African Journal of Business Management Vol. 6(3), pp. 780-786, 25 January, 2012
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM11.298

Full Length Research Paper

Inventory model with stock dependent selling rate

Jones P. C. Chuang

[1]Department of Traffic Science, Central Police University, Taiwan, R. O. C. E-mail: una050@mail.cpu.edu.tw.

Accepted 18 August, 2011

After studying the maximization of the profit of an inventory model where the selling rate is dependent on stock, we generalized the demand from a ramp type demand to an arbitrary positive demand and then examined the solution structure to locate the optimal solution. Our findings are an extension of several previous papers. Consequently, we can provide a reasonable explanation for the unsolved phenomenon that appears in the solution structure in several previous papers. Furthermore, we have also provided an intuitive derivation for the optimal solution that will illustrate why the optimal solution is independent of the demand. The paper of Wu et al. (2008) published on International Journal of Information and Management Sciences were discussed to demonstrate how our findings have significantly simplified the solution process.

Key words: Inventory models, stock dependent selling rate, ramp type demand.

INTRODUCTION

In this paper, we have studied some inventory models to show that the replenishment policy is independent of the demand pattern. Our findings will not only simplify the complicated solution procedure shown in previous papers but also provide a reasonable explanation for the previously unsolved phenomenon of why the solutions are the same when different objective functions are defined in the different domains. Recently, there is a trend to develop inventory models with ramp type demand, where the demand is increasing until a turning point, where it becomes a constant for the rest of the planning horizon.

Researchers directed a lot of attention to the development of newer and more complex inventory models with respect to ramp type demands. However, an unsolved phenomenon occurs repeatedly but researchers have yet to provide a reasonable explanation for it. Our extended inventory model from a ramp type demand to an arbitrary demand provides a simplified solution approach for the mathematical work based on different objective equations. Moreover, a rational derivation from a managerial point of view is presented, so the unsolved phenomenon can become crystal clear. Our findings will not only help the theoretical development of inventory models but also provide an intuitive approach for decision makers.

LITERATURE REVIEW

Hill (1995) was one of the pioneers in the field of inventory model with ramp type demands. There has been an ongoing trend to develop this type of inventory system to obtain maximum profit or minimal cost. For example, Mandal and Pal (1998) studied decay product, Wu et al. (1999) developed inventory models with backlogging rates relative to the waiting time, Wu and Ouyang (2000) considered inventory systems under two replenishment policies starting with or without shortage, Wu (2001) examined the problem of deteriorating items satisfying Weibull distribution and then Giri et al. (2003) generalized the model with three parameters Weibull distribution. Deng (2005) improved the results of Wu et al. (1999). Manna and Chaudhuri (2006) constructed inventory systems where the deterioration is dependent of time. Deng et al. (2007) revised the findings of Mandal and Pal (1998) and Wu and Ouyang (2000) for single period inventory models. Recently, Wu et al. (2008) studied the maximum profit problem in the inventory system with ramp type demands and stock-dependent selling rates. They developed two inventory models which were related to the turning point of the ramp type demand, and then examined the optimal solution for each case.

The unsolved phenomenon in inventory models with ramp type demand

However, there is an unsolved phenomenon that appeared in their solution structure that became visible in the paper of Deng et al. (2007) for the minimum cost problem with the objective function $TC(t_1)=\begin{cases} TC_1(t_1), & for\ \ \mu \le t_1 \le T \\ TC_0(t_1), & for\ \ 0 \le t_1 \le \mu \end{cases}$. In Mandal and Pal (1998) and Wu and Ouyang (2000), they both studied the case when $\mu < t_1$, where μ is the turning point for the ramp type demand and t_1 is the point where the inventory level drops to zero. They obtained the cost function $TC_1(t_1)$, where

