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Full Length Research Paper

Equilibrium and non-equilibrium models of the power markets

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The competition trend within the electricity segment has motivated the research community efforts and directed them towards investigations of deregulation of the electricity markets. This underlines significant research needs to insure providing appropriate design and functioning, as well as analysis support models that would fit to recent electricity market settings. Thus, this work focuses on facets of the deregulated electricity markets and on modeling the power market. It aims primarily at identifying, classifying and characterizing the quite bewildering multiplicity of the methods available in the specialized literature on the topic. This study offers review of the most appropriate works related to electricity market models, like the equilibrium and non-equilibrium models, and some other related areas of research, like optimization as an exogenic variable or firm decisions function. The agent and Cournot based supply function has non-equilibrium and equilibrium simulation models under the conditions of both imperfect and perfect competition. Lastly, it characterizes the approaches most suitable for implementing different types of market analysis and planning studies in the electricity sector for new setting.

Key words: Electricity market deregulation, equilibrium and non-equilibrium market models, market behavior.

INTRODUCTION

The success of deregulation in such industries as the transportation, airlines and telecommunications industries motivated adoption and implementation of electricity deregulation in a multitude of countries all over the world. Deregulation of the electricity market has stimulated a substantial bulk of research aiming at modeling, and consequently improving our own understanding of, how the different market sectors function and of how they interact with one another. Additionally, firms contend in supplying generation services at prices set by their corresponding markets in response to: i) the interactions between them, and (ii) market service demand. Despite the fact that the electricity markets throughout the world demonstrate considerable differences, electricity deregulation efforts almost in all these markets share in common the goal of reducing consumer prices and increasing their social welfare. Accordingly, the electricity firms susceptible to considerable risk, as well as their needs, as a consequence, for appropriate models for decision support has grown largely. However, the traditional models of electrical operation can only poorly fit to the emerging market conditions because the market behavior, which is the new stimulant of operation decisions, was not taken into consideration by these models. Thus, new areas of quite interesting electrical industry research have emerged. Some published works have already investigated how game theory-based models may be implemented for exploring the proper sides of design and deregulation in the energy markets. For example, Kahn (1998) surveyed the numerical techniques employed in analyzing the electricity market power, with particular emphasis on equilibrium models based on maximizing participant profits, which presume an oligopolistic type of competition. Hobbs (2002) published an abridged overview of the respective literature which focuses particularly on Cournot-based models. More recently, Boucher and

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Smeers (2001) discussed market equilibrium formulations.

Day et al. (2002) carried out a more elaborate survey of the literature of the power market modeling and paid special attention to the equilibrium models. Ventosa et al. (2005) overviewed the recent trends in market modeling. Moreover, a more recent survey of the critical aspects of energy market deregulation was published in 2009 by Nanduri and Das. Nevertheless, the previous literature papers do not shed light on the following: 1) the perspective of time structure in the electricity market competition and 2) market models of the behavior in the electricity market systems. Furthermore, a number of particular areas of recent research interest such as solution approaches used to obtain optimal bidding strategies and Nash equilibria are not adequately addressed. The contributions of this paper lie in filling these voids. This article aims at helping us in identifying, characterizing and classifying the somewhat confusing approaches available in the scientific literature on this topic. It provides review of the publications most related to electricity market modeling. Even though a large number of publications is found in the published literature addressing modeling of the functioning of deregulated power systems, the review only took into account the most appropriate of these publications brevity. Original ordering of the aforementioned models is provided for classifying them in terms of competition and time. These particular characteristics are fairly helpful for building an understanding of the disadvantages and advantages of each of these models. Lastly, this paper specifies the approaches most appropriate for each of the purposes of concern, and, by so doing, addresses an increasingly critical market concern.

THE TIME-DEPENDENT AND TIME-INDEPENDENT MODELS OF COMPETITION IN THE ELECTRICITY MARKET

From the perspective of time structure in the electricity market competition, the various approaches suggested in the literature so far can be categorized following the scheme portrayed by Figure 1. The time-independent and time-dependent models of the competitive electricity markets are different. The time-independent competitive market models will be investigated in the second part of this work which will cover perfectly, as well as the monopoly, competitive markets. On the other side, the oligopoly models are more appropriate when the structure of the market is time-dependent. In consequence, different modeling approaches may be adopted based on the length of time, whether it is long, medium or short. The time structure is a principal parameter for the purpose of categorizing the various electricity models because varying time-dependent and time-independent methods are associated with each time level. In the microeconomics literature, three market structures are usually investigated; the perfectly competitive, oligopoly and monopoly markets. The perfectly competitive and the monopoly markets are diametrical cases. The former involves various competing sellers whereas the latter engages one seller only. The microeconomics theory supports that perfectly competitive markets may be modeled on the basis of net benefits maximization or cost minimization. Those models based on optimization are commonly the best for modeling this kind of markets. Likewise, the monopoly may be modeled by the monopolistic firm's program of profit maximization. In such models, prices are usually drawn from the demand function. Generally, as the market heads towards perfect competition from monopoly, an increase in the level of social welfare is obtained. And it was quite this, namely: the increase in social welfare, what motivated restructuring of the electricity industry. In spite of that the ideal context for perfect competition has not been achieved yet, most of the deregulated electricity markets generally approached oligopolistic settings of the market power, which are frequently susceptible to misuse by large predominant firms, since oligopoly is by convention that market scenario corresponding to market with few sellers.

