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EPQ models for deteriorating items with linearly discounted backordering under limited utilization of facility

Hui-Ming Teng

Department of Business Administration, Chihlee Institute of Technology, No. 313, Sec. 1, Wunhua Road, Banciao City, Taipei County 220, Taiwan.

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This study presents an economic production quantity model for deteriorating items in which backorder is allowed. The selling price of backorder depends on the customers that are willing to purchase the items under the condition that they receive their orders after a certain fraction of waiting time. The utilization of facility is an important index of production efficiency in the opportunity cost perspectives. This model considers both the impact of discounted selling price of backorder and the utilization rate of facility during the production process. Numerical examples and sensitivity analysis are given to illustrate the model.

Key words: Deterioration, linearly discounted backordering, utilization of facility, opportunity cost.

INTRODUCTION

Shortage backorder issues had received much attention from researchers. Many kinds of backordering were assumed. Wee (1999) had studied that the fraction of stockout demand backordered was constant. Abad (1996) suggested that customers do not like to wait, and therefore the fraction of customers who choose to place backorders is a decreasing function of waiting time such as $B(t) = k_0 e^{-k_1 t}$ or $B(t) = k_0/(1+k_1 t)$, while t being waiting time and k_0, k_1 being parameters. You (2005) had studied the optimal replenishment policy in which the probability of customers backordered is assumed to be linearly decreasing with waiting time t and is assumed to be $\theta(t)=1-t/T$, $0 \le t<T$. However, they did not consider the suitable selling price which depended on the waiting time. In practice, the selling price always depends on the customers that are willing to purchase the items under the condition that they receive their orders after a certain fraction of waiting time. In this study, the linear fraction of waiting time for selling price is assumed. In the past, many researchers had developed inventory models (Wee, 1999; Kang and Kim, 1983; Raafat et al., 1991; Hsu et al., 2007). Goyal and Gunasekaran (1995) developed an integrated deteriorating production – inventory -marketing model for determining the economic production quantity (EPQ) and economic order quantity for raw materials in a multi-stage production system. Luo (1998) extended the model proposed by Goyal and Gunasekaran (1995) to include backorder as a decision variable. In his model, any shortages were satisfied at the beginning of the cycle, and all shortage costs were known. Teng and Chang (2005) established an EPQ model for deteriorating items when the demand rate depends not only on the on-display stock level but also on the selling price per unit. In addition, they imposed a ceiling on the number of on-display stocks because

E-mail: tenghuim@mail.chihlee.edu.tw. Tel: +886-2-22537100.

