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Review

Harmony search algorithms for inventory management problems

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In recent years, harmony search (HS) algorithms have gained significant attentions for their abilities to solve difficult problems in engineering. This paper introduces the applications of HS in inventory management problems. Four specific inventory problems in certain and uncertain environments are considered: constraint multi-product newsboy problem with fuzzy demand, economic order quantity problem with advanced payment, bi-objective newsboy problem with fuzzy costs and periodic review problem with stochastic period length and dynamic demands. The computational performance of the HS method on solving these four optimization problems will be compared with other meta-heuristic algorithms such as genetic algorithm (GA), simulated annealing (SA), and particle swarm optimization (PSO) methods. HS method is shown to achieve the best performance.

Key words: Inventory control, newsboy, economic order quantity, periodic review system, harmony search, stochastic inventory model, fuzzy inventory model.

INTRODUCTION

This paper reviews four practical inventory management problems. These four problems consider certain and uncertain environments. They are: constraint multi-product newsboy problem with fuzzy demand, bi-objective newsboy problem with fuzzy costs, economic order quantity problem with advanced payment, and periodic review problem with stochastic period length and dynamic demands. In fact this paper presents a comparison between some Meta-heuristic algorithms used to solve the four classical and the most famous and important inventory management problems. Harmony search algorithm, genetic algorithm, particle swarm optimization and simulated annealing are a group of meta heuristic algorithms which are used to solve the proposed models. These meta heuristic algorithms are commonly and widely use to solve the inventory management problems.

The newsboy problem is a periodic inventory management problem where uncertainty in demand during a single period is considered. While the probability distribution of demand is known, the actual number of demand will not be known until after the decision. The amount sold or delivered will be the minimum order or demand quantities. In the examples, HS is used to derive the order quantity under maximum benefit and service level functions.

The classical economic order quantity (EOQ) formula is the simplest model for the cycle stock. The demand, ordering and holding costs are constant over time, the batch quantity may not be an integer, the whole batch quantity is delivered at the same time and no shortage are allowed. In this case, HS is used to determining the best order quantity of the extended EOQ model.

Let us consider an inventory system that is controlled by a periodic review policy instead of continues review policy. Periodic review means that the inventory position is inspected at the beginning of each period and that all replenishments are triggered at these reviews. A period may, for example, be a day or a week. If the review period is short, there is no difference between periodic and continues review. In such a case, continues review results can be used as an approximation in periodic review systems and vice versa. Sometimes, the review period cannot be disregarded. In this case HS is used to obtain the best inventory level.

INVENTORY MANAGEMENT

The total investment in inventories is enormous and the control of capital tied up in raw material, work-in-process, and finished goods offers a very important potential for improvement. Scientific methods for inventory management can give a significant competitive advantage. Advances in information technology have drastically changed the possibilities to apply efficient inventory management techniques. Modern inventory management is based on advanced and complex decision models which may require considerable computational efforts (Axsater, 2006).

Inventory management is a common problem to all organization in any sector of the economy. The problems of inventory do not confine themselves to profit-making institutions but also encountered by social and nonprofit institutions. Inventories are common in farms, manufacturing plants, wholesaler warehouses, retailer shops, hospitals, zoos, universities, etc (Tersine, 1994).

The objective of inventory management is to balance conflicting goals. One goal is to keep stock level down to make cash available for other purposes. The purchasing manager may wish to get volume discounts for larger batches order. The production manager may prefer a large raw materials inventory to avoid production stoppage due to missing materials. Higher stock of finished goods also provides higher service level. Managing inventory involves efficiently and effectively coordinating both the information and materials flow in the supply chain (Brandimarte and Zotteri, 2007). Inventory management models help to achieve such goals. Due to uncertainty, safety stocks are required. Two types of uncertainty directly impact inventory policy. Demand Uncertainty is due to an uncertain rate of sale or demand during lead time. Performance cycle uncertainty involves inventory replenishment time variation (Bowersox et al., 2007). Uncertainties in demand and cycle length together with lead-times in production and transportation inevitably create a need for safety stocks. Most organizations can reduce their inventories using inventory management models.

There are some inventory management problems in a supply chain. We show three basic kinds: Newsboy problem, economic order quantity model, and stochastic periodic review problem with increasing and decreasing demand.

Newsboy problem

In this problem, the material manager orders at the beginning of the period based on his estimation of the demand. If his order is more than the actual demand, holding cost exists, otherwise lost sales or backorder results. The objective of this model is to maximize the expected profit through order. In the real world, many products have a limited selling period, so the newsboy problem is often used to aid decision-making. In real world, many products (fashion, sporting, and service industries) have a limited selling period, so the newsboy problem is often used to aid decision-making.

The classical newsboy problem are widely extended to different objectives and utility functions, different discount policies, multi-product multi- constraint, multi-period models and fuzzy considerations, Anvari and Kusy, 1990; Chung, 1990; Atkinson, 1979; Abdel-Malek and Montanari, 2005; Matsuyama, 2006; Alfares and Elmorra, 2005; Lushu et al., 2002; Ji and Shao, 2006; Shao and Ji, 2006; Taleizadeh et al., 2009, 2008).

