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Review

Nominal interest rate: Zero or non-zero?

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This paper aims to study zero nominal interest rates and why they are good, and the division of our economy into two, that Is, non-monetary and monetary economies. In order to achieve this, we consider one-sector neoclassical growth model. In this model, we first consider economies that have no money. We suppose that the consumption and capital stock paths in this environment are Pareto optimal. Then we enter the money with a cash-in-advance constraint into the model and obtain the consumption and capital stock paths and compare with Pareto optimal consumption and capital stock paths. To study the effect of nominal interest rate on the optimum path of these variables, we define two zero and non-zero nominal interest rate scenarios. We will see that in a standard one-sector neoclassical growth model, in which money is introduced with a cash-in-advance constraint, zero nominal interest rates are optimal.

Key words: Nominal interest rate, cash-in-advance constraint, optimal consumption, optimal capital stock, Pareto optimal.

INTRODUCTION

An interest rate is a percentage of an amount of money, which a borrower pays a lender for the privilege of borrowing it. Most interest rates are quoted as a nominal interest rate, meaning one that is not adjusted for inflation. A nominal interest rate does not take into account the fact that the value of money changes and decreases over time due to inflation. This is not necessarily a bad thing, since it is impossible to predict future inflation, and because a fixed rate of return for borrowed funds must be agreed upon beforehand.

A nominal interest rate is one such as the interest rate on a mortgage, or the interest one might earn on a high-yield savings account. It is the interest rate as stated. This is distinct from a real interest rate, which is one that has been adjusted for inflation. Real interest rates are usually lower than nominal interest rates. For example, if a bank customer puts $1,000 US Dollars (USD) in a certificate of deposit which offers a five percent rate of return, then at the end of a year, there will be $50 USD more in the account, bringing the total to $1,050 USD (Chari et al., 1996). Five percent is the nominal interest rate in the preceding example. However, if inflation for that year is calculated to be two percent, while this does not affect the nominal interest rate, it does affect the real interest rate. The customer's dollars may have increased by five percent, but if all money becomes worth two percent less over that time in terms of purchasing power, the real interest rate was actually only three percent. Of course, a three percent real interest rate is better than the two percent decrease in value that the depositor would have experienced otherwise, and this is likely why he put his money on deposit -- to maintain its purchasing power (Barsky, 1987).

In times of rapid inflation or other forms of economic turmoil, the nominal interest rate may only decrease the otherwise larger loss of a person's purchasing power. If real interest rates are zero, or even negative, even a positive nominal interest rate may deliver a negative real

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rate of return. Fortunately, conditions such as these are rare in developed countries, and even then, the nominal interest rate will still offer some comparative benefit. In addition to interest rates, the terms "nominal" and "real" can apply to many types of economic data, such as wages, government expenditures, and a country's gross domestic product, among other things (Townsend, 1987).

In this study, after expression of introduction and interest rate developments from a historical perspective, and explanation of role of interest rate in economy, we further present the model. Both preferences and technology are assumed to be well behaved. Analytical results for the effects of nominal interest rate are presented afterwards, and finally, the conclusion.

Interest rate developments from a historical perspective

In the long term, the equilibrium real interest rate will be determined by underlying structural relationships in the economy. These relationships will probably only be changed gradually so that changes in nominal rates will primarily reflect changes in expected inflation. In the short term, however, the real interest rate will vary, not least because monetary policy seeks to influence the real interest rate in the short and medium term. Changes in nominal interest rates might therefore reflect changes in both real interest rates and inflation expectations (Brzezina, 2001).

Nominal interest rates were relatively stable from the 1800s and up to the 1950s. Periodically, strong fluctuations in inflation resulted in substantial variations in the real interest rate from one year to the next, as we measure it in retrospect (Brzezina, 2001).

The gold standard set the framework for monetary policy at the end of the 1800s and up to 1929. During the gold standard era, the central bank determined the price of the national currency relative to the price of gold. Central banks had to keep stocks of gold that could be used to stabilize the gold price by actively buying and selling gold in the market.

The money supply and inflation in a country are then determined by the supply of gold on the world market. When two or more currencies are pegged to gold at a fixed price, these currencies will also have a fixed price in relation to each other. As in other systems with fixed exchange rates, this restricted the individual country's scope for using the interest rate as an instrument to stabilize the real economy (Koto and Nishiyama, 2005).

