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Full Length Research Paper

A stochastic knapsack problem with additive model of contagious distribution for the weight

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In this paper, our concern is on modelling a stochastic knapsack problem with the mixture of two known distributions (Gamma and Exponential) using additive form. The behavioural pattern of this mixed distribution is presented graphically and properly examined with different values of the parameters. It was shown that the new distribution is a proper probability density function (PDF) and its mean and variance were obtained, respectively. Also, an algebraic model was proposed for a stochastic knapsack problem with mixed (additive form) distributional weight.

Key words: Knapsack problem, contageous distribution, gamma distribution, exponential distribution.

INTRODUCTION

Literature has shown that more work has been done in the static aspect of the knapsack problem than the dynamic. Different authors have come up with different illustrations as a way of defining the knapsack problem; Martello and Toth (1990) illustrated that "the knapsack problem" can be likened to a hitch-hiker who intends to fill his knapsack by selecting from among various possible objects which will give him a maximum comfort. They formulated the problem mathematically by numbering the objects from 1 to n and introducing a vector of binary variable x_j $(j = 1,2,\ldots,n)$. Where

$x_j = \begin{cases} 1 & if\ jth\ item is select \\ 0 & otherwise \end{cases}$, then if p_j is a

measure of the comfort given by object j, w_j its size and c the size of the knapsack, the problem will now be

$$Maximize \sum_{j=1}^{n} p_j x_j \tag{1}$$

$$Subject\ to\ \sum_{j=1}^{n} w_i x_i \leq c \tag{2}$$

Whereas, Kosuch and Lisser (2008) in their own way defined knapsack problem as a combinatorial problem: each item is modelled by a binary decision variable $x \in \{0,1\}$ with x = 1 if the item is chosen and 0 otherwise; He added that the knapsack problem is generally linear, that is both the objective function and the constraints are linear.

However, in this work, we are interested in a situation where the items we are selecting in order to maximized the profit of a firm comes from two distinct population, and we employ additive model of contagious distribution to address the problem. Our reason for the contagious distribution is that, we assume the random variable x takes up distinct values, $x_1, x_2, \ldots, x_n$ with positive probabilities and also take up (assume) all values in an interval; say $a \leq x \leq b$. The probability distribution that will be obtained here will be as the result of combination of both discrete and continuous distribution (Meyer, 1965).

LITERATURE REVIEW

Kosuch and Lisser (2009) worked on a two-stage stochastic knapsack problem with probabilistic constraint. They assumed the item weight to be independently distributed following normal distribution. They showed how to obtain upper bounds on the overall problem or on sub problems and also computed high probable lower bounds on the overall problem given a first

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stage decision.

Claro and Sousa (2010) in their paper `A multiobjective for a mean-risk static stochastic knapsack problems` addressed two major challenges presented by stochastic discrete optimization problems: the multiobjective nature of the problems, once risk aversion is incorporated, and the frequent difficulties in computing exactly, or even approximately the objective function. They also proposed the use of multiobjective metaheuristics to deal with these difficulties, and apply multiobjective meta-heuristic to both exact and sample approximation versions of a mean-risk static stochastic knapsack problem.

Other researchers have provided alternative versions of the static stochastic knapsack problem. For instance, when considering problems with independent normally distributed rewards, Steinberg and Parks (1979) proposed a preference order dynamic programming algorithm, an approach further elaborated by Sniedovich (1980, 1981). Henig (1990) proposed a hybridization of dynamic programming with a search procedure, while Morton and Wood (1998) developed a Monte Carlo approximation for problem with general distributions on the random rewards. Kleywegt et al. (2002) studied a Monte Carlo simulation based approach that repeatedly solves sample average optimization problems, in which the expected value function was approximated by a sample average function, obtained by generation of a random sample.

Dizdar et al. (2011) worked on revenue maximization in the dynamic and stochastic knapsack problem where a given capacity needs to be allocated by a given deadline to sequentially arriving agents. They also derived two sets of additional conditions on the joint distribution of values and weights under which the revenues maximization policy for the case with observable weights is implementable, and thus optimal for the case with two-dimensional private information. Fujimoto and Yamada (2006) in their work on exact algorithm for the knapsack-sharing problem items developed an algorithm to solve this problem to optimality, and through a series of computational experiments, they evaluate the performance of the developed algorithm.

Cohn and Barnhart (1998) contributed to the building of a toolkit for tackling complex planning problems in a robust manner, by focusing on solving the stochastic knapsack problem with random weights (SKPRW). They also established several properties useful in solving it with a number of examples.

Han and Makino (2009) addressed the online minimization knapsack problem. They studied the removable model, where it was allowed to remove old items from the knapsack in order to accept a new item. In the course of their work, they were able to obtain the following results:

1) Derivation of a lower bound 2 for deterministic algorithms for the problem,
2) Proposition of a 2e - competitive randomized algorithm for the problem,
3) Proposed an 8 - completive deterministic algorithm for the problem, which contrasts to the result for the online maximization knapsack problem that no online algorithm has a bounded competitive ratio.

Grosan et al. (2003) developed a new evolutionary algorithm for the multiobjective 0/1 knapsack problem. Their algorithm used an E-dominance relation for direct comparison of two solutions and their experimental results show that the new proposed algorithm out performs the existing evolutionary approaches to this problem. Fidanova (2005) in his work on heuristics for multiple knapsack problems compared four types of heuristics, statics and dynamic for A C O algorithms to solve multiple knapsack problems. He observed using static heuristics result in improved performance. Fricke (2006) worked on example of new facets for the precedence constrained knapsack problem where he considered the polyhedral structure of the problem equally known as the partially ordered knapsack problem. His concept was, given a set of items N along with a partial order, or set of precedence relationships, on the items, denoted by $S \subseteq NXN$. A precedence relationship $(i, j) \in S$ exists if item i can be placed in the knapsack only if item j is in the knapsack. He derived a new approach for determining facets of the precedence constraint knapsack (PCK) polyhedron based on clique inequalities.

Djamnaty and Doostdar (2008) in their paper ''A hybrid genetic algorithm for the multidimensional knapsack problem'' observed that genetic algorithm is hybridized with a good initial population generated by Danzig algorithm solution of single knapsack problem. They also developed a number of novel penalty functions that can drive infeasible solutions towards feasibility with an incredible speed. They proposed that these penalty functions can be applied to any optimization problem having linear constraints and nonlinear objective functions, such as quadratic assignment problem. Bazgan et al. (2009) worked on implementing an efficient fully polynomial time approximation scheme (FPTAS) for the 0 to 1 multiobjective knapsack problem and proposed a methodology that makes use of very general techniques (such as dominance relations in dynamic programming) and thus may be applicable in the implementation of FPTAS for other problems as well. They showed that by using several complementary dominance relations, and sharing the error appropriately among the phases; they obtain an FPTAS, which is experimentally extremely efficient.

CONTAGIOUS DISTRIBUTION

From the brief overview of the mixed distribution earlier mentioned, it is worth noting that this mixture of distribution can take any form; mixture of two or more continuous distribution or discrete distribution or even both through either multiplicative or additive model. From Equation 3

$$\Pr(X \le x) = F(x) = PF_1(x) + (1-P)F_2(x) \tag{3}$$

Where P is a factor used to weigh the relative contribution of each population $(0 < P < 1)$, and $F(x)$ is the composite exceedance probability. $F_1(x)$ and $F_2(x)$ are the component. Escalante-Sandoval (2007) adopted the model and used it in his work on a mixed distribution with Extreme value distribution (EV1) and General extreme value component (GEV) component for analyzing heterogeneous samples as:

$$F(x) = P\exp^{-\exp^{-\left(\frac{x-v}{\alpha}\right)}} + (1-P)\exp\left\{-\left[1-\left(\frac{x-w}{\lambda}\right)\beta\right]^{\frac{1}{\beta}}\right\} \tag{4}$$

The assumption was that the first and second populations behave as Gumbel distribution (EV1) and (GEV) distributions, respectively.

THE CONTAGIOUS DISTRIBUTION MODEL

Literature has shown that most studies on the static stochastic knapsack problem concentrates on normally distributed rewards (Morton and Wood, 1998; Goel and Indyk, 1999; Sniedovich, 1980; Carraway et al., 1993) because the normality assumption covers a wide range of practical applications and at the same time, makes the static problem more tractable. In this work, we are deviated completely from the use of normal distribution to a mixture of a standard Gamma and Exponential distribution.

We employ the additive model of the contagious distribution as shown in Equation 3.

$$\Pr(X \le x) = F(x) = PF_1(x) + (1-P)F_2(x)$$

Let $F_1(x)$ be a standard Gamma distribution and $F_2(x)$ be an Exponential distribution. Therefore

$$F(x) = P\frac{x^{n-1}e^{-x/\beta}}{\beta^n\Gamma(n)} + (1-P)e^{-\lambda x} \tag{5}$$

The graphical presentations of our new distribution with different values of $P, \lambda, n, and\, ranges\, of$ x are shown in the work.

From the graphs shown (a mixture of Gamma and Exponential distribution), the parameter λ was chosen to be as small as 1, 2, and 3 for Figures 1 to 3 with the weighing factor P being kept as constant (0.2) and the second parameter β was varied as 0.2, 0.5 and 0.8, respectively. It was observed that, although they were all decreasing function graphs, the one with the least value of β (that is, 0.2) with λ being (1 and 2) began as an increasing function graph at $0 \le x \le 0.3$ before it began to decay as x increases.

In Figures 4 to 6, the parameter λ was still kept at 1, 2 and 3, while the weighing factor P was made constant (0.5) but the parameter β was varied as 0.2, 0.5 and 0.8, respectively. It was observed that when β was 0.2, the graph started as an increasing function to the point where $x = 0.4$ and then began to decrease (almost the form of a normal distribution but skewed to the right), whereas in Figures 5 and 6, the graph with $\lambda = 1$ and $\beta = 0.5$ was almost platykurtic in shape while the rest decayed as x increases.