Example 1: Multi-product newsboy with fuzzy demand

To define a problem for multi-product newsboy with fuzzy demand, we assume there are T products and the newsboy makes order at the beginning of the period. The demands are assumed to be fuzzy and the newsboy orders are based estimation. The orders can be taken at the beginning of each period and no opportunity is allowed to replenish the stock in that period. The order quantity should be a multiplier of a defined batch size. The demand rate for the company is assumed to be a triangular fuzzy number. There are restrictions on warehouses capacity, budget and service level. Shortage lost sale is allowed. The costs of shortage and holding and the remained items are assumed to be deterministic. The holding cost is a fraction of purchasing cost and the goal is to determine the optimal order quantity of each product. The model of this problem is formulated by Taleizadeh et al. (2009) as follows:

$$Max: Z=\sum_{j=1}^{T}\left[P_j\tilde{\xi}_j-h_j(Q_j-\tilde{\xi}_j)\right]X_j+\sum_{j=1}^{T}\left[P_jQ_j-\hat{\pi}_j(\tilde{\xi}_j-Q_j)\right](1-X_j)-\sum_{j=1}^{T}\sum_{i=1}^{n}W_{nj}\lambda_{nj}Q_j$$

$$S.t: \quad Q_j \geq \tilde{\xi}_j\text{-}MX_j \qquad \forall j\ ,\ j=1,2,\cdots,T$$

$$Q_j < \tilde{\xi}_j+M(1-X_j) \qquad \forall j\ ,\ j=1,2,\cdots,T$$

$$\sum_{j=1}^{T} f_j.m_j \leq F$$

$$\sum_{j=1}^{T}\left(\sum_{i=1}^{n} C_{nj}\lambda_{nj}\right)Q_j \leq W$$

$$\sum_{j=1}^{T}\left(\sum_{i=1}^{n} C_{nj}\lambda_{nj}\right)Q_j \leq W$$

$$\alpha_j\left[\frac{1}{4}(\xi_{1j}+2\xi_{2j}+\xi_{3j})\right] \leq Q_j \leq UL_j \ \forall j\ ,\ j=1,2,\cdots,T$$

$$Q_j = K_j m_j \qquad \forall j,\ j=1,2,\cdots,T$$

$$0 < Q_j \leq q_{1j}\lambda_{1j} \qquad \forall j,\ j=1,2,...,T$$

$$q_{i-1,j}\lambda_{i-1,j} < Q_j \leq q_{ij}\lambda_{i-1,j} \ \forall i,j,\ j=1,2,...,T, i=2,3,\cdots,n$$

$$q_{nj}\lambda_{nj} < Q_j \qquad \forall j,\ j=1,2,...,T$$

$$X_j = 0,1 \ \ \forall j,\ j=1,2,\cdots,T$$

$$\sum_{i=1}^{n}\lambda_{ij}=1 \ \ \forall j,\ j=1,2,...,T$$

$$\lambda_{ij}=0,1;\ \forall j,\ j=1,2,...,T, \forall i=1,2,\cdots;n$$

$$Q_j, m_j \geq 0\ ,\ \text{integer} \ \ \forall j\ ,\ j=1,2,\cdots,T \ ; \text{Where, M is a big positive number} \qquad (1)$$

Table 1. Best results of objective function by different hybrid Algorithms.

Hybrid algorithms	Products' order quantity						Maximum profit ($)
	1	2	3	4	5	6	
Simulated annealing and fuzzy simulation	190	212	112	94	79	42	9006
Genetic algorithm and fuzzy simulation	197	215	103	99	70	41	9218
Particle swarm optimization and fuzzy simulation	198	216	103	98	69	38	9433
Harmony search and fuzzy simulation	198	221	98	99	64	36	9722

Since the model in 1 is integer in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). So we need to use meta heuristic algorithms. The proposed model for six products is solved using meta-heuristic approach, four hybrid intelligent algorithms of harmony search-fuzzy simulation (HS-FS) Taleizadeh and Niaki (2009), particle swarm optimization-fuzzy simulation (PSO-FS) Taleizadeh et al. (2009), simulated annealing- fuzzy simulation (SA-FS) Taleizadeh et al. (2009), and genetic algorithm-fuzzy simulation (GA-FS) Taleizadeh et al. (2009).

A comparison of the results in Table 1 shows the PSO-FS hybrid algorithm performs better than the GA-FS and SA-FS algorithms in terms of the objective function values, and the proposed HS-FS method of this research performs the best. In the term of CPU time the expected values of SA-FS, GA-FS, PSO-FS and HS-FS are respectively 67, 59, 59 and 52 seconds showing HS-FS performs better than other do.

Example 2: Bi-objective multi-product newsboy with fuzzy cost

Consider a company that orders products only once and only at the start of a period. The customer demand for each product follows a Poisson distribution. There is no enforced constraint on the supplier to supply an order. The entire capacity of the warehouse is assigned to the products. The shortage and holding costs are fuzzy and deployed at the end of the period and increase in quadratic fashion. The transportation cost to carry the products has two components: fixed cost for each shipment and variable cost for each unit of the products. Discount for purchasing items is allowed and follows incremental discount rule. Since the transportation and the order-processing times are relatively small as compared to the cycle length, the lead-time is considered equal to zero, which is a common practice in the newsboy problems. Similarly to example 1, the order quantity of each product should only be integer multiples of packets and shortage lost sale is considered. The goal is to determine the order quantity of each product such that the constraints are satisfied and both the expected profit and service rate are maximized. The model of this problem is formulated by Taleizadeh et al. (2009) as follows:

