Full Length Research Paper

Estimating the maximum probable annual losses due to flooding

Emad A. A. Ismail¹,²* and Fuad A. A. Awwad¹

¹Department of Quantitative Analysis, College of Business Administration, King Saud University, Saudi Arabia.
²Mathematics and Insurance Department, Faculty of Commerce, Cairo University, Egypt.

Received 26 October, 2018; Accepted 7 February, 2019

This paper aims to estimate the maximum probable annual losses to the risks of flooding. While examining the frequency of the event, it was observed that, at least one accident occurred periodically, which resulted in maximum losses. This is an empirical study and it is based on maximum losses due to floods per year, as obtained from the International Disaster Database and Munich Re. The analysis is based on 66 extreme flood events in the world between 1906 and 2015. The complementary risk method has been used in deriving a mixed probability distribution, which expresses the number of floods and the maximum losses realized, where zero-truncated Poisson distribution is used for frequency distribution and Last order Weibull distribution for losses. The maximum of flood losses was fitted with compound truncated Poisson-Weibull distribution. Probabilities have been calculated for extreme flood losses, which are less than specific descriptive measures, and expected values have been calculated for extreme flood losses, which exceed the specific descriptive measures. The results from the study indicate that the maximum probable losses are greater than the maximum actual losses. This paper contributes to the risk of extreme floods pricing; and it also helps the governments of the countries in calculating the financial reserves to cover the extreme flood losses.

Key words: Risk management, extreme flood, complementary risk, truncated poisson distribution, last order weibull distribution, expected losses.

INTRODUCTION

A number of natural disasters occur around the world year after year. These natural disasters usually happen unexpectedly. Even for those that are expected, such as floods, hurricanes, and volcanos, their expectation usually comes only a few days or hours before the occurrence. Consequently, there is often no sufficient time to prepare to manage or mitigate the damages and losses resulting from the disaster. Undoubtedly, natural disasters are considered the most dangerous risks that threaten human lives and properties. The impact of these disasters may even threaten the existence and survival of property and even countries, due to the heavy losses that usually result from them. In addition, there are some natural disasters, such as earthquakes, that are difficult to predict in time and place, and this worsens the damages and losses that result from them. Even if we are capable of expecting the occurrence of some natural disasters and we can control the amount of expected

*Corresponding author. E-mail: emadali@ksu.edu.sa.

Disclaimer: Author(s) agree that this article remain permanently open access under the terms of the Creative Commons Attribution License 4.0 International License.
damage, especially the loss of human lives, the actual losses will still be huge and may exceed the potentials of some countries and continents.

The Centre for Research on the Epidemiology of Disasters (CRED) defines natural disaster as "the event that overburdens the local capabilities and necessitates the importance of seeking national or foreign aid. It is an unexpected event and is often unpredictable. It causes huge damages and heavy losses as well as human suffering."

An event is considered to be a disaster if at least one of the following criteria applies to it:

1) The death of 10 persons or more;
2) Harm to a hundred persons or more;
3) Announcement of a state of emergency;
4) Calling for international aid.

As a matter of fact, 2004 was the costliest natural catastrophe year so far in insurance history. The most expensive losses were those caused by hurricanes in the Caribbean and the United States and typhoons in Japan. The overall economic losses amounted to over US$ 145bn. Almost two-thirds of this total is attributable to windstorms and a third to geological events, in particular the Niigata earthquake in Japan and the earthquake and tsunami catastrophe in South Asia [Munich Re, 2005]. In 2005, insured losses from Hurricanes Katrina, Rita, and Wilma alone are estimated at over $85 billion (including the $23 billion for flood claims paid by the government-run and-funded National Flood Insurance Program). The U.S. federal government provided over $120 billion in federal relief which is another historical record [Michel-kerjan and Morlaye, 2008]. Natural disasters pose a serious risk to humans and represent a huge economic challenge to the state. They may also sometimes lead to destruction and human suffering whose impact may last for many years. Table 1 shows the events of losses of the 10 costliest floods between 1980-2015. Maximum economic losses are US$ 59,000 million and insured losses are US$ 16,000 million, where the overall losses are US$ 43,000 million, without coverage by traditional insurance. This gap needs coverage by securitization tools in the capital markets or any suitable methods.