According to the definition provided by the microeconomics literature, the market power (MP) is the capability of seller to preserve prices beyond the competitive levels for a prolonged time period. The time structure is a chief characteristic for classification of the electricity models because every time domain entails various decision parameters and diverse modeling techniques. For instance, when studies of long-time planning are implemented, decisions on capacity-investment are the principal decision parameters involved while, in the meantime, decisions on unit-commitment are frequently overlooked. Contrarily, the start-ups and shut-downs grow important decision parameters and each generator's maximum capacity is considered as constant in studies of short-time scheduling (Ventosa et al., 2005). Specifically, under the condition of short-time functioning, a single day to seven days the conclusions derived from that literature so far surveyed by this paper illustrate that the leader-in-price model which originated from the microeconomics theory, stands as the best method for representing the market (Anderson and Philpott, 2002; Baillo, 2002; Baíllo et al., 2002; García et al., 1999). According to this model, the firm runs after maximum profit achievement, meanwhile taking into account the firm's residual demand function which associates the price with the corresponding energy outcome where medium-time (a month to a year) studies. Most of the models are established simultaneously upon the supply function equilibrium (Bushnell, 1998; Kelman et al., 2002; Otero-Novas et al., 2002; Rivier et al., 2001; Scott and Read, 1996). Eventually, the microeconomics theory proposes that in problems of long-time investment decisions, the Stackelberg equilibrium may be more fitting than other oligopolistic models

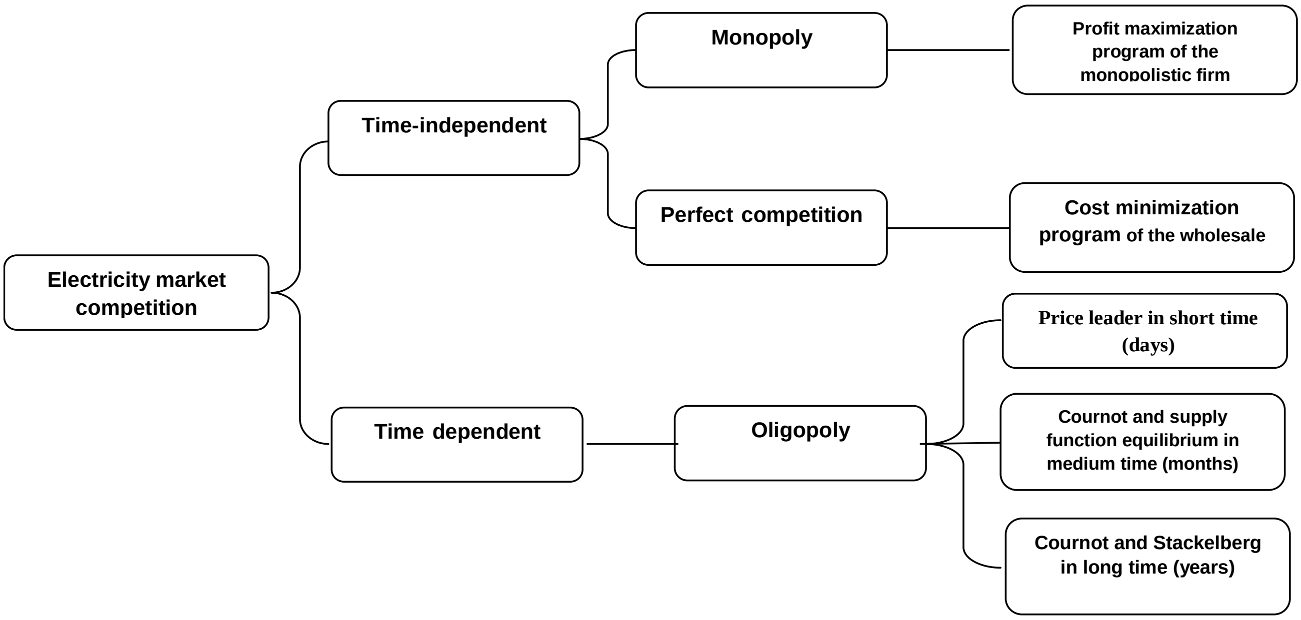

Figure 1. A scheme for classification of the time construction of electricity market competition.

because of the sequential nature of its decision-making process. Normally, a leader firm exists and decides firstly on its individual optimum capacity, and follower firms next make their optimum decisions in light of knowledge of the leader firm's capacity (Varian, 1992). Thus far, only few articles devoted to a representation of investment in the imperfect electricity markets are available in the scientific literature.

In many of the accessible related works, comparisons between the Stackelberg and the Cournot equilibrium for investment decision modeling had been held. One of the main conclusions derived from these studies is that albeit, from a theoretical stand point, the two models build on dissimilar assumptions, only minor differences, from a practical perspective, are found between these models in most of the results they produce. The Stackelberg model created by Ventosa et al. (2002) proves to possess a so-called mathematical programming with equilibrium structure (MPEC) as a consequence to the fact that it corresponds to a single leader firm only. Contrariwise, the model of Murphy and Smeers (2002) which is based on the Stackelberg model, possesses a structure of equilibrium problem with equilibrium constraints (EPEC) because many, rather than one, leader firms are assumed to co-exist. A final notice, the MPEC model is less common than the EPEC one though the former is easier to manage.

