

Performance and measurement are two familiar words in the literature of evaluation. Performance is predetermined parameters and measurement sounds like the ability to monitor events and activities in a meaningful way, so performance measurement can be defined as the process of quantifying the effectiveness and efficiency of action (Neely et al., 1995). Several approaches are used in order to measure the performance, some of them include: balanced scorecard (Kaplan and Norton, 1992), the performance measurement matrix (Keagan et al., 1989), performance measurement questionnaire (Dixon et al., 1990), criteria for measurement system design (Globerson, 1985) and computer aided manufacturing approaches. Although, effective, we can pointout some shortages such as: lack of strategic focus, forcing managers to encourage local optimization rather than seeking continuous improvement, and they are also unable to provide adequate information about competitors.

The main aim of this paper is to study bank profitability efficiency, effectiveness and overall performance by utilizing two stage DEA model. Charnes et al. (1978) proposed the DEA model to produce the efficiency frontier based on the concept of Pareto optimum. DEA is a nonparametric powerful tool in analyzing efficiency with multiple inputs and outputs which can consider both qualitative and quantitative measures.

The two stage concept in DEA dates back to the work by Schinnar et al. (1990) to evaluate the performance of mental health care programs (Kao and Hwang, 2008) for a recent survey). Kao and Hwang (2008) modify the standard DEA model by determining the relationship of the two stages within the whole process. Under their framework, the whole efficiency can be decomposed into the product of the efficiencies of the two sub-processes. Note that such efficiency decomposition is not available in the standard DEA approach of Seiford and Zhu (1999) and the two stage approach of Chen and Zhu (2004) and Chen et al. (2009).

The two stage DEA model keeping each stage independent from one another was applied by Wang et al. (1997), Rho and An (2007), and Tsolas (2010). Other works appearing in the banking literature are those by Seiford and Zhu (1999) to divide the production process of a commercial bank into marketability and profitability, analyze the banking operation by Chen (2002), evaluate the profitability and marketability efficiencies of large banks by Luo (2003), and measure the performance of Taiwan’s commercial banks by Ho and Zhu (2004). Related studies analyze the financial efficiency of firms by Zhu (2000), measure the efficiency of the American Major Baseball League by Sexton and Lewis (2003), measure information technology’s indirect impact on firm performance by Chen and Zhu (2004), decompose the efficiency of non-life insurance companies in Taiwan into the product of the efficiencies of the two sub-processes by Kao and Hwang (2008), and examine relations and the equivalence between the approach of Chen and Zhu (2004) and Kao and Hwang (2008) by Chen et al. (2009).

These studies have several problems, so we have reviewed some of them. Kao and Hwang (2008) proposed the two stage model based on DEA for evaluating the efficiencies of non-life insurance companies in Taiwan. Their models and approach do not present overall information about DMUs. For example, in their model, none of the DMUs are relatively efficient, and there is no efficient unit. Their model doesn’t determine the projection unit. On the other hand, the produced projection is not efficient either, and the model in variable return to scale (VRS) mood is nonlinear.

Chen et al. (2009) modified the Kao and Hwang (2008) model, but their modal has the same problem. The model proposed by Chen et al. (2009) does not determine the efficient projection unit, and the produced projection is not efficient. Also, in this model results, none of the DMUs are relatively efficient.

Chen et al. (2009) in an article entitled "additive efficiency decomposition in two-stage DEA" proposed the additive model to evaluate efficiency. They developed their model in constant and variable return to scale (CVR and VRS) mood. Their model is linear in both mood, but none of the DMUs are relative efficient, and the model does not present information about the projection unit. This model was modified by Cook et al. (2010), who proposed a multi stage model by modifying the Chen et al. (2009) model. But this model cannot calculate the relative efficiency.

The model proposed by Tone and Tsutsui (2009), determine the efficient projection units, but none of the DMUs are relative efficient. So the main aim of this article is to build the models that do not have these problems. These problems are solved in our model; the efficient projection presented, the relative efficiency presented for one of the DMUs, the model can be developed into the CRS and VRS mood, and the model is linear.

Different sections of this paper can be structured as: a brief description on efficiency, effectiveness and overall performance, followed by a DEA review and related applications and concepts with performance evaluation. The next section contains an explanation of methodology and two stage DEA models to overall performance and demonstrates their applications in bank branches in Iran. Finally, some conclusions are provided about the research.