This paper aims to estimate the maximum probable losses as a result of the occurrence of the risk of extreme flooding. The estimated expected losses from the occurrence of flooding contribute to determine the gap between the non-covered losses to the economy and insurance losses. This helps determine the amount of loss that will be covered by the capital market such as Insurance-linked Securities (ILS). In addition to the development of the ability of insurers to price extreme floods, it also rationalizes the reinsurance agreements. Michel-Kerjan and Morlaye (2008) have discussed some of the main drivers of the radical shift that happened in the insurance-linked securities (ILS) market after the 2005 hurricane season in the Atlantic basin, which has rapidly become one of the world peak zones in terms of exposure? They explain why, despite this very encouraging evolution, the market has not expanded significantly (contrary to credit derivatives, for instance).

**LITERATURE REVIEW**

Having good planning and estimating the expected losses, as a result of natural disasters, can be challenging. Countries throughout the world need to estimate the necessary financial reserves required to cope with these losses. Extreme weather-related events (such as hurricanes, floods, and ice storms) are certainly important elements of the "insurance and finance meeting with climate change" phenomenon [Michel-kerjan and Morlaye, 2008]. According to the Fourth Assessment Report prepared by the Intergovernmental Panel on Climate Change (2007) [Climate Change, 2007], warming of the climate system is "unequivocal" and extreme events have increased in frequency and/or intensity over the last 50 years. It is also stated that there is high agreement and much evidence that with current climate change mitigation policies and related sustainable development practices, global greenhouse gas emissions will continue to grow over the next few decades. This highlights the need for improving the methods for estimating the maximum possible losses as a result of occurrence of the risk of extreme events as well as the associated expected losses. Noy (2009) and Strobl (2012) have examined the macroeconomic implications of natural disasters and finds that natural disasters considerably deteriorate the welfare of society [Noy, 2009; Strobl, 2012].

Born and Klimaszewski-Blettner (2013) have analysed the crucial factors that drive insurers’ willingness to offer coverage in catastrophe-prone lines of business (Born and Klimaszewski-Blettner, 2013). They have suggested certain policy implications for overcoming availability constraints with regard to improving insurance against catastrophic threats. Based on the disaster risk management programs in Mexico, Saldana-Zorrilla (2015) indicates that there is a deficit of central planning from the Mexican public sector to manage disaster risks. He has provided a comprehensive view of government risk management and also put forth a set of policy suggestions for integrating risk management and increasing risk reduction measures and planning. Chang and Berdiev (2013) examined the relationship between natural disasters, political risk and insurance market development in a panel of 39 countries over the period 1984-2009 using a dynamic panel two-step system generalised method of moments model. They have established that that the incidences of natural disasters and deaths caused by natural disasters lead to greater total insurance, as well as life insurance and non-life
insurance consumption. Further, there seems to exist an inverse relation between the levels of political risk and insurance consumption. The incidences of natural disasters and deaths attributable to natural disasters contribute to insurance market development under the tenure of a government with lower levels of political risk. Therefore, it should be emphasised that natural disasters, political risk, and their interaction effects are important determinants of insurance market development.

The high complexity of insurance markets with equally high potential for catastrophic loss, calls for improved estimation of the probable losses as a result of the occurrence of natural disasters. Gao et al. (2016), while discussing the difficulties in precisely estimating catastrophe risk, have applied the modeling framework to a full-scale case study for hurricane risk (flood and wind combined) for residential buildings in eastern North Carolina. The results indicated that the level of concentration in the primary insurance market can lead to significant differences in the firm's operational decisions (for example choice in reinsurance and retained or capped surplus). Further, results suggested that encouraging catastrophe reserves for insurance companies can reduce their likelihood of insolvency. Davidson (1998) presented alternative approaches to funding US natural catastrophe exposures; existing and evolving private and public funding arrangements are evaluated, and public policy changes are identified and recommended to encourage insurers to pre-fund catastrophe losses, use a broader array of capital, and encourage loss prevention to minimize the tragic consequences of natural disasters. Changes in federal tax policy are recommended to encourage "policyholder safety reserves" to enhance existing private market efforts to fund catastrophe losses. Bouriaux and MacMinn (2009) discussed the technical and regulatory issues that could be crucial to market growth and recommended new private and public initiatives aimed at boosting the use and efficiency of CAT-linked securities and derivatives.