In the interwar years, nominal interest rates were relatively high. This may primarily be ascribed to the policy of gold parity that was pursued in many countries. The aim of a number of countries, including Norway, was to strengthen the value of the national currency against gold, so as to reestablish the gold parity of the pre-First World War period (Lucas, 1987).

Nominal interest rates increased markedly from 1950 to 1985, reflecting accelerating inflation in this period. After 1985, nominal interest rates fell again and are now at about the same level as in the interwar period. At the same time, short-term variations in nominal interest rates seem to have increased. This may be related to central banks' more active use of interest rate policy to combat inflation (Brzezina, 2001).

Developments in inflation since 1870 have been remarkably similar from country to country. This is not a new feature that has coincided with the market globalization we have witnessed in recent years. It would appear that inflation is particularly low in periods when monetary policy has a clearly defined nominal anchor. Up to the First World War, the gold standard was that anchor, providing a direct link between the supply of gold and inflation. Whenever gold was discovered, the gold price dropped. In order to fulfill its commitment to maintaining the fixed price between a country's currency and gold, the central bank was obliged to buy gold using the national currency as payment. The supply of money then increased, which contributed to inflation. In periods when no new gold finds were made, prices tended to be stable or falling. The tendency for prices to fall was amplified in periods of strong economic growth. Economic growth is accompanied by higher turnover. Unless new money is printed to accommodate this, prices must fall (Wilson, 1979).

After 1945, with the introduction of the Bretton Woods system, a number of countries' currencies were pegged to the US dollar, which was in turn pegged to gold until 1971. In many ways, US monetary policy and nominal interest rate levels functioned as a nominal anchor, and to a large extent governed global interest rates and inflation developments (Wilson, 1979).

Inflation has been particularly high in times of war. The 1970s and 1980s were characterized by fairly high inflation following the collapse of the Bretton Woods agreement, the oil price shocks and counter-cyclical policy. From the end of the 1980s, an inflation target for monetary policy has functioned as a credible nominal anchor in a number of countries, curbing the pace of inflation. To calculate historical real interest rates, an estimate must be made of expected inflation. Figures for actual inflation are normally used. Inflation expectations may deviate from actual inflation, especially in periods when the latter is subject to considerable fluctuation. From 1870 to today, developments in global real interest rates can be roughly divided into 5 periods. In the period 1870 to 1896, the real interest rate ranged between 4 and 5%, and inflation fluctuated around zero. No new major gold finds were made in this period and the economy in general was exhibiting brisk growth (Cole and Stockman, 1992). The real interest rate fell to about 1 to 3% in the period 1897 to 1913, reflecting some acceleration in inflation of this period. New discoveries of gold were again made in this period. In the 1800s and up to the

First World War, the gold standard and price fluctuations in the agricultural sector contributed to alternating inflation and deflation (Cole and Stockman, 1992).

In the interwar period, 1918 to 1940, real interest rates were fairly high; sometimes well above 5%. This was partly due to the policy of gold parity and the reestablishment of the gold standard, which resulted in high nominal interest rates in a number of countries (Brzezina, 2001).

The post-Second World War era can be divided into two periods. The postwar period up to the collapse of the Bretton Woods agreement in 1971 was characterized by unusually low real interest rates, ranging from 0 to 2%. Real interest rates were low despite high returns in the stock market, strong growth in output and in stocks of real capital. According to most economic growth theories, this should have been accompanied by a high real interest rate. From a more short-term perspective, however, low real interest rates led to high investment activity and growth. However, this probably contributed to the tensions that led to the marked changes in economic policy in the 1970s and 1980s, internationally and in Norway. To a certain extent, a negative real interest rate characterized the 1970s and most of the 1980s in Norway. In retrospect, we might ask whether the interest rate was kept too low in relation to a reasonable long-term equilibrium level. In particular, this was a period of substantial fluctuations in the Norwegian economy, with high and variable inflation. The absence of a nominal anchor was one of the main reasons behind these pronounced swings in the Norwegian economy. Because of the extensive use of credit regulations, nominal interest rates remained low. With a policy of low interest rates and devaluations, inflation took root. Nominal interest rates were kept at a low level even though inflation and the value of tax- deductible interest expenses rose. The wide fluctuations culminated in a credit boom in the mid-1980s, followed by a deep recession and high unemployment towards the end of the 1980s.

