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Full Length Research Paper

Bianchi Type I tilted perfect fluid cosmological model in general relativity

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In this paper, we have investigated a tilted Bianchi Type I cosmological model filled with dust of perfect fluid in general relativity because tilted dust models are quite homogeneous and expanding. To get a determinate solution, we have assumed a linear relation between shear and expansion, that is, $\frac{\sigma}{\theta}$ = constant (σ is shear tensor and θ is expansion of the model), which leads to A = $(BC)^n$, where A and B are metric potentials and n is constant. Also, we have assumed that fluid is pressure less, that is, p=0. The physical and geometrical aspects of the model together with singularities in the model are also discussed.

Key words: Tilted, dust perfect fluid, Bianchi Type I universe.

INTRODUCTION

In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic universe in which the matter does not move orthogonally to the hypersurface of homogeneity. These are called tilted universe. The general dynamics of tilted universe have been studied in detail by King and Ellis (1973), Ellis and King (1974), and Collins and Ellis (1979). Tilted Bianchi Type I models have been obtained by Dunn and Tupper (1978) and Lorenz (1981). Mukherjee (1983) has investigated tilted Bianchi Type I universe with heat flux in general relativity. He has shown that the universe assumes a pancake shape. Bradley (1988) obtained all tilted spherically symmetric self similar dust models. The equations for tilted cosmological models are more complicated than those of non-tilted ones. Ellis and Baldwin (1984) have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. A tilted cold dark matter cosmological scenario has been discussed by Cen et al. (1992). Bali and Sharma (2002) investigated tilted Bianchi Type I dust fluid and shown that model has cigar type singularity when T = 0.

In this paper, we have investigated tilted Bianchi Type I dust fluid of perfect fluid in general relativity. To get a determinate solution, a supplementary condition P = 0, A = $(BC)^n$ between metric potential is used. The behavior of the singularity in the model with other physical and geometrical aspects of the models is also discussed.

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THE METRIC AND FIELD EQUATIONS

We consider metric in the form:

$$ds^2 = -dt^2 + A^2\,dx^2 + B^2\,dy^2 + C^2\,dz^2\,, \quad (1)$$

Where A, B and C are functions of 't' alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis (1971) is taken into the form:

$$T_i^j = (\in + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j\,, \quad (2)$$

Together with

$$g_{ij}\, v_i\, v^j = -1\,, \quad (3)$$

$$q_i q^j > 0\,, \quad (4)$$

$$q_i v^i = 0, \tag{5}$$

Where p is the pressure, $\in$ the density and q_i the heat conduction vector orthogonal to v^i. The fluid flow vector has the components $\left(\frac{\sinh\lambda}{A}, 0, 0, \cosh\lambda\right)$ satisfying Equation 3 and λ is the tilt angle.

The Einstein field equation

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j, \text{ (units such that c = G = 1)} \tag{6}$$

For the line, element of Equation 1 are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi\left[(\in + p)\sinh^2\lambda + p + 2q_1 \frac{\sinh\lambda}{A}\right], \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi p, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi p, \tag{9}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = -8\pi\left[-(\in + p)\cosh^2\lambda + p - 2q_1 \frac{\sinh\lambda}{A}\right], \tag{10}$$

$$(\in + p) A\sinh\lambda \cosh\lambda + q_1 \cosh\lambda + q_1 \frac{\sinh^2\lambda}{\cosh\lambda} = 0, \tag{11}$$

Where the suffix ‘4’ stands for ordinary differentiation with respect to cosmic time ‘t’ alone.

SOLUTION OF FIELD EQUATIONS

Equations 7 to 11 are five equations in seven unknown A, B, C, $\in$, p, λ and q_1; therefore to determine the complete solution we require two more conditions:

1) We assume that the model is filled with dust of perfect fluid which leads to

$$p = 0 \tag{12}$$

2) Relation between metric potential as:

$$A = (BC)^n \tag{13}$$

Where n is constant.