**EFFICIENCY, EFFECTIVENESS AND PERFORMANCE EVALUATION**

Drucker (1977) distinguished efficiency and effectiveness.
He defined efficiency as “doing things right” and effectiveness as “doing the right things”. In this terminology, efficiency is defined as the ability of an organization to attain the outputs with the minimum level of inputs, and effectiveness is defined as the ability of an organization to reach its goals and objectives. So we can say that efficiency and effectiveness are two components of overall performance. Ho and Zhu (2004) introduced overall performance as a product of efficiency and effectiveness. He defined Return on assets (ROA) as performance (Ho and Zhu, 2004), and obtained performance from the product of Profit margin ratio as effectiveness and Total assets turnover ratio as efficiency. Ho and Zhu (2004) introduced a model with two stages to obtain overall performance. Other researchers have combined this model with the DEA model such as Tsolas (2010), Kumar and Gulati (2010), Keh et al. (2006), and Garcia-Sanchez (2007). In these studies researchers designed a model with two stages, one stage from efficiency and another from effectiveness. Efficiency and effectiveness score were then obtained with a DEA model separately. Finally, they determined the overall performance by product the efficiency and effectiveness score.

We can categorize measurement tools into 2 types which are used to evaluate different tools. Gap based techniques like Spider and Radar diagram and Z chart are used in parametric analysis. These tools are very graphical in nature, which makes them more understandable; however, when an analyst needs to integrate different elements into one complete picture they are not be useful.

Ratio is another parametric method which has been used in different areas and can compute and calculate the relative efficiency of output versus input easily. But different ratios provide different interpretations and integrating the entire set of ratios into a single one would be difficult.

Another robust tool in performance measurement to analyze data is analytical hierarchy process. In order to convert various weighted scores into a single score, AHP utilizes the personal views of experts. Although it provides managerial insights in quantifying measures, it is subjugated to a high degree of subjectivity.

Regression is a statistical method which provides meaningful relationships for decision makers, but its main limitation is analyzing one single output at a time, which means by adding another criterion the approach has to be repeated. Also, it considers average values instead of real ones that may not occur in the real world. This means regression considers one method in combining different factors in all firms.

The other non parametric tool in evaluation is data envelopment analysis (DEA). DEA considers both qualitative and quantitative measures. This is why managers would be able to provide reasonable judgments based on resource usage. DEA works based on the concept of efficient frontier suggested by Farrell (1957) and can consider multiple inputs and outputs by dividing the weighted sum of outputs by a weighted sum of inputs. Efficiency numbers are between 0 and 1. In ranking, DMU with an efficiency number of 1 is the most efficient, and the DMU with an efficiency number of 0 is the least efficient. Although it has a wide range of covering shortages of other tools, utilizing DEA reveals some limitations as well:

1. Meaningful results in DEA require the availability of data about different inputs and outputs. But sometimes some of these data are unreachable or sometimes firms are not interested in sharing data.
2. Number of DMUs being compared cannot be less than a certain limit. By decreasing the number of DMUs, we would have an increase in the number of efficient DMUs too.
3. Since DEA considers the same strategic goals and objectives for all of the DMUs in the firm, we cannot compare DMUs which are different in these aspects.
4. It is so critical to interpret the results of DEA precisely. Although it provides reasonable rankings for efficient DMUs, DEA is unable to address the reasons for inefficiencies or provide suitable solutions.

As mentioned before, each tool has its own profits and shortages together, so it is difficult to choose the best one, however, the distinguished advantages of DEA can convince us to utilize this tool. First of all, it is important to choose a simple tool that is easy to work with (Maskell, 1991; Sheridan, 1993; Detoro, 1995; Blossom and Bradley, 1998). Also, its reliability in supporting the process of decision making is so important. DEA is a robust, standardized and transparent methodology that can fulfill all the requirements above. It also provides some other features which make it an appropriate tool in order to measure performance. Some of them are listed as follows:

1. DEA can process multiple elements (Charnes et al., 1978).
2. DEA does not need to specify the relationships among the performance measures.
3. It utilizes the concept of efficient frontier, which is an empirical standard of excellence.
4. This toll can analyze both the qualitative and quantitative measures simultaneously.
5. There is no need to assume priority estimates, so the reliability of the results increases.
6. It can provide valuable information about inefficient DMUs as well as efficient ones.
7. Its flexibility enables the researcher to mold it to other analytical methods such as statistical analysis and other multi criteria decision making techniques (Zhu et al., 2004; Golany, 1988; Spronk and Post, 1999).