MARKET BEHAVIOR IN THE POWER SYSTEM MODELS

According to the market behavior in power systems stand point, the various approaches proposed in the respective literature may be categorized following the scheme shown in Figure 2. It summarizes the findings of a review of the most appropriate published works related to market behavior in the power system modeling. Deregulation of the electricity market is directing the production of power energy in the direction of competitive market environment. Development of the electricity markets has necessitated analysis of market behavior and modeling. The restructured electric power system reviews the most recent developments in the models of the electricity market and discusses application of these models to practical analysis and evaluation of the electricity markets.

From an equilibrium point of view, models of behavior of the market power industry are generally divided into two types: non-equilibrium and equilibrium models. And these problem structures are attracting substantial attention recently because of their broad applicability to many models, like the equilibrium and the non-equilibrium models, in the power systems economics. On account of, this article tends to present a general formula for the power system problem where π_i represents the profit of each firm i (for optimization models of non-equilibrium models is i = 1 i = 1 and for equilibrium models is $i\grave{o}\{1,\ldots,n\}$); x_i x_i correspond to firm decision variables whereas attendant constraints are given by $R(x_i)$, S(x_i). Subsequently, the mathematical structure can be expressed as:

$$Maximize : \Pi_i\left(x_i\right)$$

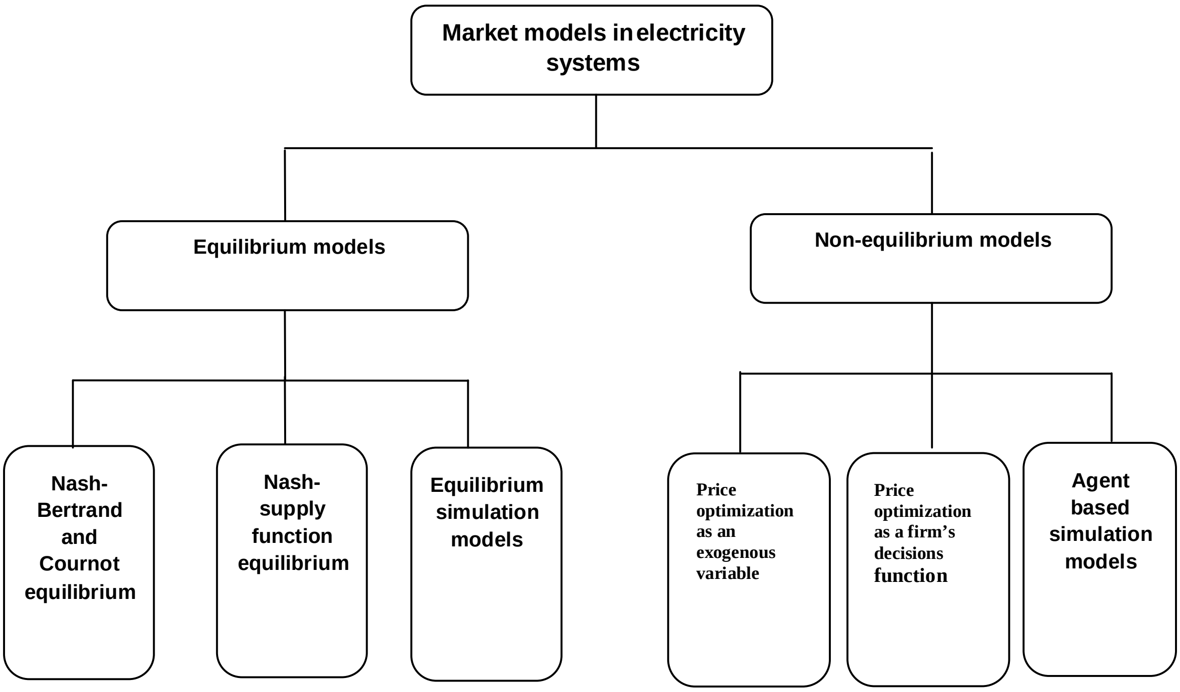

Figure 2. The market models of the behavior in the electricity market systems.

$$Subject\,to : R_i(x_i) = 0$$
$$S_i(x_i) \le 0$$
$$S(x_i)$$
$$Maximize: \Pi_i(x_i)$$
$$S_i(x_i) \le 0$$

Non-equilibrium models of market behavior

As far as the non-equilibrium models of market behavior are concerned, many researchers have, for a considerable period of time, been developing models addressing problems like the agent-based simulation models, and optimizing scheduling of electricity generation or establishment of offer curves within the context of imperfect as well as perfect competition. From the other point of view, prices in the various optimization models may be classified and modeled either as a function of a firm's decision or as an external variable.

Optimization considering price as an exogenous variable

Optimization models concentrate on the problem of maximization of profit for a firm competing in the market. Additionally, models which are founded on optimization grounds are frequently established as an optimization program such that one firm seeks to achieve its maximum potential revenue. In such case, there exists one objective function to optimize and this function is prone to a group of economic and technical constraints. On the other side, optimization models tend to represent a single firm only. Accordingly, the market is assembled in the latter models in a demonstration of the process of price-clearing which is amenable to modeling because it depends on the particular quantity provided by that firm of concern or as if it were exterior to the optimization program. Therefore, optimization models are suitable for short-time (days) planning and analysis of the market power on the basis that they only take one participant into consideration. In contrast with multiple-firm competitions, well-known and robust optimization algorithms offering further detailed modeling capabilities may be utilized for Solving optimization-based models fora single firm competing in the market.