$$TC_1(t_1)=\frac{\mu D_0}{T}\left[\left(C_d+\frac{C_h}{\theta}\right)\left(\frac{e^{\theta t_1}}{\theta}-\frac{e^{\theta\mu}-1}{\theta^2\mu}-t_1+\frac{\mu}{2}\right)+\frac{C_s}{2}(T-t_1)^2\right] \quad (1)$$

for $t_1 \in [\mu, T]$ and then solved for $\frac{d}{dt_1}TC_1(t_1)=0$, which yields

$$\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s(T-t_1)=0 \quad (2)$$

under the restriction $t_1 \in (\mu, T)$. Deng et al. (2007) also considered the case when $\mu \ge t_1$ to derive the cost function $TC_0(t_1)$, where

$$TC_0(t_1)=\frac{D_0}{T}\left\{\left(C_d+\frac{C_h}{\theta}\right)\left[\frac{t_1\theta e^{t_1\theta}-e^{t_1\theta}+1}{\theta^2}-\frac{t_1^2}{2}\right]+C_s\left(\frac{\mu T^2}{2}-\frac{\mu^2 T}{2}-\frac{Tt_1^2}{2}+\frac{t_1^3}{3}+\frac{\mu^3}{6}\right)\right\} \quad (3)$$

for $t_1 \in [0, \mu]$ and then solved for $\frac{d}{dt_1}TC_0(t_1)=0$. It follows that

$$\left(C_d+\frac{C_h}{\theta}\right)\left(e^{\theta t_1}-1\right)-C_s(T-t_1)=0 \quad (4)$$

under the restriction of $t_1 \in (0, \mu)$. However, Deng et al. (2007) did not explain why when trying to locate the critical point (the minimum point) using two different cost functions, $TC_1(t_1)$ defined for $\mu \le t_1 \le T$, and $TC_0(t_1)$ defined for $0 \le t_1 \le \mu$, within different domains, the two identical functions in Equations 2 and 4, appear in different domains ($t_1 \in (\mu, T)$ and $t_1 \in (0, \mu)$).

The same unsolved phenomenon also happened in a maximum profit problem in Wu et al. (2008). We will provide a managerial explanation for this unsolved phenomenon and then we will present a detailed discussion of how to simplify Wu et al. (2008) with numerical examples. Motivated by the above observations, we predict that the optimal solution will be independent of the turning point, μ, of the ramp type demand. Consequently, we may also predict that the optimal solution is the same for all ramp type demands. Following this paradigm shift, we will try to prove that the optimal solution is independent of the demand pattern. Our findings will significantly simplify the solution process and provide a reasonable explanation for the unsolved phenomenon that occurs in previous papers, for example, Deng et al. (2007), Wu et al. (2008), and Wou (2010).

ANALYSES

Notations and assumptions

To develop a maximum profit inventory model under stock-dependent selling rate, we used the following assumptions and notation:

(1) The replenishment rate is infinite so that replenishments are instantaneous and the lead time is zero
(2) T is a finite time horizon
(3) A is the set up cost
(4) s is the selling price per unit
(5) C_h is the inventory holding cost per unit per unit of time
(6) C_d is the deterioration cost per unit. We extend the model of Wu et al. (2008) to include the deterioration cost such as Wu et al. (2008) is a special case of our proposed model with (a) $C_d = 0$, and (b) ramp type demand
(7) C_s is the shortage cost per unit per unit of time
(8) c is the purchasing cost per unit
(9) θ is the constant fraction of the on-hand inventory deterioration per unit of time
(10) $I(t)$ is the on-hand inventory level at time t over the ordering cycle $[0, T]$

(11) Shortage is allowed and fully backordered

(12) The theoretical demand, $D(t)$, is a positive continuous function

(13) $R(t)$ is the actual selling rate, that is, stock dependent, where α is a positive constant with

$$R(t)=\begin{cases} D(t)+\alpha\, I(t), & I(t)>0,\\ D(t), & I(t)\le 0.\end{cases}$$