In Figures 7 to 9, the weighing factor P was increased to 0.8 and was kept as constant, while the parameter β was varied at 0.2, 0.5 and 0.8, respectively. The parameter λ was still 1, 2 and 3. It was observed that with high value of P, the graphs were almost of a normal distribution but positively skewed except Figure 8 with $\beta = 0.5$ which decays below zero at $x = 1.7$ and 1.8, thus the graph tends to negative as $x \ge 1.7$. Figures 10 to 12 were graph with a little increment in the parameter λ (2, 4 and 6) with P being kept as constant (0.2) and the parameter β was varied at 0.2, 0.5 and 0.8. It was observed that the graph were all decreasing function up to the point where $0.5 \le x \le 0.7$ and then decayed simultaneously as x increases. In Figures 13 to 15, λ was still kept at 2, 4 and 6 and the weighing factor P was made constant at 0.5, whereas β was varied at 0.2, 0.5 and 0.8. We observed that in Figure 13 with $P = 0.2$, graphs began as an increasing function for the smaller values of λ and later at $x = 0.4$ it begins to decay as x increases; but as λ increases, the graph tends to a complete decreasing function without any increase at the beginning. However, Figures 14 and 15 exhibited a decay property as x increases. But in Figure15 with β = 0.8, the graph tends to be negative at $1.3 \le x \le 1.6$. Figures 16 to 18 had λ still being kept as 2, 4 and 6, but β was varied at 0.2, 0.5 and 0.8 whereas the weighing factor P was constant (0.8). We observed that Figure 16 with β = 0.2 produces a graph which appears almost as a normal distribution graph but skewed to the right. In Figure 17, the graph with λ = 2 and 4 were platykurtic

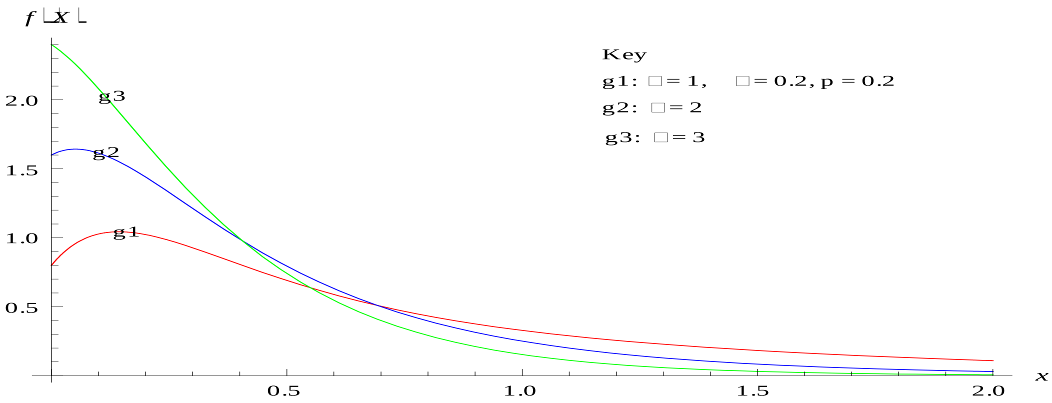

Figure 1. p = 0.2; β = 0.2; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

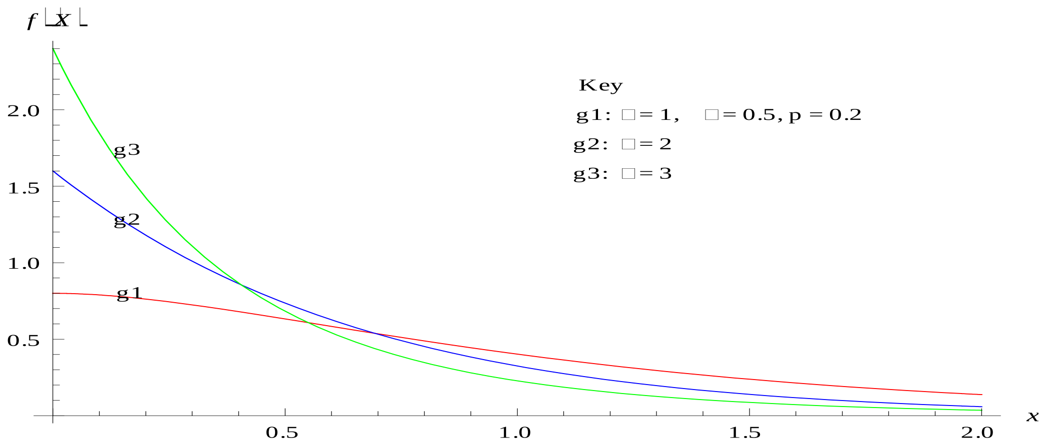

Figure 2. p = 0.2;β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

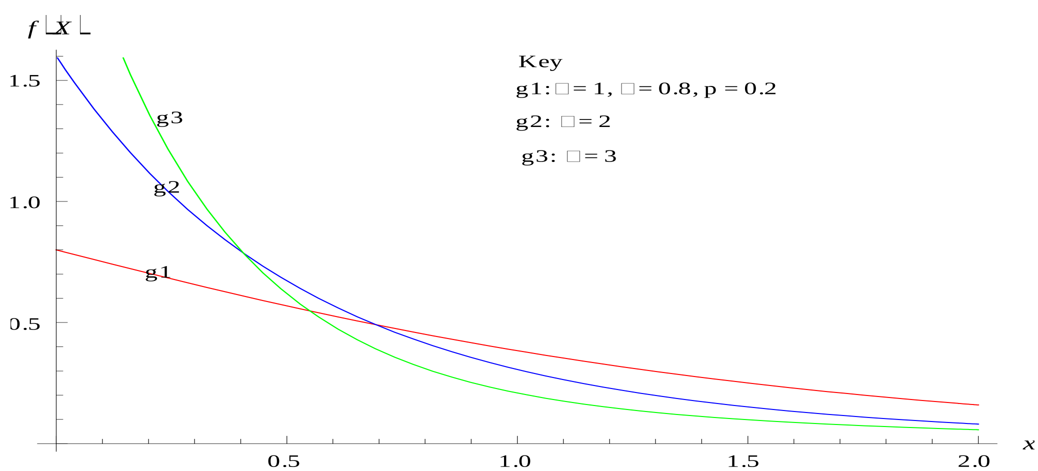

Figure 3. p = 0.2;, β = 0.8; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