Gross and Finlay (1996) showed that the optimization problem of a particular firm may be fragmented into a group of sub-problems and thus price may be modeled in the sense of being exogenous variable. As is expected in the perfect competition case, convex costs and deterministic prices, comparisons between the market prices and marginal costs of each generator dictate each generator's production. Subsequently, the generation unit's best offer corresponds to bidding of its marginal cost. Rajaraman et al. (2002) recounted and delivered an answer or the generation firm's self-commitment problem under the condition of external prices uncer-tainties. The definite objective function that requires to be magnified is firm's revenue in light of energy prices and the reserves at those nodes whereat the units of the very firm are positioned. And these are at the same time presumed to be non-certain and determined externally. In a mode that compares with the Gross and Finlay's approach, the authors advocate that within the scope of this context, each generating unit's scheduling problem can be handled independently. This makes the task of ending up with a solution, significantly, and accordingly allows for an elaborate representation of every unit. The problem can be resolved by backward dynamic programming and a multitude of numerical examples spotlight the potentials of this approach. A number of the late models express electricity prices as uncertain exogenous variables for deciding on the functioning of the diverse generating units while meantime assuming risk-hedging standards. Fleten et al. (1997, 2002) handled risk management problem that was encountered by the electricity producers participating in Nord Pool at the medium-time (months) scale. These firms used to confront some considerable uncertainness in hydraulic inflow and in the contract and the spot market price. By assuming that the inflows and prices were highly associated, they suggested a stochastic programming model coordinating hedging through the forward market with the resources of physical generation. They modeled risk detestation via penalizing risk by means of a target piecewise, linear, deficiency cost function. Lately, Unger (2002) modified Fleten's approach through explicit measurement of the risk within the framework of conditional value at risk. Consequently, these problems can be resolved by means of linear programming.

Optimization considering price as a decision function of firm

As opposed to the foregoing approaches wherein the process of clearing of price is thought of as non-contingent upon the decisions which the corporation's decisions, another group of models, which plainly consider the effects of production of a firm on produce price, exists. The amount of electricity in these models which a firm of interest can sell at each given or potential price is dictated by the associated residual demand function of which. Behavior of corporation targeting maximum possible revenue/profit while in the meantime accepting supply and demand curves of its competitors as they are given is described in the microeconomics theory by the model known as the leader-in-price model (Varian, 1992). García et al. (1999) addressed the problem of unit commitment that a specific firm may face in the form of linear residual demand function. Bearing in our minds that the revenues of markets are quadratic functions of total product of the firm, a procedure of piecewise linearization of market revenues would be proposed so as to enable use of solvers of the powerful mixed integer linear programming type. Anderson and Philpott (2002) used a probability distribution to express the uncertitude accompanying the residual demand function. The thesis of Baillo (2002) advanced the approach of Anderson and Philpott through incorporation of elaborate methodology of generator systems modeling, which means that hourly offer curves are more of dependent than of independent.

Agent-based simulation models

The sophisticated array of interdependencies of, and interactions between, the participants in the decentralized and deregulated, electricity markets of nowadays are quite similar to those addressed by the game theory (Picker, 1997). Nevertheless, strategies employed by numerous participants in the power market are usually more complicated than can be modeled by the techniques of the standard game theory conveniently. Specifically, the capability of the market participants to repetitively explore the markets and to quickly adjust their strategies accordingly imparts further complexity. Simulation hence provides a somewhat more flexible structure for an exploration of the effects that the repetitive participants' interactions impose on evolution of large scale electricity markets. It seems that the static models overlook the fact that the majority of agents build their decisions on historic information cumulated on the foundations of the daily functioning of market mechanisms. That's to say, the majority of agents seem to be learning from past experiences and, by so doing, refine their individual decision-making and tune to alterations attendant to such environments like alterations in demand, competitor's actions, and uncertainty in hydro inflows. This proposes that the adaptive, agent-based simulation approaches may highlight those characteristics of the electricity markets that the static models tend to ignore. Many electricity market agent-based simulation models have so far been created. Examples include the models established by Bower and Bunn (2002), Petrov and Sheblé (2002), Lai et al. (2002), North et al. (2002), Veselka et al. (2002) and Skoulidas et al. (2002). And this type of models was used in analyzing the new; by that time, electricity trading arrangements for England and Wales (Bunn and Oliveira, 2002). On the other hand, Bower and Bunn (2002) put forward an agent-based simulation model in which the generation enterprises are viewed as adaptive, autonomous agents who participate in a daily, recurrent market and who look for strategies that maximize their benefits on the grounds of the findings gained from the past session. Every company utters its own strategic decisions on account of the prices at which its plant products are offered. The companies are posited to every day target the goals of minimizing the utilization rates of their generation portfolio and securing a benefit that is greater than the benefit it achieved a day before. All the information at hand for every one generation corporation constitutes its profits and the products that its generating units release hour by hour. Compliant with what is common to this set of models, the demand component is illustrated by means of a linear demand curve. This framework provided the authors with the ability to examine several market designs fitting the modifications that took place lately in the large scale market of electricity. Specifically, the authors will compare the market output obtained under the assumption of uniform pricing with that attendant to the pay-as-bid one. North (2001a, b) utilized some agent-based models in identifying those infrastructure variables in electric power generation and transmission resulting in local price spikes. These researchers additionally illustrated viability of employing agent simulation in quantifying the degree of interdependencies between the natural gas and the electric power and infrastructures. Thomas et al. (2003) formulated a conceptual modeling framework aiming at an examination of infrastructure interdependencies. And the various models have shown that agent-based simulation models are appropriate for repetitive experimentation under controlled conditions.