(14) t_1 is the time when the inventory level drops zero

(15) t_1^* is the optimal solution for t_1

(16) $f(t_1)$ is an auxiliary function defined as $f(t_1)=\left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)\,t_1}-1\right)+C_s(T-t_1)$

where $\Omega=\alpha(s-c)-(C_h+\theta c+\theta C_d)$

(17) $Z(t_1)$ is the total profit that consists of selling price, set up cost, purchasing cost, holding cost, deterioration cost, and shortage cost

Replenishment occurs at time $t=0$ when the inventory level attains its maximum. From $t=0$ to t_1, the inventory level reduces due to both selling rate, $R(t)$, and deterioration. At t_1, the inventory level drops to zero, after which shortages are allowed during the time interval (t_1,T), and is completely backlogged at T.

Our proposed inventory

The inventory levels of our model are described by the following equations:

$$\frac{d}{dt}I(t)+\theta\, I(t)=-R(t),\quad 0<t<t_1 \tag{5}$$

and

$$\frac{d}{dt}I(t)=-R(t),\quad t_1<t<T \tag{6}$$

We directly solve Equations 5 and 6 to imply that

$$I(t)=e^{-(\theta+\alpha)t}\int_t^{t_1}D(x)e^{(\theta+\alpha)x}dx\text{, for } 0\le t\le t_1 \tag{7}$$

and

$$I(t)=\int_t^{t_1}D(x)\,dx\text{, for } t_1\le t\le T \tag{8}$$

Assuming that it is completely backordered, the purchase quantity at $t=0$ is

$$I(0)-I(T)=\int_0^{t_1}D(x)e^{(\theta+\alpha)x}dx+\int_{t_1}^{T}D(x)\,dx \tag{9}$$

Using integration by part, the holding cost during $[0,t_1]$ is evaluated:

$$C_h\int_0^{t_1}I(t)\,dt=C_h\int_0^{t_1}D(x)\frac{e^{(\theta+\alpha)x}-1}{\theta+\alpha}dx \tag{10}$$

The amount of deteriorated items during $[0,t_1]$ is evaluated:

$$I(0)-\int_0^{t_1}R(x)dx=I(0)-\int_0^{t_1}\left(D(x)+\alpha I(x)\right)dx$$

$$=\frac{\theta}{\theta+\alpha}\int_0^{t_1}D(x)\left(e^{(\theta+\alpha)x}-1\right)dx \tag{11}$$

The shortage cost during $[t_1,T]$ is evaluated through integration by part

$$C_s\int_{t_1}^{T}-I(t)dt=C_s\int_{t_1}^{T}(T-x)D(x)dx \tag{12}$$

The sale revenue per cycle is computed as

$$s\int_0^T R(t)dt=s\int_0^T D(x)dx+\frac{s\alpha}{\theta+\alpha}\int_0^{t_1}D(x)\left[e^{(\theta+\alpha)x}-1\right]dx \tag{13}$$

Therefore, the total profit per cycle is expressed as (sale revenue – order cost –purchase cost – holding cost – shortage cost – deterioration cost)

$$Z(t_1)=\frac{1}{T}\left\{s\left[\int_0^T D(x)\,dx+\frac{\alpha}{\theta+\alpha}\int_0^{t_1}D(x)\left[e^{(\theta+\alpha)x}-1\right]dx\right]-A-c\left[\int_0^{t_1}D(x)e^{(\theta+\alpha)x}dx+\int_{t_1}^{T}D(x)\,dx\right]-C_h\int_0^{t_1}D(x)\frac{e^{(\theta+\alpha)x}-1}{\theta+\alpha}dx\right.$$
$$\left.-C_s\int_{t_1}^{T}(T-x)D(x)dx-\frac{C_d\theta}{\theta+\alpha}\int_0^{t_1}D(x)\left(e^{(\theta+\alpha)x}-1\right)dx\right\} \tag{14}$$

The location of the maximum solution

From Equation 14, it follows that

$$\frac{d}{dt_1}Z(t_1) = \frac{R(t_1)}{T}\left[\left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)t_1}-1\right)+C_s(T-t_1)\right] \quad (15)$$

where $\Omega = \alpha(s-c)-(C_h+\theta c+\theta C_d)$.

