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Multivariate generalized autoregressive conditional heteroscedasticity (GARCH) modeling of sector volatility transmission: A dynamic conditional correlation (DCC) model approach

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This paper employs a multivariate generalized autoregressive conditional heteroscedasticity dynamic conditional correlation (GARCH-DCC) model to simultaneously estimate the mean and conditional variance of Brazilian financial and consumer sectors using daily returns from January 2, 2008 to September 30, 2010 of the indexes that represent these sectors. Since different financial assets are traded based on these sector indexes, it is important for financial market participants to understand the volatility transmission mechanism over time and across sectors in order to make optimal portfolio allocation decisions. We find significant bilateral transmission of volatility between the sectors. These findings support the idea of cross-market hedging and sharing of common information by investors in these sectors in Brazil.

Key words: Volatility spillover, dynamic conditional correlation, sector indexes, Brazilian market.

INTRODUCTION

Managing and monitoring major financial assets are routine for many individuals and organizations. Therefore careful analysis, specification, estimation and forecasting the dynamics of returns of financial assets, construction and evaluation of portfolios are essential skills in the toolkit of any financial planner and analyst (Caporini and McAleer, 2010).

Within this context, the knowledge of the stochastic behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management (Baur, 2006). The study of volatility is therefore of great importance in finance, particularly in derivative pricing and risk management of investments. Traditionally the calculation of financial returns volatility estimates, as well its application in determining the value at risk (VaR), rely on the daily changes in asset prices (Goodhart and O'hara, 1997).

The literature on models of measuring volatility provides various methods with varying degrees of implementation difficulty. From the temporal unconditional standpoint, the simplest estimator is the historical standard deviation, which gives uniform weight to all observations. The advantage of the standard deviation is the ease of calculation and interpretation; however, it has two major drawbacks, which are its symmetry and the fact that it is constant. In contrast, the autoregressive conditional heteroscedasticity (ARCH) introduced by Engle (1982) and its generalization generalized autoregressive conditional heteroscedasticity (GARCH) proposed by Bollerslev (1986) are more widely applied to model the volatility of financial series as these models do not have the same problems. Nevertheless, many variations of the GARCH model have been proposed, such as, the exponential GARCH (EGARCH) proposed by Nelson (1991), for example, that allows the inclusion of asymmetric effects in conditional volatility.

Although unconditional correlations can be easily estimated, the same does not happen with stochastic correlations. Thus, it is possible to extend the concepts

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about measurement of conditional volatility for a multivariate approach, using a multivariate GARCH (MVGARCH). This type of model is very interesting because it can identify and use common movements between different asset volatilities.

Regarding to economic sectors, the transmission of shocks from one sector returns to another was documented by Ewing (2002), among others.The finding of spillover of shocks from sector variances brings a whole new set of implications. Additionally, since different financial assets are traded based on these sector indexes, it is important for financial market participants to understand the volatility transmission over time and across sectors in order to make optimal portfolio allocation decisions (Hassan and Malik, 2007).

Thus, this paper examines the volatility and shock transmission mechanism between two sectors, that is, financial and consumer, of Brazilian market. To achieve this goal, daily returns were used from January 2, 2008 to September 30, 2010 of sector indexes IFNC and ICON, which are traded as assets in BM&F/Bovespa, with a sample of 680 observations.

MATERIALS AND METHODS

This area is subdivided into: i) multivariate volatility modeling, which presents the evolution of the conditional covariance estimation; and ii) data and methodology, which expose the utilized data and the methodological procedures adopted in order to fulfill the proposed objective.

Multivariate volatility modeling

Multivariate models of volatility have attracted considerable interest during the last decade. This may be associated with increased availability of financial data, increasing the processing capacity of computers, and the fact that the financial sector began to realize the potential advantages of these models.

But when it comes to the specification of a multivariate GARCH model, there is a dilemma. On the one hand, the model should be flexible enough to be able to represent the dynamics of variance and covariance. Moreover, as the number of parameters in a multivariate GARCH model often increases rapidly with the size of assets, the specification must be parsimonious enough to allow the model to be estimated with relative ease, as well as allowing a simple interpretation of its parameters (Silvennoinen and Teräsvistra, 2008).