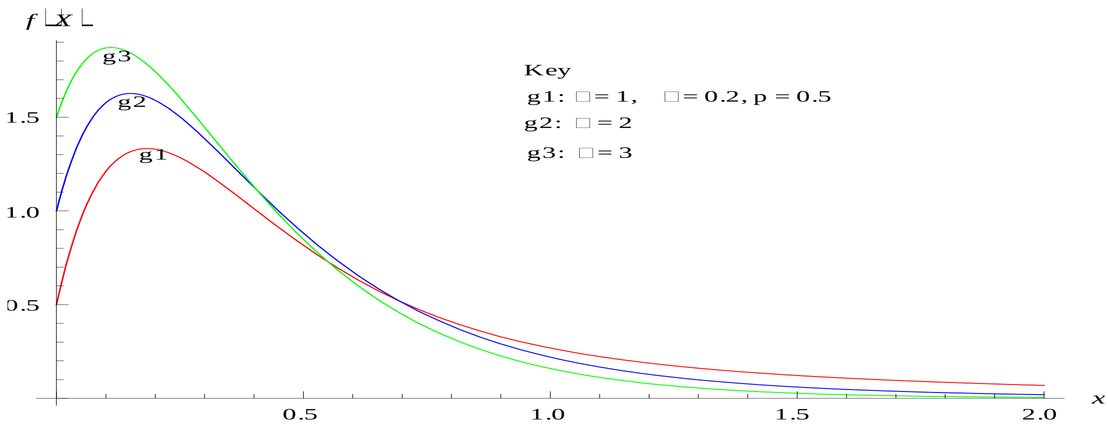

Figure 4. p = 0.5; β = 0.2; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

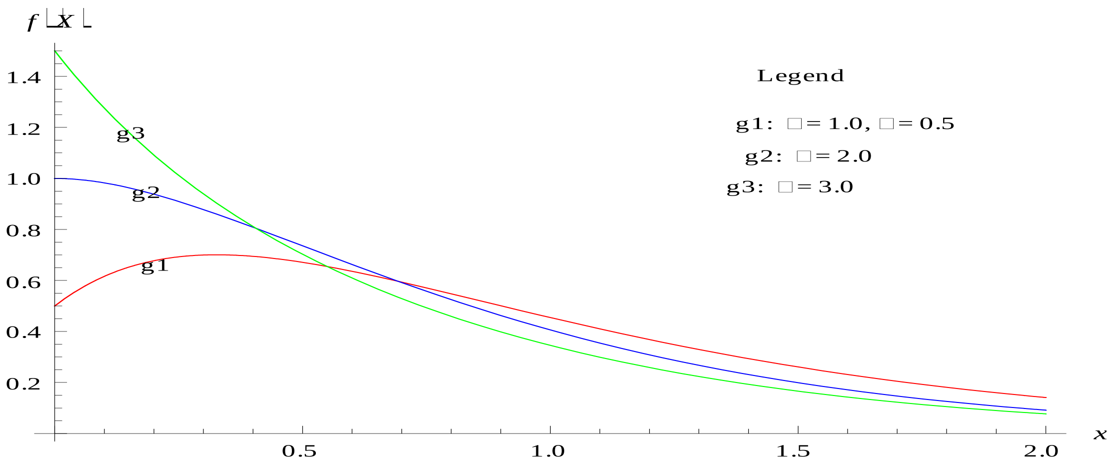

Figure 5. p = 0.5; β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

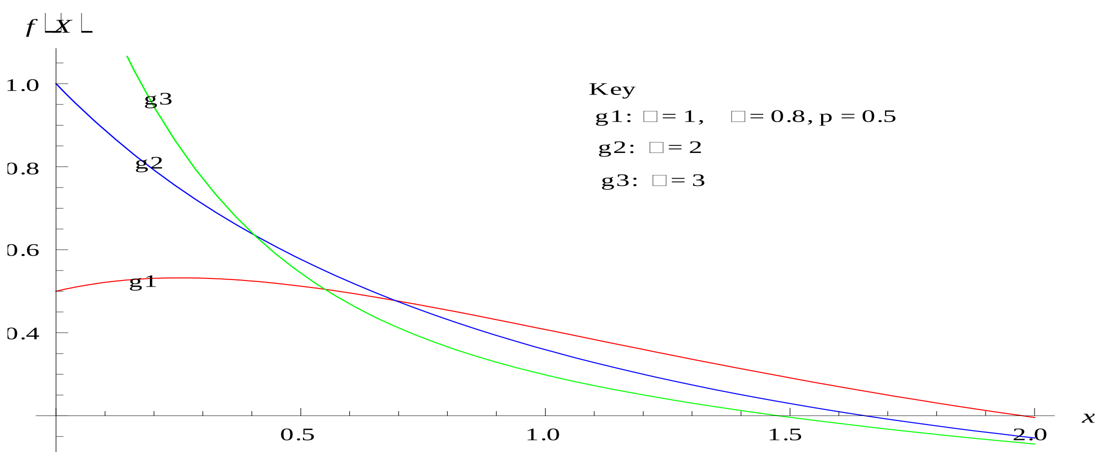

Figure 6. p = 0.5; β = 0.8; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

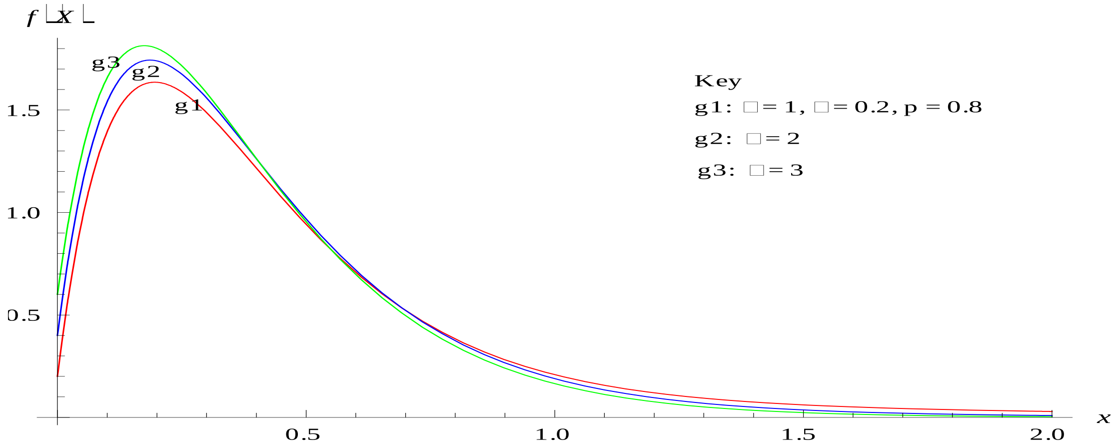

Figure 7. p = 0.8; β = 0.2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

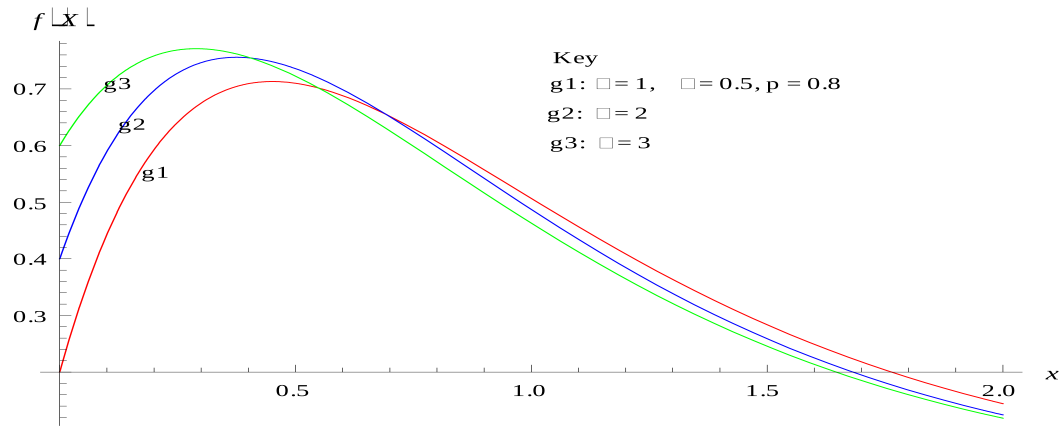

Figure 8. p = 0.8; β = 0.5; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

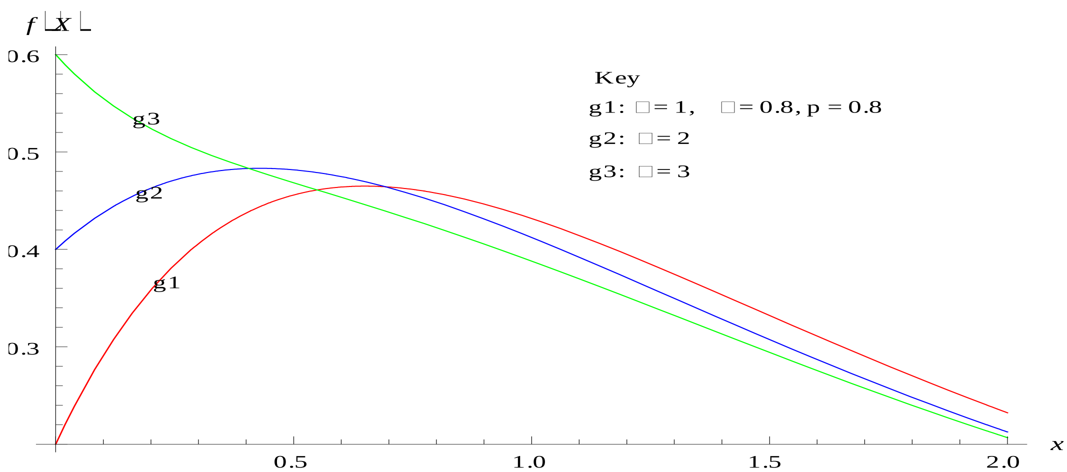

Figure 9. p = 0.8; β = 0.8; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

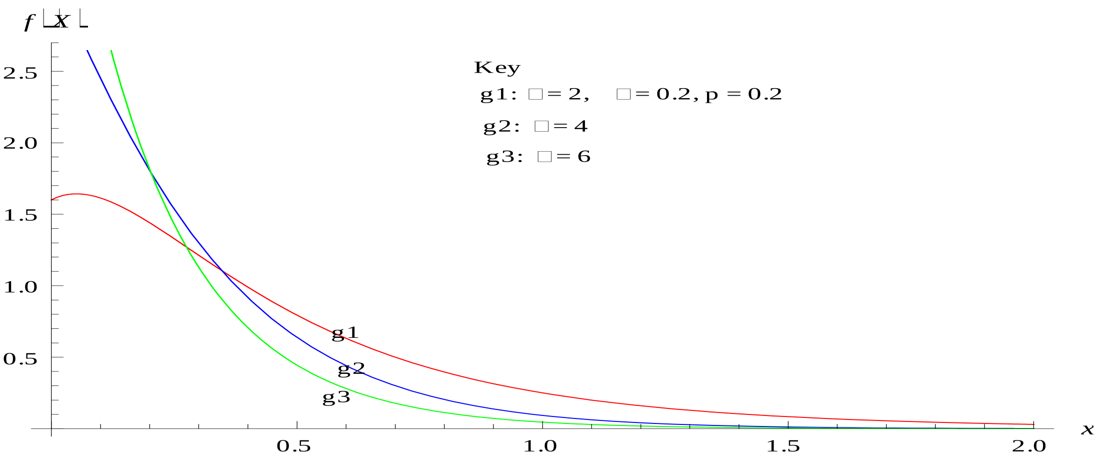

Figure 10. p = 0.2; β = 0.2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

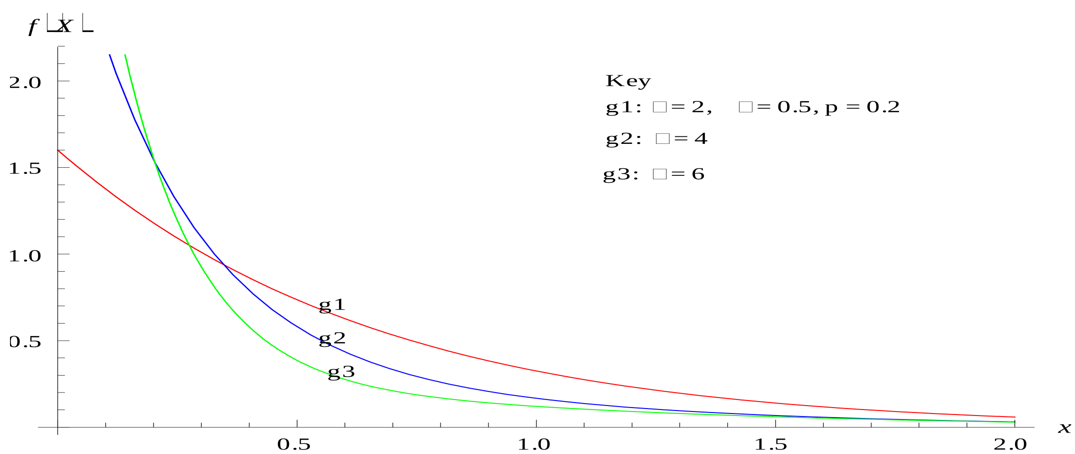

Figure 11. p = 0.2; β = 0.5; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; .
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

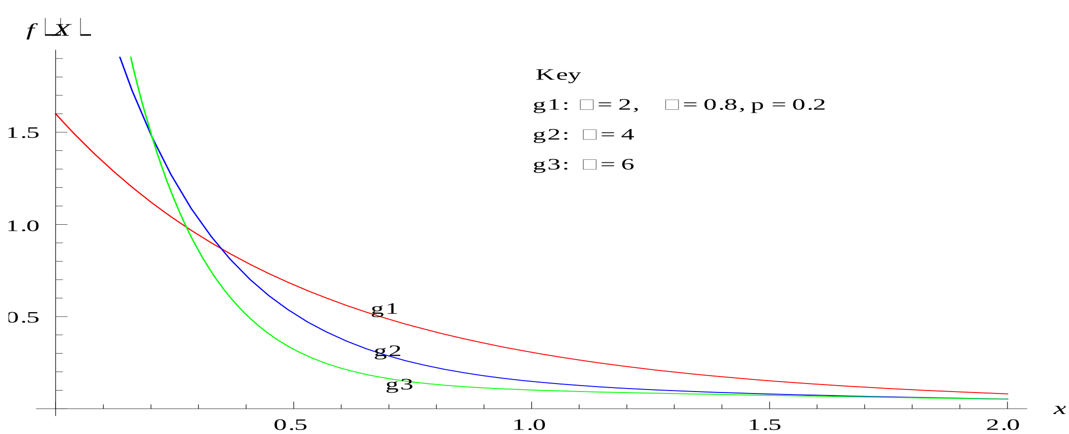

Figure 12. p = 0.2; β = 0.8; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

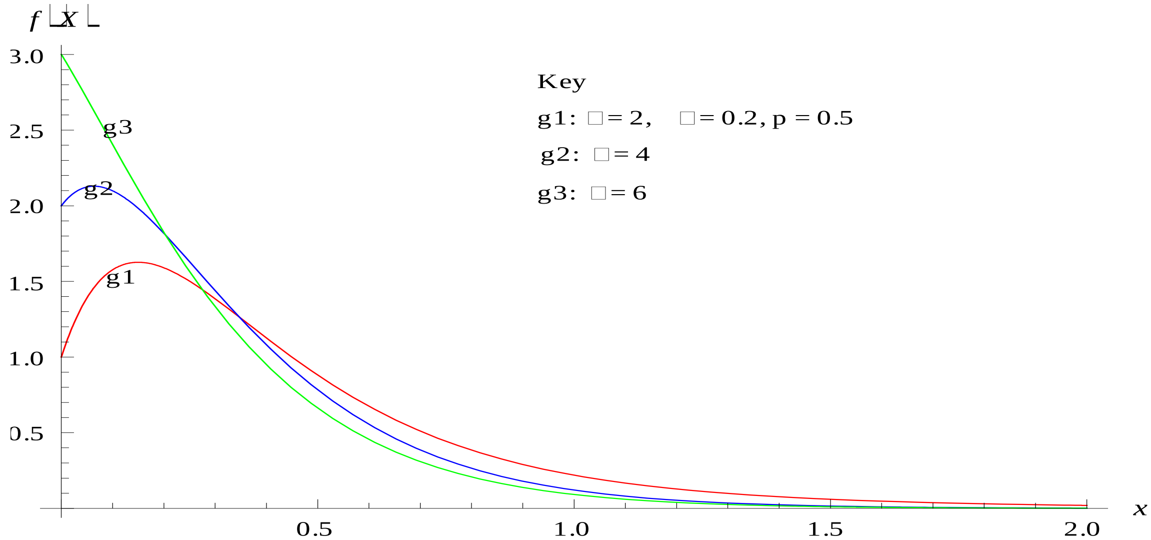

Figure 13. p = 0.5; β = 0.2; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

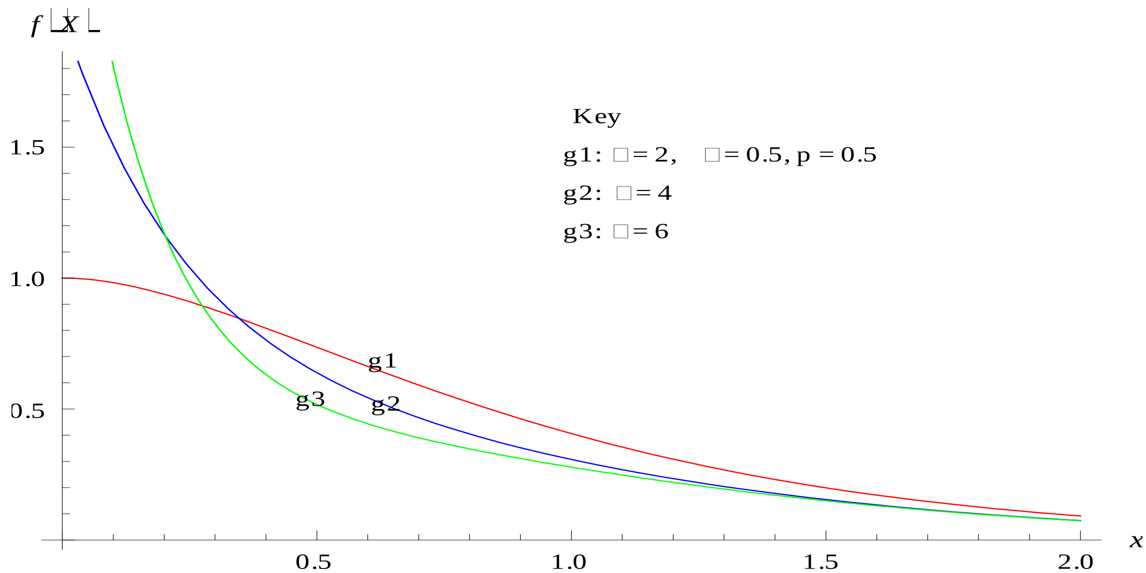

Figure 14. p = 0.5; β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

and positively skewed, whereas with $\lambda = 6$ it was nearly a decreasing function graph. Figure 18 was a decreasing function graph except the graph with $\lambda = 2$ was platykurtic but rightly skewed.