Substantial growth in coastal populations has led to a dramatic increase in the consequences of natural disasters (Roth and Kunreuther, 1998). Hence, the risks due to extreme flooding needs emphasis. Management of the catastrophes of extreme floods depends mainly on the efficiency of the governmental administration at all levels in a country. When overwhelming floods take place, the governmental financial aid and social donations can be used to compensate people for the losses resulting from the catastrophe. However, these compensations usually cover only a small part of the losses resulting from such catastrophes. Catastrophe losses tend to be highly correlated in space and characterized by "fat tail" distributions, making it especially difficult for an insurer to avoid the possibility of insolvency (Kousky and Cooke, 2012). Designs and simulations on the pricing based on the extreme flood data in China during 1961 to 2009, using quantitative analysis method were carried out, combining with the non-life insurance actuarial method and the Wang-double-factor model (Chen et al., 2013). The results show that the price of the catastrophe bond is increasing with the increase of the value for triggering points. The results provide guidance for the pricing of extreme flood catastrophe bonds.

There is an urgent need to redistribute the flood disaster risk in the social system. Here the importance of the securitization of the catastrophe risks that protects the capital and the insurance market could be realized as the risk can be transferred to the capital market via the catastrophe bonds, which are considered an effective method of distributing risks. Pricing catastrophe bonds is considered the most important technical process for the issuance of these bonds. Nevertheless, scientific researches related to the designing and issuing of the destructive flood catastrophe bonds are limited. Risks with large maximum probable losses also stress the capacity of traditional insurance and reinsurance markets. For such risks, securitization may be the most efficient solution. As the costs resulting from covariability, skewness, and high-potential losses increase, securitization begins to substitute for reinsurance but, for the very highest level of risk, reinsurance may be uneconomic and hence reinsurance and securitization are complementary [Cummins and Trainar, 2009].

Cummins and Weiss (2009) provided a survey and overview of the hybrid and pure financial market instruments and provide new information on the pricing and returns on contracts such as industry loss warranties and Cat bonds. Bouriaux and Tomas (2014) analyzed the reasons for failure of exchange-traded insurance-linked derivatives like catastrophe insurance futures and options to attract interest from financial market participants. They also showed that, when analyzing large storm estimates, a long development period may not be crucial to the success of exchange-traded derivatives.

Since growth in coastal populations has led to a dramatic increase in the consequences of natural disasters due to extreme floods and also since there is a paucity of studies on estimates of natural disaster losses, this paper proposes models for the number of floods and the maximum losses. This paper contributes to the risk of extreme floods pricing; and it also helps the governments of the countries in calculating the financial reserves to cover the extreme flood losses.

**METHODOLOGY**

Extreme flooding has a low recurrence rate but huge economic losses that may overwhelm individuals and nations. Empirically, the distribution of damage amount from disasters, as a rule, is governed by laws of the heavy tail of the distribution (Rodkin and Pisarenko, 2008).

Ismail (2016) has been used the complementary risk method to determine a mixed probability distribution to express the number of accidents and the maximum realized losses (Ismail, 2016). The
where, $Y = \max(1, 2, \ldots, N)$. The distribution within the research sample was selected from the flood losses recorded in each year. Table 2 shows the descriptive statistics of maximum losses resulting from floods. It can be observed that highest losses were about $40.317 billion.

### Frequency distribution

Conceição et al. (2014) envisaged modified truncated zero poison distribution for determining the probability of occurrence of a flood. The probability distribution is given by:

$$P(N) = \frac{e^{-\lambda N}}{N^\alpha}, \quad N = 0, 1, \ldots \text{and } \lambda > 0 \quad (1)$$

Where, N is discrete random variable represents the number of floods and $\lambda$ is average number of floods.

The probability of occurrence of at least a flood is given by:

$$\sum_{N=1}^{\infty} P(N) = 1 - P(0) = 1 - e^{-\lambda} \quad (2)$$

Thus, the truncated Poisson probability distribution is given by:

$$P(z) = \frac{e^{-\lambda z}}{z!(1-e^{-\lambda})}, \quad z = 1, 2, \ldots, \infty \quad (4)$$

This probability distribution is used for predicting occurrence of at least one flood.

### Losses distribution

Considering the random variable $y$ expressing the maximum losses due to floods, Nadarajah et al., 2013 and Ismail 2016 have established that it follows the last order Weibull (Maximum Risk) distribution and has a probability density function given by:

$$f(y) = \alpha \frac{y^{\alpha-1}}{\theta}$$

Where, $\theta$ is the scale parameter and $\alpha$ is the shape parameter. Let:

$$Y = \max(y_1, y_2, \ldots, y_n)$$

In general, the last distribution for any continuous variable is given by:

$$f_1(y/z) = z f(y) [F(y)] z-1 \quad (6)$$

Where, $f(y)$ is the probability density function and $F(y)$ is the cumulative density function.

