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The underground economy: What is the influence of the discount factor?

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Underground economy is a continuous concern of economists all over the world and finding determinants and factors acting on its size is crucial in the fight to control and stop this negative phenomenon. In this paper, there is investigated a mathematical model founded on the basis of the inter-action between underground economy and official economy. As starting point, it used the logistic model of Verhulst, completed with the effect of the discount factor. Finally, the findings suggest that, at a small growth rate of the underground economy, it is possible not to exist any stable equilibrium points, but excessive size of the underground economy or too high growth rate cause the appearance of stable equilibrium points. This fact may be explained by the assumption that appreciable variations of the underground economy (in size or in growth rate) generate punitive and/or administrative measures from the authorities in order to counteract them.

Key words: Discount factor, underground economy, definitional approach, behavioural approach, equilibrium solution, Lambert function.

INTRODUCTION

The underground economy is, as a reality, a globally spread kind of production and distribution of goods and services; it is so extended that some scholars speak about a “natural rate of underground economy” (Castellucci and Bovi, 1999).

The goal of this paper is to investigate the influence of the discount factor on the existence of the equilibrium points in the dynamics of the underground economy.

THE UNDERGROUND ECONOMY: CONCEPT AND ESTIMATING METHODS

As a concept, the term “underground economy” is confusingly defined, having assigned many different understandings and meanings. As synonyms, there are used terms as: Unrecorded/unreported/undeclared economy, hidden economy, unofficial/informal economy, second/parallel economy, black or grey economy, shadow economy, illegal economy, and maybe others.

Some of the common definitions are listed as follows:

1. All unregistered economic activities that contribute to the officially calculated (or observed) Gross National Product (Feige, 1989, 1994; Schneider, 2003, 2005; Frey and Pommerehne, 1984);
2. The informal economy is “unregulated by the institutions of society, in a legal and social environment in which similar activities are regulated” (Portes et al., 1989);
3. “Market-based production of goods and services, whether legal or illegal, which escapes detection in the official estimates of GDP” (Thomas, 1999);
4. Activities that circumvent or otherwise government regulation, taxation or observation (Feige, 1989; Del’Anno, 2003);
5. “The shadow economy includes unreported income from the production of legal goods and services, either from monetary or barter transactions, and so includes all productive economic activities that would generally be taxable were they reported to the state (tax) authorities”
Table 1. A taxonomy of types of underground economic activities.

<table>
<thead>
<tr>
<th>Kind of activity</th>
<th>Monetary transactions</th>
<th>Non-monetary transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illegal activities</td>
<td>Trade in stolen goods; drug dealing and manufacturing; prostitution; gambling; smuggling, fraud and so forth.</td>
<td>Barter: drugs, stolen goods, smuggling and so forth. Produce or growing drugs for own use. Theft for own use</td>
</tr>
<tr>
<td>Legal activities</td>
<td>Tax evasion - Unreported income from self-employment; Wages, salaries and assets from unreported work related to legal services and goods</td>
<td>Tax avoidance - Employee, discounts, fringe benefits</td>
</tr>
<tr>
<td></td>
<td>Tax evasion</td>
<td>Tax avoidance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Barter of legal services and goods</td>
</tr>
<tr>
<td></td>
<td></td>
<td>All do-it-yourself work and neighbor help</td>
</tr>
</tbody>
</table>


(Feld and Schneider, 2010).

The confusion does not limit to the name of this part of the economy, but, more important, it also includes its nature. Some scholars (Feige, 1990) identify four specific types of underground economy: The illegal economy, the unreported economy, the unrecorded economy and the informal economy. The coverage of every concept is related to the activities which are “included” in the respective concept.

A primary necessary distinction operates between legal and illegal activities.

1. Legal activities are driven in the underground economy by the existing high tax rates, meaning they are legal and honest activities per se, but operating “underground” in order to keep the entire profit for the entrepreneurs. Actually, the main causes identified, beside excessive taxation, are: Excessive regulations, bureaucracy and corruption (Schneider and Enste, 2000). Legal activities include production and distribution of goods and services that can be found in the official economy, provided by legal, tax-payers entrepreneurs or firms. High tax rates – tax evasion and tax avoidance, are the push factors attracting these activities in the unofficial sector.

2. Illegal activities consist of activities illegal per se, goods and services not provided anywhere else that in the black economy, as: Drug production and distribution, contraband goods, prostitution and so forth.