In Figures 19 to 21, the parameter λ was made as large as 5, 10 and 15, while P was made constant ($P = 0.2$) and β was varied as 0.2, 0.5 and 0.8. Figure 19 with $\beta = 0.2$ exhibited a sharp decrease to the point $x = 0.5$ and decayed to the point $x = 1.0$ and continued at zero then got terminated at $x = 1.8$; whereas Figure 20 with β = 0.5 exhibited a rapid decrease between, $0.2 \leq x \leq 6.0$ but at f(x) = 0.2 and x = 0.5 the graph began to decay slowly as x increases. Figure 21 exhibited the same characteristics with that of Figure 20 except that at x = 0.5 and f(x) = 0.2, the graphs were constant as x increases.

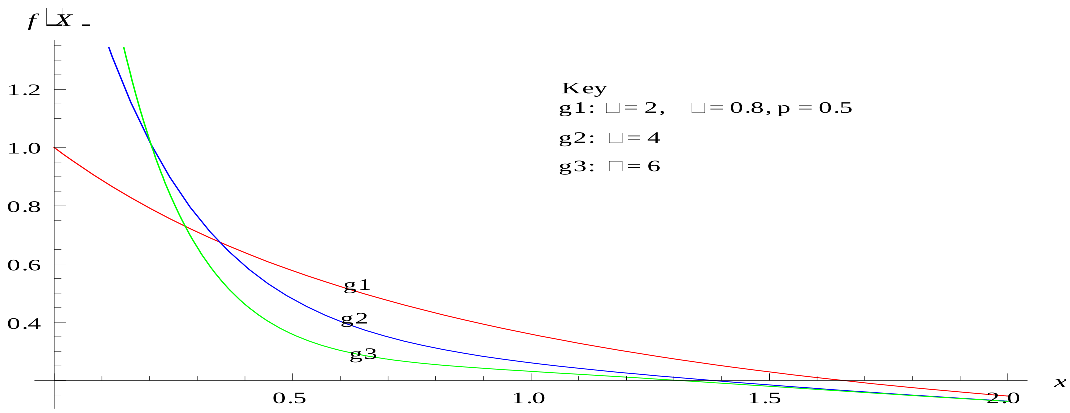

Figure 15. p = 0.5; β = 0.8; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

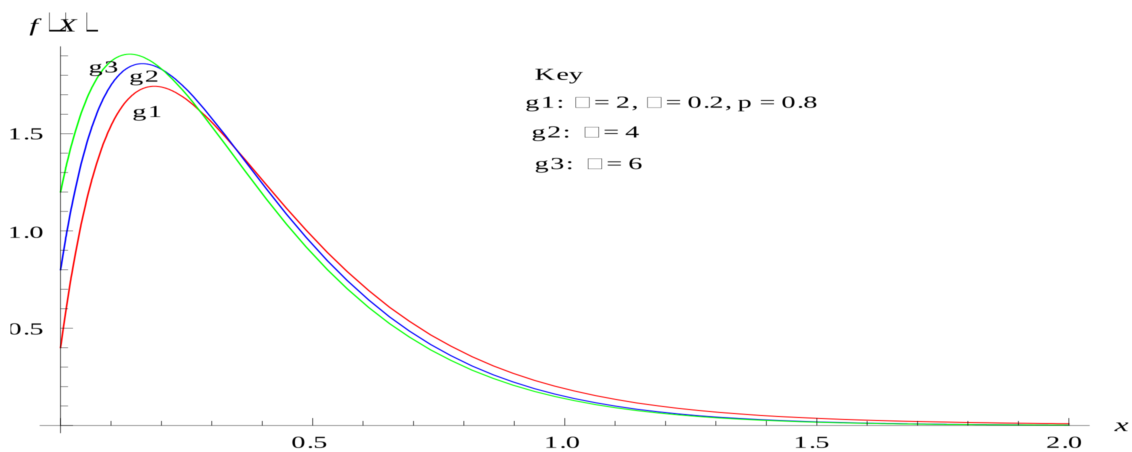

Figure 16. p = 0.8; β = 0.2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

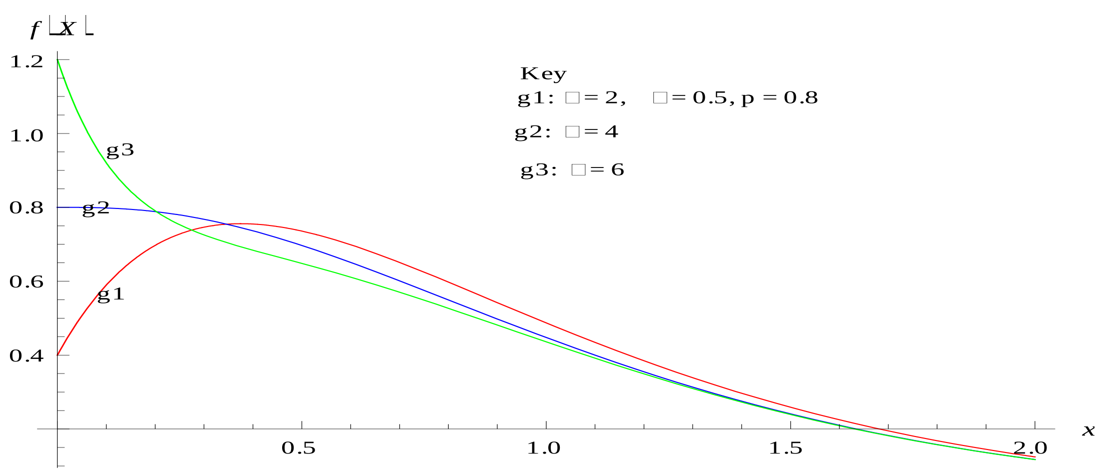

Figure 17. p = 0.8; β = 0.5; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

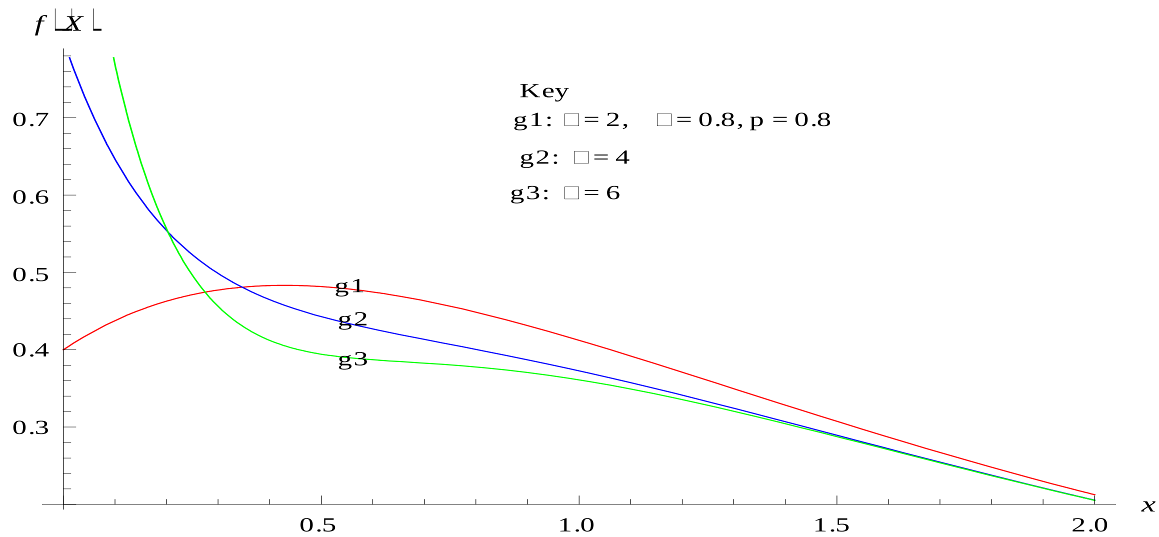

Figure 18. p = 0.8; β = 0.8; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

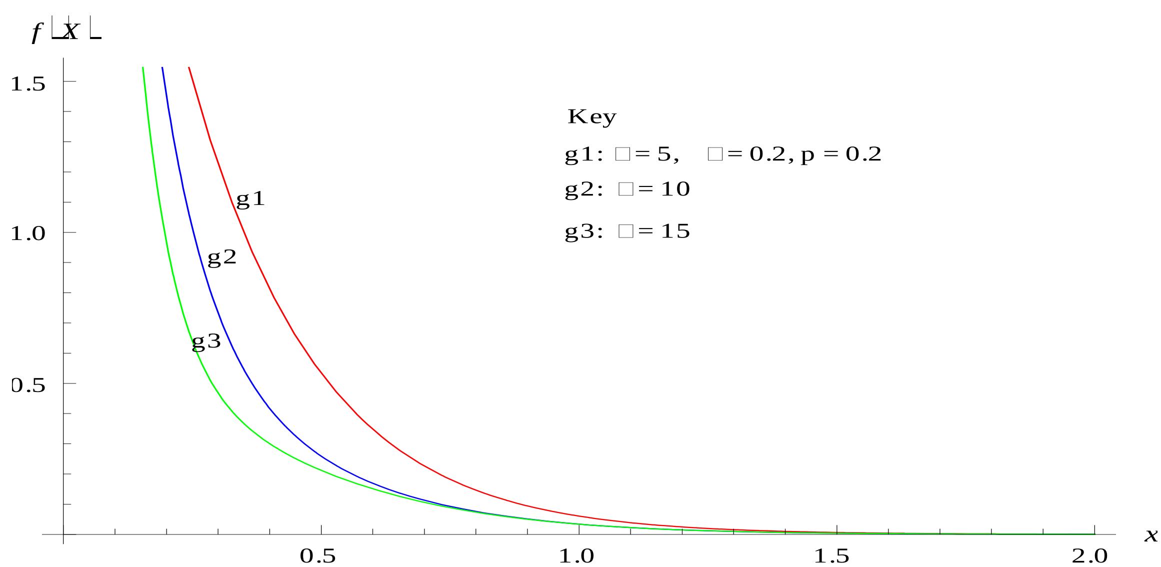

Figure 19. p = 0.2; β = 0.2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

From Figures 22 to 24, λ was still 5, 10 and 15 and $P=0.5$, whereas β was varied as 0.2, 0.5 and 0.8. We observed that in Figure 22 with β= 0.2, the graph was a decreasing function graph which decays gradually as x increases. Figure 23 decreases sharply between $0.2 \leq x \leq 0.5$ and the one with λ = 10 and 15 converges as one graph at x= 0.6 and began to decay gradually as x increases. At x = 1.0, the graph with λ = 5 then converge with the previous two as one graph and continued decreasing, whereas, Figure 24 exhibited the same characteristics with that of Figure 23 except that they converge at different point; the first one (λ=10 and 15) converge at x = 0.7, while λ= 5 converge with them at x= 1.2 but they decayed to negative at x= 1.4 and continued as x increases.