Market behavior considering equilibrium models

Equilibrium models address market behavior with consideration of competitions between all the participants, and for this reason they are in general more fitting planning and analysis of the market power at the long-time level. On the other hand, the approaches considering equilibria of this market using traditional mathematical programming settings plainly are grouped into two classes of models categories. The most common category is the one based on Cournot and Bertrand competition theorem wherein competition between firms takes place in quantity and in price strategies. The other category, which is more complex and consequently less common than the former one, builds on the supply function equilibria wherein firms rivalize in the strategies of their offer curves. Nonetheless, these approaches contrast in the strategic variables support has grown largely. However, each of them considers, both find roots in the concept of Nash equilibrium which states that markets reach equilibrium when the strategy of each and every firm is the ideal reaction to the strategies its contenders/rivals follow. On the other side, the equilibrium simulation models comprise a replacement to the equilibrium models in the situations of sophisticated problems that it can not be handled within the framework of formal equilibrium. The equilibrium simulation models take into account those concurrent programs of profit maximization that each firm competing in the market implements and impart a more resilient approach of addressing the market problem than the other equilibrium models though the former are in general established upon assumptions characteristic of each study.

The Nash- Bertrand and Cournot equilibrium

The Cournot assumption holds that the firms in general compete with quantities only. Each firm presumes that the quantities its opponents hold are invariable and then makes its quantity decision accordingly. Afterwards, the game is resolved for Nash-Cournot equilibrium where no one firm wins by unilateral deviation from its own bid quantity. Contrarily, Bertrand posits that firms compete in prices. Each generator presumes that its opponents' prices are fixed and hence makes its individual price bid. The Nash equilibrium achieved under this competition scenario is called the Bertrand-Nash equilibrium. In spite of that theoretical support for adapting the Cournot equilibrium type of model to the electricity market is controversial, the economists research group exhibits a consensus on that in the situations when competition is not perfect, this model is nonetheless suitable market model is often employed in supporting the studies of market power. A contemplative compilation of essays related to Cournot equilibrium which incorporates this approach can be found elsewhere (Daughety, 1988e).

The various publications dedicated to the aforementioned models focus on certain areas such as influences of the transmission networks, risk assessment, hydrothermal coordination, and market power analysis. Cournot models' mathematical structures appear to be a framework of algebraic equations besides Cournot equilibrium whereby the firms specify their optimum outputs. Due to that, the majority of the equilibrium-based models derive from the concepts of Cournot solution.

Measurement of the market power was the first application of Cournot-based models to the electricity market. Borenstein et al. (1995) utilized this type of theoretical market models in analyzing the electricity market power of California, in place of employing the more traditional Lerner index (LI) and the Hirschman-Herfindahkl index (HHI) which respectively estimate margins of the price-cost and market shares. Scott and Read (1996) employed the dual dynamic programming (DDP) approach in studying the New Zealandian electricity market. This approach entails superimposition of the hydro optimization problem on the Cournot market equilibrium at each stage. Bushnell (1998) suggested a comparable model for the purpose of studying the Californian market. The most important contribution of the latter model was its deliberation on the meaning of the marginal water value of the firm within a deregulated electricity context. The approach of Rivier et al. (2001) differs from both the Bushnell and the Scott and the Read models in that it makes use of the solid fact of that optimum conditions may be, in a direct manner, resolved by virtue of its mixed complementary problem (MCP) construct which enables the utilization of distinct complementarity approaches for factual resolvement of the sized problems. Kelman et al. (2002) united the Stochastic Dynamic Programming technique and the Cournot concept to deal with the problems of uncertainty in hydraulic inflows.

The pricing of congestion in transmission networks is an additional field wherein the Cournot-based models have additionally played a substantial role. Hobbs (2002) models adulterate the competition between the electricity producers in the POOLCO-based and the reciprocal power markets in the form of a Linear Complementarity Problem. This kind of electric network model is called the DC model and contrary to the foregoing models, it employs a variational inequality (Jing-Yuan and Smeers, 1999) while the approaches the former models adopt enable coping with big problems. All of these models assume that the firm's generation units are positioned at a single node only in the network each. Subsequently, as was highlighted by Neuhoff (2003), a pure equilibrium strategy never exists.