$$Max: Z_P = \sum_{j=1}^{T} \sum_{X_j=0}^{Q_j-1} P_j . X_j . f_{X_j}(x_j) + \sum_{j=1}^{T} \sum_{X_j=Q_j}^{+\infty} P_j . Q_j . f_{X_j}(x_j) - \sum_{j=1}^{T} \sum_{i=1}^{n} C_{ij} W_{ij}$$

$$- \sum_{j=1}^{T} \sum_{X_j=0}^{Q_{j-1}} \left(\hat{h}_{1j}(Q_j - X_j) + \hat{h}_{2j}(Q_j - X_j)^2 \right) \frac{e^{-\lambda_j} \lambda_j^{X_j}}{X_j !}$$

$$- \sum_{j=1}^{T} \sum_{X_j=Q_j+1}^{\infty} \left(\hat{\pi}_{1j}(X_j - Q_j) + \hat{\pi}_{2j}(X_j - Q_j)^2 \right) \frac{e^{-\lambda_j} \lambda_j^{X_j}}{X_j !} - \sum_{j=1}^{T} K_j Q_j - \sum_{k=1}^{m} kAY_k$$

$$Max: Z_{SR} = \frac{1}{T} \sum_{j=1}^{T} \sum_{X_j=0}^{Q_j-1} \frac{(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j}$$

s.t:

$$\sum_{j=1}^{T} f_j B_j \le F$$

$$Q_j = V_j B_j \quad \forall j, \quad j = 1, 2, ..., T$$

$$q_{1j} \lambda_{2j} \le W_{1j} \le q_{1j} \lambda_{1j} \quad \forall j, \quad j = 1, 2, ..., T$$

$$\vdots$$

$$\left(q_{ij} - q_{i-1,j} \right) \lambda_{ij} \le W_{ij} \le \left(q_{ij} - q_{i-1,j} \right) \lambda_{i-1,j}$$

$$\forall i, \quad i = 2, ..., n_j - 1 \text{ and } \forall j, \quad j = 1, 2, ..., T$$

$$0 \le W_{n_j j} \le M \lambda_{n_j j} \quad \forall j, j = 1, 2, ..., T, \text{M is a big number}$$

$$0 < \sum_{j=1}^{T} f_j B_j \le f \hat{Y}_1$$

$$(k-1) f \hat{Y}_i < \sum_{j=1}^{T} f_j B_j \le k f \hat{Y}_k \; ; \forall k = 2, 3, \cdots, m$$

$$\sum_{k=1}^{m} Y_k = 1 \quad , \quad Y_k = 0, 1 \; ; \; \forall k = 1, 2, ..., m$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{n_j j} \quad \forall j, \quad j = 1, 2, \cdots, T$$

$$\lambda_{ij} = 0, 1 \quad \forall j, \; j = 1, 2, \cdots T \text{ and } \forall i, i = 1, 2, \cdots, n$$

$$B_j \ge 0 \text{ and integer} \quad \forall j, \quad j = 1, 2, \cdots, T \qquad (2)$$

Since the model in 2 is integer in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). So we need to use meta heuristic algorithms. To solve the model with ten products under

Table 2. Best results of objective functions by different hybrid algorithms.

Hybrid algorithms	Products' order quantity										Maximum profit ($)	Maximum service level
	1	2	3	4	5	6	7	8	9	10		
Simulated annealing, fuzzy simulation and goal programming	88	72	81	53	66	79	83	54	54	**101**	2926	0.7926
Genetic algorithm, fuzzy simulation and goal programming	90	70	80	48	58	76	79	55	52	**98**	3012	0.8103
Genetic algorithm, fuzzy simulation and pareto selecting	90	72	85	47	57	78	80	55	52	**95**	3103	0.8109
Harmony search and fuzzy simulation and goal programming	93	72	85	49	60	78	80	57	54	**95**	3250	0.8726

meta-heuristic approach, four hybrid intelligent algorithms of harmony search, fuzzy simulation and goal programming (HS-FS-GP) Taleizadeh and Niaki (2009), simulated annealing, fuzzy simulation and goal programming (SA-FS-GP) Taleizadeh et al. (2009), Taleizadeh and Niaki (2009), genetic algorithm, fuzzy simulation and goal programming (GA-FS+GP) Taleizadeh et al. (2009), and genetic algorithm, fuzzy simulation and Pareto selecting (GA-FS+PS) Taleizadeh et al. (2009), are used. A comparison of the results in Table 2 shows the proposed HS-FS+GP method performs the best.

Results show that HS has the best optimal solutions. The profit and service level are $3250 and 0.8726 respectively. It should be also noted that hybrid method of GA-FS-pareto selecting performs better than the hybrid method of GA-FS-GP and SA-FS-GP. In the term of CPU time the expected values of SA-FS-GP, GA-FS-GP, GA-FS-PS and HS-FS+GP are respectively 122, 108, 108 and 104 seconds showing HS-FS+GP performs better than other do.

Economic order quantity model

The classical economic order quantity (EOQ) formula is the simplest model for cycle stock. The square-root-formula for the economic order quantity (EOQ) has been cited in the inventory literature since 1915. This formula is based on the assumption of a constant demand. The discrete case of the dynamic version of EOQ was first discussed by Wagner and Whitin (1958). Regarding the continuous-time dynamic EOQ models, Silver and Meal (1969) were the first to suggest a simple modification of the classical square-root-formula in the case of time-varying demand.