From Figures 25 to 27, λ= (5, 10 and 15), respectively and $P=0.8$ all through, while β was varied as 0.2, 0.5 and 0.8, respectively. We observed that the graph with

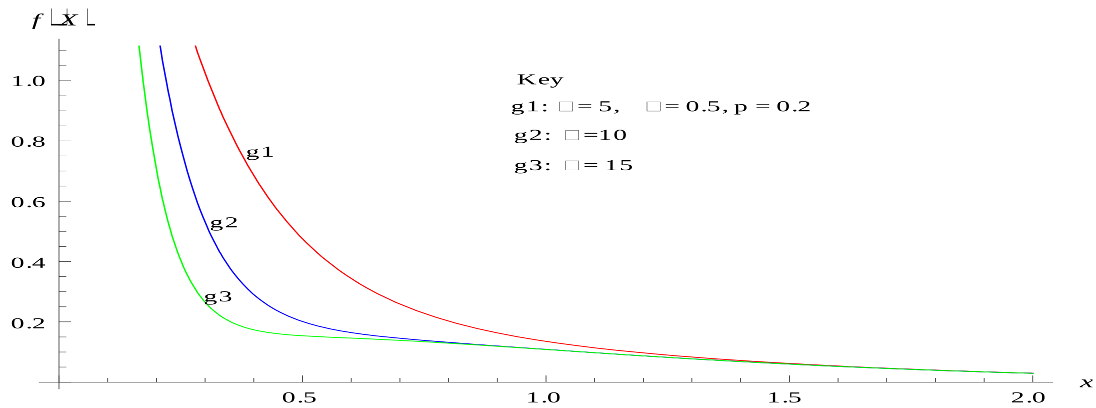

Figure 20. p = 0.2; β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

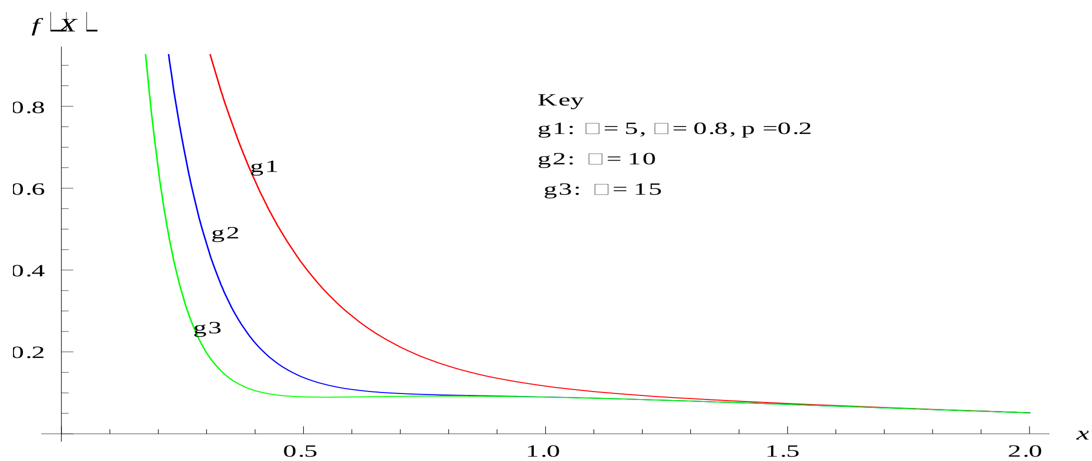

Figure 21. p = 0.2; β = 0.8; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

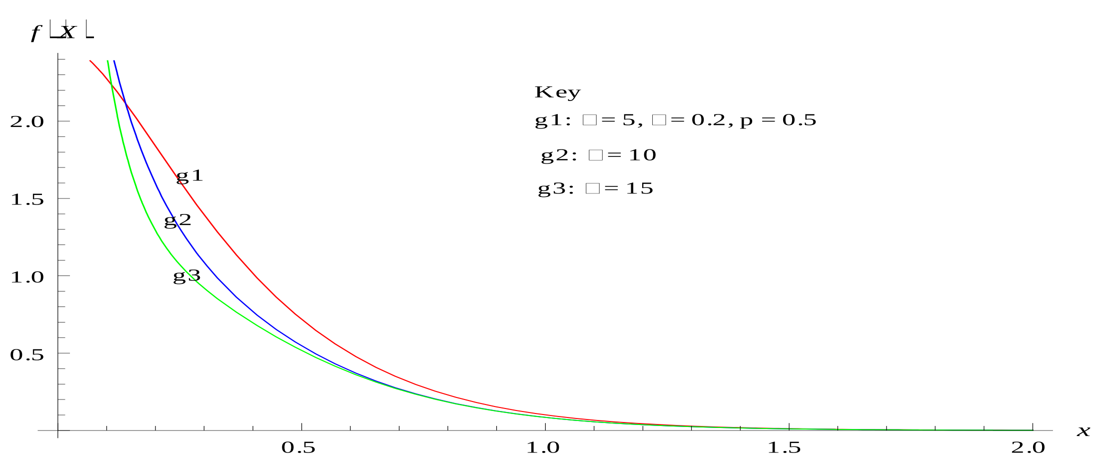

Figure 22. p = 0.5; β = 0.2; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

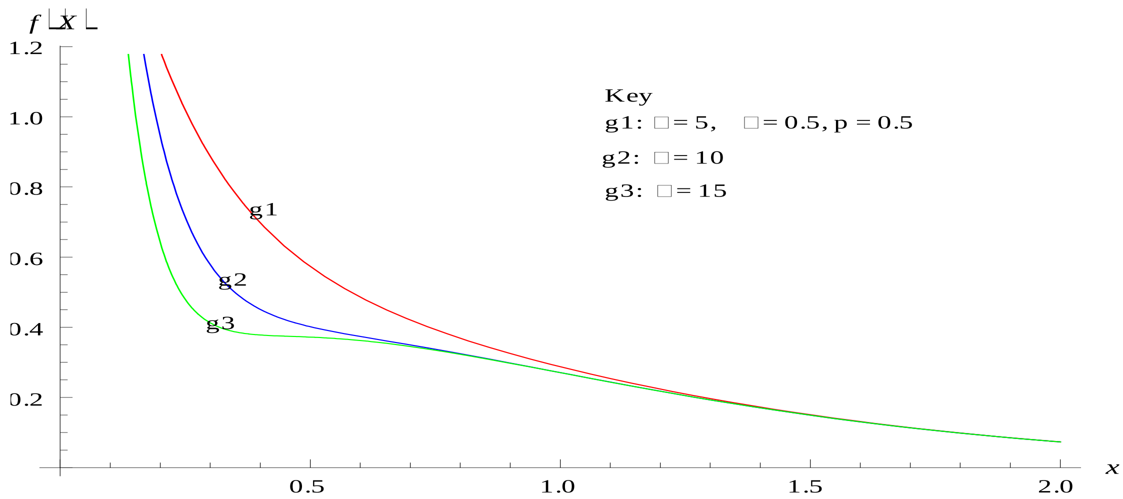

Figure 23. p = 0.5; β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

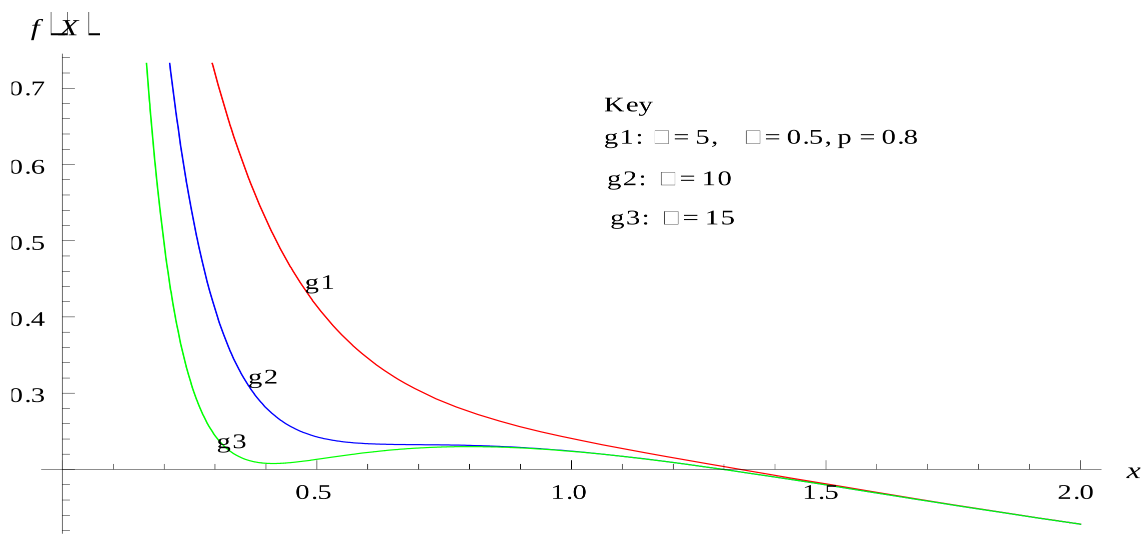

Figure 24. p = 0.5; β = 0.8; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

λ= 5 began as an increasing function to the point x = 0.3 and started to decay with the other two. Figures 26 and 27 exhibited the same characteristics by decreasing sharply and converging together as one at x= 0.6 and 0.9 and the later at 0.6 and 1.1. Then, continued as a decreasing function.

In Figure 28 to 30, λ= (2, 4 and 6) and the weighing parameter $P = 0.2$, while β was made an integer and was varied as 2, 3 and 4. It was observed that they were all decreasing function graph that decays gradually as x increases. Also, in Figures 31 to 33, λ = (2, 4 and 6) and the weighing parameter P was made constant as 0.5, while β was varied as 2, 3 and 4. It was observed that they were all decreasing function graphs that decay as x increases, except the one with the least value of β (Figure 31) which tends to negative at $0.5 \le x \le 1.0$ and continued to decay as x increases.