There are numerous studies on bank efficiency evaluation...

Several researchers have worked on profitability efficiency such as Athanassopoulos (1997), Oral et al. (1992), Oral and Yolalan (1990), Soteriou and Zenios (1999), and Manandhar and Tang (2002). Profitability efficiency evaluates the ability of branches to minimize inputs for the level of outputs generated. Profitability efficiency is obtained from the ratio of the weighted sum of revenue to the weighted sum of expenses (Giokas, 2008a, 2008b).

In recent years, few studies have been completed which explicitly recognize the efficiency and effectiveness as two mutually exclusive components of the overall performance of an organization. For example, Schinnar et al. (1990), Karlaftis (2004), Ho and Zhu (2004, 2007), Mouzas (2006), Keh et al. (2006), Garcia-Sanchez (2007), Yu and Lin (2007), Rho and An (2007), and Kao and Hwang (2008).

Data envelopment analysis

DEA is a linear programming based methodology which can calculate multiple inputs and outputs and can also evaluate DMUs both qualitatively and quantitatively. DMU, which can be related to different firms or the condition of the same firm over time, stands for decision making unit.

DEA was first proposed by Charnes, Cooper and Rhodes (CCR) in 1978. The evolutionary form of CCR model was suggested by Banker et al. (1984). In subsequent years, several models were developed by a large number of researchers. Orientation, disposability, diversification, and return to scale are different aspects that can be seen in these models.

DEA utilizes frontier function in measuring efficiency. It then introduces a set of efficient and inefficient units. The analysis of inefficient units has two aspects. First, it can show the maximum input level in order to attain a given amount of outputs. Secondly, it can also show the highest output level attained for a given amount of inputs. These approaches are called "minimal principle of efficiency" and "maximum principle of efficiency" respectively.

CCR and BCC models

Efficiency is derived from physical and engineering science and demonstrates the relationships between inputs and outputs. Ratio definition of efficiency was introduced by Charnes et al. (1978), and is also known as the CCR ratio definition. CCR is the classical science of applying single-input to single-output ratio definition to multiple outputs and inputs without requiring pre-assigned weights.

We have been dealing with the pairs of positive input and output vectors \((x_j, y_j)\) of n DMUs. We will call a pair of such semi positive input \(x \in R^n\) and output \(y \in R^n\) an activity and express them by notation \((x, y)\). The set of feasible activities is called the production possibility set and is denoted by \(P\). We postulate the following properties of \(P\): The observed activities \((x_j, y_j)\) belong to \(P\).

If activities \((x, y)\) belong to \(P\), then the activities \((tx, ty)\) belong to \(P\) for any positive scalar \(t\). we call this property the constant return to scale assumption. For activities \((x, y)\) in \(P\), any semi positive activity \((\bar{x}, \bar{y})\) with \(x \geq \bar{x}\) and \(\bar{y} \leq y\) is included in \(P\). That is, any activity with input of no less than \(x\) in any component and with output no greater than \(y\) in any component is feasible.

Any semi positive linear combination of activities in \(P\) belongs to \(P\).

Arranging the data sets in matrices \(x = (x_i)\) and \(y = (y_i)\), we can define the PPS satisfying 1 through 4 by:

\[
P = \{(x, y) | x \geq x_\lambda, y \leq y_\lambda, \lambda \geq 0\}
\]  

(1)

Where \(\lambda\) is a semi positive vector in \(R^n\). Based on the matrix \((X, Y)\), the CCR model with row vector \(v\) for input and row vector \(u\) as output was formulated.