Hence, the classical economic order quantity model is widely extended in Different ways by Hariga (1994), Chung and Ting (1994), Kim (1997), Lin et al. (2000), Grubbstrom and Erdem (1999), Taleizadeh et al. (2008) etc.

Example 3: Joint constraint EOQ with advanced payment

Economic order quantity (EOQ) model with advanced payment is usually developed to purchase high-price raw materials. A joint policy of replenishments and pre-payments is employed to supply the materials. The rate of demand and lead time are taken to be constant and it is assumed that shortage does not occur. The cycle is divided into three parts; the first part is the time between the previous replenishment-time to the next order-time (t_0), the second part is the period between t_0 to a payment-time (t_{lc}), and the third part is the period between t_{lc} to the next replenishment-time.

At the start of the second part (t_0), $\alpha\%$ of the purchasing cost is paid. The $(1-\alpha)\%$ remaining purchasing cost is paid at the start of the third part (t_{lc}) (Figure 1). The cost of the model is purchasing with incremental discount for each order, clearance cost, fixed-order cost, transportation cost, holding and capital costs. Holding cost is for on hand inventory and capital cost is for capital that is paid for the next order. The constraints of the problem are space, budget and upper limit for the number of orders per year. Also lead-time is considered less than a cycle time. The model of this problem is formulated by Taleizadeh et al. (2008) as follows:

$$Min: Z = C_T + C_P + C_C + C_A + C_H$$

$$= \frac{1}{T}\sum_{k=1}^{K} kA_T Y_k + \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} + \sum_{j=1}^{P} C_j^c D_j + \frac{A}{T} + \sum_{j=1}^{P}\frac{h_j^1 D_j T}{2} + \sum_{j=1}^{P} h_j^2 D_j (t_{lc} - t_0) + \sum_{j=1}^{P} h_j^3 D_j \left[L - (t_{lc} - t_0)\right]$$

$$= \frac{1}{T}\left[A + \sum_{k=1}^{K} kA_T Y_k + \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}\right] + \left[\sum_{j=1}^{P}\frac{h_j^1 D_j}{2}\right]T + \left[\sum_{j=1}^{P} C_j^c D_j + \sum_{j=1}^{P} h_j^2 D_j (t_{lc} - t_0) + \sum_{j=1}^{P} h_j^3 D_j \left[L - (t_{lc} - t_0)\right]\right]$$

$$S.t: \quad \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \le TB$$

$$\sum_{j=1}^{P} f_j Q_j \le F$$

$$T \ge \frac{1}{N_T}$$

$$0 < \sum_{j=1}^{P} f_j m_j \le f\hat{Y}_1$$

$$f\hat{Y}_2 < \sum_{j=1}^{P} f_j m_j \le 2f\hat{Y}_2$$

$$\vdots$$

$$(K-1)f\hat{Y}_k < \sum_{j=1}^{P} f_j m_j \le Kf\hat{Y}_k$$

$$TD_j = n_j m_j$$

$$Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj}$$

$$q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j}$$

$$\left(q_{2j} - q_{1j}\right)\lambda_{3j} \le Q_{2j} \le \left(q_{2j} - q_{1j}\right)\lambda_{2j}$$

$$\vdots$$

$$0 \le Q_{nj} \le M\,\lambda_{nj}$$

$$Y_1 + Y_2 + \cdots + Y_k = 1$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj}$$

$$\lambda_{ij} = 0,1 \quad ; \quad Y_k = 0,1 \quad \forall k = 1,2,\cdots,K \;, \;\; \forall j\,,\; j = 1,2,\cdots,P \;, \forall i\,,\; i = 1,2,\cdots,m$$

$$\lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj} \qquad (3)$$

Since the model in 2 is integer in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). So we need to use meta heuristic algorithms. To solve the model under meta-heuristic approach, four hybrid intelligent algorithms of harmony search Taleizadeh et al. (2008), simulated annealing, Taleizadeh et al. (2008), Taleizadeh et al. (2009a), genetic algorithm Taleizadeh et al. (2008), Taleizadeh et al. (2009a), and particle swarm optimization Taleizadeh et al. (2009a), are used. Table 3 shows the proposed HS-FS+GP method performs the best.

Results show that HS reached the optimal solutions with best objective function value (Profit=124,820,000). It should be also noted that particle swarm optimization performs better than the genetic algorithm and simulated annealing approaches based on objective function values. In the term of CPU time the expected values of SA, GA, PSO and HS are respectively 16, 17, 17 and 15 seconds showing HS performs better than other do.