Nash-supply function equilibrium

The supply functions are in fact price-quantity curves delivered by the generators to the independent system operator. The supply function competition, on the other hand, is thought of as representing the functioning of independent-type power markets more tightly than either the Bertrand or even the Cournot type competitions. The considerateness most critical stems from that generators' strategies are conveyed as quantities rather than offer curves in the Cournot approach. Thus, prices at equilibrium conditions are set by the respective demand function only. Since the demand function is largely vulnerable to the representation of demand, then the equilibrium prices will be higher usually than the real ones observed in the markets. This drawback appears to support the conception that the approach of supply function equilibrium provides quite more satisfactory an alternative for the purpose of representing competitiveness in the electricity market (Rudkevich, 1999). Dissimilar with the afore-mentioned three Nash games in specific oligopolistic circumstances, the supply function equilibrium approach presumes that a player has more information than all others. The Cournot equilibrium, in which firms select their respective optimum outputs, is easier to calculate than the supply function equilibrium because the mathematical construct of Cournot models corresponds to a group of algebraic equations whereas that of the supply function equilibrium models is a combination of differential equations. And this assumption brings about the model known as Stackelberg game. In the Stackelberg game, a ''leader" firm makes decision first then the ''followers" take their own decisions in light of knowledge of the decision of the "leader." A competition like this has been proved to be of use in modeling those oligopolistic markets having a large dominant firm and some smaller contending firms. Albeit the foregoing presumptions have been used somewhat extensively in the literature of bidding strategies, it can be noticed that the assumption of availability of complete information on bids of the competitor before a firm takes its own bidding decision does not represent the non-cooperative power market games. Despite variations in the modeling proposals available, we can recognize a set of features that may be employed in constructing a contrast between the varying techniques of the supply function equilibrium. Certain traits of these pertain to that representation of market which the researcher takes on representation of the market, for example, the potential consideration of the non-symmetrical corporations and supposition regarding the configuration of demand and supply functions, and the marginal cost curves. Additional characteristics pertain to the models applicable to the transmission networks (for example, transmission constraints) and the generation systems (for example, capacity constraints). Lastly, the methods used to solve the problems of concern by the different researchers and the numerical cases covered are two pertinent features too.

The approaches of supply function equilibrium were in the first place created by Klemperer and Meyer (1989). They demonstrated being some exceedingly appealing stream of research for equilibrium analysis in electricity wholesale markets. Calculations of these models call for resolving a group of differential equations in place of those algebraic classic set of equations that is involved in the classic equilibrium models wherein strategic parametric quantities take forms of price or quantities. Green and Newbery (1992) scrutinized duopoly behavior distinguishing electricity markets of England and Wales in their first operation years under the equilibrium approach of the supply function. In the asymmetric duopoly case, it has been illustrated that large firms find increases in prices more profitable and accordingly having a larger motivation for submitting a supply function with a higher slope. Small firms hence face a residual demand curve that is less elastic and tending to depart from the corresponding marginal costs. Rudkevich et al. (1998) has produced an expression of closed-form that gives price to a supply function equilibrium given realization of demand compliant with n-firm symmetric oligopoly distinguished by uniform pricing and inflexible demand.

As such, the convergence problems emanating from numerical integration of the supply function equilibrium set of differential equations were accordingly overcome.

This approach in addition allowed a consideration of the stepwise marginal cost functions. This is somewhat more realistic than the convex and differentiable type cost functions which are quite representative to the models earlier to this one.

Green and Newbery (1992) reflected on the scenario of non-symmetric (asymmetric), n-firm oligopoly of linear marginal costs against a linear demand curve with an invariable slope over time. A linear supply function equilibrium articulated by an affine supply function was attained. Grant and Kahn (2000) expanded the foregoing outcomes to the situation of capacity constraints and affine marginal cost functions. They implemented this approach in predicting the degree to thatstructural changes imposed on the electricity industry of England and Wales could influence spot prices of the wholesale electricity industry. Hogan (2001) comprehensively reviewed the approach of supply function equilibrium and showed the exceptional complexity of earning solution for the set of differential equations which derive. And they particularly highlighted how much difficult it is to discard impractical solutions, like the equilibria attendant to decreasing supply functions, for example.

Generally, approaches founded on the SFE models require the demand function to be rigid, and this is the hypothesis most appropriate to the case of electricity. Berry et al. (1999) utilized a model of supply function equilibrium to forecast the output of a specified market structure, encompassing clear representation of transmission networks. Hobbs (2002) created a model wherein every firm's strategy assumes the form of nodal affine supply function. Henceforth, the problem is constructed at twooptimization and the procedure for solving this problem is set up on MPEC approach.

The approach of supply function equilibrium offers specific advantages over the more traditional imperfect competition models. Particularly, it seems to be a model relevant to prediction of the medium-term electricity prices, provided that it, in a similar fashion to that of the Cournot model, is not contingent upon the demand function, however instead on the configuration of the respective equilibrium supply function set by the particular firms. Besides, the strategies of a firm that it will not be contingent upon the demand function, in a manner similar to that of the Cournot model, however on the outline of the equilibrium supply function which the firm defined. In contrast, supply functions are especially thought of as be representing the behaviors of firms under various scenarios of demand. This flexibility is nevertheless associated with significant practical restrictions regarding the numerical docility. For different reasons, like some of these models being difficult to prove in simple forms; being rarely gotten in closed-forms; and being non-clear in general multiple forms, the numerical methods are required for solving the set of differential equations, and this hence increases the computational requirements and complexity of this method. Besides this, a number of these solutions which are offered by this system may disobey the restraint of non-decrease that the supply functions ought to abide with. Ultimately, this results in heuristic procedures which themselves introduce convergence problems.

The approach of conjectural variations is easy to incorporate with the Cournot-based models. It modifies the conjectures which generators are anticipated to presume about the strategic decisions of their competitors as regards the possible future reactions. Three late publications (Day et al., 2002; García-Alcalde et al., 2002; Garcia et al, 2005) argued that this technique be taken into account for improvement of the Cournot pricing in the electricity markets. Within the domain of the electricity market, this method is referred to as the conjectured supply function (CSF) approach. Moreover, the calculation or estimation of conjectures may be influenced by various practical questions like the technical limitations of the generation units, for example, unit start-up, ramps, and units shut-down, and/or the functional representation of the generation costs, for example, step-wise, piecewise linear, or simply linear cost functions. Lambertini and Mantovani (2004) examined the strategic properties of dominant strategies, best reply functions, and substitutability or complementarity for differential games applied to Cournot firms experiencing static demands and following investment policies of the Ramsey-type. The research work of Cellini and Lambertini (2008) flows in this direction and examines the concepts of strong and weak time consistencies in the differential games as well as their relationships with the game structures, specifically, their relationships with the attendant Hamiltonians.