Let \(x_{ij}, i = 1, ..., m\) and \(y_{rj}, r = 1, ..., s\) be the \(i\)th input and \(r\)th output, respectively, of the \(j\)th DMU, \(j = 1, ..., n\). The DEA model for measuring the relative efficiency of \(DMU_o\) under an assumption of constant returns to scale is the CCR model (Charnes et al., 1978):

\[
\text{Max} \sum_{r=1}^{s} u_r y_{r o}
\]  

(2)
We can say that the dual model is:

\[ \begin{align*}
    \sum_{i=1}^{m} v_i x_{io} &= 1 \\
    \sum_{r=1}^{k} u_r y_{rf} - \sum_{i=1}^{m} v_i x_{ij} &\leq 0 \quad j = 1, \ldots, n \\
    u_r &\geq 0 \quad r = 1, \ldots, s \\
    v_i &\geq 0 \quad i = 1, \ldots, m
\end{align*} \] (3)

The following BCC input oriented value-based model (Banker et al., 1984) can be used to assess efficiencies.

\[ \begin{align*}
    \text{Min } \theta \\
    \sum_{j=1}^{n} \lambda_j x_{ij} &\leq \theta x_{io} \quad i = 1, \ldots, m \\
    \sum_{j=1}^{n} \lambda_j y_{rj} &\geq y_{ro} \quad r = 1, \ldots, s \\
    \lambda_j &\geq 0 \quad j = 1, \ldots, n
\end{align*} \] (4)

The dual model is:

\[ \begin{align*}
    \text{Max } \sum_{r=1}^{k} u_r y_{ro} + w \\
    \sum_{i=1}^{m} v_i x_{io} &= 1 \\
    \sum_{r=1}^{k} u_r y_{rf} - \sum_{i=1}^{m} v_i x_{ij} + w &\leq 0 \quad j = 1, \ldots, n \\
    u_r &\geq 0 \quad r = 1, \ldots, s \\
    v_i &\geq 0 \quad i = 1, \ldots, m
\end{align*} \] (5)

We can say that the dual model is:

\[ \begin{align*}
    \sum_{j=1}^{n} \lambda_j y_{rj} &\geq y_{ro} \quad r = 1, \ldots, s \\
    \sum_{j=1}^{n} \lambda_j &= 1 \\
    \lambda_j &\geq 0 \quad j = 1, \ldots, n
\end{align*} \]

Benchmarking is particularly practical when there is no objective standard available for defining effective performance. So it is used in managing services because it is difficult to define the service standard in comparison with the manufacturing standard.

One of the ways a benchmark is presented is by applying a DEA model. Consider the set of DMUs including DMUj j=1,...,n with m inputs and s outputs such that \(X_i=(x_{i1},...,x_{im}), Y_j=(y_{j1},...,y_{js})\) are inputs and outputs, respectively. Let \(\theta^*\) be optimal solution of the CCR model and \(DMU_0\) is under evaluation, and \(S^-\) and \(S^+\) be optimal solution of model (6).

\[ \begin{align*}
    \text{Max } &\ 1s^- + 1s^+ \\
    \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- &= \theta^* x_{io} \quad i = 1, \ldots, m \\
    \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ &= y_{ro} \quad r = 1, \ldots, s \\
    \lambda_j &\geq 0 \quad j = 1, \ldots, n \\
    s_i^- &\geq 0 \quad i = 1, \ldots, m \\
    s_r^+ &\geq 0 \quad r = 1, \ldots, s
\end{align*} \] (6)

Then the reference set is defined as:

Reference set=\{ \(\lambda_i^+\) \} and \(\tilde{X}_o = \theta^* X_o - S^-\) and \(\tilde{Y}_o = Y_o + S^+\) are benchmarks or improved activities gained from DEA.

**PROPOSAL MODEL**

Consider a two stage model that is shown in Figure 1. Suppose, we have n DMUs. Each DMU has m inputs to the first stage, \(X_{ij}\), \(i = 1,2,\ldots, m\), and D outputs to the first stage, \(Z_{di}\), \(d = 1,2,\ldots, D\). These outputs then
Figure 1. A two stage model.

\[ X_{ij}, \quad i = 1, 2, ..., m \] 
\[ Z_{dj}, \quad d = 1, 2, ..., D \] 
\[ Y_{rf}, \quad r = 1, 2, ..., s \]

Property 1. The observed activities \((x_j, y_j)\) \((j = 1, ..., n)\) belong to \(P\). According to this property, we have:

\[ P_{two-stage} = \begin{cases} 
(x_j, y_j) & \text{if } i = 1, ..., n 
\end{cases} \]

\[ P_{two-stage} = \begin{cases} 
(x_j, y_j) & \text{if } i = 1, ..., n 
\end{cases} \]