Periodic review model

In multi-periodic inventory control models, the continuous review and the periodic review are the major vastly used policies. However, the underlying assumptions of the proposed models restrict their correct utilization in real-world environments. In continuous review policy, the user has the freedom to act at anytime and replenish orders based upon the available inventory level. While in the periodic review policy, the user is allowed to replenish the orders only in specific and predetermined times. The

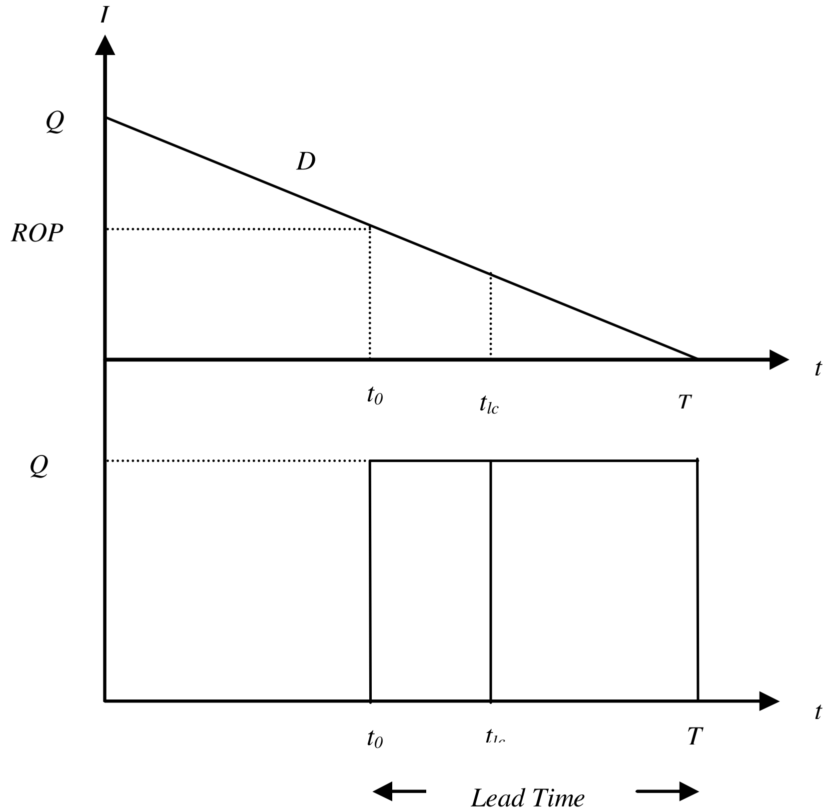

Figure 1. Inventory control picture.

Table 3. Best results of objective function by different algorithms.

Hybrid algorithms	T	Minimum cost ($)
Simulated annealing	0.2134	131,456,000
Genetic algorithm	0.2256	128,956,000
Particle swarm optimization	0.2312	126,450,000
Harmony search	0.2383	124,820,000

multi-periodic inventory control problems have been investigated in depth in different researches by Chiang Chiang (2003), Bylka (2005), Chiang (2006), Feng and Rao (2007), Eynan and Kropp (2007) and Taleizadeh et al. (2008, 2009a, 2009).

Example 4: Stochastic period length with dynamic demand

Consider a periodic inventory control model with stochastic replenishment time. Let the time-periods between two replenishments be identical and independent random variables; in case of shortage, a fraction is considered back-order and a fraction as lost-sale. The demands of the products are dynamic and varying. The costs associated with the inventory control system are holding, back-order, lost-sales and purchasing costs. Furthermore, the service level of each product, warehouse space and budget are limited and the decision variables are integer digits. The goal is to identify the inventory levels in each cycle such that the expected profit is maximized.

Taleizadeh et al. (2009) introduced an inventory management model with stochastic period length with varying demand functions. Figures 2 and 3 show the inventory diagram in decreasing demand case. In the second case, the time period between replenishments is greater than the amount of time required for the inventory level to reach zero. The increasing and decreasing demand functions are given in Equations 4 and 5 by Taleizadeh et al. (2009), respectively.

$$D_i(t) = D_i t^{\alpha_i} \quad ; \quad \alpha_i \geq 1,\ D_i > 0 \tag{4}$$

$$D_i(t) = D_i^{\frac{t_i}{t_i - 1}} \quad ; \quad D_i > 1 \tag{5}$$

The models of this problem for increasing and decreasing demands are formulated by Taleizadeh et al. (2008) as follows:

$$Max: Z(R)=\sum_{i=1}^{n}(P_i-W_i)\left[\int_{T_{Min_i}}^{t_{D_i}}\left(\int_{0}^{t_i}D_i t_i^{\alpha_i}dt_i\right)\frac{1}{T_{Max_i}-T_{Min_i}}dt_i+\int_{t_{D_i}}^{T_{Max_i}}\left(R_i+\beta_i\left(\int_{t_{D_i}}^{T_{Max_i}}D_i t_i^{\alpha_i}dt_i\right)\right)\frac{1}{T_{Max_i}-T_{Mini}}dt_i\right]$$

$$-\sum_{i=1}^{n}h_i\left[\int_{T_{Min_i}}^{t_{D_i}}\left(\int_{0}^{t_i}(R_i-\frac{D_i}{\alpha_i+1}t_i^{\alpha_i+1})dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i+\int_{t_{Di}}^{T_{Max_i}}\left(\int_{0}^{t_{D_i}}(\frac{D_i}{\alpha_i+1}t_i^{\alpha_i+1})dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]$$

$$-\sum_{i=1}^{n}\pi_i\beta_i\left[\int_{t_{D_i}}^{T_{Max_i}}\left(\int_{t_{D_i}}^{t_i}D_i(t_i)dt_i\right)f_{T_i}(t_i)dt_i\right]-\sum_{i=1}^{n}\hat{\pi}_i(1-\beta_i)\left[\int_{t_{D_i}}^{T_{Max_i}}\left(\int_{t_{D_i}}^{t_i}D_i t_i^{\alpha_i}dt\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]$$