Usually, the majority of the conjectural models are run using historic data (Barquín et al., 2004; Centeno et al., 2007; Song et al., 2003). Nonetheless, this kind of estimation may seriously be affected either by structural changes or by regulation, as it is more appropriate for short-term investigations and analysis. Accordingly, fundamental approaches for computing the conjectures' values from equilibrium models can too be found in the literature, however incorporating further hypotheses and/or constraints (Arriaga et al., 2005; Daxhelet, 2008; Sánchez et al., 2007) and thus leading to a kind of supply function equilibrium or of Conjectured Supply Function equilibrium in the majority of cases. Contrarily, demand is ideally presented in the long-term models as block levels where temporal coupling is wasted (Contreras and Pozo, 2009). Moreover, the technical constraints are regarded only in some implied manner at intermediary scale through modifying some of the input data adequately. For instance, ramps may be taken into account by statistical production restrains between blocks (Villar et al., 2010) and the costs of shutdown and startup may be encircled, in an implicit fashion, so raising marginal cost margins of the units of production (Contreras and Pozo, 2009).

Equilibrium simulation models

In general, those models founded on simulation are a substitute to the equilibrium models if the problem under investigation is more complex that what may be treated in the framework of formal equilibrium. In a large number of situations, simulation models proved to associate tightly with an equilibrium model family or another. For instance, when firms are presumed by a simulation model to make their own decisions on the basis of quantities, the researchers will ideally allude to the Cournot equilibrium model for supporting the competence of the approach they follow. The big merit of the particular simulation account resides on the extent of flexibility it can provide for implementing nearly any type of strategic behavior. Anyway, this freedom additionally entails the assumptions attendant to the simulation model be defended on theoretical foundations. Borenstein and Bushnell (1999) broadened this method by means of creating an empirical simulation model that determines, iteratively, the Cournot type equilibrium each firm's output of profit-maximizing is acquired and productions of the rest firms are assumed to be fixed. Then, for each supplier this step is reiterated till no firm is able refine its profits. Albeit this model applied successfully to California's markets, it demonstrated few algorithmic defects concerning the convergence properties, and oversimplified depiction of the hydroelectric plant functioning. Otero-Novas et al. (2002) produced a simulation model which takes into consideration the maximization of profit objective of generation firms each while explaining the technical restrains which influence the hydro-, and thermal-generating units. In this mode, the decisions that generation firms take are obtained from iterative procedure of a decision process of the two-level type. Firstly, the single firm refreshes its outcomes associated with the planning periods each through problem of the profit maximization type which implies that the market clearing prices are fixed and a Cournot constraint in included, thus limiting the output very company. Afterwards, a descending rule is used to modify that price at it the firm tends to offer the produce of the generating units each in every planning period. New clearing prices of markets are usually computed on the basis of such offerings and on progression of demand which is presumed not to be resilient. Day and Bunn (2001) suggested a simulation model that constructs the optimum supply functions for the purpose of analyzing the market power in England and Wales Pool. This model posits that every production corporation surmises that its own contenders will tend to maintain the very supply functions they rendered a day before. Doubtfulness about the demand curve of the residuals is explained by daily variations in demand. This way is quite analogous to the equilibrium system of supply functions, nonetheless it offers a framework further flexible to the extent that it allows us to take into consideration the factual, non-symmetric corporations and marginal costs information.

MODEL APPLICATIONS TO POWER MARKETS

This part sums up those experiences as well as the conclusions extracted from the scientific works reviewed so far and cited in the foregoing parts of this article in the context of the significant goals of market equilibrium and non-equilibrium models. As can be concluded, dissimilarities in model arithmetic structures and in market competition and modeling render beneficial knowledge for an identification of the applications of each model class. On the contrary, upon implementation of modeling studies which are time-dependent, the more appropriate class of models is the equilibrium one because as study scope extends further in time, the higher important grows competitors' response and the bigger becomes the necessity for elaborate modeling capacities. Contrariwise, optimization models entail demand-part bids and probabilistic expression of the offers of competitors, and additionally a greater degree of details. Accordingly, most models dedicated to hydrothermal equilibration and economic planning on annual scale turn out to be Cournot-based model type since these models render greater truthfulness in the interpretation of the material limitations than supply function equilibrium type of approaches which in fact exhibit numeric tractableness restrictions. Subsequently, equilibrium models seem to be the optimum substitute for the classic anti-trust instruments as evidenced by such indices like the Lerner index (LI) and the Hirschman-Herfindahl index (HHI). Lastly, as to investigation of the management of congestion in electric transmission networks, both equilibrium models of the supply function and Cournot are capable of synchronously taking into account contention between different corporations and the power flow restrains and at every node. The principal features of the models, of those cited so far by this work, demonstrating the highest significance are presented by Table 1. Depending on market model kind, the various models have been categorized into two groups. And models of each category have been arranged chronologically with reference to the publication year. In brief, this table summarizes the models and their behaviors and applications.