Equations 7 and 10 lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4 C_4}{BC} + \frac{A_4 C_4}{AC} + \frac{A_4 B_4}{AB} = 8\pi(\in - p) \tag{14}$$

From Equations 12 and 14, we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4 C_4}{BC} + \frac{A_4 C_4}{AC} + \frac{A_4 B_4}{AB} = 8\pi \in \tag{15}$$

Equations 8 and 9 lead to

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \tag{16}$$

This leads to

$$\frac{\nu_4}{\nu} = \frac{a}{\mu^{n+1}} \tag{17}$$

Where BC = μ, $\frac{B}{C} = \nu$ and ‘a’ is constant of integration.

Again from Equations 8 and 9, we have

$$\frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{A_4 B_4}{AB} = -16\pi p \tag{18}$$

From Equations 12 and 18, we have

$$\frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{A_4 B_4}{AB} = 0 \tag{19}$$

Equation 19 gives

$$2(1+2n)\frac{\mu_{44}}{\mu} - (4n^2 - 2n - 1)\left(\frac{\mu_4}{\mu}\right)^2 + \left(\frac{\nu_4}{\nu}\right)^2 = 0 \tag{20}$$

Where A = μ^n.

From Equations 17 and 20, we have

$$2ff^1 - \frac{(4n^2 - 2n - 1)}{(1+2n)}\frac{f^2}{\mu} = \frac{-a^2}{(1+2n)\mu^{2n+1}} \tag{21}$$

Where $\mu_4 = f(\mu)$.

Equation 21 leads to

$$f^2 = \frac{1}{(4n+1)\mu^{2n}}[a^2 + b(4n+1)\mu^{4n+1/2n+1}] \tag{22}$$

Where ‘b’ is a constant of integration.

$$\log\nu = a\sqrt{4n+1}\int \frac{d\mu}{\mu\sqrt{a^2 + b(4n+1)\mu^{4n+1/2n+1}}} \tag{23}$$

Hence, the metric of Equation 1 reduces to the form

$$ds^2 = -\frac{d\mu^2}{f^2} + \mu^{2n}dx^2 + \mu\nu dy^2 + \frac{\mu}{\nu}dz^2 \tag{24}$$

Where ν is determined by Equation 23.

By introducing the following transformations

$$\mu = T, \qquad x = X, \qquad y = Y, \qquad z = Z$$

The metric of Equation 24 reduces to the form

$$ds^2 = \left[\frac{-(4n+1)T^{2n}}{a^2 + b(4n+1)T^{4n+1/2n+1}}\right] dT^2 + T^{2n} dX^2 + T\nu dY^2 + \frac{T}{\nu} dZ^2 \tag{25}$$

Where

$$\log \nu = a\sqrt{4n+1}\int \frac{dT}{T\sqrt{a^2 + b(4n+1)T^{4n+1/2n+1}}} \tag{26}$$

SOME PHYSICAL AND GEOMETRICAL FEATURES

The density for the model of Equation 25 is given by:

$$8\pi \in = \frac{(4n+1)b}{2(2n+1)T^{\frac{4n^2+2n+1}{2n+1}}} \tag{27}$$

The tilt angle λ is given by:

$$\cosh\lambda = \frac{1}{2}\sqrt{\frac{2n+1}{n}} \tag{28}$$

$$\sinh\lambda = \frac{1}{2}\sqrt{\frac{1-2n}{n}} \tag{29}$$

The reality conditions

(i) $\in + p > 0$,
(ii) $\in + 3p > 0$, lead to

$$\frac{b(4n+1)T^{\frac{-(4n^2+2n+1)}{(2n+1)}}}{2(2n+1)} > 0 \tag{30}$$

Where

$$\frac{b(4n+1)}{2(2n+1)} > 0$$

The scalar of expansion θ calculated for the flow vector ν^i is given by:

$$\theta = \frac{(n+1)}{2T^{n+1}}\sqrt{\frac{(2n+1)\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}}{(4n+1)T^{n+1}}} \tag{31}$$