Property 2. Any semi positive linear combination of activities in \(P\) belongs to \(P\). According to the convexity axiom, we have:

\[ P_{two-stage} = \begin{cases} 
(\sum_{j=1}^{n} \lambda_j x_j, \sum_{j=1}^{n} \lambda_j y_j) & \text{if } \sum_{j=1}^{n} \lambda_j = 1 
\end{cases} \]

Property 3. For activities \((x, z)\) in \(P\), any semi positive activity \((\bar{x}, \bar{z})\) with \(x \leq \bar{x}\) and \(z \leq \bar{z}\) is included in \(P\) and for an activities \((z, y)\) in \(P\), any semi positive activity \((\bar{z}, \bar{y})\) with \(z \leq \bar{z}\) and \(y \leq \bar{y}\) is included in \(P\). With this property we have:

\[ P_{two-stage} = \begin{cases} 
(x_j, y_j) & \text{if } i = 1, ..., n 
\end{cases} \]

Property 4) If activities \((x, y)\) belong to \(P\), then the activities \((tx, ty)\) belong to \(P\) for any positive scalar \(t\). We call this property the constant return to scale assumption. With accept this assumption the PPS to become changed into:

\[ P_{two-stage} = \begin{cases} 
(x_j, y_j) & \text{if } i = 1, ..., n 
\end{cases} \]

Now, to build the model with the use of PPS definition, we have:

\[ (\theta x, z, y) \in P_{two-stage} \]

With these properties and the definition of PPS, the two stage DEA model is:

\[ \text{Min } \theta \]
The dual of this model is:

\[
\begin{align*}
\text{Max } & \quad uy_0 + wz_0 \\
\text{S.t.} & \quad \Sigma_{r=1}^2 u_r y_{rj} + \Sigma_{d=1}^m w_d z_{dj} - \Sigma_{i=1}^n v_i x_{ij} \leq 0 \\
& \quad u_r \geq 0, v_i \geq 0
\end{align*}
\]  

(13)

Theorem 1. If \( DMU_0 \) is under evaluation, then at least one of the constraints in model 13 in optimality is binding.

Proof: Let \((\bar{v}, \bar{w}, \bar{u})\) be an optimal solution for model 13. It is evident that if an optimal solution exists, then an optimal extreme point exists. Let \((v^*, w^*, u^*)\) be an optimal extreme solution for model 13, so it lies on \((D+m+s)\) linearly independent hyper plans.

\(v x_0 = 1\) is binding at every feasible solution, so this constraint is tight at optimal solution. Therefore, we are searching for \(D+m+s-1\) binding hyper plan.

Every \(v^*_i\) cannot be equal to zero, because in this case \(v^* x_0 = 0 \times x_0 = 1\).

So at most, the \((m+s-1)\) of variables in optimality are binding; consequently the \(d-1\) of constraints \(\Sigma_{r=1}^2 u_r y_{rj} + \Sigma_{d=1}^m w_d z_{dj} - \Sigma_{i=1}^n v_i x_{ij} \leq 0\) in optimality should be binding.

Theorem 2: If in evaluation the \( DMU_0 \) constraint related to \( DMU_i \) is binding, then \( DMU_i \) is efficient.

Proof: Let \((\bar{v}, \bar{w}, \bar{u})\) be an optimal solution for model 13 in evaluating \( DMU_0 \) and \( u^* y_1 + w^* z_0 - v^* x_i = 0 \). The efficiency of \( DMU_i \) is gained by solving the following model:

\[
\begin{align*}
\text{Max } & \quad uy_1 + wz_i \\
\text{S.t.} & \quad \begin{cases}
vy_i = 1 \\
u > 0, w > 0, v > 0
\end{cases}
\end{align*}
\]  

(14)

If \(v^* x_0 = 1\), then \((v^*, w^*, u^*)\) is a feasible solution for model 13 and the proof is completed.