From the mid-1980s, during and after the credit bubble, it was recognized that a substantial revision of economic policy would be necessary and that the problems created by inflation had to be taken seriously. The exchange rate was chosen as the nominal anchor. The real interest rate gradually rose and was relatively high until the mid-1990s, partly as a result of the tight monetary policy that was conducted in order to reduce inflation. The real interest rate has fallen markedly since the beginning of the 1990s (Cole and Stockman, 1992).

Neither the neutral interest rate nor the equilibrium interest rates are variables that can be observed. Economic theory can contribute insight into the factors that determine the equilibrium rate, but attempts to quantify the equilibrium interest rate based on theory yield a broad interval for a possible level. Quantification also presents problems, as estimates must be made for unobserved variables such as consumer impatience. An estimate of the equilibrium interest rate in industrialized countries presented in the publication (Kreditt, 2003), indicate an interval of 3 to 9%. The upper limit in particular seems unreasonable in relation to observed real interest rates, for example Lardic and Mignon (2003).

An alternative is to use historical average interest rates on the basis of an assumption that actual real interest rates will vary around the equilibrium interest rate. This method also poses problems. Each historical era will, for example, be affected by the specific characteristics of that era. Credit rationing; for example, has influenced interest rate formation for long periods. In addition, both population and productivity growth have varied over time.

Throughout the period 1870 to 2003, the average for long-term real interest rates in Norway has remained close to the average in the UK and the US: Norway 2.2%, the UK 2.0% and the US 2.8%. The economy has changed substantially in this period. Thus, the average for this period probably provides little indication of what a long-term equilibrium interest rate is today.

In order to comment on whether the real interest rate today is low or high, it may therefore be more meaningful to look at a “representative” period when the structure of the economy was more similar to the current structure. It is most reasonable to examine a period in the recent past, for example the past 10 to 15 years. Markets were deregulated in this period, for example. For Norway, it would also be natural to study the period after 1994. In the period 1994 to 2002, the average real interest rate for Norway was 4.2%, the UK 4.7% and the US%. Since 1994, the long-term real interest rate in Norway has varied between 3 and 6%, reaching its highest level at the beginning of the period and its lowest in the years 1997 to 2001. Both the Norwegian and the global economy have now entered a period of low real interest rates. The low interest rate level must be viewed in the light of the possibility of persistently low inflation both in Norway and other countries. However, the real interest rate is not by any means as low as when inflation in Norway was high in the 1970s and 1980s Ireland, (2003).

The interest rate has several functions in an economy

Many financial decisions involve a trade-off between present and future consumption. One example of this is households’ decisions on saving and borrowing. Income from employment normally varies in the course of a lifetime. Earnings are low when we are young, rising in our middle years, before falling again as we reach retirement age. Many people raise loans in early adulthood, repay their loan and build up positive net wealth as they reach middle age, then draw on their savings towards the end of their lives. This results in consumption that is more evenly spread over a lifetime than implied by the flow of annual income. A company’s investment choices also involve the choice between present and future consumption. Owners may give priority to present consumption by taking out dividends, or they can invest profits in the company and thereby lay the basis for larger profits later. By placing capital at the disposal of others, in other words by saving, one's own consumption is postponed. People require compensation for this, both because human beings are impatient and because there is a certain probability we may not live to see the future. The interest rate provides this compensation. If we want to use our money for consumption now or to finance investments by raising loans, the interest rate is the price we must pay in order to do so. The interest rate is therefore a key variable in our choice between consumption now or in the future (Brzezina, 2001).

Interest rates are the terms at which money or goods today may be traded off for money or goods at a future date. The interest rate is also the price of money. We can choose to store our savings in the form of cash or in a current account. The price we pay is the return other alternatives would have provided. Bank deposits and bonds are examples of investments that provide a reliable return – interest income. If we choose to store money, we lose this income. But in contrast to bonds, money can be used directly to purchase goods and services. Interest is therefore also the price we pay in order to have liquid holdings (Peter et al., 1992).