The joint distribution between $y$ and $z$ are obtained by multiplying (4) and (6) as follows:

$$f_2(y, z) = P(z) f_1(y/z) \quad (7)$$

**Table 1.** Comparison between overall losses and insured losses of flood loss events worldwide 1980-2015.

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>Overall losses in US$ m</th>
<th>Insured losses in US$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-15.11.2011</td>
<td>Floods, landslides</td>
<td>43,000</td>
<td>16,000</td>
</tr>
<tr>
<td>12-22.8.2002</td>
<td>Floods, flash floods</td>
<td>16,500</td>
<td>3,400</td>
</tr>
<tr>
<td>25-30.6.2007</td>
<td>Floods, severe storms</td>
<td>4,000</td>
<td>3,000</td>
</tr>
<tr>
<td>30.5-19.6.2013</td>
<td>Floods</td>
<td>12,500</td>
<td>3,000</td>
</tr>
<tr>
<td>20-23.7.2007</td>
<td>Floods</td>
<td>4,000</td>
<td>3,000</td>
</tr>
<tr>
<td>10-14.1.2011</td>
<td>Floods, flash floods</td>
<td>3,200</td>
<td>1,900</td>
</tr>
<tr>
<td>20-28.8.2005</td>
<td>Floods</td>
<td>3,300</td>
<td>1,800</td>
</tr>
<tr>
<td>19-24.6.2013</td>
<td>Floods, severe storms</td>
<td>5,700</td>
<td>1,600</td>
</tr>
<tr>
<td>October-November 2000</td>
<td>Floods</td>
<td>2,000</td>
<td>1,500</td>
</tr>
<tr>
<td>27.6-15.8.1993</td>
<td>Floods</td>
<td>21,000</td>
<td>1,300</td>
</tr>
</tbody>
</table>

Source: Munich Re, NatCatSERVICE (2016).

**Table 2.** Descriptive statistics of floods maximum losses ($\$ Billion$).

<table>
<thead>
<tr>
<th>Minimum</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Maximum</th>
<th>Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00023</td>
<td>0.3875</td>
<td>1.4650</td>
<td>4.9704</td>
<td>8.0456</td>
<td>40.317</td>
<td>90%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>95%</td>
</tr>
</tbody>
</table>

Thus, the truncated Poisson probability distribution is given by:
Table 3. Probabilities of extreme flood losses Classes.

<table>
<thead>
<tr>
<th>Classes of maximum losses</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than minimum</td>
<td>0.00291</td>
</tr>
<tr>
<td>Less than 1st Qu.</td>
<td>0.16595</td>
</tr>
<tr>
<td>Less than Median</td>
<td>0.38159</td>
</tr>
<tr>
<td>Less than mean</td>
<td>0.70028</td>
</tr>
<tr>
<td>Less than 3rd Qu.</td>
<td>0.81844</td>
</tr>
<tr>
<td>Less than Percentiles 90%</td>
<td>0.93175</td>
</tr>
<tr>
<td>Less than Percentiles 95%</td>
<td>0.95099</td>
</tr>
<tr>
<td>Less than maximum</td>
<td>0.99225</td>
</tr>
</tbody>
</table>

\[
 f_2(y, z) = \frac{e^{-\lambda y}}{z(1-e^{-\lambda})} \times z f(y)[F(y)]^{z-1} 
\]

\[
 f_2(y, z) = \frac{\lambda e^{-\lambda y} [F(y)]^{z-1}}{(z-1)(1-e^{-\lambda})} 
\]

The Marginal distribution for Y is given by:

\[
 g(y) = \frac{\lambda e^{-\lambda y} (y^a e^{-\lambda y})^{z-1}}{(z-1)!}, y > 0 
\]

Where,

\[
 \sum_{z=1}^{\infty} \frac{[F(y)]^{z-1}}{(z-1)!} = e^{F(y)} 
\]

\[
 g(y) = \frac{\lambda e^{-\lambda y} (y^a e^{-\lambda y})^{z-1}}{(z-1)!} 
\]

Where,

\[
 f(y) = a \left(\frac{y}{b} \right)^{a-1} e^{-\frac{y}{b}} 
\]

\[
 F(y) = 1 - e^{-\frac{y}{b}} 
\]

Thus, the PDF of a compound truncated Poisson Weibull distribution is given by:

\[
 g(y) = \frac{\lambda a \left(\frac{y}{b} \right)^{a-1} e^{-\frac{y}{b}} e^{-\lambda y}}{(1-e^{-\lambda})} 
\]

Where, \( \theta \) is the scale parameter and \( \lambda, \alpha \) are the shape parameters.