A feature that must be taken into account in the specification is the restriction of positivity (covariance matrices must necessarily take its determinants defined as positive). Based on this idea, Bollerslev et al. (1988) proposed and consider the multivariate GARCH model with parameterization VECH. The disadvantage of this model is that it has a large number of parameters and in order to ensure the positivity, restrictions must be imposed.

Thus, emerges as an alternative the BEKK parameterization, as suggested by Engle and Kroner (1995). The BEKK parameterization essentially takes care of the afore-mentioned problems about the model VECH. However,it has the disadvantage of estimated parameters with hard interpretation. Even for the case of bivariate modeling, the interpretation of the coefficients can be confusing because there are no parameters that are governed exclusively by an equation (Baur, 2006).

Thus, as emphasized by Peters (2008), an approach to circumvent the problem of parameters interpretation is the model with conditional covariance matrix, observed indirectly through the matrix of conditional correlations. The first such model was the constant conditional correlation (CCC) proposed by Bollerslev and Wooldridge (1992). The conditional correlation was assumed to be constant and only the conditional branches are variable in time.

However, according to Bauwens et al. (2003), the assumption that the conditional correlation is constant over time is not convincing, since, in practice, the correlation between assets undergoes many changes overtime. Thus, Engle and Sheppard (2001) introduced the model of dynamic conditional correlation (DCC). The DCC model is a two-step algorithm to estimate the parameters which makes it relatively simple to use in practice. In the first stage, the conditional variance is estimated by means of univariate GARCH model, respectively, foreach asset. In the second step,the parameters for the conditional correlation, given the parameters of the firsts tage, are estimated. Finally, the DCC model includes conditions that make the covariance matrix positive definite at all points in time and the covariance between assets volatility a stationary process.

The DCC model is represented by the formulation 1.

$$H_t = D_t R_t D_t. \tag{1}$$

Where,

$$R_t = diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right) Q_t diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right) \tag{2}$$

Since the square matrix of order *N* symmetric positive defined

$Q_t = \left(q_{ij,t}\right)$ has the form proposed in (3).

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1} u_{t-1}^{'} + \beta Q_{t-1}. \tag{3}$$

In 9, $u_{i,t} = \varepsilon_{i,t} / \sqrt{h_{i,t}}$;$\bar{Q}$ is the *N x N* matrix composed by unconditional variance of u_t; *α* and *β* arenon-negative scalar parameters satisfying *α+β<1*.

Data and procedure

In order to verify the volatility transmission mechanism between financial and consumer sectors in Brazilian market, we collected data IFNC and ICON prices from BM&F/Bovespa, comprising the period from January, 2, 2008 to september, 30, 2010, totaling 680 observations for each index. The stocks of these indexes are selected for their liquidity and size, and the portfolios are weighted by market value of shares available for trading.

Through the prices of indices studied, we calculated the variation of the natural logarithm of the series to eliminate non-stationarity problems. Firstly, were calculated the descriptive statistics of both the indexes. Subsequently, by *Q* statistic of Ljung and Box (1978), represented for (4), which tests the null hypothes is that the data are random against the alternative of non-randomness of these, we sought to identify the presence ofcorrelation serial number on the returns of the indices.

$$Q = n(n+2)\sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{n-k}. \tag{4}$$

In 4, *n* is the size of sample; $\hat{\rho}_k^2$ is the autocorrelation of sample in lag *k*; *h* is the number of lags being tested; The Ljung-Box *Q*

Table 1. Descriptive statistics of daily financial and consumer sector returns.

Statistics	Financial	Consumer
Mean	0.0005	0.0005
Median	0.0006	0.0015
Minimum	-0.1285	-0.0967
Maximum	0.1899	0.1183
Standard deviation	0.0277	0.0189
Skewness	0.5336	0.1119
Kurtosis	5.8135	5.8227

The sample covers January, 2, 2008 to September, 30, 2010. The total number of observations is 680.

Table 2. Ljung-Box *Q* statistic for daily financial and consumer sector returns.