The interest rate is also used as an instrument in economic policy. Setting the interest rate to achieve a monetary policy objective, often price stability or low and stable inflation is usually the responsibility of the central bank. The central bank sets a very short-term nominal interest rate. In Norway, this is the interest rate on banks' overnight deposits in Norges Bank, the sight deposit rate. This rate determines the very short interest rates in the money market with maturities from one day upwards, normally up to Norges Bank's next monetary policy meeting. Longer-term rates are determined by expectations concerning Norges Bank's use of instruments in the future and by the degree of confidence in monetary policy. The real interest rate, that is the nominal interest rate minus expected inflation, is the rate that influences decisions concerning saving and investment (Lucas and Stokey, 1984).

The interest rate influences inflation indirectly via domestic demand for goods and services and via its effect on the exchange rate. When the interest rate falls, it is less profitable for households to save, and they will therefore increase their consumption now rather than wait until later. Borrowing also becomes less costly, with an associated rise in investment. Higher demand in turn leads to a higher rise in prices and wages. Lower interest rates make it less attractive to invest in NOK and less attractive for Norwegian enterprises and households to raise loans in other currencies. Lower interest rates will therefore normally result in reduced capital inflows and a weaker krone. This makes imported goods more expensive. In addition, a weaker krone increases activity, profitability and the capacity to pay in the internationally exposed sector (Brzezina, 2001).

The equilibrium interest rate and the neutral interest rate are closely related concepts. The neutral interest rate is the rate that does not in itself result in an increase or a reduction in price and cost inflation in the economy in the course of a business cycle. An assessment of whether interest rate setting is expansionary or contractionary, involves comparing short-term market rates with the neutral rate. A real interest rate in the interval 3 to 4% is often regarded as neutral in economies such as the Norwegian economy. A quantification of this level will be reverted later.

In the longer term, the interest rate level influences capital accumulation in the economy and the potential for economic growth. The equilibrium interest rate is the rate that ensures that capital accumulation corresponds to saving in the economy. These result in an output potential that over time satisfies demand without generating pressures in the economy.

The equilibrium interest rate is determined by long-term phenomena associated with the structure of the economy, while the neutral rate is defined on the basis of its influence on pressures in the economy and thereby on inflation. In the long term, the neutral interest rate will correspond to the long-term equilibrium interest rate in the economy (Nobuyuki and Kyoichi, 2005). The long-term equilibrium interest rate is determined by fundamental structural relationships in the economy, such as consumer impatience and the economic growth rate. Rising population growth means that a larger labor force must be equipped with real capital. Fixed investment and saving must increase. Higher population growth will therefore require a higher equilibrium interest rate. The higher productivity growth is the higher future gains from today's investments will be. This also provides the basis for a higher equilibrium interest rate (Cole and Kocherlakota, 1998). The long-term equilibrium interest rate cannot deviate too much between countries over time. With liberalized capital markets, capital will move towards those countries that can provide the highest return. Substantial interest rate differentials between countries cause fluctuations in the exchange rate and will not be compatible with a long-term equilibrium. Thus, we may refer to a global equilibrium interest rate for open economies, although perhaps with an added national risk premium (Fisher, 1907). The interest rate has thus several roles to play in the economy and these roles should be fairly and closely linked. The interest rate shall in the short and medium term contribute to stable inflation and stable developments in production. At the same time, it shall in the long term also contribute to equilibrium in the market for real capital. Capital accumulation shall over time correspond to saving. To achieve this, the real interest rate must not over time deviate substantially from the return on real capital. Substantial deviations can give rise to undesirable fluctuations in the markets for real capital that have no basis in economic fundamentals (Enders, 1995).

The economic situation varies over time. Monetary policy will set an interest rate that is alternately above and below the neutral rate. Consequently, the interest rate level will probably not deviate substantially from the long-term equilibrium rate over time. By taking a gradualist approach to interest rate setting, it is also possible to assess whether imbalances are developing in capital markets (Clower, 1967).