In general, the CDF is defined as follows:

\[
 G(y) = \frac{e^{-\lambda(1-F(y))} - e^{-\lambda}}{(1-e^{-\lambda})} 
\]

Where,

\[
 1 - F(y) = e^{-\frac{y}{b}} 
\]

\[
 G(y) = \frac{e^{-\lambda e^{-\frac{y}{b}}} - e^{-\lambda}}{(1-e^{-\lambda})} 
\]

Where, \( G(0) = 0 \) and \( G(\infty) = 1 \)

\[
 E(y) = \int_0^\infty y \cdot g(y) \cdot dy 
\]

\[
 Var(y) = \int_0^\infty [y - E(y)]^2 \cdot g(y) \cdot dy 
\]

RESULTS AND DISCUSSION

Estimation of parameters

The maximum likelihood estimators of the parameters are \( \hat{\lambda} = 2.93275 \), \( \hat{\alpha} = 0.4841 \) and \( \theta \) is the scale parameter equal 1000000000 and \( \lambda, \alpha \) are the shape parameters.

Goodness of fit

Kolmogorov-Smirnov test was used to test the following hypothesis:

H0: Maximum of maximum losses of flood fit with Compound Truncated Poisson-Weibull Distribution.

The results of the test are presented in the Appendix (Ismail, 2016). It can be observed that the value of the test statistic is 0.161, Critical Value is 1.311619 and P-Value is 0.064; which results in non-rejection of the null hypothesis at 5% level.

Application of model

Using Equation 18, the probabilities of extreme flooding losses as well as the quantum of expected extreme flooding losses for a given cumulative probability can be calculated. Table 3 shows the probabilities for extreme flood losses for various classes of maximum losses. It can be observed that the probability that the extreme flood losses are less than median losses (less than $1.4650 billion) is 0.38159, and the probability that the extreme flood losses are less than actual maximum losses (less than $40.317 billion) is 0.99225. Further, it is observed that the probability of maximum probable annual losses of flooding, equaling $188.852 billion, is 0.99999.

Using Equation 19, the expected value that exceeds some descriptive measures or any other values of extreme flooding losses can be calculated. Table 4 illustrates the expected losses for extreme flood losses with a maximum actual loss of $40.317 billion and with a maximum probable loss of $188.852 billion, which is less than specific descriptive measure. For example, the expected extreme flood losses more than median loss (more than $1.4650 billion) is $4.317 million under actual maximum losses and $4.755 million under probable maximum losses. In addition, Table 4 shows the expected losses more than maximum actual losses equal $0.376 billion under maximum probable losses.

Descriptive statistics showed that the maximum actual loss of extreme flooding losses was $40.317 billion and using the proposed model, it was found that the probability of extreme flooding losses less than this value was 0.99225. Conversely, it was found that the maximum probable loss of extreme flooding losses, which corresponds to a cumulative probability of 0.99999, was $188.852 billion. The maximum probable loss of extreme
floodings losses are greater than actual because they correspond to a higher cumulative probability. Table 4 shows the expected values of maximum flood losses with a maximum actual loss of $40.317 billion, and one more time considering the maximum probable loss $188.852 billion. The differences between expected values actual vs probable losses are the gaps between the non-covered losses.

On the practical level, the actual contribution of this paper is estimating the maximum probable loss of extreme flooding losses and comparing them to the actual losses and then determining the losses that are not covered. Which governments must cover by creating financial reserves or financing them to the capital market.

On the scientific knowledge, the actual contribution of this paper is to propose a statistical model is mixed probability distribution to express the number of floods and the maximum realized losses. The zero-truncated Poisson distribution represents frequency and Last order Weibull distribution and goodness of fit was conducted.