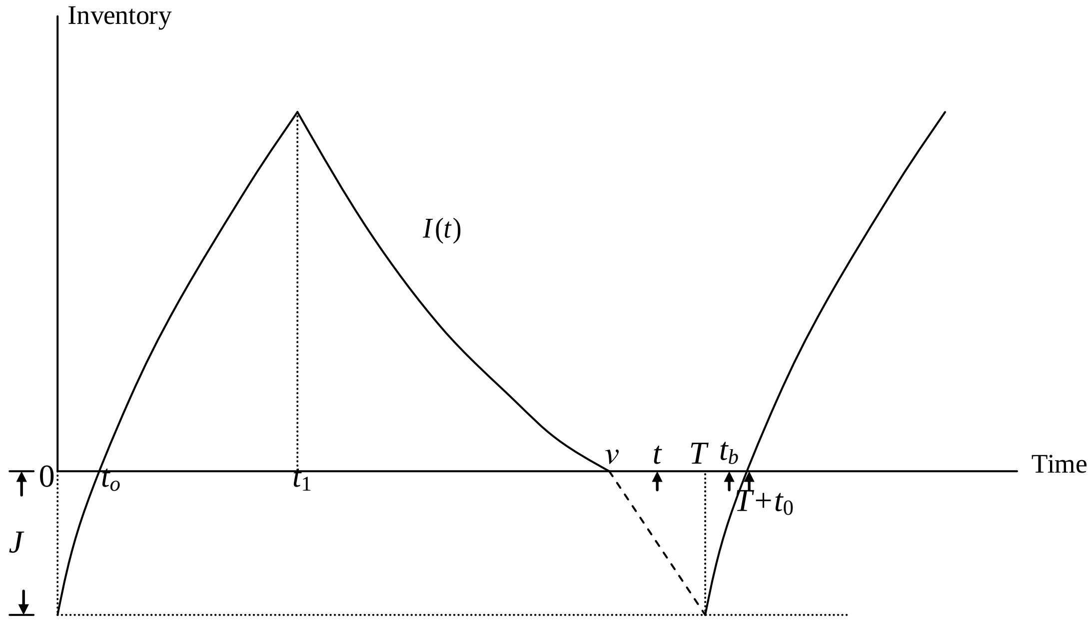

Figure 1. Inventory system of deteriorating items with backordering.

having too much stock left a negative impression on the buyer and the amount of display space was limited. Huang (2010) investigated that if an item is out of stock in an inventory system in which shortage is allowed, the supper may offer a negotiable price discount to the loyal, tolerant and obliged customers to pay off the inconvenience of backordering. Hung (2011) developed a ramp type demand rate and Weibull deterioration rate to arbitrary demand rate and arbitrary deterioration rate in the consideration of partial backorder. Hou et al. (2011) developed an inventory model for deteriorating items that the shortages are allowed and the unsatisfied demand is partially backlogged at the exponential rate with respect to the waiting time. Lin (2012) investigated the impact of setup cost reduction on an inventory policy for a continuous review mixture inventory model involving controllable backorder rate and variable lead time with a service level constraint. However, the inventory model with linear discounted backordering (that is, the backordered selling price of customer is assumed to be linearly decreasing with waiting time) has received little attention in past years.

The utilization of facility is an important issue in recent literature (Einhorn, 1987; Hou, 2007; Chakraborty et al., 2008). Many managers treat the utilization of facility as an index of production efficiency or expect the industries will run their production facility at an optimum level (Sharma, 2008).

Opportunity cost is the cost incurred by choosing one option over the other best alternative. Clark and Easaw (2007) studied optimal access pricing for natural monopoly networks with large sunk costs and uncertain revenues when considering the opportunity cost. Azaiez (2002) integrated opportunity costs for the unsatisfied demand to develop a multi-stage decision model for the conjunctive use of ground and surface water with an artificial recharge. In this paper, an EPQ model for deteriorating items with backordering is developed. This model considers linearly discounted selling price of backordered items and the constraint on the utilization rate. The aim of this model is to determine the production run time and shortage demand so that the average profit per unit time is maximized.

MATHEMATICAL MODEL AND ANALYSIS

A constant production rate K starts at t=0 to both finish the last backorder and the demand, then continues up to $t=t_1$ where the inventory level reaches the top. Production then stops at $t=t_1$, and the inventory gradually depletes to v (with $t_1<v\leq T$) due to deterioration and consumption. A graphical representation of this inventory system is depicted in Figure 1. The objective is to determine the optimal values for the shortage demand, J, and the production run time, t_1, such that the average profit per unit time is maximized. From the above assumptions and notation, we know that the inventory level I (t) at time t satisfies the following two differential equations:

$$dI(t)/dt + \theta I(t) = K - (d+rt), \quad 0 \leq t \leq t_1 \tag{1}$$

with initial condition $I(0) = -J$, and

$dI(t)/dt + \theta I(t) = -(d+rt)$, $t_1 < t \leq \nu$. (2)

with initial conditions $\lim_{t \to t_1^+} I(t) = I(t_1)$ and $I(\nu) = 0$. Solving the equations gives

$$I(t) = \frac{K-d-rt}{\theta} + \frac{r}{\theta^2} + \frac{e^{-\theta t}(-K\theta + d\theta - r - J\theta^2)}{\theta^2},$$