Both legal and illegal activities may refer to monetary or non-monetary transactions, as revealed by the taxonomy in the Table 1.

Despite the fact of dealing with little agreement on the concept and its meanings, the efforts of economists are directed to the measure and estimation of the underground economy’s size compare with the official recorded economy. Here also we encounter a variety of methods. A comprehensive inventory of them has been advanced by Easton (2001), who identifies the following methods used to estimate the underground economy:

1. The national accounts reconciliation method based on statistical discrepancies;
2. The audit method;
3. The labor market and household survey method;
4. The monetary aggregates or currency ratio method;
5. The transactions method;
6. Sensitivity analysis;
7. Electricity consumption estimates; and
8. Econometric modeling”.

According to Frey and Schneider (2001), these methods can be synthesized into three different approaches.

The direct approaches are based on surveys and interviews with the providers and the clients of the underground activities. Another direct approach concerns the auditing of tax returns undertaken by tax collection and social security administrations. Even both types of methods enable only point estimates, underestimates, and do not capture all the underground activities, they provide detailed information about the unrecorded activities and the structure of people involved in them.

The indirect approaches estimate the size of the underground economy by investigating the discrepancies appearing in various markets. Discrepancies may involve different markets:

1. Discrepancies between the official personal (households) incomes and the amounts actually spent;
2. Discrepancies in the monetary market (the unofficial transactions are in their majority cash transactions);
3. Discrepancies in the labor market;
4. Discrepancies concerning physical inputs, especially the use of electricity (the excess over the amount of electricity normally used to produce the official national income).

Each approach has limits, because not all the difference can be attributed and relates to the underground economy.
Table 2. Estimations on the size of the underground economies in % of official GDP for 1990 and 2005 of selected countries.

<table>
<thead>
<tr>
<th>Group of countries</th>
<th>2004/2005 (DYMIMIC approach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing countries</td>
<td></td>
</tr>
<tr>
<td>Africa</td>
<td>(Average): 42.8. Ranging from: 28.2, South Africa to 64.6, Zimbabwe</td>
</tr>
<tr>
<td>Asia</td>
<td>(Average): 29.8. Ranging from: 12.1, S, gapore to 53.6, Thailand</td>
</tr>
<tr>
<td>Central and South America</td>
<td>(average): 42.2. Ranging from: 19.4, Chile to 67.2, Bolivia</td>
</tr>
<tr>
<td>Transition countries (East and Central European and Former Soviet Union countries)</td>
<td>(average): 25. Ranging from: 18.2, Slovak Republic to 66.4, Georgia</td>
</tr>
<tr>
<td>Developed OECD economies</td>
<td>(Average): 14.8. Ranging from: 7.9, United States to 26.3, Greece</td>
</tr>
</tbody>
</table>

Source: Schneider (2007).

The integrated model approach focuses on the causes and effects of the underground economy, by elaborating a model starting from the main motivators of engaging in underground activities. There is an arbitrage between the losses caused by tax and contributions payments, or complying with government regulations (especially on labor market) and the anticipated punishment of acting in the underground economy. This approach is almost comprehensive but requires a large amount of data, which reduces its application (Frey and Schneider, 2001).

According to the electricity demand approach, the size of the underground economy varied, at the beginning of the 1990s, from less than 10% of the GDP in some developed OECD economies to 50 to 60% or even more in some Asian and African developing countries (Schneider and Enste, 2000).

Similar figures were estimated by Schneider (2007) for 2004/2005, but using the DYMIMIC (dynamic multiple indicators multiple causes) approach (latent estimation approach).

A more complete picture may be seen in the Table 2:

RESULTS AND DISCUSSION

The variation of the variable $X$ is given by:

$$X' = \frac{x'y - xy'}{y^2}$$

(Arnold, 1980), but, according to the used assumption, $\frac{y'}{y^2} = 0$ and thus $X' = \frac{x'}{y}$.

Because, as noticed already, $x'$ is directly proportional with $x$, also $\frac{x'}{y}$ is proportional with $\frac{x}{y}$. In these conditions, $X'$ is directly proportional with $X$.

It is obvious that the size of the underground economy is influenced by the decision of the fiscal authorities to adopt punitive measures.