THE ECONOMIC ENVIRONMENT

Among several desirability function used in optimization problems that our features required, utility function used in this article, is Cobb-Douglas utility function that is easier, more appropriate, and flexible than the other functions and in this form of utility function optimum conditions can be maintained. In economic literature, it is always assumed that consumer is rational individual that maximize your utility with regard to facilities available and the constraints and obstacles ahead. Courses intended to determine the optimal values of objective variables, is unbounded. First, we consider the problem lies in finding the optimum for the economy that does not have any money in it. Explicitly, it is assumed that the obtained optimum values from optimum solution, are Pareto optimal. Then enter the money in economy with a cash-in-advance constraint on consumer goods. Then we compare optimum values obtained from finding the optimum solution with the state before Ireland, (2003).

Firms

The problem of the representative firm is a sequence of static maximization problems, since the firm simply seeks to maximize profits in each period by renting labor and capital to produce output which it sells to households. The static problem of the firm, then, is to

$$\max\ (p_t\, a k_{t-1}^{b} n_t^{1-b} - W_t n_t - r_t k_{t-1}) \tag{1}$$

The firm's first-order conditions are

$$p_t a(1-b) k_{t-1}^{b} n_t^{-b} = W_t \tag{2}$$

$$p_t abk_{t-1}^{b-1} n_t^{1-b} = r_t \tag{3}$$

Households

Economy without money

Here, we set out the physical environment in which agents interact, and we characterize efficient allocations in that environment. We consider an infinite-horizon environment with a continuum of identical households. Each household has a unit of time in every period; this time can be split between leisure l_t and work n_t. There is a single consumption good. Objective function is as follows:

$$U = \sum_0^\infty \beta^t\, [c_t^{\alpha_1} l_t^{\alpha_2}] \tag{4}$$

Where $\alpha_1 > 0$ and $\alpha_2 > 0$ are preference parameters and β is discount rate and $0 < \beta < 1$. The utility function u is strictly concave and continuously differentiable and satisfies the conditions that $u_c > 0, u_l > 0, u_{cc} < 0, u_{ll} < 0$. If the total time a person in authority be divided between work and leisure, and if the total time available, we normalized to one, we will have:

$$n_t + l_t = 1 \tag{5}$$

Therefore, the utility function of representative household will be as follows:

$$U = \sum_0^\infty \beta^t\, [c_t^{\alpha_1} (1-n_t)^{\alpha_2}] \tag{6}$$

At the beginning of period 1, there are k0 > 0 units of capital (All quantities are written in per capita terms). In period t, capital and labor can be used to produce output according to the Cobb-Douglass production function:

$$f(k_{t-1}, n_t) = a k_{t-1}{}^{b} n_t{}^{1-b} \tag{7}$$

The production function f is continuously differentiable, homogeneous of degree one, and concave.

Output y_t can be split between consumption c_t and investment x_t. Therefore for we have:

$$y_t = c_t + x_t \tag{8}$$

Law of motion of capital accumulates is as follow:

$$k_t = (1-\delta) k_{t-1} + x_t \tag{9}$$

Where δ is depreciation rate and Capital must satisfy the no negativity constraint that

$$k_t \geq 0 \tag{10}$$

Given this description of the environment, what is the symmetric Pareto optimal allocation of resources? The utility maximization problem of an entrepreneur is therefore described as:

$$\max \sum_{0}^{\infty} \beta^t \left[c_t^{\alpha_1} (1 - n_t)^{\alpha_2} \right]$$

Subject to

$$c_t + k_t \leq a k_{t-1}{}^b n_t{}^{1-b} + (1 - \delta) k_{t-1}$$

$$k_t \geq 0$$

First order conditions resulting from finding the optimum solution would be as follows:

$$\beta^t \alpha_1 c_t{}^{\alpha_1 - 1} (1 - n_t)^{\alpha_2} = \lambda_t \quad (11)$$

$$\beta^t \alpha_2 c_t{}^{\alpha_1} (1 - n_t)^{\alpha_2 - 1} = \lambda_t (1 - \mathrm{b}) \mathrm{a} k^b{}_{t-1} n^{-b}{}_t \quad (12)$$

$$-\lambda_t + \lambda_{t+1} \mathrm{ab} k^{b-1}{}_t n^{1-b}{}_{t+1} + \lambda_{t+1} (1 - \delta) = 0 \quad (13)$$

And the transversality condition is

$$\lim_{t \to \infty} \inf \ \beta^t \alpha_1 c_t{}^{\alpha_1 - 1} (1 - n_t)^{\alpha_2} k_t = 0 \quad (14)$$

Note that the transversality condition is restrictions only on the limit infima of the relevant sequences, not on the limits. This condition shows that when we go towards infinity, the discount factor β^t will be smaller and thus infinite in extent, the value $\beta^t \alpha_1 c_t{}^{\alpha_1 - 1} (1 - n_t)^{\alpha_2} k_t$ will be zero. This is important that in the infinite limit, this term will not be negative in any way.