$0 \leq t \leq t_1$. (3)

$$I(t) = \frac{-d-rt}{\theta} + \frac{r}{\theta^2} + e^{-\theta(t-t_1)}\left[\frac{d+rt_1}{\theta} - \frac{r}{\theta^2} + I(t_1)\right],$$

$t_1 < t \leq \nu$. (4)

Solving $I(t_0) = 0$ and $I(\nu) = 0$, with $e^x \approx 1 + x + x^2/2$, using software Maple, one has

$$t_0(J) = \frac{K - d + J\theta - \sqrt{K^2 - 2Kd + d^2 - 2Jr - J^2\theta^2}}{r + (K-d)\theta + J\theta^2}. \quad (5)$$

$$\nu(t_1, J) = \frac{d + \left[t_1 d + t_1{}^2 r + I(t_1)\right]\theta + t_1 I(t_1)\theta^2 - \sqrt{d^2 + t_1{}^2 r^2 + 2rI(t_1) + 2rt_1 d - I(t_1)^2\theta^2}}{-r + (d + rt_1)\theta + I(t_1)\theta^2}. \quad (6)$$

The shortage backordered quantity is

$$J = \int_\nu^T d\,dt = d(T - \nu). \quad (7)$$

One has

$$T(t_1, J) = \nu + \frac{J}{d}. \quad (8)$$

When the customer agrees to place backorder at time t, the backorder satisfied time, t_b, can be approximated as follows:

Using linear interpolation, $\frac{T-t}{T-\nu} \approx \frac{T + t_o - t_b}{T + t_o - T}$, one has

$$t_b = T + t_o - \frac{(T-t)t_o}{T-\nu}. \quad (9)$$

From the definition of $p_b(\eta)$, the backordered revenue is

$$\int_\nu^T d(t_b - t)p_b dt = dp\int_\nu^T 1 - \frac{\delta(t_b - t)}{T-\nu} dt. \quad (10)$$

From the analysis above, the average profit per unit time, AP, is

$AP = \frac{1}{T}$ [normal revenue + backordered revenue - production cost - inventory holding cost - ordering cost]

$$= \frac{1}{T}\left[\int_0^\nu (d+rt)p\,dt + \int_\nu^T d(t_b - t)p_b dt - \int_0^{t_1} Kc_p dt - \int_{t_o}^\nu I(t)c_h dt - c_o\right]. \quad (11)$$

Where

$$\int_0^\nu (d+rt)p\,dt = (d\nu + r\nu^2/2)p. \quad (12)$$

$$\int_\nu^T dp_b(t_b - t)dt = \{J/d - \delta(T + t_o - Tt_o d/J) - \delta(t_o d/J - 1)(T+\nu)/2\}dp. \quad (13)$$

$$\int_0^{t_1} Kc_p dt = Kt_1 c_p. \quad (14)$$

$$\int_{t_o}^\nu I(t)c_h dt \approx I(t_1)(\nu - t_o)c_h/2. \quad (15)$$

Since t_o is a function of J, and ν is a function of t_1 and J, therefore, AP is a function of t_1 and J. When the backordering quantity is J, the production run time t_1 needs to satisfy the production quantity which meets the customer's demand, the backordering quantity and