Lag	Financial	P-value	Consumer	P-Value
1	1.9064	(0.167)	1.9189	(0.166)
2	3.7441	(0.154)	4.6633	(0.097)
3	16.3591	(0.001)	8.0212	(0.046)
4	16.3597	(0.003)	8.0362	(0.090)
5	16.4034	(0.006)	8.6898	(0.122)
6	21.2289	(0.002)	9.5079	(0.147)
7	26.5453	(0.000)	9.7805	(0.201)
8	27.1073	(0.001)	11.0876	(0.197)
9	27.3434	(0.001)	11.1418	(0.266)
10	27.5841	(0.002)	13.0801	(0.219)
11	28.1325	(0.003)	13.1849	(0.281)
12	28.1347	(0.005)	13.4741	(0.336)

Bold values are significant at 5% level.

Table 3. Checking the number of lags to be used in the VAR, based on AIC, BIC and HQC.

Lag	AIC	BIC	HQC
1	-10.5024	-10.4621	-10.4868
2	-10.4973	-10.4301	-10.4713
3	-10.5464	-10.4522	-10.5099
4	-10.5358	-10.4147	-10.4889
5	-10.5380	-10.3901	-10.4808
6	-10.5375	-10.3626	-10.4698
7	-10.5414	-10.3396	-10.4632
8	-10.5456	-10.3168	-10.4570
9	-10.5342	-10.2785	-10.4352
10	-10.5303	-10.2478	-10.4209

Bold values represent the chosen lag.

statistics follows a chi-squared (χ^2) distribution with *k* degrees of freedom.

Since there is serial dependence in the series, the results may contain biaso f estimation. Thus, as shown by Karmakar (2008), to enable the filtering of the serial dependence of the estimated residuals, we used a vector autoregressive (VAR) to obtain the average estimate of the return series of each index. The mathematical form of theVAR model used is represented by (5)

$$VAR(f,c) = \begin{cases} \Delta f_t = \beta_0 + \sum_{i=1}^{n} \beta_i \Delta f_{t-i} + \sum_{j=1}^{m} \beta_j \Delta c_{t-j} + \varepsilon_{1,t} \\ \Delta c_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i \Delta c_{t-i} + \sum_{j=1}^{m} \alpha_j \Delta f_{t-j} + \varepsilon_{2,t} \end{cases} \quad (5)$$

In 5, Δf_t and Δc_t are, respectively the daily returns of financial and consumer sector; β_k and α_k are regression parameters; $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are, correspondingly, the estimated residuals.

Thus, to choose the number of lags used in estimating the VAR model, we reapplied the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and Hannan-Quinninformation criterion (HQC) (Akaike, 1974; Hannan and Quinn, 1979). The mathematical representations of such criteria are set out in the formulations 6, 7 and 8, respectively.

$$AIC = \log\left[\frac{1}{N}\sum_{i=1}^{N}(y_t - \hat{y}_t)^2\right] + \frac{2}{k}. \quad (6)$$

$$BIC = \log\left[\frac{1}{N}\sum_{i=1}^{N}(y_t - \hat{y}_t)^2\right] + \frac{k}{N}\log(N). \quad (7)$$

$$HQC = \log\left[\frac{1}{N}\sum_{i=1}^{N}(y_t - \hat{y}_t)^2\right] + 2k\ln[\ln(N)]. \quad (8)$$

Where, *k* is the number of parameters in the model; *N* are the total of observations; y_t is the observed value of the dependent variable at time*t* ;$\hat{y}_t$ is the estimated value of the dependent variable at time*t*.

Subsequent to this initial empirical analysis, using the residuals that were obtained through the VAR applied to the series, we used the multivariate DCC GARCH model, exposed previously in this article, to identify the volatility transmission mechanism.

RESULTS AND DISCUSSION

Initially we discuss the descriptive statistics of the financial and consumer sector daily returns. As seen in Table 1, both indexes showed similar behavior with respect to descriptive characteristics, with mean and median of daily returns close to zero, positive skewness and high kurtos is (fat tails). This leptokurtic behavior is well documented in literature (Longin and Solnik, 2001). It is noteworthy the fact that the value of the financial sector standard deviation is about the double of consumer. This can be a vestige of the 2007/2008 sub-prime crisis, which strongly affected the financial sector.

Subsequently, conforming to the explaination in section data and methodology, we identified the presence of serial correlation in the daily returns, selected the best VAR model and verified the dependence in the VAR residuals.The results of this step are shown inTables 2, 3 and 4. The results of these tables appoint for the use of a VAR (3) to model the conditional means. Further, this model successfully filtered information the linear dependence with past.