We use the term optimal allocation to refer to the aforementioned unique solution to the social planner's problem. We assume that the utility function u and the production function f are such that the optimal allocation is globally stable.

Solving first orders conditions, optimal consumption and capital stocks in economy that does not have any money it, will be as follows:

$$c^* = \frac{\alpha_1}{\alpha_2} a (1 - b) k_{t-1} n_t{}^{-b} (1 - n_t) \quad (15)$$

$$a k^{*b} n^{1-b} - \delta k^* = c^* \quad (16)$$

Where c^* and k^* are the optimal consumption and capital stock in an economy that does not have any money it. We know that in an economy with no money in it, allocation of resources is Pareto optimal. Hence, the optimal path of both consumption and capital stock are Pareto optimal. In other words, with any other allocation, the optimality of state benefits to beam out and the welfare of society as a whole will decrease.

Economy with money

Here, we add to the physical environment described earlier, a particular monetary trading arrangement that households use to allocate resources among themselves, and we characterize the equilibrium that arise under this arrangement for different monetary policies. The key feature of the trading arrangement is that households are required to use previously accumulated money balances to buy consumption goods. This cash-in-advance feature generates a transaction demand for money (Ireland, 2003; Friedman, 1963).

To describe the monetary trading arrangement, we first specify the ownership of the various goods. There is a continuum of firms, each of which is endowed with a constant returns-to-scale technology that allows the firm to produce output according to the aforementioned production function. Households begin life with equal claims to the profits of these firms. Households also own their time endowment, k_0 units of capital, and M_0 units of money. Finally, there is an entity called the government which can give money to or take it from households. Before trade begins, the government specifies a sequence of monetary taxes and transfers χ_t; this transfer sequence implies a sequence of money supply levels by the accumulation equation:

$$M_t = M_{t-1} + \chi_t \quad (17)$$

Trading works as follows. Each household starts period t with M_{t-1} units of money, b_{t-1} units of bonds, and k_{t-1} units of capital. At the beginning of the period, a competitive goods market opens. Let money be the numeracies good in this market. Firms buy labor at wage rate w_t and rent capital at rental rate r_t from households and use these inputs to produce consumption and investment goods. Households buy consumption and investment goods from firms at price p_t. In the goods market, households face two restrictions on their ability to purchase goods. One is that households do not receive their wage and rental payments until after the goods market has closed (this can be understood intuitively: a firm cannot pay its workers until the firm has sold its goods). The other restriction is that households cannot use credit or bonds to purchase consumption goods (although households can use credit to buy investment goods). These two restrictions together imply that all consumption purchases have to be made using the original money holdings M_{t-1}. This restriction is termed cash-in-advance constraint; it is meant to capture the idea that money can be used to buy more goods than can be bought with credit.

After the goods market closes, the asset market opens. In the asset market, households receive $1 + i_{t-1}$ units of money for every unit of bonds with which they started the period. A household also receives its nominal labor income $w_t n_t$ and capital income $r_t k_{t-1}$ less its expenditures on new capital $p_t x_t$ and receives a net transfer of money from the government. The household divides its nominal wealth in the asset market among money holdings and one-period bonds. Then the asset market closes, and the period ends.

The utility maximization problem of an entrepreneur, is therefore described as:

$$\max \sum_{0}^{\infty} \beta^t \left[{c_t}^{\alpha_1} (1-n_t)^{\alpha_2} \right]$$

Subject to

$$p_t c_t \leq m_{t-1} \tag{18}$$

$$m_t + b_t \leq r_t k_{t-1} + W_t n_t + b_{t-1}(1+i_t) + m_{t-1} + \chi_t - p_t(c_t + x_t) \tag{19}$$

$$k_t = (1-\delta)k_{t-1} + x_t \tag{20}$$

$$k_t \geq 0,\ m_t \geq 0,\ b_t \geq -B \tag{21}$$

Indeed, in the household's first constraint, $p_t c_t$ is total expenditure on consumption goods and therefore is demand for money. In the other hand, $m_{t-1}$is money supply. In equilibrium, the money supply is equal to money demand, hence $p_t c_t = m_{t-1}$. Now if $p_t c_t \leq m_{t-1}$, in fact, some money will remain in the form of additional money that will be held and accumulated. In other words, as will be shown, coefficient of constraint in Hamiltonian functions (μ_t) is opportunity cost of money held. It is important that, this interest rate is different with the interest rate bonds that are without risk.