**Table 4.** Expected maximum losses of flooding ($Billion).

<table>
<thead>
<tr>
<th>Classes of maximum losses</th>
<th>with maximum actual loss of 40.317</th>
<th>with maximum probable loss of 188.852</th>
</tr>
</thead>
<tbody>
<tr>
<td>More than minimum</td>
<td>4.531</td>
<td>4.968</td>
</tr>
<tr>
<td>More than 1st Qu.</td>
<td>4.506</td>
<td>4.944</td>
</tr>
<tr>
<td>More than median</td>
<td>4.317</td>
<td>4.755</td>
</tr>
<tr>
<td>More than mean</td>
<td>3.388</td>
<td>3.826</td>
</tr>
<tr>
<td>More than 3rd Qu.</td>
<td>2.640</td>
<td>3.077</td>
</tr>
<tr>
<td>More than percentiles 90%</td>
<td>1.388</td>
<td>1.825</td>
</tr>
<tr>
<td>More than percentiles 95%</td>
<td>1.060</td>
<td>1.498</td>
</tr>
<tr>
<td>More than maximum</td>
<td>0</td>
<td>0.376</td>
</tr>
</tbody>
</table>

**Conclusion**

The proposed model is based on a complementary risk for estimating the maximum probable loss of extreme flooding losses, where zero-truncated Poisson distribution represents the frequency and Last order Weibull distribution for losses. Using the model, the maximum probable losses as a result of the occurrence of the extreme flooding was estimated and the results indicate that the maximum probable losses are greater than the maximum actual losses. Applying the proposed model, the gap between the non-covered losses of the economy and insurance losses can be estimated and also determine the amount of loss that will be covered by the capital market. This paper contributes to the risk of extreme floods pricing and helps the governments of the countries in calculating the financial reserves to cover the extreme flood losses.

**CONFLICT OF INTERESTS**

The authors have not declared any conflict of interests.

**ACKNOWLEDGEMENT**

The authors would like to extend their sincere appreciation to the Deanship of Scientific Research at King Saud University for funding this Research group NO (RG -1435-088).

**REFERENCES**


Louzada F, Cancho VG, Roman M, Leite JG (2012). A new long-term...


Appendix. Goodness of fit with Compound Truncated Poisson-Weibull Distribution Kolmogorov-Smirnov test

\[ F(i) = \frac{e^{-\lambda_l \left(1 - \text{pweibull}(Y_1, \alpha_l)\right)} - e^{-\lambda_l}}{1 - e^{-\lambda_l}} \]

<table>
<thead>
<tr>
<th>i</th>
<th>0.0029</th>
<th>0.1555</th>
<th>0.3083</th>
<th>0.6145</th>
<th>0.9366</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0244</td>
<td>0.1694</td>
<td>0.3526</td>
<td>0.6346</td>
<td>0.9389</td>
<td></td>
</tr>
<tr>
<td>0.0277</td>
<td>0.1694</td>
<td>0.3715</td>
<td>0.7569</td>
<td>0.9484</td>
<td></td>
</tr>
<tr>
<td>0.0285</td>
<td>0.1795</td>
<td>0.3914</td>
<td>0.7979</td>
<td>0.9523</td>
<td></td>
</tr>
<tr>
<td>0.0363</td>
<td>0.1903</td>
<td>0.4004</td>
<td>0.8184</td>
<td>0.9851</td>
<td></td>
</tr>
<tr>
<td>0.0476</td>
<td>0.1953</td>
<td>0.416</td>
<td>0.8186</td>
<td>0.9923</td>
<td></td>
</tr>
<tr>
<td>0.0626</td>
<td>0.1966</td>
<td>0.4558</td>
<td>0.829</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0715</td>
<td>0.2027</td>
<td>0.4558</td>
<td>0.8478</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0715</td>
<td>0.2146</td>
<td>0.4726</td>
<td>0.8488</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1023</td>
<td>0.2335</td>
<td>0.4915</td>
<td>0.8629</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.109</td>
<td>0.2452</td>
<td>0.5064</td>
<td>0.8892</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1092</td>
<td>0.2529</td>
<td>0.5415</td>
<td>0.8985</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1159</td>
<td>0.2675</td>
<td>0.5428</td>
<td>0.9266</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.141</td>
<td>0.2836</td>
<td>0.5961</td>
<td>0.9268</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1547</td>
<td>0.2981</td>
<td>0.6024</td>
<td>0.9295</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_i := |Z_i - F(i)| \]
\[ D := \max(d) \]
\[ D = 0.161 \]
\[ K := D \sqrt{n} \]
\[ K = 1.311619 \]
\[ PV(K) = 0.064 \]