Otherwise

\(0 < v^* x_i = \alpha\)

So we have:

\[
\frac{v^*}{\alpha} x_i = 1 \text{ and } \frac{1}{\alpha} u^* y_j + \frac{1}{\alpha} w^* z_j - \frac{1}{\alpha} v^* x_i \leq 0 \quad \forall j
\]

These relations imply that \((\frac{v^*}{\alpha}, \frac{w^*}{\alpha}, \frac{u^*}{\alpha})\) is a feasible solution for model 13 and the objective value is equal to 1. So \( DMU_i \) is efficient.

Theorem 3: The improved activating \((\bar{x}_0, \bar{z}_0, \bar{y}_0)\) is efficient.

Proof: The efficiency of \((\bar{x}_0, \bar{y}_0)\) is evaluated by solving the following model:

\[
\begin{align*}
\text{Min } & \quad \theta \bar{x}_0 \\
\text{S.t.} & \quad \begin{cases}
x \lambda \leq \theta \bar{x}_0 \\
z \lambda = \bar{z}_0 \\
y \lambda \geq \bar{y}_0 \\
\lambda \geq 0
\end{cases}
\end{align*}
\]  

(15)

Let the optimal objective value be \(\hat{\theta}\), so we solve model 16 to gain the max-slack solution for improved activating.

\[
\begin{align*}
\text{Max } & \quad 1s^- + 1s^+ \\
\text{S.t.} & \quad \begin{cases}
s^- - \hat{\theta} \bar{x}_0 - x \lambda = 0 \\
s^+ = y \lambda - \bar{y}_0 \\
\bar{z}_0 = z \lambda \\
\lambda \geq 0, s^- \geq 0, s^+ \geq 0
\end{cases}
\end{align*}
\]  

(16)
Suppose that an optimal solution for \((x^*_0, z^*_0, y^*_0)\) is \((\tilde{\theta}, \tilde{\lambda}, \tilde{s}^-, s^+)\). By using the definition of improved activities, we have:

\[
\tilde{\theta}^* x_0 = x\tilde{\lambda} + \tilde{s}^- + \tilde{\theta}s^-
\]

\[
z_0 = z\tilde{\lambda}
\]

\[
y_0 = y\tilde{\lambda} - \tilde{s}^+ - s^+
\]

Now we rewrite the above formula:

\[
\tilde{\theta} x_0 = x\tilde{\lambda} + \tilde{s}^-
\]

\[
y_0 = y\tilde{\lambda} - \tilde{s}^+ - s^+
\]

Where \(\tilde{\theta} = \tilde{\theta}^*\), \(\tilde{s}^- = \tilde{s}^- + \tilde{\theta}s^-\), and \(\tilde{s}^+ = \tilde{s}^+ + s^+\).

If \(\tilde{\theta} < 1\) then \(\tilde{\theta}^* < \theta^*\) so \(\tilde{\theta} < \theta^*\) so, this is a contradiction. Therefore \(\tilde{\theta} = 1\).

With \(\tilde{\theta} = 1\) we have:

\[
es\tilde{s}^- + es\tilde{s}^- = (es\tilde{s}^- + es\tilde{s}^-) + (es\tilde{s}^- + es\tilde{s}^-) \leq es\tilde{s}^- + es\tilde{s}^- + es\tilde{s}^- + es\tilde{s}^- + es\tilde{s}^-
\]

Since \(es\tilde{s}^- + es\tilde{s}^-\) is maximal, we must have \(e\tilde{s}^- + e\tilde{s}^- = 0\) so \(\tilde{s}^- = 0, \tilde{s}^+ = 0\) and \(\tilde{\theta} = 1\).

Hence improved activity is efficient.

A TWO STAGE PERFORMANCE EVALUATION MODEL FOR IRANIAN BANK BRANCHES

In order to appraise the efficiency, effectiveness, and performance of Iranian banks, we apply a two-stage model as proposed in this paper. So in this section we calculate the overall performance directly. This performance is a combination of profitability efficiency and effectiveness. Based on the above definition of performance, this paper evaluates the performance of 37 Iranian bank branches via a two stage evaluation process that separates profitability and effectiveness. A two stage evaluation process is defined as that level of assets required to generate profitability in the first stage, and generate effectiveness in the second stage.

Determination of inputs and outputs to calculate profitability efficiency and effectiveness are an important step in performance evaluation. Since we want to evaluate profitability efficiency and effectiveness, we must have two sets of inputs and outputs.