The second constraint says that available wealth can be split between money and bonds in the asset market.

The third constraint shows the direction of capital. The last constraint ensures that both capital and monetary assets to be non-negative and imposed a lower bound on debt to No-Ponzi game condition be stated. This condition says that the Households cannot be owed at the end of his life. If we suppose that B is large enough, therefore this constraint does not cause any restrictions. b_t Is asset and therefore is positive. On the other hand, B is a type of debt that has negative value. According to this constraint, total of asset and debt must be positive; that is, the amount of assets must be greater than debt. In other words, this constraint says that the debt should be a ceiling. Therefore, a limit for bonds will be created so that, the total of asset and debt is positive. The other case is that if the B value is small, maybe there is no balance. In other words, low B, possible balance makes canceled.

First order conditions resulting from finding the optimum solution would be as follows:

$$\beta^t \alpha_1 {c_t}^{\alpha_1 - 1}(1-n_t)^{\alpha_2} = p_t(\lambda_t + \mu_t) \tag{22}$$

$$\beta^t \alpha_2 {c_t}^{\alpha_1}(1-n_t)^{\alpha_2 - 1} = \lambda_t W_t \tag{23}$$

$$\lambda_{t+1}[r_{t+1} + (1-\delta)p_{t+1}] - \lambda_t p_t = 0 \tag{24}$$

$$-\lambda_t + \lambda_{t+1}(1+i_t) = 0 \tag{25}$$

$$\mu_{t+1} + \lambda_{t+1} = \lambda_t \tag{26}$$

And the transversality conditions are:

$$\lim_{t\to\infty} \inf \lambda_t p_t k_t = 0 \tag{27}$$

$$\lim_{t\to\infty} \inf \lambda_t (b_t + B) = 0 \tag{28}$$

$$\lim_{t\to\infty} \inf \lambda_t m_t = 0 \tag{29}$$

Solving first order condition, optimal consumption and capital stocks in the economy in which money is kept as a cash-in-advance constraint on the purchase of consumer goods, will be as follows:

$$c^{**} = \frac{\alpha_1}{\alpha_2} \frac{W_t}{p_t} \frac{(1-n_t)}{(1+i)} \tag{30}$$

$$a{k^{**}}^{b} n^{1-b} - \delta k^{**} = c^{**} \tag{31}$$

Where c^{**} and k^{**} are the optimal consumption and capital stock in economy which money is kept as cash-in-advance constraint on purchase of consumption goods.

If we consider, equilibrium consumption and capital stock in case that there is not any money in the economy is quite different with the case that the money is kept as a cash-in-advance constraint on purchases of consumption goods.

Money itself adds no new possibilities for resource reallocations to the environment, so no equilibrium with money can make all households better off relative to the optimal allocation characterized above. In fact, because households must use a low-yield asset (money) for their purchases of consumption goods, equilibrium allocations are typically Pareto inferior to the above optimal allocations. In fact, it can be said that, Pareto optimal is the optimal allocation that cannot be re-allocated to improve the welfare of society as a whole unless at least welfare of one individual is reduced. As seen, when there is no money in an economy, allocation of resources are Pareto optimal. So there is no need to re-allocate. Therefore, because our allocation was optimized, consequently injecting money into the environment reduces the total welfare of society. In other words, if money adds new possibilities and new sources, consequently the possibility of new production and consumption will be provided. However, because money adds no new possibilities for resource reallocations to the environment, the consumption and capital stock in equilibrium would not be Pareto optimal (Friedman, 1969).

NOMINAL INTEREST RATE

Here, to study the effect of nominal interest rate on the optimum path of consumption and capital stock we define two non-zero and zero nominal interest rate scenarios.