The implementation of these measures is determined by the degree of visibility of the effects of the underground economy. The visibility increases with the growth of the underground production in disfavor of the official production. Thus, the regulating effect intervenes in the variation of $X$ by introducing the term $$X(t)^{\text{not} x(t) = \frac{x(t)}{y(t)}},$$ where $x(t)$ designates the level of the underground economy and $y(t)$ the GDP level.

We will use the hypothesis that are not important variations of the GDP (related to its annual level), which means that $\frac{y'}{y}$ takes negligible values. In these conditions, $\frac{y'}{y^2}$ may be approximated to zero.

Another observation is that the underground economy growth rate develops in direct proportion with its size.
The variation law of the underground economy becomes:

\[ X'(t) = aX(t) - bX^2(t) - X(t)e^{-X(t)} \quad (1) \]

\[ a > 0, \quad b > 0 \]

The equilibrium solutions are given by the roots of the equation:

\[ aX(t) - bX^2(t) - X(t)e^{-X(t)} = 0 \quad (2) \]

The set of admissible solutions is \([0,1]\).

The solutions are:

\[ x_1 = 0, \text{ which represents an unstable equilibrium point and} \]

\[ x_2 = \frac{a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right)}{b}, \]

where \(W(\cdot)\) is Lambert function.

In order that \(x_2\) represents an admissible solution it must be satisfied two conditions:

1. First condition is that \(x_2\) be a real number. As \(W(x)\) is real only for \(x \geq -\frac{1}{e}\), it must satisfied the inequality:

\[ -\frac{1}{b} \cdot e^{\frac{a}{b}} \geq -\frac{1}{e} \Leftrightarrow a \geq b(1 - \ln b) \quad (\text{condition 1}) \]

2. \(x_2 \in [0,1]\), the real values of \(W(x)\) are greater than \(-1\) that implies

\[ x_2 = \frac{a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right)}{b} \geq \frac{a - b}{b}. \]

If \(a \geq b\) then \(x_2 \geq 0\) (condition 2)

In order that \(x_2 \leq 1\), it must be that

\[ a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) \leq b \quad (\text{condition 3}) \]

In order to determine the stability of the equilibrium, we must analyze the sign of the eigenvalue in \(x_2\). The eigenvalue is:

\[ \lambda = a - 2b \cdot \left( \frac{a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right)}{b} \right) - e^{\frac{a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right)}{b} + \frac{b}{b} \cdot e^{\frac{a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right)}{b}}} \]

The stable equilibrium condition is \(\lambda < 0\) [20].

From condition 3 we derive:

\[ a + bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) \leq b, \text{ so:} \]

\[ \lambda < a - 2a - 2bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) - e^{\frac{b}{b} + \frac{b}{b}} = \]

\[ -a - 2bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) - e + e = -a - 2bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) \]

If \(-a - 2bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) < 0\) then \(x_2\) represents a point of stable equilibrium.

\[ -a - 2bW\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) < 0 \Leftrightarrow W\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) > -\frac{a}{2b} \]

**CONCLUSION**

Concluding, we find that for \(W\left(-\frac{1}{b} \cdot e^{\frac{a}{b}}\right) > -\frac{a}{2b}\) there is a
stable equilibrium solution, and for $W\left( -\frac{1}{b}b - e^{-\frac{a}{b}} \right) < -\frac{a}{2b}$ there is an unstable equilibrium solution.

In the modeling of the underground economy's dynamics, the component with the significance of the discount factor increases as the underground economy's level increases, too. This is natural given that an increase in the discount factor means a decrease of the interest rate, which generates an increase in the consumption. Consumption's growth drives also an increase in the underground economy. One of the hypothesis taken in this article is that the growth rate of the underground economy is proportional with its size. Consequently, an increase in the discount factor will determine an increase in the growth rate of the underground economy.

As we found in Badulescu and Caus (2010), the results suggest that, at a small growth rate of the underground economy, it is possible not to exist any stable equilibrium points. This fact may be explained by the variations too small to determine the authorities to adopt punitive measures. Nevertheless, excessive size of the underground economy or too high growth rate causes the appearance of stable equilibrium points. This fact is motivated by the assumption that appreciable variations of the underground economy (in size or in growth rate) automatically generate a reaction of the authorities and the adoption of economic and/or administrative measures to counteract them.

Beyond estimations and estimating methods, the underground economy reveals to be a reality that cannot be ignored and a phenomenon which should be counteracted. The fight against it may be achieved by mainly by two opposite approaches: Increasing deterrence or motivating to act in the official economy.

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