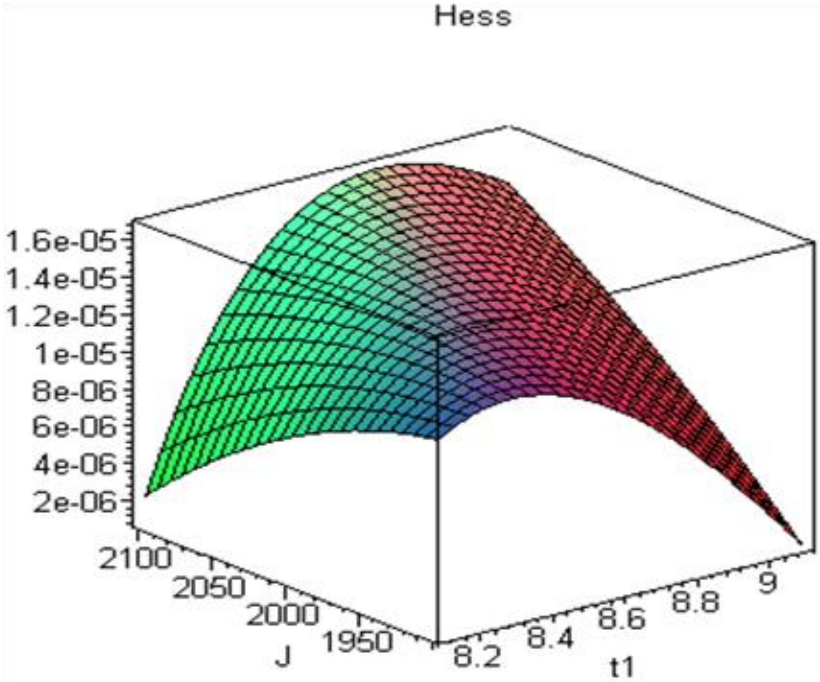

Figure 2a. Graph of the Hessian matrix function of AP(t1, J) on [8.1, 9.1] [1900,2100].

deterioration. That is, the following inequality is satisfied:

$$t_1 \geq t_o = \frac{K-d+J\theta-\sqrt{K^2-2Kd+d^2-2Jr-J^2\theta^2}}{r+(K-d)\theta+J\theta^2} \quad . \qquad (16)$$

If the idle time of facility is limited and the utilization rate, $\frac{t_1}{T}$, is more than a fraction of ω, then the problem can be formulated as follows:

Max: $AP(t_1, J)$

Subject to: (a) $t_1 \geq t_o$, (b) $1 \geq \frac{t_1}{T} \geq \omega$, $0<\omega<1$ (17)

From Equation 17(a) and (b), the domain of the problem is closed and bounded, which means the optimum of the problem occurs at either relative maximum of $AP(t_1, J)$ in the interior of the domain or at the boundary of the domain [20,21]. Since the complexity of $AP(t_1, J)$, the closed form of the solution is hard to find, the following solution procedure is used.

Solution procedure

Step 1. Check the concavity of $AP(t_1, J)$.(Hessian matrix function of $AP(t_1, J)$ is positive)
Step 2. Find both the relative maximum of $AP(t_1, J)$ in the interior of the domain and at the boundary of the domain.
Step 3. Find the maximal value of Step 2, the optimum is obtained.
Stop.

NUMERICAL EXAMPLE

The proposed model can be illustrated with the following numerical example.

Example 1

Let K=300, θ=0.05, d=75, r=0.5, c_o=100, c_p=1.5, c_h=0.65, p=4, δ =0.9 and ω =0.1.

Figure 2a (Using software Maple) shows the graph of the Hessian matrix function of AP (t_1, J) on [8.1, 9.1]×[1900, 2100]. That means Hessian matrix of AP (t_1, J) on [8.1, 9.1]×[1900, 2100] is positive. A graphical representation showing the concave function AP is given in Figure 2b. With the given data, the optimal decision for the retailer is obtained by using software MATHCAD. The optimal shortage demand J=2018, the optimal production run time t_1=8.325, the total cycle time T=37.445, the facility utilization rate t_1/T =0.222, and the average profit per unit