$$S.t: \quad \sum_{i=1}^{n}f_i R_i \le F$$

$$\sum_{i=1}^{n}W_i Q_i \le TB$$

$$\int_{\sqrt[\alpha_i+1]{\frac{(\alpha_i+1)}{D_i}R_i}}^{T_{Max_i}}\frac{1}{T_{Max_i}-T_{Min_i}}dt_i \le 1-SL_i \qquad \forall i: \ i=1,2,\cdots,n$$

$$R_i \ge 0, Integer \qquad \forall i: \ i=1,2,\cdots,n \tag{6}$$

And

$$Max: Z(R)=\sum_{i=1}^{n}\left[(P_i-W_i)\int_{T_{Min_i}}^{t_{D_i}}\left(\int_{0}^{t_i}D_i^{\frac{t_i}{t_i-1}}dt_i\right)\frac{1}{T_{Max_i}-T_{Min_i}}dt_i+\int_{t_{D_i}}^{T_{Max_i}}\left(R_i+\beta_i\left(\int_{t_{D_i}}^{T_{Max_i}}D_i^{\frac{t_i}{t_i-1}}dt_i\right)\right)\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]$$

$$-\sum_{i=1}^{n}h_i\left[\int_{T_{Min_i}}^{t_{D_i}}\left(\int_{0}^{t_i}(R_i-t_i)dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i+\int_{t_{D_i}}^{T_{Max_i}}\left(\int_{0}^{t_{Di}}t_i dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]$$

$$-\sum_{i=1}^{n}\pi_i\beta_i\left[\int_{t_{Di}}^{T_{Max_i}}\left(\int_{t_{D_i}}^{t_i}D_i^{\frac{t_i}{t_i-1}}dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]-\sum_{i=1}^{n}\hat{\pi}_i(1-\beta_i)\left[\int_{t_{D_i}}^{T_{Max_i}}\left(\int_{t_{D_i}}^{t_i}D_i^{\frac{t_i}{t_i-1}}dt_i\right).\frac{1}{T_{Max_i}-T_{Min_i}}dt_i\right]$$

$$S.t: \quad \sum_{i=1}^{n}f_i R_i \le F$$

$$\sum_{i=1}^{n}W_i Q_i \le TB$$

$$\int_{\sqrt[\alpha_i+1]{\frac{(\alpha_i+1)}{D_i}R_i}}^{T_{Max_i}}\frac{1}{T_{Max_i}-T_{Min_i}}dt_i \le 1-SL_i \qquad \forall i: \ i=1,2,\cdots,n$$

$$R_i \ge 0, Integer \qquad \forall i: \ i=1,2,\cdots,n \tag{7}$$

Since the model in 2 is integer in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). So we need to use Meta heuristic algorithms. To solve the models under meta-heuristic approach, four hybrid intelligent algorithms of harmony search (Taleizadeh et al., 2009), simulated annealing (Taleizadeh, 2008, 2009), genetic algorithm Taleizadeh (2008, 2009), and particle swarm optimization (Taleizadeh et al., 2009) are used. A comparison of the results in Tables 4 and 5 for increasing and decreasing demand show that HS method performs the best.

From Tables 4 and 5, for increasing and decreasing demand functions, in term of objective function's values HS performs better than other algorithms do. Similarly, in other examples, particle swarm optimization method has a better solution than genetic algorithm and simulated

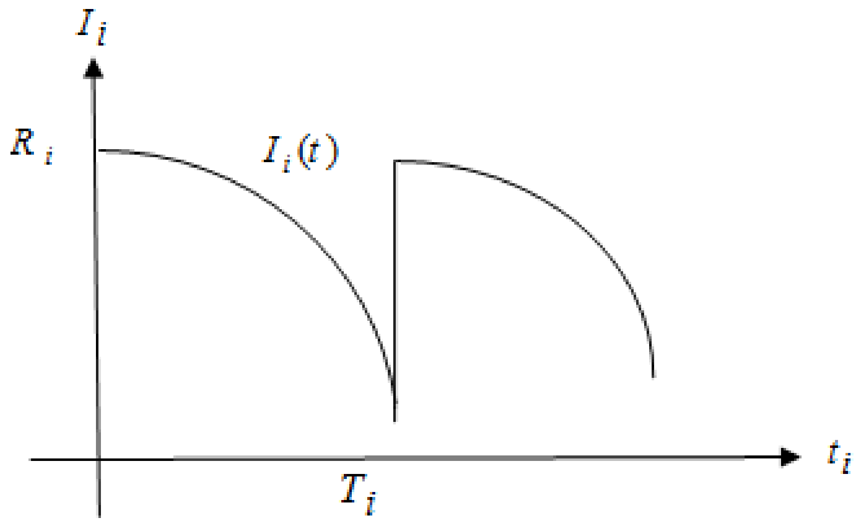

Figure 2. Presenting the inventory cycle when $T_{Min_i} \leq T_i \leq t_{D_i}$.