In Figure 34 to 36, λ was still 2, 4 and 6 and the weighing parameter P was increased to 0.8, while β was varied as 2, 3 and 4, respectively. It was observed that they were all decreasing function graphs. But in Figure 34

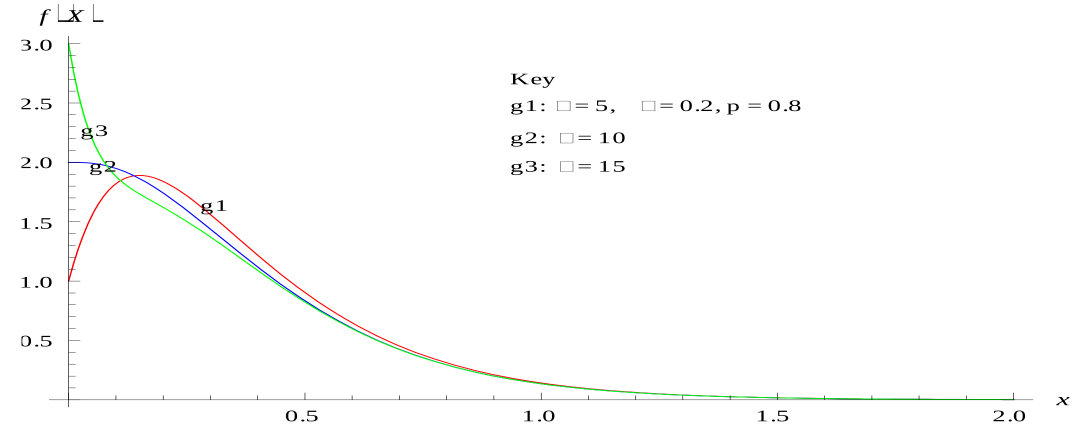

Figure 25. p = 0.8; β = 0.2; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

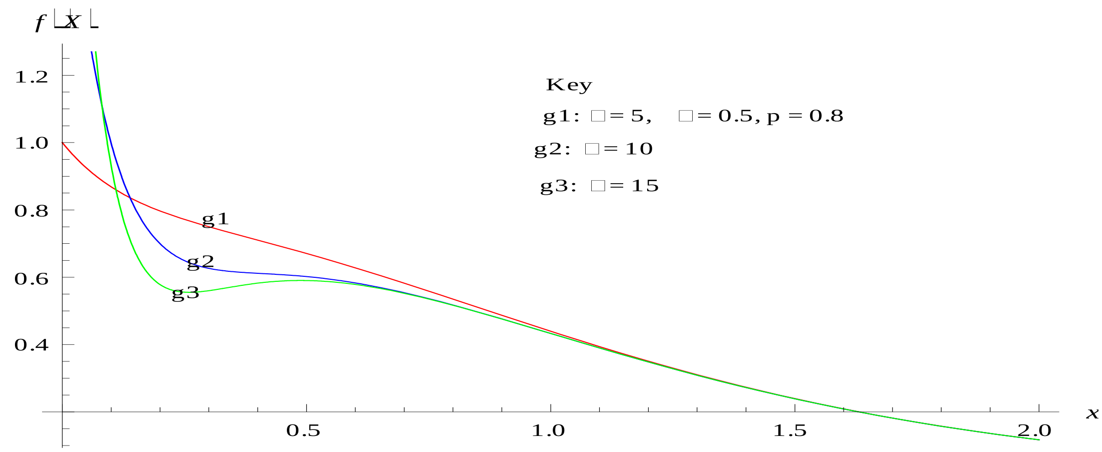

Figure 26. P = 0.8;β = 0.5; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

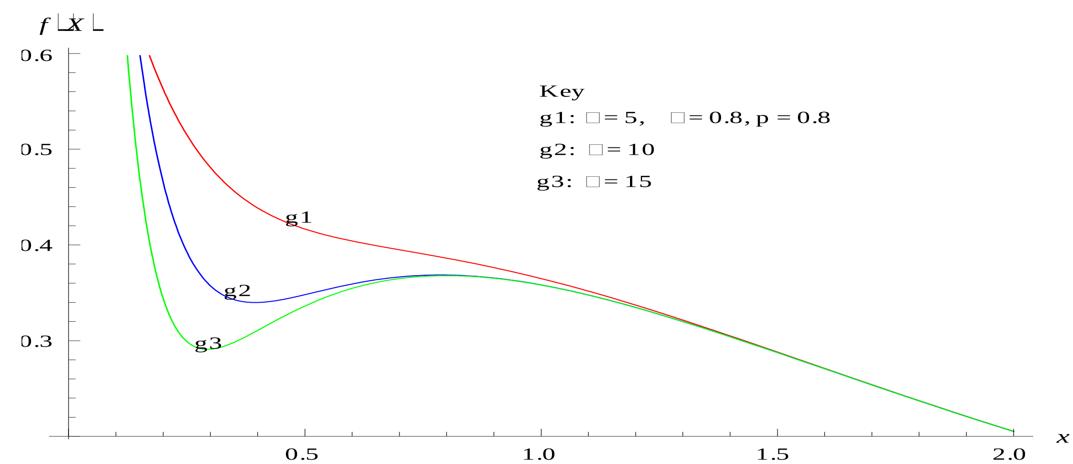

Figure 27. p = 0.8; β = 0.8; (*λ = 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);

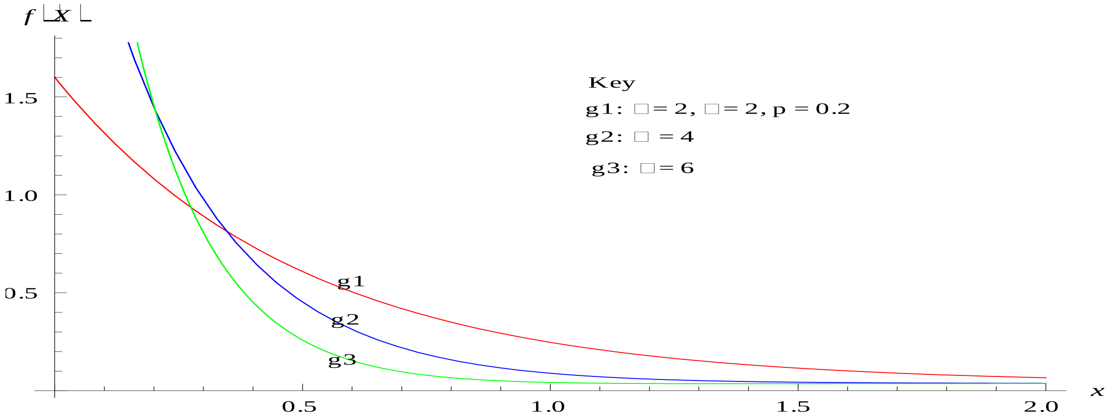

Figure 28. p = 0.2; β = 2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

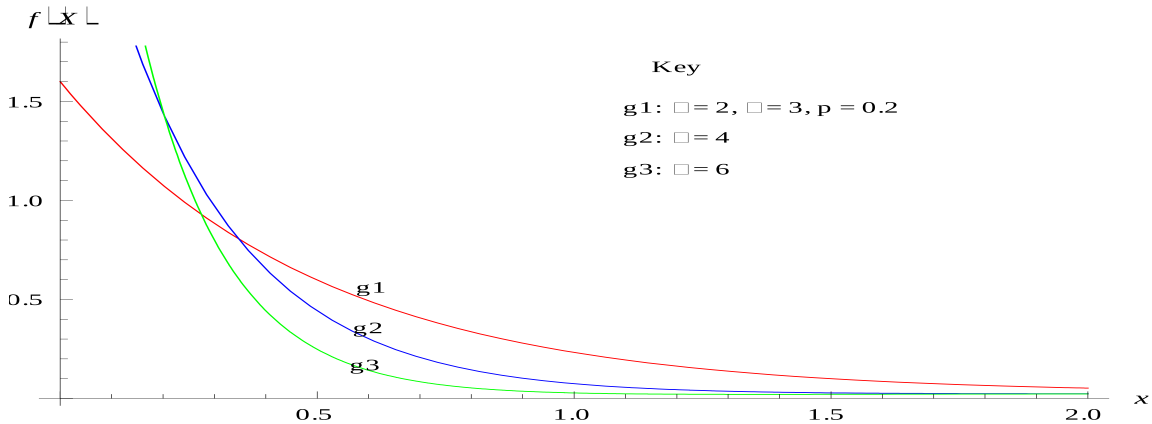

Figure 29. p = 0.2; β = 3; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

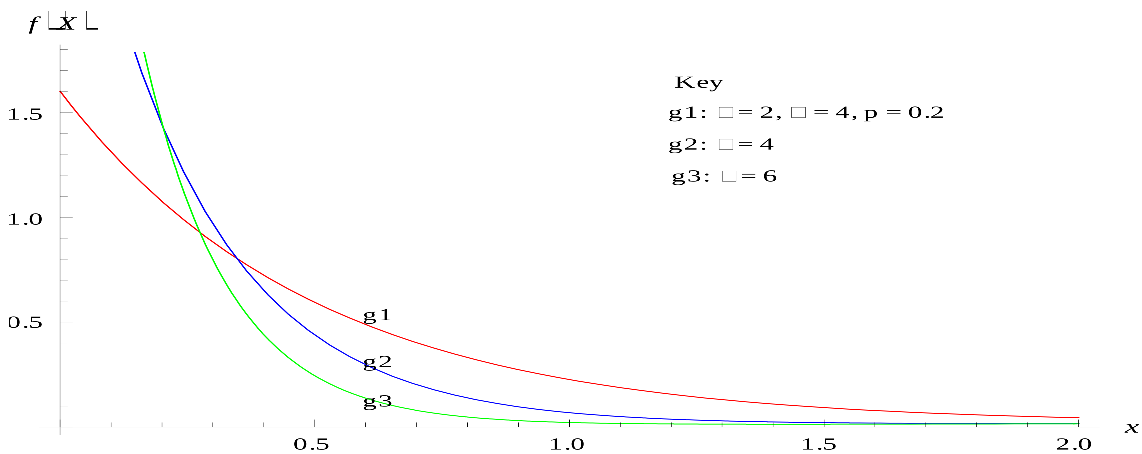

Figure 30. p = 0.2; β = 4; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

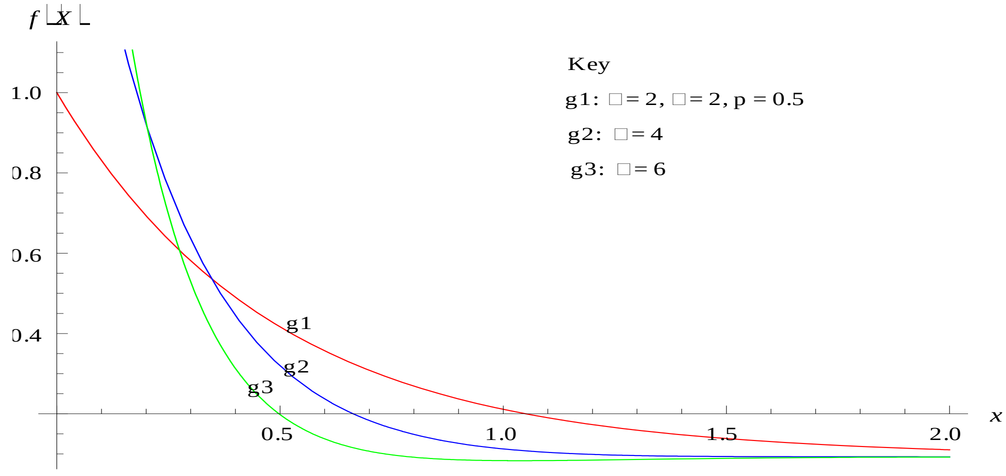

Figure 31. p = 0.5; β = 2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

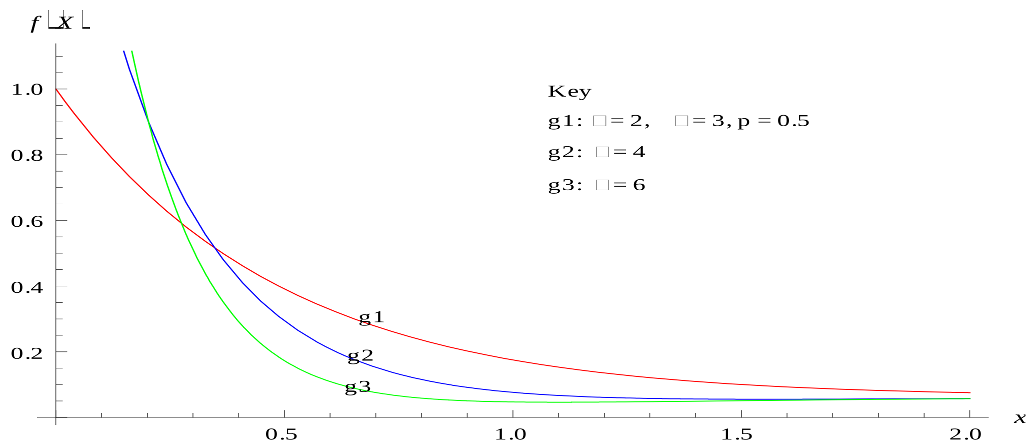

Figure 32. p = 0.5; β =3; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]);.