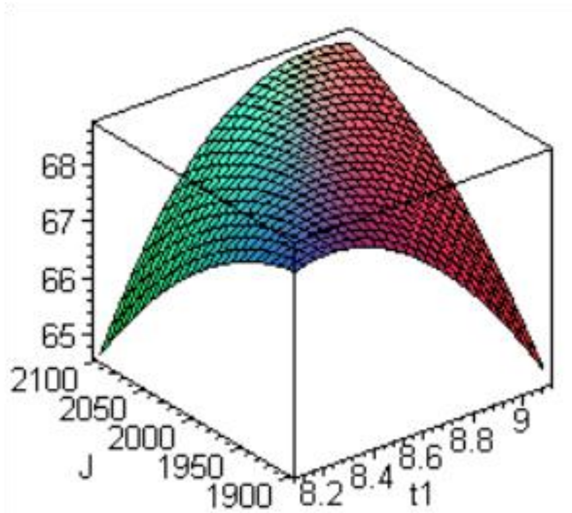

Figure 2b. Graph of the average profit AP(t1, J) on [8.1, 9.1] [1900,2100].

K=290, c_o=100, d=75, c_p=1.5, θ=0.05, δ=0.9, r=0.5						
c_h	ω	J	t_1	T	t_1/T	AP
0.6	0.1	1737	7.716	33.055	0.233	61.272
0.6	0.25	3631	20.30	81.202	0.25	32.785
0.65	0.1	1857	8.104	35.008	0.231	61.083
0.65	0.25	3705	20.708	82.832	0.25	32.106
0.7	0.1	1933	8.339	36.219	0.23	60.931
0.7	0.25	3758	21	83.998	0.25	31.553

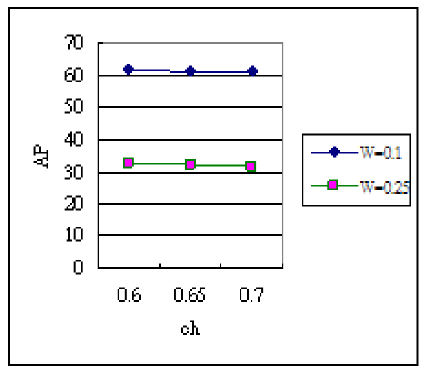

Figure 3. The effect of constant production rate K=290 on the average profit AP.

time AP=\$61.25.

SENSITIVITY ANALYSIS

Sensitivity analysis is carried out and shows the changes in J, t_1, T, t_1/T, and AP for variables K, c_h, and ω.

Figures 3, 4 and 5 show that as the production rate K increases, the average profit AP increases. However, as ω increases, the average profit AP decreases. It is shown that as the holding cost c_h increases, the production run time t_1 increases, the utilization rate of facility t_1/T and the average profit AP decreases.

CONSIDERING THE OPPORTUNITY COST

Other than the limited utilization rate of production facility discussed earlier, we consider the opportunity cost of the production facility as a linear function of utilization rate in the current section. That is, the opportunity cost of production facility is assumed to be $(1-\frac{t_1}{T})c_u$, where c_u is the unit opportunity cost. Then, the average profit of considering opportunity cost per unit time, APU, is

$APU = \frac{1}{T}$[normal revenue + backordered revenue - production cost - inventory holding cost-ordering cost - opportunity cost]

$$= \frac{1}{T}\left[\int_0^V [d+rt]p\,dt + \int_V^T dp_b(t_b-t)\,dt - \int_0^{t_1} Kc_p\,dt - \int_{t_o}^{V} I(t)c_h\,dt - c_o - \left(1-\frac{t_1}{T}\right)c_u\right] \quad (14)$$

K=300, c_o=100, d=75, c_p=1.5, θ=0.05, δ=0.9, r=0.5						
c_h	ω	J	t_1	T	t_1/T	AP
0.6	0.1	1927	8.056	36.007	0.224	61.414
0.6	0.25	3840	22.214	88.855	0.25	23.716
0.65	0.1	2018	8.325	37.445	0.222	61.25
0.65	0.25	3884	22.477	89.909	0.25	22.982
0.7	0.1	2082	8.51	38.45	0.221	61.115
0.7	0.25	3919	22.687	90.748	0.25	22.346

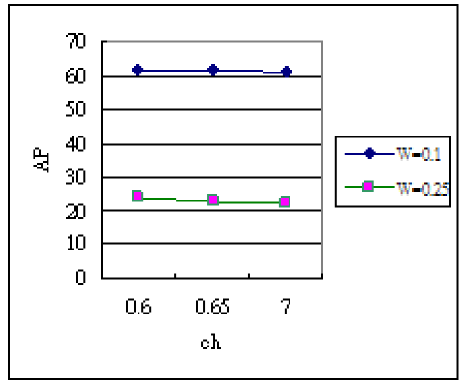

Figure 4. The effect of constant production rate K=300 on the average profit AP.