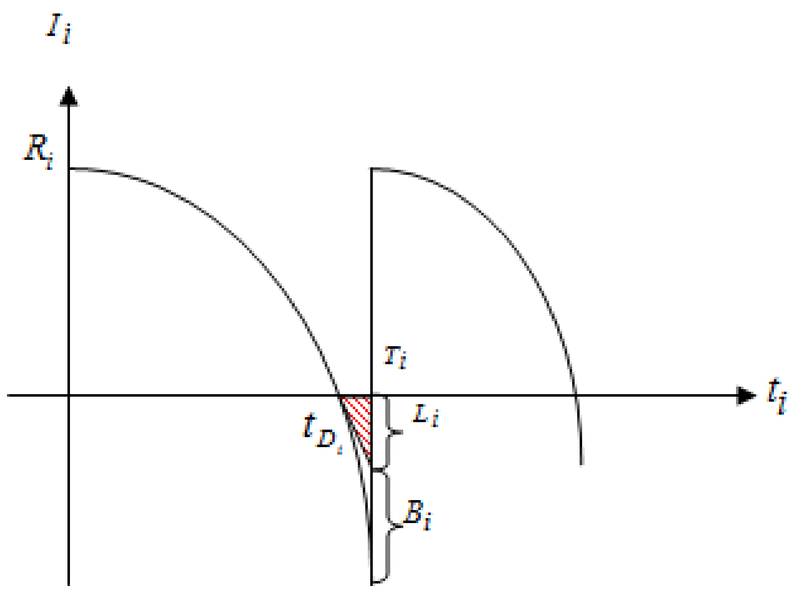

Figure 3. Presenting the inventory cycle when $t_{D_i} < T_i \leq T_{Max_i}$.

Table 4. Best results of objective function by different algorithms for increasing demand.

Hybrid algorithms	Products' maximum inventory level								Maximum profit ($)
	1	2	3	4	5	6	7	8	
Simulated annealing	1456	1617	6545	5696	3856	5109	19341	15845	604,239
Genetic algorithm	1393	1734	6144	5495	4147	5247	18448	16200	609,157
Particle swarm optimization	1369	1750	6146	5407	4146	5243	18446	16230	613,833
Harmony search	1350	1734	6144	5400	4050	5202	18432	16200	625,190

Table 5. Best results of objective function by different algorithms for decreasing demand.

Hybrid algorithms	Products' maximum inventory level								Maximum profit ($)
	1	2	3	4	5	6	7	8	
Simulated annealing	59	15	81	67	35	39	69	49	118,320
Genetic algorithm	30	36	70	60	30	222	64	60	122,270
Particle swarm optimization	30	34	64	60	30	233	64	60	155,644
Harmony search	47	24	64	60	30	35	64	60	204,950

annealing approaches do. In the term of CPU time the expected values of SA, GA, PSO and HS are respectively 8, 8, 7 and 7 seconds showing HS performs better than other do.

SUMMARY AND CONCLUSION

This paper has investigated various HS applications in inventory management. Specific examples include constraint multi-product newsboy problem with fuzzy demand, joint economic order quantity problem with advanced payment, constraint bi-objective newsboy problem with fuzzy costs, and periodic review problem with stochastic period length and dynamic demands. The goals of the first and third examples were to identify the order quantity of each product in uncertain and certain environment respectively. The goals of second example were to determine the order quantity of each product so as to maximize the total expected profit and service level. In the fourth example, the inventory manager determined the optimal inventory level of each product and maximized the total expected profit.

To solve the models, four kinds of meta-heuristic algorithms or meta-heuristic hybrid algorithms: harmony search, particle swarm optimization, genetic algorithms, simulated annealing, fuzzy simulation, goal programming and Pareto selecting were used.

Computational results showed that HS, hybrid of HS and fuzzy simulation, and hybrid method of HS, fuzzy simulation and goal programming performed the best, based on objective function values respect to other kind of the algorithms. Other applications of HS in inventory and supply chain managements can be extended to consider pricing problems.

REFERENCES

Abdel-Malek LL, Montanari R (2005). An analysis of the multi-product newsboy problem with a budget constraint. Int. J. Prod. Econ., 97: 296-307.

Alfares HK, Elmorra HH (2005). The distribution free newsboy problem: Extension to the shortage penalty case. Int. J. Prod. Econ., 93-94: 465-477.

Anvari M, Kusy M (1990). Risk in inventory models: review and implementation. Eng. Costs Prod. Econ., 19: 267-272.

Atkinson AA (1979). Incentives, uncertainty, and risk in the newsboy problem. Decis. Sci., 10: 341-357.

Axsater S (2006). Inventory control. Second ed., Springer, New York, USA.

Bowersox DJ, Closs DJ, Cooper MB, (2007). Supply chain logistics management. Second Ed., Mc Graw. Hill, New York, USA.

Brandimarte P, Zotteri G (2007). Introduction to distribution Logistics. First ed., John wily & Sons, New Jersey, USA.

Bylka S (2005). Turnpike Policies for Periodic Review Inventory Model with Emergency Orders. Int. J. Prod. Econ., 93-94: 357-373.

Chiang C (2003). Optimal Replenishment for a Periodic Review Inventory System with Two Supply Modes. Eur. J. Oper. Res., 149: 229-244.

Chiang C (2006). Optimal Ordering Policies for Periodic-Review Systems with Replenishment Cycles. Eur. J. Oper. Res., 170: 44-56.

Chung KH (1990). Risk in inventory models: the case of the newsboy problem–optimality conditions. J. Oper. Res. Soc., 41: 173-176.

Chung KJ, Ting PS (1994). On replenishment schedule for deteriorating items with time proportional demand. Prod. Plan. Control, 5: 392-396.