CONCLUSIONS AND SUGGESTIONS FOR PERSPECTIVE RESEARCH

This paper gives a review of a number of the critical areas in power market research. With the exception of a number of already available related reviews, the majority of the other cited works laid emphasis on narrow areas related to the power markets. This work, however, takes

Table 1. Summary of the most common power market models.

Market system	Authors	Year	Application of model	Market behavior	Market model
Equilibrium competition	Green and Newberry	1992	Market power analysis	Oligopoly	Supply function equilibrium
Equilibrium competition	Green	1996	Market design	Oligopoly	LSFE
Non-equilibrium competition	Fleten et al.	1997	Stochastic prices	Perfect competition	Price optimization (EV)
Equilibrium competition	Rudkevich et al.	1998	Market power analysis	Oligopoly	SFE
Equilibrium competition	Bushnell	1998	Hydrothermal coordination	Oligopoly	Cournot equilibrium
Non-equilibrium competition	Pereira et al.	1999	Risk management	Perfect competition	Price optimization (EV)
Equilibrium competition	Borenstein and Bushnell	1999	Market power analysis	Oligopoly	Simulation equilibrium model
Equilibrium competition	Otero-Novas et al.	2000	Yearly economic planning	Oligopoly	Simulation equilibrium model
Non-equilibrium competition	Bower and Bunn	2000	Market design	Oligopoly	Agent-based model
Equilibrium competition	Baldick et al.	2000	Market power analysis	Oligopoly	Linear supply function equilibrium
Non-equilibrium competition	Rajamaran et al.	2001	Price uncertainty	Perfect competition	Price optimization (EV)
Equilibrium competition	Day and Bunn	2001	Market power analysis	Oligopoly	Simulation Equilibrium Model
Equilibrium competition	Hobbs	2001	Congestion management	Oligopoly	Conjectural variations
Equilibrium competition	Ventosa et al.	2002	Capacity expansion planning	Oligopoly	Stackelberg game
Equilibrium competition	Murphy and Smeers	2002	Capacity expansion planning	Oligopoly	Stackelberg game
Equilibrium competition	García-Alcalde et al.	2002	Price forecasting	Oligopoly	Conjectural variations
Non-equilibrium competition	Anderson and Philpott	2002	Strategic bidding	Leader in price	Price optimization (FDF)
Equilibrium competition	Barquín et al.	2003	Hydrothermal coordination	Oligopoly	Cournot equilibrium
Equilibrium competition	Barquín et al.	2004	Demand function	Oligopoly	Conjectured supply function
Equilibrium competition	Garcia et al.	2005	Price forecasting	Oligopoly	Conjectural variations
Equilibrium competition	Anderia	2006	Demand function	Oligopoly	Conjectured supply function
Equilibrium competition	Sánchez et al.	2007	Inelastic demand	Oligopoly	Conjectured supply function
Equilibrium competition	Centeno et al.	2007	Price elasticity from historical data	Oligopoly	Conjectured supply function
Equilibrium competition	Daxhelet	2008	Elastic demand	Oligopoly	Conjectured Supply Function
Equilibrium competition	Cellini and Lambertini	2008	Demand function	Oligopoly	Conjectured Supply Function
Equilibrium competition	Contreras and Pozo	2009	Marginal cost functions	Oligopoly	Conjectured Supply Function
Equilibrium competition	Villar et al.	2010	Marginal cost functions	Oligopoly	Conjectured Supply Function

over a wider approach via examining many of the most popularly researched themes in the energy field. It sheds light on several comparatively recent research areas like the electricity markets time-dependent and the time-independent competitions models. In addition, it dedicates profound consideration of some of the critical issues most frequently researched, like the equilibrium and non-equilibrium models of markets behaviors. Several widely employed application methods have too been discussed. Under the non-equilibrium models scenario, many researchers have been creating models which are deemed to cover problems like simulation models of the agent-based type and decisions functions of corporations within the context of imperfect as well as of perfect competition conditions, and optimization of price under the assumption of price being an exterior variable. Currently, most of these researchers join one of two distinguished research groups striving each to non-mutually tackle one of two serious challenges. The price optimization modelers are trying to develop risk management models which aim at helping the firms in setting their optimum positions in the respective markets and handling the forefront challenge of transforming the offer curves of the electricity-generating corporations. However, the static modelers appear to be overlooking the fact that the vast majority of the agents understructure their decisions on historic knowledge about the daily functioning of the mechanisms of the markets. That's to say, the majority of agents seem to be learning from past experiences and, by so doing, refine their decision-making and adjust to changes concomitant to this environment (for example, variations in demand, competitors' actions, and uncertainty in hydro inflows). This proposes that the adaptive, agent-based simulation approaches may highlight those characteristics of the electricity markets that the static models tend to ignore. The models which assess the firms' interactions in the power markets have persistently originated from the perception of equilibrium characteristic of the game-theory. A number of these models express equilibrium within the framework of equilibrium simulation models and the Cournot-based supply function equilibrium in order to provide a system enabling analysis of actual cases that encircle an elaborate expression of the systems generating electricity and of the transmission networks. In recent times, many researchers availed themselves of improving the already present Cournot-based models. The conjectural variations methodology of overcoming the elevated sensitivity of the process of price-clearing, as regards representation of demand, is representative of these models. Many questions still seek answers. For instance, pure strategy solutions may not be found once transmission retrains are existent, and supply functions with non-decreasing trends may be unsteady when the generating capacity falls short. Accordingly, these two issues, amongst some other, are deemed to lay foundations for further future research.

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