The analytical solution procedure of our model

Motivated by Equation 15, we assume an auxiliary function $f(t_1)$, with

$$f(t_1) = \left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)t_1}-1\right)+C_s(T-t_1). \quad (16)$$

Motivated by Equation 16, we divide the problem into two cases: Case (a), $\Omega<0$, and Case (b), $\Omega \geq 0$.

Case (a), yields

$$\frac{d}{dt_1}f(t_1) = \Omega e^{(\theta+\alpha)t_1} - C_s < 0 \quad (17)$$

so $f(t_1)$ is a decreasing function. From

$$f(0) = c_s T > 0 \quad (18)$$

and

$$f(T) = \left(\frac{\Omega}{\theta+\alpha}\right)\left(e^{(\theta+\alpha)T}-1\right) < 0 \quad (19)$$

due to the intermediate value theorem for continuous functions (Thomas and Finney, 1996), there is a unique point, t_1^* that satisfies $f(t_1^*)=0$. We know that $f(t_1)>0$, for $0<t<t_1^*$ and $f(t_1)<0$, for $t_1^*<t<T$. According to Equations 15 and 16, $\frac{d}{dt_1}Z(t_1)$ and $f(t_1)$ have the same sign, it yields that $Z(t_1)$ is increasing for $0<t<t_1^*$ and is decreasing for $t_1^*<t<T$, so t_1^* is the optimal solution (maximum point).

Case (b), from Equation 15, $\frac{d}{dt_1}Z(t_1)>0$ so the increasing function $Z(t_1)$ attains its maximum at T. We summarize our findings in the next theorem.

Theorem

Under the assumption $\Omega = \alpha(s-c)-(C_h+\theta c+\theta C_d)$, we divide the inventory model into two cases: Case (a), $\Omega<0$, where $Z(t_1)$ attains its maximum at t_1^* which satisfies $f(t_1^*)=0$; Case (b), $\Omega \geq 0$, where $Z(t_1)$ attains its maximum at T.

MANAGERIAL VIEW OF OUR FINDINGS

We intend to provide an explanation from the managerial point of view to discuss our findings. If there is an item with demand, Q that takes place at time t, then there are two replenishment policies: (a) fulfill the demand from the stock, or (b) satisfy the demand from the backorder. If we decide to fulfill the demand, Q, at time t, from the stock, we will store $Qe^{(\theta+\alpha)t}$ at time $t=0$. Owing to the condition of stock dependent selling rate, the solution of $\frac{d}{dt}I(t)+(\theta+\alpha)\ I(t)=0$ is $I(t)=I(0)e^{-(\theta+\alpha)t}$ so that the beginning stock is $I(0)=Qe^{(\theta+\alpha)t}$. The inventory level, $I(x)=I(0)e^{-(\theta+\alpha)x}=Qe^{(\theta+\alpha)(t-x)}$, for $0\leq x\leq t$, and then the holding cost can be calculated by

$$C_h\int_0^t Qe^{(\theta+\alpha)t}e^{-(\theta+\alpha)x}dx = C_h\frac{Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right) \quad (20)$$

The deteriorated items can be computed as $\int_0^t \theta I(x)dx = \frac{Q\theta}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$ and those stock-dependent selling items, $\int_0^t \alpha I(x)dx = \frac{Q\alpha}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$. It follows that, when time is t after deteriorated items, $\frac{Q\theta}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$ and those stock dependent selling items, $\frac{Q\alpha}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)$ are depleted, the remaining stock Q is just enough to meet the demand.