Eynan A, Kropp D (2007). Effective and Simple EOQ-like Solutions for Stochastic Demand Periodic Review Systems. Eur. J. Oper. Res., 180: 1135-1143.

Feng K, Rao US (2007). Echelon-Stock (R-nT) Control in Two Stage Serial Stochastic Inventory System. Oper. Res. Lett., 35: 95-104.

Gen M, Cheng R (1997). Genetic Algorithm and Engineering Design. John Wiley & Sons, New York, U.S.A.

Grubbstrom RW, Erdem A (1999). The EOQ with backlogging derived without derivatives. Int. J. Prod. Econ., 59: 529-530.

Hariga M (1994). The inventory lot-sizing problem with continuous time-varying demand and shortages. J. Oper. Res. Soc., 45: 827-837.

Ji X, Shao ZH (2006). Model and algorithm for bi-level newsboy problem with fuzzy demands and discounts. Appl. Math. Comput., p. 172:

Lin C, Tan B, Lee WC (2000). An EOQ model for deteriorating items with time-varying demand and shortages. Int. J. Syst. Sci., 3: 391-400.

Lushu L, Kabadi SN, Nair KPK (2002). Fuzzy models for single-period inventory problem. Fuzzy sets Syst., 132: 273-289.

Matsuyama K (2006). The multi period inventory problem. Eur. J. Oper. Res., 171: 170-188.

Shao ZH, Ji X (2006). Fuzzy multi-product constraint newsboy problem. Appl. Math. Comput., 180: 7-15.

Silver EA, Meal HC (1969). A simple modification of the EOQ for the case of a varying demand rate. Prod. Inventory Manage., 10: 52-65.

Taleizadeh AA, Aryanezhad MB, Niaki STA (2008). Optimizing Multi-Product Multi-Constraint Inventory Control Systems with Stochastic Replenishment. J. Appl. Sci., 8: 1228-1234.

Taleizadeh AA, Moghadasi H, Niaki STA, Eftekhari A (2008). An EOQ-Joint Replenishment Policy to Supply Expensive Imported Raw Materials with Payment in Advance. J. Appl. Sci., 8: 4263-4273.

Taleizadeh AA, Niaki AA, Shafii N, Ghavamizadeh-Meibodi R, (2009) A Particle Swarm Optimization Approach for Constraint Joint Single Buyer-Single Vendor Inventory Problem with Changeable Lead-Time and (r,Q) Policy in Supply Chain. Int. J. Adv. Manuf. Syst., 51(9-12): 1209-1223.

Taleizadeh AA, Niaki ST, Aryanezhad MB (2009). A Hybrid Method of Pareto, TOPSIS and Genetic Algorithm to Optimize Multi-Product Multi-Constraint Inventory Control Systems with Random Fuzzy Replenishments. Math. Comput. Model., 49: 1044-1057.

Taleizadeh AA, Niaki ST, Hosseini SV (2008). Multi products, Multi

Constraints Newsboy Problem with Batch Order and Discount. Asian J. Appl. Sci., 1: 110-122.
Taleizadeh AA, Niaki ST, Hosseini V (2009). Optimizing Multi Product Multi Constraints Bi-objective Newsboy Problem with discount by Hybrid Method of Goal Programming and Genetic Algorithm. Eng. Optimiz., 4: 437-457.
Taleizadeh AA, Niaki STA (2009). A Hybrid Method to Optimize Bi-Objective Single Period Newsboy Problem with Fuzzy Cost and Incremental Discount. J. Ind. Eng., Qazvin Islamic Azad University, 3(3): 1-14.
Taleizadeh AA, Niaki STA, Aryanezhad MB (2009). Multi-Product Multi-Constraint Inventory Control Systems with Stochastic Replenishment and Discount under Fuzzy Purchasing Price and Holding Costs. Am. J. Appl. Sci., 6(1): 1-12.
Taleizadeh AA, Niaki STA, Aryanezhad MB, (2009a) Multi-product multi-constraint inventory control systems with stochastic period length and total discount under fuzzy purchasing price and holding costs. Int. J. Syst. Sci., 41(10): 1187-1200.
Taleizadeh AA, Niaki STA, Hosseini SV (2009). Optimizing multi-product multi-constraint bi-objective newsboy problem with discount by hybrid method of goal programming and genetic algorithm. Eng. Optimiz., 41: 437-457.
Taleizadeh AA, Niaki STA, Seyed Javadi MH. (2009). a harmony search algorithm to optimize chance constraint stochastic inventory systems with dynamic demand. J. Manuf. Syst., In Press.
Taleizadeh AA, Shavandi H, Amiri M (2009). Joint Replenish-up-to multi products EOQ with fuzzy rough demand. 6th international industrial engineering, Tehran, Iran.
Taleizadeh AA, Shavandi H, Aslani S (2009). Developing a Multi-products Newsboy Problem with a Hybrid Method of Fuzzy Simulation and Simulated Annealing, Proceeding of 39th International Conference on Computers and Industrial Engineering, Paris, France, 6-8 July 1805-1805.
Tersine RJ (1994). Principles of inventory and materials management. Fourth ed., Prentice Hall, New Jersey, USA.
Wagner HM, Whitin TM (1958). Dynamical version of the economic lot size model, Manag. Sci., 5: 89-96.