Table 4. Ljung-Box *Q* statistic for daily financial and consumer sector returns estimated by VAR.

Lag	Financial	P-value	Consumer	P-value
1	0.0002	(0.988)	0.0101	(0.920)
2	0.0080	(0.996)	0.0164	(0.992)
3	0.0570	(0.996)	0.0323	(0.998)
4	0.0592	(1.000)	0.0408	(1.000)
5	0.1277	(1.000)	0.1106	(1.000)
6	3.6264	(0.727)	0.2294	(1.000)
7	7.8610	(0.345)	0.3701	(1.000)
8	9.1871	(0.327)	2.7823	(0.947)
9	9.3816	(0.403)	2.8635	(0.969)
10	9.7032	(0.467)	6.7362	(0.750)
11	9.7535	(0.553)	6.7469	(0.819)
12	9.7607	(0.637)	7.3661	(0.833)

None of the values are significant at 5% level.

Table 5. Estimated coefficients for the multivariate GARCH model assuming dynamic conditional correlation for the relationship between daily financial and consumer sector returns.

Variable	Coefficient	Standard error	T-Statistic	Significance
C(1)	3.2331e-04	1.0290e-05	31.41867	0.0000
C(2)	1.7438e-04	2.2746e-06	76.6646	0.0000
A(1,1)	0.1514	0.0315	4.7989	0.0000
A(1,2)	-0.0526	0.0461	1.1401	0.2543
A(2,1)	0.0137	0.0419	0.3261	0.7443
A(2,2)	0.2020	0.0679	2.9738	0.0029
B(1,1)	0.5703	0.0195	29.2992	0.0000
B(1,2)	-0.1610	0.0446	3.6090	0.0003
B(2,1)	-0.0821	0.0257	3.2003	0.0014
B(2,2)	0.5767	0.0330	17.4515	0.0000

Bold values are significant at 5% level.

Thus, after this initial data preparation, it became possible to estimate the multivariate GARCH-DCC model to check volatility transmission mechanism between financial and consumer sectors daily returns. The estimated values for the model coefficients are presented in Table 5.

Firstly, one can note that the conditional volatility of both is affected by the squared errors shocks from the previous trading day, as evidenced by the parameters A (1,1) and A (2,2) statistical significance. However, there was not any kind of interaction with respect to such innovations among the studied sector during thes ample period.

About the impact of lagged conditional volatilities, all the parameters B (1,1), B(1,2), B(2,1) and B (2,2) coefficients reached statistical significance. This finding implies in conditional volatility of both sectors to be affected during the period analyzed by the previous trading day. Nevertheless, the results indicate that there was bilateral volatility transmission between the sesectors during the sample period. These results are similar to those obtained by Hassan and Malik (2007) who investigated the volatility transmission between U.S. sectors from January 1992 to June 2005, through modeling BEKK.

This volatility transmission is usually attributed to cross-market hedging and changes in common information, which may simultaneously alter expectations across sectors (Fleming et al., 1998). Thus, these results could be interpreted as an outcome of cross-market hedging undertaken by financial market participants within these sectors (Hassan and Malik, 2007).

It follows as the main consequence of these results the fact that could be more difficult to diversify unsystematic portfolio risk by means of these sector indexes, and, in general, companies belonging to both sectors. Table 6 presents the Q statistics for the residuals, indicating that there is no dependence with past information. Complementing, Figure 1 contains plots of the daily estimated conditional volatilities, as well as its dynamic conditional correlation.

Based on Figure 1, we find that the estimated volatility for the financial sector is about twice that obtained for the consumption sector, reinforcing that previously mentioned. Further, both sectors show volatility peaks during the 2007/2008 sub-prime crisis turbulence apex. Nonetheless, it appears that the dynamic conditional correlation between the estimated volatilities remained around 0.7. Thus, the sectors dependence did not suffered any sudden change. This result emphasize that these sectors did not follow the tendency of correlation breakdown, that is, a statistically significant increase in correlation during the crash period (Kenourgios et al., 2011).

Conclusion

This paper aimed to verify the volatility transmission mechanism between financial and consumer sectors. The results show that during the sample period, there was bilateral volatility transmission. This means that the conditional volatility of both sectors is significantly influenced by the occurrence in the previous trading day in the two assets in question. It is worth noting that, with respect to shocks from squared errors, the volatilityof each sector is impacted only by their own negotiation activity.