K=310, c_o=100, d=75, c_p=1.5, θ=0.05, δ=0.9, r=0.5						
c_h	ω	J	t_1	T	t_1/T	AP
0.6	0.1	2093	8.285	38.516	0.215	61.565
0.6	0.25	3564	21.76	87.041	0.25	7.446
0.65	0.1	2169	8.496	39.698	0.214	61.418
0.65	0.25	3574	21.856	87.424	0.25	5.181
0.7	0.1	2226	8.649	40.573	0.213	61.296
0.7	0.25	3.581	21.925	87.702	0.25	2.947

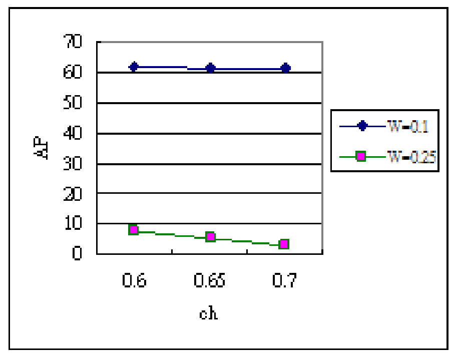

Figure 5. The effect of constant production rate K=310 on the average profit AP.

Then the problem can be formulated as follows:

Max: $APU(t_1, J)$

Subject to: $t_1 \geq t_o$. (15)

Example 2

Let K=300, θ=0.05, d=75, r=0.5, c_o=100, c_p=1.5, c_h=0.65, p=4, δ =0.9 and c_u =500.

Figure 6a (Using software Maple) shows the graph of the Hessian matrix function of $APU(t_1, J)$ on [9, 12]$\times$[2700,3000]. That means Hessian matrix of $APU(t_1, J)$ on [9, 12]$\times$[2700,3000] is positive. A graphical representtation showing the concave function *APU* is given in Figure 6b. With the given data, the optimal decision for the retailer is obtained by using software MATHCAD. The optimal shortage demand J=2842, the optimal production run time t_1=11.213, the total cycle time T=52.321, the facility utilization rate t_1/T =0.214, and the average profit per unit time APU=\$52.792.

CONCLUSION

This study develops a deteriorating EPQ model. We develop the model by considering the linearly discounted selling price of backorder and the utilization rate of facility. In practice, the selling price usually depends on the customers that are willing to purchase the item under the condition that they receive their orders after a certain fraction of waiting time.

The utilization of facility issue has drawn attentions in the recent years. Many managers treat the utilization of facility as an index of production efficiency. In this study, sensitivity analysis shows that as the holding cost c_h increases, the production run time t_1 increases, however, the utilization rate of facility t_1/T and the average profit AP decreases. This can be used as a reference for the decision-makers.