In this study we want to obtain the overall performance directly. The overall performance obtains from net income/total cost. The overall performance can be divided into two concepts. The first concept is profitability that obtains from Total income/Total cost, and the second concept is effectiveness that obtains from Net income/Total income (Ho and Zhu, 2004, Ho, 2007; Tsolas, 2010, Kumer and Gulati’s, 2010).

Net income/total income (effectiveness) could be treated as a metric of effectiveness in this study and is defined as the ability to achieve the expected level of income generation (Tsolas, 2010).

Total income/total cost (profitability) assesses the ability of the branch to generate income with the available resources expressed in monetary values and it is an index of profitability (Tsolas, 2010).

The choice of inputs and outputs is influenced by the literature on the DEA application in the banking industry. The inputs and outputs used in earlier DEA applications studies are listed in Table 1. Based on the literature of DEA application in the banking industry, only the inputs and outputs in banking efficiency that there are in the literature of bank efficiency are listed. Then, with expert supervision and the bank’s staff ideal, the most important indexes of profitability and effectiveness in Iranian banks are selected. In fact, we determine two sets of indexes. The first set of indexes is for profitability efficiency evaluation in stage 1 and the second set for effectiveness evaluation in stage 2. The inputs and outputs set for profitability efficiency consisted of personal expenses, equipment expenses and operational expenses as inputs and non-operational income, sum of deposits, and commissions as outputs. These inputs and outputs are shown in the figure below (Figure 2).

The inputs and outputs set for effectiveness evaluation consisted of non-operational income, sum of deposits, and commissions as inputs and net-income as output. These inputs and outputs are shown in the figure below (Figure 3).

As seen, the outputs in profitability efficiency evaluation then become the inputs of effectiveness evaluation in stage 2. So we have a two stage model for overall performance evaluation (Figure 4).

In this study, we used the inputs and outputs data from April 2010. In this step we apply the proposal model for performance evaluation in Iranian bank branches. As previously mentioned, this method calculates the overall performance directly. The results of this model are shown in Table 2.

As seen in Table 2, the DMU 1, 2, 3, 6, 8, 13, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 33 and 34 are efficient, and the other DMUs are non efficient.

Conclusion

A general framework to evaluate the overall performance in terms of profitability efficiency and effectiveness
**Table 1. An overview of inputs and outputs variables used in some of the studies.**