With respect to literature, the results are very similar to those obtained by Hassan and Malik (2007), who analyzed the volatility transmission volatility based on the BEKK model. As a result of such bilateral transmission of volatility, we stress the difficulty of minimizing portfolio unsystematic risk through diversification among assets which belong to these sectors. Regarding to the variances and correlation estimates, we find that the

Table 6. Ljung-Box *Q* statistic for daily financial and consumer sector residuals estimated by GARCH-DCC.

Lag	Financial	P-value	Consumer	P-value
1	0.260	0.610	0.046	0.830
2	0.288	0.866	0.215	0.898
3	0.336	0.953	0.358	0.949
4	0.337	0.987	0.435	0.980
5	0.630	0.987	0.444	0.994
6	0.697	0.995	0.951	0.987
7	0.914	0.996	3.384	0.847
8	0.992	0.998	4.773	0.782
9	1.930	0.993	5.023	0.832
10	2.041	0.996	6.340	0.786

None of the values are significant at 5% level.

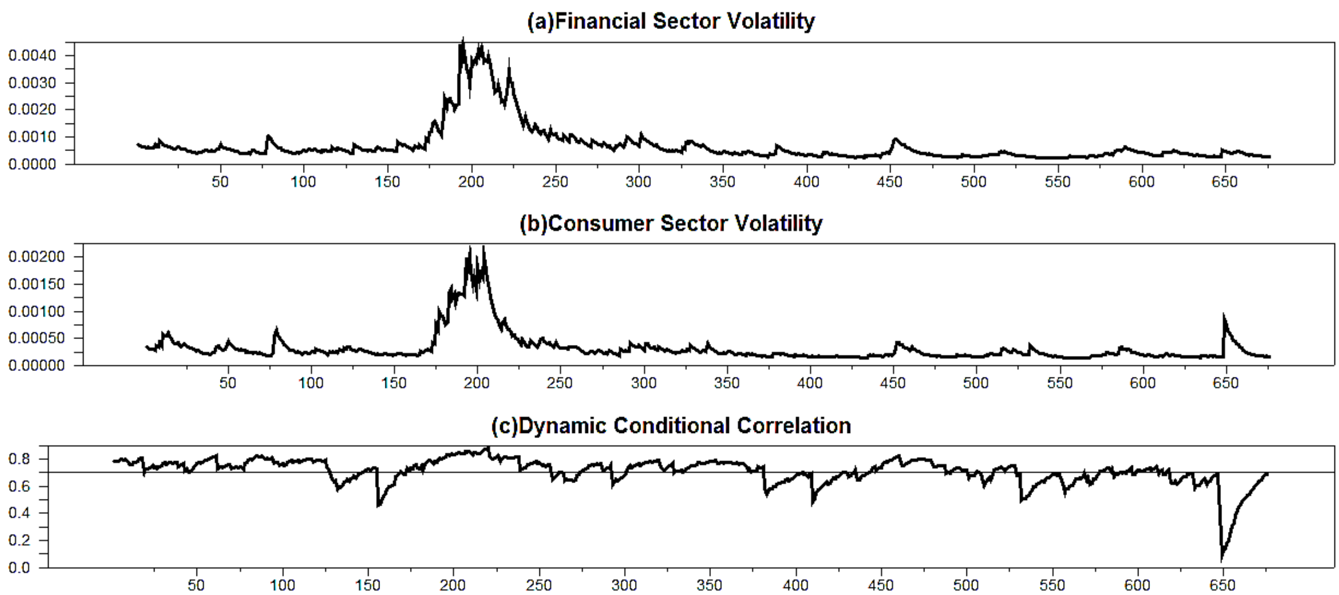

Figure 1. Volatility estimated by the DCC model for (a) the financial sector, (b) Consumer sector, and (c) their dynamic conditional correlation.

estimated volatility for the financial sector is about twice that was obtained for the consumption sector, which can be related with the 2007/2008 sub-prime crisis turbulence that affected this sector. Moreover, the sectors dependence did not suffered any sudden change.

Finally, we suggest for future researches the application of GARCH-DCC model in different assets belonging to Brazilian capital market, as well as in different sectors of other countries, especially those that are traditionally used for diversification in order to optimize the portfolio resource allocation.

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