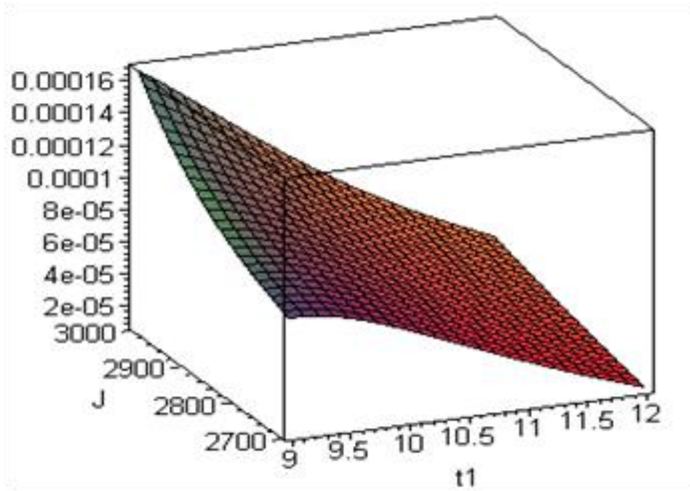

Figure 6a. Graph of the Hessian matrix function of APU(t1, J) on [9, 12] [2700,3000].

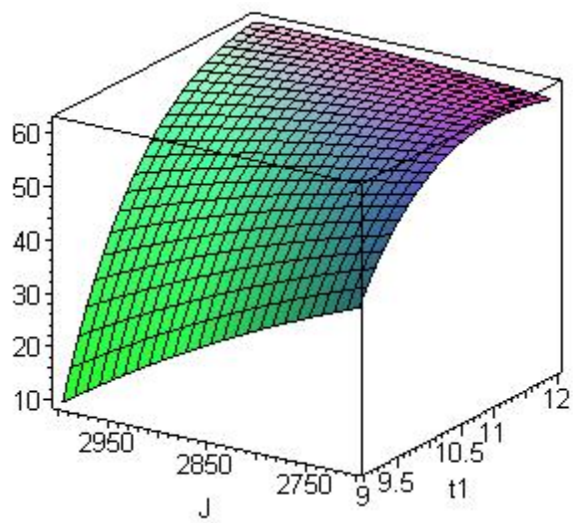

Figure 6b. Graph of the average profit APU(t1, J) on [9, 12] [2700,3000].

ASSUMPTIONS AND NOTATION

The following notations were used throughout this paper.

T	total cycle time
t_1	the production run time ; t_1 is a decision variable
t_0	a production time to make up shortage from the previous cycle, $0<t_0\leq t_1$
v	a critical time at which inventory level reaches zero after t_1, $t_1<v\leq T$
t_b	the backorder satisfied time when the customer agrees to place backorder at time t
$I(t)$	the inventory level at time t
K	the constant production rate
θ	the constant deterioration rate, where $0<\theta\leq 1$
c_o	the set-up cost per cycle
c_p	the production cost per unit

c_h the holding cost per unit per unit time
c_u the unit opportunity cost
p the constant selling price per unit
p_b the backordered selling price during shortages
δ the discount factor
ω the lower bound of facility utilization rate
J the shortage demand; J is a decision variable
AP the average profit per unit time
APU the average profit of considering opportunity cost per unit time

In developing the model, the following assumptions were made:

1. The demand rate $D(t)$ for the product follows a deterministic function of time t such that:

$$D(t)=\begin{cases} d+rt, & 0 \le t < v, \\ d, & v \le t \le T, \end{cases}$$

where d, r>0 are constants. That means the customers' demand increases before time v for the advertisement effect, and the demand is constant during v and T since the customers are willing to be backordered due to discount and new items.
2. Backorder is allowed, and the backlogged demand is satisfied sequentially at the beginning of each cycle which depends on the customer's waiting time.
3. The product deteriorates with time, and there is no replacement or repair of deteriorated items during a given cycle.
4. The backordered selling price of customer is assumed to be linearly decreasing with waiting time η such that: $p_b(\eta) = (1-\frac{\delta\eta}{T-v})p$, $t_o \le \eta < T-v$, where the discount factor, δ, is a constant with , $0<\delta \le 1$.
5. The capacity of the warehouse is unlimited.
6. The production rate is higher than the consumption and deterioration rate combination.
7. The opportunity cost of production facility is a decreasing function of utilization rate which is assumed to be $(1-\frac{t_1}{T})c_u$, where $\frac{t_1}{T}$ is the utilization rate, c_u is the unit opportunity cost.

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