Hence, the total profit for demand Q fulfilled from the stock is computed as the sum of selling price + stock dependent selling price – holding cost – deterioration cost –purchase cost:

$$sQ + s\frac{Q\alpha}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right) - C_h\frac{Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right) - C_d\frac{\theta Q}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right) - cQ\left(e^{(\theta+\alpha)t}-1\right) \quad (21)$$

On the other hand, if the backlog policy is adopted then the shortage cost is $QC_s(T-t)$, and the profit is computed as

$$sQ - QC_s(T-t) \quad (22)$$

We computed the difference between Equations 21 and 22 which is equivalent to

$$Q\left(\frac{\Omega}{\theta+\alpha}\left(e^{(\theta+\alpha)t}-1\right)+C_s(T-t)\right) \quad (23)$$

that is $Q\,f(t)$ (24)

according to Equation 16.

The replenishment policy

If $\Omega \ge 0$, then the result in Equation 23 is always positive, so to replenish from the stock is more profitable than replenishing from the backlog. We know that t_1 should be as large as possible so $t_1 = T$.

On the other hand, if $\Omega < 0$, by Equation 24, we know that if $f(t)>0$, then the item, Q, should be provided from stock, and if $f(t)<0$, then the item, Q, should be provided from backlog. By the same argument as Equations 17, 18 and 19, there is a unique point, t_1^* with $f(t_1^*)=0$ such that $f(t)>0$, for $0<t<t_1^*$ and $f(t)<0$ for $t_1^*<t<T$.

Hence, t_1^* is the optimal solution.

In our managerial approach, we did not construct the objective profit function for the inventory model. Our managerial approach comes up with the same results as our analytical method which shows that this alternative approach may provide an intuitive point of view.

Review of Wu, Ouyang and Yang

Here, we will demonstrate that our finding can simplify the solution process of Wu et al. (2008). They considered a ramp type demand with

$$D(t)=\begin{cases} D_0 t, & \text{if } 0\le t\le \mu, \\ D_0\mu, & \text{if } \mu \le t \le T \end{cases} \quad (25)$$

According to the relation between μ and t_1, they divided the problem into two cases: Case 1: $t_1 \ge \mu$ and Case 2: $t_1 \le \mu$.

Case 1

They derived the total profit, $Z_1(t_1)$, where

$$Z_1(t_1)=\frac{D_0\mu}{T}\left\{(s-c)T-\frac{s\mu}{2}+(\alpha s-C_h)\left\{\frac{(\theta+\alpha)\mu e^{(\theta+\alpha)\mu}-e^{(\theta+\alpha)\mu}+1}{(\theta+\alpha)^3\mu}\right.\right.$$
$$\left.+\frac{e^{(\theta+\alpha)\mu}-1}{(\theta+\alpha)^2}\left[e^{(\theta+\alpha)(t_1-\mu)}-1\right]-\frac{\mu}{2(\theta+\alpha)}+\frac{e^{(\theta+\alpha)(t_1-\mu)}-(\theta+\alpha)(t_1-\mu)-1}{(\theta+\alpha)^2}\right\}-c\left[\frac{1-e^{(\theta+\alpha)\mu}}{(\theta+\alpha)^2\mu}+\frac{e^{(\theta+\alpha)t_1}}{\theta+\alpha}-t_1\right]-\frac{c_s(T-t_1)^2}{2}-\left.\frac{A}{D_0\mu}\right\} \quad (26)$$

and then they used $\frac{d}{dt_1}Z_1(t_1)=0$ to imply that

$$\frac{[\alpha(s-c)-(C_h+\theta c)]}{\theta+\alpha}\left[e^{(\theta+\alpha)t_1}-1\right]+c_s(T-t_1)=0 \quad (27)$$

Table 1. Reproduction of Table 1 of We et al. (2008) for Example 1.