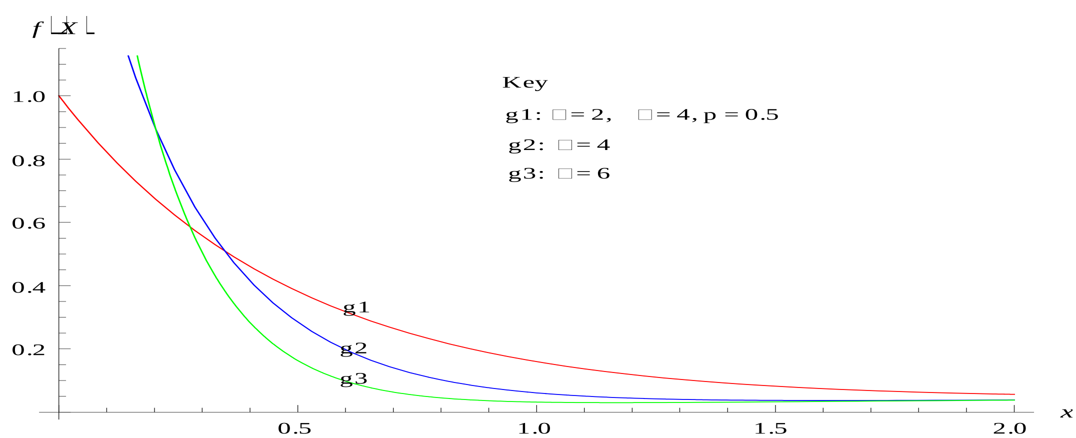

Figure 33. p = 0.5; β = 4; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!;
f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]).

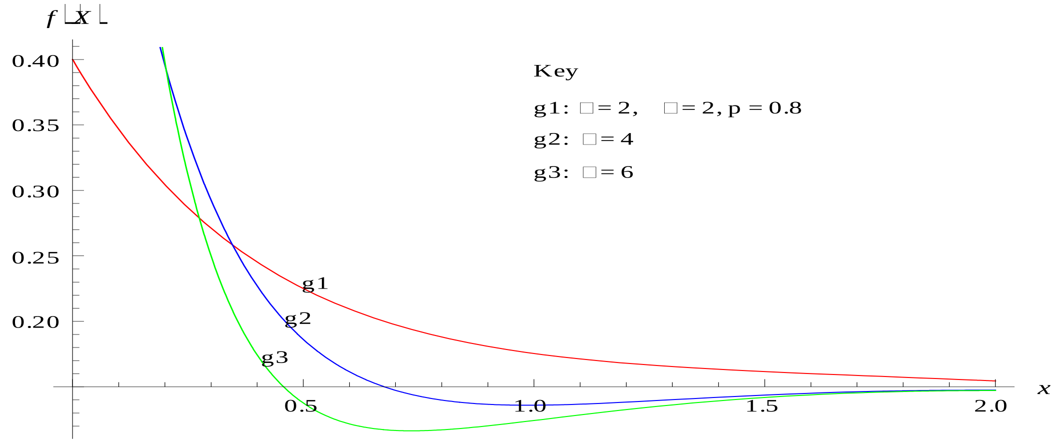

Figure 34. p = 0.8; β = 2; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]).

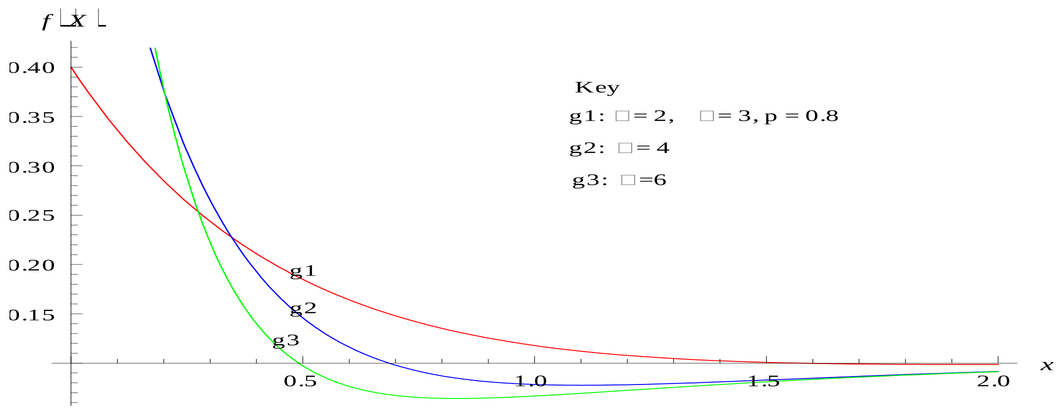

Figure 35. p = 0.8; β = 3; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]).

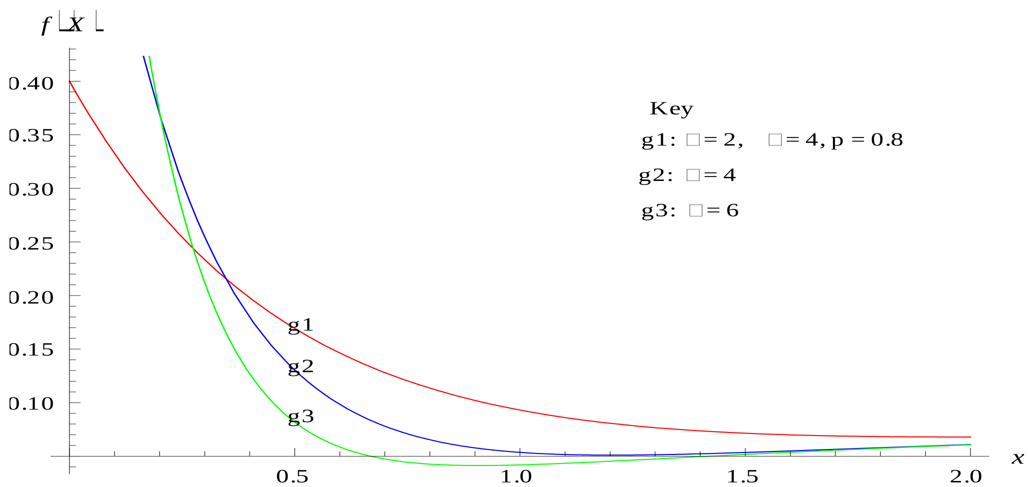

Figure 36. p = 0.8; β= 4; (*λ= 0.1;*) n = 2; Γ[n]=(n-1)!; f[λ_,β_,x]=(p*x^(n-1)*Exp[-x/β])/β^n*Γ[n]+((1-p)*λ*Exp[-λ*x]).

and 35, the graphs with λ = 4 and 6 decreased to negative at x = 0.45 and 0.7, respectively and later increased gradually towards f(x) = 0 as x increases.

PARAMETER ESTIMATION FOR THE MIXED DISTRIBUTION

$$\frac{PX^{n-1}e^{-\frac{x}{\beta}}}{\beta^n\Gamma(n)} + (1-P)\lambda e^{-\lambda x} \tag{6}$$

By reducing the sample size to two we have:

$$L(\lambda,;x_1,x_2) = \prod_{i=1}^{n} f(x_i;\lambda) = \prod_{i=1}^{2}\left[\frac{P{x_i}^{n-1}e^{-\frac{x_i}{\beta}}}{\beta^n\Gamma(n)} + (1-P)\lambda e^{-\lambda x_i}\right] \tag{7}$$

$$= \left(\frac{[P{x_1}^{n-1}e^{-\frac{x_1}{\beta}} + (1-P)\lambda e^{-\lambda x_1}]*[P{x_2}^{n-1}e^{-\frac{x_2}{\beta}} + (1-P)\lambda e^{-\lambda x_2}]}{[\beta^n\Gamma(n)]^2}\right)$$

$$= \frac{\left[P^2 x_1^{n-1}x_2^{n-1}e^{-\frac{(x_1+x_2)}{\beta}} + P(1-P)\lambda x_1^{n-1}e^{-\left(\frac{x_1}{\beta}+\lambda x_2\right)} + P(1-P)\lambda x_2^{n-1}e^{-\left(\lambda x_1+\frac{x_2}{\beta}\right)} + (1-P)^2\lambda^2 e^{-\lambda(x_1+x_2)}\right]}{[\beta^n\Gamma(n)]^2}$$

$$= \frac{P^2 x_1^{n-1}x_2^{n-1}e^{-\frac{(x_1+x_2)}{\beta}} + \lambda P(1-P)x_1^{n-1}x_2^{n-1}e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]} + (1-P)^2\lambda^2 e^{-\lambda(x_1+x_2)}}{[\beta^n\Gamma(n)]^2} \tag{8}$$

$$\frac{dL()}{d\lambda} = \frac{\left[-\lambda(x_1+x_2)e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]} + e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]}\right]P(1-P)x_1^{n-1}x_2^{n-1} - \lambda^2(x_1+x_2)e^{-\lambda(x_1+x_2)} + 2\lambda e^{-\lambda(x_1+x_2)}}{[\beta^n\Gamma(n)]^2} \tag{9}$$

$$= \frac{P(1-P)x_1^{n-1}x_2^{n-1}[1-\lambda(x_1+x_2)]e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]} + [2\lambda-\lambda^2(x_1+x_2)]e^{-\lambda(x_1+x_2)}}{[\beta^n\Gamma(n)]^2} = 0$$

$$\Rightarrow P(1-P)x_1^{n-1}x_2^{n-1}[1-\lambda(x_1+x_2)]e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]} + [2\lambda-\lambda^2(x_1+x_2)]e^{-\lambda(x_1+x_2)} = 0$$

$$\Rightarrow P(1-P)x_1^{n-1}x_2^{n-1}[1-\lambda(x_1+x_2)]e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]} = -[2\lambda-\lambda^2(x_1+x_2)]e^{-\lambda(x_1+x_2)}$$