Non-zero nominal interest rate

We saw that in the economy which money is kept as cash-in-advance constraint on purchase of consumption goods, optimal consumption and capital stock are as follow:

$$c^{**} = \frac{\alpha_1}{\alpha_2}\frac{W_t}{p_t}\frac{(1-n_t)}{(1+i)}$$
$$ak^{**b}n^{1-b} - \delta k^{**} = c^{**}$$

We also saw that because money adds no new possibilities for resource reallocations to the environment, the consumption and capital stock in equilibrium would not be Pareto optimal. Solving the optimal tracking solution we have as follows:

$$p_t a(1-b)k_{t-1}^{b}n_t^{-b} = W_t$$

Therefore, we have:

$$a(1-b)k_{t-1}^{b}n_t^{-b} = w_t \tag{32}$$

Where, $\frac{W_t}{p_t} = w_t$.

If we use the $a(1-b)k_{t-1}^{b}n_t^{-b} = w_t$ in optimal consumption that is above, we have:

$$c^{**} = \frac{\alpha_1}{\alpha_2}a(1-b)k_{t-1}n_t^{-b}\frac{(1-n_t)}{(1+i)} \tag{33}$$

In Equation 33, if $i \neq 0$, therefore $1+i>0$ and we have:

$$c^{**} < c^{*} \tag{34}$$

And

$$k^{**} < k^{*} \tag{35}$$

As we see, optimal path of both consumption and capital stock in economy that money is kept as cash-in-advance constraint on purchase of consumption goods and nominal interest rate be non-zero, are less than optimal path of both consumption and capital stock in an economy in which there is no money (in this environment which there is no money, allocations are Pareto optimal). Therefore, in economy which nominal interest rate is non-zero the allocations are not Pareto optimal.

Zero nominal interest rate

We saw that in the economy which there is no money, optimal consumption and capital stock are Pareto optimal and are as follows:

$$c^{*} = \frac{\alpha_1}{\alpha_2}a(1-b)k_{t-1}n_t^{-b}(1-n_t)$$
$$ak^{*b}n^{1-b} - \delta k^{*} = c^{*}$$

Also the optimal path of consumption and capital stock in the economy which money is kept as cash-in-advance constraint on purchase of consumption goods are as follows:

$$c^{**} = \frac{\alpha_1}{\alpha_2}\frac{W_t}{p_t}\frac{(1-n_t)}{(1+i)}$$
$$ak^{**b}n^{1-b} - \delta k^{**} = c^{**}$$

Solving the optimal tracking solution we have as follows:

$$a(1-b)k_{t-1}^{b}n_t^{-b} = w_t$$

Where, $\frac{W_t}{p_t} = w_t$.

And we have:

$$c^{**} = \frac{\alpha_1}{\alpha_2}a(1-b)k_{t-1}n_t^{-b}\frac{(1-n_t)}{(1+i)}$$

In the preceding equation if $i=0$, therefore $1+i=1$ and we have:

$$c^{**} = c^{*} \tag{36}$$

And

$$k^{**} = k^{*} \quad (37)$$

It is obvious that in the economy that money holds on consumption goods (money enter in the economy), if nominal interest rate be zero, therefore allocations are Pareto optimal. Hence, the optimal path of both consumption and capital stock in economy which nominal interest rate be zero are Pareto optimal.

Conclusion

In this article we first use the model to assess the characteristics of interest rates when monetary policy is optimal. The cash-in-advance constraint implies that households have to wait until next period to use their current wage earnings to buy goods.

In this study, we investigate the resource allocations in both economies in which there is no money and economy in which money is introduced with a cash-in-advance constraint. We have shown that in a standard one-sector neoclassical growth model, in which money is introduced with a cash-in-advance constraint, zero nominal interest rates are optimal. In other words, zero nominal interest rates are necessary for efficient resource allocation. The result is that this rate of return on money (nominal interest rate) should be zero. In fact, interest rate is transaction costs of now with future and therefore when it will be optimal that is zero. Therefore, the nominal interest rate should be zero. Our results have a striking policy implication. Zero nominal interest rates are consistent with a large set of monetary policies. This means that the optimality of monetary policy can be verified only by looking at interest rates, not by looking at the growth rates of the money supply.

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