	μ	$\Delta = f(\mu)$	t_1^*	Q^*	Z^*
$\Omega < 0$	0.4	Δ>0	0.5953	132.36	443.62
	0.5953	Δ=0	0.5953	171.60	594.15
	0.6	Δ<0	0.5953	172.36	597.19

to solve for the solution interval (μ, T).

Case 2

They obtained the total profit, $Z_2(t_1)$, where

$$Z_2(t_1) = \frac{D_0\mu}{T}\left\{(s-c)\left(T-\frac{\mu}{2}\right)+(\alpha s - C_h)\left[\frac{(\theta+\alpha)t_1e^{(\theta+\alpha)t_1} - e^{(\theta+\alpha)t_1}+1}{(\theta+\alpha)^3\mu} - \frac{t_1^2}{2(\theta+\alpha)\mu}\right]\right.$$
$$\left. -c\left[\frac{(\theta+\alpha)t_1e^{(\theta+\alpha)t_1} - e^{(\theta+\alpha)t_1}+1}{(\theta+\alpha)^2\mu} - \frac{t_1^2}{2\mu}\right] - c_s\left(\frac{\mu^2}{6}+\frac{t_1^3}{3\mu}-\frac{\mu T}{2}-\frac{Tt_1^2}{2\mu}+\frac{T^2}{2}\right)-\frac{A}{D_0\mu}\right\} \quad (28)$$

$$\frac{[\alpha(s-c)-(C_h+\theta c)]}{\theta+\alpha}\left[e^{(\theta+\alpha)t_1}-1\right]+c_s(T-t_1)=0 \quad (29)$$

and then they used $\frac{d}{d\,t_1}Z_2(t_1)=0$ to yield that to solve for the solution interval $(0,\mu)$.

They knew that Equations 27 and 29 are identical, but they cannot provide an explanation as to why these two equations are the same. It is an unsolved phenomenon in their paper. It follows that Equation 27, defined for $\mu < t_1 < T$ and Equation 29, defined for $0 < t_1 < \mu$ are in fact the same function of our derived Equation 15, with $C_d = 0$, defined for $0 < t_1 < T$. We have a function Equation 16 defined for $0 < t_1 < T$ so it is not surprising that the two functions Equations 27 and 29 defined for $0 < t_1 < \mu$ and $\mu < t_1 < T$ are identical. Our contribution shows that (a) the optimal solution is independent of the demand pattern, (b) previous papers that construct the objective function piece by piece results in a complicated solution process that is unnecessary, and (c) a reasonable explanation for the unsolved phenomenon in previous papers.

Numerical examples

We consider the same numerical examples as Example 1 in Wu et al. (2008), $D_0 = 400$, $A = 50$, $s = 20$, $c = 15$, $C_h = 3$, $C_s = 5$, , $\alpha = 0.01$ $\theta = 0.05$ and $T = 1$. In Wu et al. (2008), the deterioration cost was not considered, so to compare with their results, we have to assume that $C_d = 0$. They have considered three different cases, $\mu = 0.4$, $\mu = 0.5953$ and $\mu = 0.6$. We quote their computation results in Table 1.

For a ramp type demand with the turning point, μ, with three different cases of $\mu = 0.4$, $\mu = 0.5953$ and $\mu = 0.6$, we know that the optimal solution, $t_1^* = 0.5953$ is independent of demand pattern and it has no relation to μ. For these three different cases, they have the same optimal solution we expected.

DISCUSSION AND CONCLUSION

In this paper, we extended the model of Wu et al. (2008) from a ramp type demand to an arbitrary demand and showed that the optimal solution is independent of the demand rate. Moreover, we also included the deterioration cost in our maximum profit inventory model. Our generalization not only handles a maximum range of all possible demand patterns but also provides an improved

improved approach to significantly simplify the solution process. Our intuitive approach used to solve the optimal problem can provide a clear explanation as to why the unsolved phenomenon occurs. For future purpose, researchers should focus on finding an algebraic method to solve this problem and develop model with several replenishments during the finite planning horizon.

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