The components of fluid flow vector ν^i and heat conduction vector q^i for the model of Equation 25 are given by:

$$\nu^1 = \frac{1}{2T^n}\sqrt{\frac{1-2n}{n}} \tag{32}$$

$$\nu^4 = \frac{1}{2}\sqrt{\frac{2n+1}{n}} \tag{33}$$

$$q^1 = \frac{-(4n+1)b}{64\pi T^{\frac{6n^2+3n+1}{2n+1}}}\sqrt{\frac{1-2n}{n}} \tag{34}$$

$$q^4 = \frac{-(4n+1)(1-2n)b}{64\pi(2n+1)T^{\frac{4n^2+2n+1}{2n+1}}}\sqrt{\frac{1+2n}{n}} \tag{35}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by

$$\sigma_{11} = \frac{(4n^2-1)}{24nT^{1-n}}\sqrt{\frac{(2n+1)\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}}{n(4n+1)}} \tag{36}$$

$$\sigma_{22} = \frac{\nu}{12T^n}\sqrt{\frac{(2n+1)}{n(4n+1)}}\left[(1-2n)\sqrt{\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}} + 3a\sqrt{4n+1}\right] \tag{37}$$

$$\sigma_{33} = \frac{1}{12\nu T^n}\sqrt{\frac{(2n+1)}{n(4n+1)}}\left[(1-2n)\sqrt{\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}} - 3a\sqrt{4n+1}\right] \tag{38}$$

$$\sigma_{44} = -\frac{(2n-1)^2}{24n}\sqrt{\frac{(1+2n)\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}}{n(4n+1)}} \tag{39}$$

$$\sigma_{14} = -\frac{-(26n^2+3n-2)}{24nT}\sqrt{\frac{(1-2n)\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}}{n}} \tag{40}$$

$$\omega_{14} = \frac{(6n+1)}{16T}\sqrt{\frac{(1-2n)\left\{a^2 + b(4n+1)T^{4n+1/2n+1}\right\}}{n(4n+1)}} \tag{41}$$

The rates of expansion H_i in the direction of x, y and z axes are given by

$$H_1 = \frac{n}{T^{8n^2+4n+1/2(2n+1)}}\sqrt{\frac{a^2T^{4n^2-2n-1/(4n+1)} + b(4n+1)T^{2n}}{4n+1}} \tag{42}$$

$$H_2 = \frac{1}{2T^{8n^2+4n+1/2(2n+1)}}\sqrt{\frac{a^2T^{4n^2-2n-1/(4n+1)} + b(4n+1)T^{2n}}{4n+1}} + \frac{a}{2T^{n+1}} \tag{43}$$

$$H_3 = \frac{1}{2T^{8n^2+4n+1/2(2n+1)}}\sqrt{\frac{a^2T^{4n^2-2n-1/(4n+1)} + b(4n+1)T^{2n}}{4n+1}} - \frac{a}{2T^{n+1}} \tag{44}$$

DISCUSSION

The model started with a big-bang at T = 0 and the expansion in the model decrease as time T increases and it stopped at T = ∞. The model has point type singularity at T = 0 (MacCallum, 1971). The model represents shearing and rotating universe in general and rotation goes on decrease as time increases. Since $\lim_{T \to \infty} \frac{\sigma}{\theta} \neq 0$, then the model does not approach isotropy for large value of T.

Density $\in \to 0$ as $T \to \infty$ and $\in \to \infty$ as $T \to 0$. When $T \to 0$. $q^1 \to \infty$ and $q^4 \to \infty$. Also, q^1 and q^4 tend to zero as $T \to 0$. At T = 0, the Hubble parameters tend to infinite at the time of initial singularity of vanish as $T \to \infty$.

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