<table>
<thead>
<tr>
<th>S/N</th>
<th>Author</th>
<th>Year</th>
<th>Country of study</th>
<th>Input indexes</th>
<th>Output indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sherman and Gold</td>
<td>1985</td>
<td>USA</td>
<td>Employees, expenses, space</td>
<td>No. of transactions</td>
</tr>
<tr>
<td>2</td>
<td>Parkan</td>
<td>1987</td>
<td>Canada</td>
<td>Employees, expenses, space, rent, terminals</td>
<td>No. of transactions, customer response, corrections</td>
</tr>
<tr>
<td>3</td>
<td>Oral and Yolalan</td>
<td>1990</td>
<td>Turkey</td>
<td>Employee, terminals, no. of accounts, credit applications</td>
<td>No. of transactions</td>
</tr>
<tr>
<td>4</td>
<td>Giokas</td>
<td>1991</td>
<td>Greece</td>
<td>Employee, expenses, rent</td>
<td>No. of transactions</td>
</tr>
<tr>
<td>5</td>
<td>Bhattacharya et al.</td>
<td>1997</td>
<td>India</td>
<td>Interest expenses, operating expenses</td>
<td>Advances, deposits, investment</td>
</tr>
<tr>
<td>6</td>
<td>Jackson et al.</td>
<td>1998</td>
<td>Turkey</td>
<td>Employees, non-labor operating expenses</td>
<td>Total loans, total demand deposits, total time deposits</td>
</tr>
<tr>
<td>7</td>
<td>Chen and Yeh</td>
<td>1998</td>
<td>Taiwan</td>
<td>Assets, branches, operating cost, deposits, interest expenses</td>
<td>Interest income, non interest income</td>
</tr>
<tr>
<td>8</td>
<td>Seiford and Zhu</td>
<td>1999</td>
<td>USA</td>
<td>Employees, assets, capital stock</td>
<td>Revenues, profits</td>
</tr>
<tr>
<td>9</td>
<td>Athanassopoulos and Giokas</td>
<td>2000</td>
<td>Greece</td>
<td>Labor hours, branch size, computer terminals, operating expenditure</td>
<td>No. of transactions, easiest, medium-easy, most-difficult, credit transactions, deposit transactions, foreign receipts</td>
</tr>
<tr>
<td>10</td>
<td>Ho</td>
<td>2001</td>
<td>Taiwan</td>
<td>Assets, interest expenses, employee, fixed assets</td>
<td>Interest income, non interest income</td>
</tr>
<tr>
<td>11</td>
<td>Mukherjee et al.</td>
<td>2002</td>
<td>India</td>
<td>Net worth, borrowings, operating expenses, employee, branches</td>
<td>Deposit, net income, advance, non-interest income, interest spread</td>
</tr>
<tr>
<td>12</td>
<td>Galagedera and Edirisuriya</td>
<td>2004</td>
<td>India</td>
<td>Customer and short term funding and total operating expenses</td>
<td>Loans and other earning assets</td>
</tr>
<tr>
<td>13</td>
<td>Sturm and Williams</td>
<td>2004</td>
<td>Australia</td>
<td>Employee, deposits and equity capital</td>
<td>Loans and off-balance sheet items</td>
</tr>
<tr>
<td>14</td>
<td>Kumbhakar and Lozano-Vivas</td>
<td>2005</td>
<td>Spain</td>
<td>Purchase funds and core deposits, labor, and physical capital</td>
<td>Loans and securities</td>
</tr>
<tr>
<td>15</td>
<td>Chambers and Cifter</td>
<td>2006</td>
<td>Turkey</td>
<td>No. of branches, personal members per branch, share in total assets, share in total loans, share in total deposits</td>
<td>Non-interest income, total assets and net interest income, total operating income</td>
</tr>
<tr>
<td>16</td>
<td>Rezitis</td>
<td>2006</td>
<td>Greece</td>
<td>Labor, capital expenses, value of deposits</td>
<td>Value of loans, advances and value of investment assets</td>
</tr>
<tr>
<td>17</td>
<td>Sturm and Williams</td>
<td>2007</td>
<td>Australia</td>
<td>Interest expenses, non-interest expenses</td>
<td>Net interest income and non-interest income</td>
</tr>
<tr>
<td>18</td>
<td>Gikas</td>
<td>2008a</td>
<td>Greece</td>
<td>Personal costs, running and other operating costs</td>
<td>Value of loan portfolio, value of deposits, non interest income, loan transactions, deposit transactions, remaining transactions</td>
</tr>
<tr>
<td>19</td>
<td>Gikas</td>
<td>2008b</td>
<td>Greece</td>
<td>Interest costs, non interest costs, personal costs, running costs, operating expenses</td>
<td>Interest income, non interest income, value of deposits, value of loans, non interest income</td>
</tr>
<tr>
<td>20</td>
<td>Kumar and Gulati</td>
<td>2009</td>
<td>India</td>
<td>Physical capital, loanable funds, labor</td>
<td>Advances investment, net interest income,</td>
</tr>
<tr>
<td>21</td>
<td>Tsolas</td>
<td>2010</td>
<td>Greece</td>
<td>Rental expenses, personal expenses, other operating expenses, depreciation</td>
<td>Origination income, outcome of a predetermined function mapping loans selling branch performance, commissions, other non interest income, net income</td>
</tr>
</tbody>
</table>
by means of two stage DEA model is proposed. There are several studies about two stage DEA model in the literature of DEA. The most of these models have several problems, such as, they cannot determine the relative efficiency of DMUs, nor can they determine an efficient projection unit. In our model, these problems have been solved. The proposed model can determine the relative efficiency and an efficient projection unit.

In the next step we apply the performance for a sample of the Iranian bank. We used our two stage DEA model to evaluate the Iranian bank performance. In this case study, we have 3 inputs for the first stage, 3 outputs in the second stage and 3 immediate measures. The results show that the DMU 1, 2, 3, 6, 8, 13, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 33 and 34 are efficient.

We believe that our proposed model can be modified into a multi-stage model. Future researchers can extend this improvement. Note that, in the proposed model, the values of immediate measures are fixed so we propose to the future researchers that modify this model into the changeable immediate value.

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