From Equation 6 we obtain the partial derivative with respect to β

$$\frac{dL(.)}{d\beta}=\frac{\left[\beta^{n}\Gamma(n)\right]^{2}\left\{\frac{P^{2}x_{1}^{\,n-1}x_{2}^{\,n-1}(x_{1}+x_{2})e^{-\frac{(x_{1}+x_{2})}{\beta}}}{\beta^{2}}+\frac{\lambda P(1-P)x_{1}^{\,n-1}x_{2}^{\,n-1}(x_{1}+x_{2})e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}}{\beta^{2}}\right\}-}{\left[\beta^{2}\Gamma(n)\right]^{4}}$$

$$\frac{\left\{P^{2}x_{1}^{\,n-1}x_{2}^{\,n-1}e^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda P(1-P)x_{1}^{\,n-1}x_{2}^{\,n-1}e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}+(1-P)^{2}\lambda^{2}e^{-\lambda(x_{1}+x_{2})}\right\}2\left(n\beta^{n-1}\Gamma(n)\right)}{\left[\beta^{2}\Gamma(n)\right]^{4}}=0 \qquad (10)$$

$$\Rightarrow \quad \frac{\left[\beta^{n}\Gamma(n)\right]^{2}\left\{Px_{1}^{\,n-1}x_{2}^{\,n-2}(x_{1}+x_{2})\left[Pe^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda(1-P)e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}\right]\right\}}{\beta^{2}}=$$

$$\left[\beta^{n-1}\Gamma(n)\right]\left\{Px_{1}^{\,n-1}x_{2}^{\,n-1}\left[Pe^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda(1-P)e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}\right]+(1-P)^{2}\lambda^{2}e^{-\lambda(x_{1}+x_{2})}\right\}$$

$$\Rightarrow \left[\beta^{n}\Gamma(n)\right]^{2}\left\{Px_{1}^{\,n-1}x_{2}^{\,n-1}(x_{1}+x_{2})\left[Pe^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda(1-P)e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}\right]\right\}=$$

$$\left[n\beta^{n+1}\Gamma(n)\right]\left\{Px_{1}^{\,n-1}x_{2}^{\,n-1}\left[Pe^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda(1-P)e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}\right]+(1-P)^{2}\lambda^{2}e^{-\lambda(x_{1}+x_{2})}\right\} \qquad (11)$$

Again, from Equation 6, we obtain the partial derivative with respect to n. But we first of all have to factorize the function with respect to n.

$$=\frac{P^{2}x_{1}^{\,n-1}x_{2}^{\,n-1}e^{-\frac{(x_{1}+x_{2})}{\beta}}}{\left[\beta^{n}\Gamma(n)\right]^{2}}+\frac{\lambda P(1-P)x_{1}^{\,n-1}x_{2}^{\,n-1}e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}}{\left[\beta^{n}\Gamma(n)\right]^{2}}+\frac{(1-P)^{2}\lambda^{2}e^{-\lambda(x_{1}+x_{2})}}{\left[\beta^{n}\Gamma(n)\right]^{2}} \qquad (12)$$

$$=\frac{x_{1}^{\,n-1}x_{2}^{\,n-1}}{\left[\beta^{n}\Gamma(n)\right]^{2}}\left[P^{2}e^{-\frac{(x_{1}+x_{2})}{\beta}}+\lambda P(1-P)e^{-(x_{1}+x_{2})\left[\frac{1}{\beta}+\lambda\right]}\right]+\frac{(1-P)^{2}\lambda^{2}e^{-\lambda(x_{1}+x_{2})}}{\left[\beta^{n}\Gamma(n)\right]^{2}}$$

$$= \frac{x_1{}^n x_2{}^n}{x_1 x_2} * \frac{1}{\left[\beta^n \Gamma(n)\right]^2}\left[P^2 e^{-\frac{(x_1+x_2)}{\beta}} + \lambda P(1-P) e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]}\right] + \frac{(1-P)^2 \lambda^2 e^{-\lambda(x_1+x_2)}}{\left[\beta^n \Gamma(n)\right]^2}$$

$$= \frac{(x_1 x_2)^n}{x_1 x_2 \left[\beta^n \Gamma(n)\right]^2}\left[P^2 e^{-\frac{(x_1+x_2)}{\beta}} + \lambda P(1-P) e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]}\right] + \frac{(1-P)^2 \lambda^2 e^{-\lambda(x_1+x_2)}}{\left[\beta^n \Gamma(n)\right]^2}$$

$$= \left(\frac{x_1 x_2}{\beta^2}\right)^n \left(\frac{1}{x_1 x_2 \Gamma(n)^2}\right)\left[P^2 e^{-\frac{(x_1+x_2)}{\beta}} + \lambda P(1-P) e^{-(x_1+x_2)\left[\frac{1}{\beta}+\lambda\right]}\right] + \left(\frac{1}{\beta^{2n}\Gamma(n)^2}\right)\left[(1-P)^2 \lambda^2 e^{-\lambda(x_1+x_2)}\right] \tag{13}$$

At this point, we observe that differentiating with respect to n is very difficult if at all possible.

Proof of the mixture of gamma and exponential distribution as a proper probability density function (PDF)

$$\int_0^\infty \left[\frac{P X^{n-1} e^{-\frac{x}{\beta}}}{\beta^n \Gamma(n)} + (1-P)\lambda e^{-\lambda x}\right] dx \tag{14}$$

$$= \int_0^\infty \frac{P X^{n-1} e^{-\frac{x}{\beta}}}{\beta^n \Gamma(n)} dx + \int_0^\infty (1-P)\lambda e^{-\lambda x} dx \tag{15}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty X^{n-1} e^{-\frac{x}{\beta}} dx + (1-P)\lambda \int_0^\infty e^{-\lambda x} dx \tag{16}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty (\beta u)^{n-1} e^{-u} \beta du + (1-P)\lambda \int_0^\infty e^{-\lambda x} dx \tag{17}$$

$$= \frac{P \beta^{n-1} \beta}{\beta^n \Gamma(n)} \int_0^\infty u^{n-1} e^{-u} du + (1-P)\lambda \int_0^\infty e^{-\lambda x} dx \tag{18}$$

$$= \frac{P}{\Gamma(n)} \int_0^\infty u^{n-1} e^{-u} du + (1-P)\lambda \int_0^\infty e^{-\lambda x} dx \tag{19}$$

But recall that $\Gamma(\alpha) = \int_0^\infty X^{\alpha-1} e^{-x} dx$

$$\therefore \frac{P\Gamma(n)}{\Gamma(n)} + (1-P)\lambda \int_0^\infty e^{-u}\, du/\lambda \tag{20}$$

$$= \frac{P\Gamma(n)}{\Gamma(n)} + \frac{(1-P)\lambda}{\lambda} \int_0^\infty e^{-u} du$$

$$= P + (1-P)$$

$$= 1 \ (proper\ \ pdf)$$

To obtain the mean

$$E(X) = \int_0^\infty x\, f(x) dx \tag{21}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty x x^{n-1} e^{-\frac{x}{\beta}} dx + \int_0^\infty (1-P)\, x \lambda e^{-\lambda x} dx \tag{22}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty x^n e^{-\frac{x}{\beta}} dx + (1-P) \int_0^\infty x \lambda e^{-\lambda x} dx \tag{23}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty (\beta u)^n e^{-u} \beta du + (1-P)\lambda \int_0^\infty x e^{-\lambda x} dx \tag{24}$$

$$= \frac{P \beta^n \beta}{\beta^n \Gamma(n)} \int_0^\infty u^n e^{-u} du + (1-P)\lambda \int_0^\infty x e^{-\lambda x} dx \tag{25}$$

$$= \frac{P\beta\Gamma(n+1)}{\Gamma(n)} + (1-P)\lambda \int_0^\infty \left(\frac{u}{\lambda}\right) e^{-u}\, du/\lambda \tag{26}$$

$$= \frac{P\beta\Gamma(n+1)}{\Gamma(n)} + \frac{(1-P)\lambda}{\lambda^2} \int_0^\infty u e^{-u} du \tag{27}$$

$$= \frac{P\beta n\Gamma(n)}{\Gamma(n)} + \frac{(1-P)}{\lambda} \tag{28}$$

$$E(X) = \beta nP + \frac{(1-P)}{\lambda} \tag{29}$$

To obtain the variance

$$Var(x) = E[x^2] - [E(x)]^2 \tag{30}$$

$$but \quad E[x^2] = \int_0^\infty x^2 f(x)dx$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty x^2 X^{n-1} e^{-\frac{x}{\beta}} dx + (1-P)\lambda \int_0^\infty x^2 e^{-\lambda x} dx \tag{31}$$

$$= \frac{P}{\beta^n \Gamma(n)} \int_0^\infty (\beta u)^{n+1} e^{-u} \beta du + (1-P)\lambda \int_0^\infty x^2 e^{-\lambda x} dx \tag{32}$$

$$= \frac{P\beta^{n+1}\beta}{\beta^n \Gamma(n)} \int_0^\infty (u)^{n+1} e^{-u} du + (1-P)\lambda \int_0^\infty x^2 e^{-\lambda x} dx \tag{33}$$

$$= \frac{P\beta^2 \Gamma(n+2)}{\Gamma(n)} + (1-P)\lambda \int_0^\infty \left(\frac{u}{\lambda}\right)^2 e^{-u}\, du/\lambda \tag{34}$$

$$= \frac{P\beta^2 \Gamma(n+2)}{\Gamma(n)} + \frac{(1-P)\lambda}{\lambda^3} \int_0^\infty u^2 e^{-u} du \tag{35}$$

$$= \frac{P\beta^2 \Gamma(n+2)}{\Gamma(n)} + \frac{2(1-P)\lambda}{\lambda^3} \tag{36}$$

$$= \frac{P\beta^2 (n+1)n\Gamma(n)}{\Gamma(n)} + \frac{2(1-P)}{\lambda^2} \tag{37}$$

$$\therefore \; E[x^2] = np\beta^2(n+1) + \frac{2(1-P)}{\lambda^2} \tag{38}$$

and

$$Var(x) = np\beta^2(n+1) + \frac{2(1-P)}{\lambda^2} - \left[\beta nP + \frac{(1-P)}{\lambda}\right]^2 \tag{39}$$

$$= nP\beta^2(n+1) + \frac{2(1-P)}{\lambda^2} - (\beta nP)^2 + 2\beta nP\frac{(1-P)}{\lambda} + \left(\frac{1-P}{\lambda}\right)^2 \tag{40}$$

$$= nP\beta^2(n+1) - (\beta nP)^2 + \frac{2\beta nP(1-P)}{\lambda} + \frac{2(1-P)}{\lambda^2} - \left(\frac{1-P}{\lambda}\right)^2 \tag{41}$$

$$= nP\beta^2(n+1) - (\beta nP)^2 + \frac{(1-P)}{\lambda}\left[\frac{2}{\lambda} + \frac{(1-P)}{\lambda} + 2\beta nP\right] \tag{42}$$

THE MODEL

To formalize our model, let x_i equal to 1 if item i is selected and zero otherwise. Also, for convenience, let $R(x)$ denote our contagious distribution; that is,

$$R(x) = P\frac{X^{n-1}e^{-\frac{x}{\beta}}}{\beta^n \Gamma(n)} + (1-P)\lambda e^{-\lambda x}$$

We are interested in maximizing the value of the accepted (selected) items given that they do not exceed the capacity of the firm. In an effort to maximize this value, we are at the same time considering the lost of goodwill by those costumers whose items constitute an overflow (exceeded capacity). If k is the penalty cost per unit of the overflow item, then we have

$$Maximize \sum_{i=1}^{n} V_i x_i - k\sum_{i=c+1}^{n}(i-C)R(x)$$

$$Subject\ to \sum R(x) \le C$$

This can as well be written as:

$$Maximize \sum_{i=1}^{n} V_i x_i - k\sum_{i=c+1}^{n}(i-C)R(x)$$

$$Subject\ to \sum P\frac{(X^{n-1}e^{-x})}{\Gamma(n)} + (1-P)\lambda e^{-\lambda x} \le C$$

Where V_i is the profit for the ith selected item. C is the capacity of the mill (firm).

Conclusion

An algebraic stochastic knapsack model was proposed for the item weight following a contagious distribution of a standard Gamma and Exponential distribution using an additive model.

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