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Multi parameter fuzzy soft set approach to decision making problem

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In this paper, a new approach for decision making problem is introduced by extending the definition of fuzzy soft set for multiple parameter sets and is called extended fuzzy soft set. Also, few operations such as "AND" and "$MaxMin$" are defined on extended fuzzy soft sets and illustrated with examples. Further an algorithm for decision making using the concept of extended fuzzy soft set is presented. The decision making process includes construction of comparison matrix and ranking strategy is based on the row sum of comparison matrix. Finally an application of proposed algorithm for decision making is presented.

Key words: Fuzzy soft set, decision making, comparison matrix, ranking.

INTRODUCTION

To handle the uncertainties in the imprecise data arising in most of real life problems in engineering, social and medical science, economics, environment, etc., Zadeh (1965) introduced the concept of fuzzy set and fuzzy set operations and further various researchers have extended it to an intuitionistic fuzzy set (Atanassov and Gargov, 1989), interval intuitionistic fuzzy set (Atanassov, 1986) in which information to handle the uncertain information is more precise. Under these environments, different types of approaches are discussed by the researchers to solve the decision-making problems (Xu, 2007; Garg, 2018a, b; Xu and Yager, 2006; Garg, 2017; Garg and Kumar, 2018). As there is an inadequacy of the parameterizations tool associated with these approaches, Molodtsov (1999) introduced soft sets as general mathematical tool for dealing with objects which have been defined using a very general set of characteristics and applied the soft theory into several directions, such as, game theory, operations research, Riemann integration, theory of probability, theory of measurement, and so on. Maji et al. (2002) presented application of soft sets in decision making problems. Maji et al. (2001) introduced the concept of the fuzzy soft sets by using the ideas of fuzzy sets (Zadeh, 1965) and then many interesting applications of fuzzy soft set theory have been proposed by various researchers. Roy and Maji (2007) presented applications of fuzzy soft sets for decision making problem. Som (2006) defined soft relation and fuzzy soft relation on the theory of soft sets. Mukherjee and Chakraborty (2008) worked on intuitionistic fuzzy soft

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relations. Aktas and Cagman ((2007) compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. (2007) worked on operations on fuzzy soft sets. Zou and Xiao (2008) introduced the soft set and fuzzy soft set into the incomplete environment. Yang et al. (2009) presented the combination of interval-valued fuzzy set and soft set. Kong et al. (2008) introduced the normal parameter reduction in the fuzzy soft sets. Majumdar and Samanta (2010) presented generalized fuzzy soft sets for decision making problems. Zhao and Jia (2015) presented decision making method based on Cartesian products of fuzzy soft sets. Garg et al. (2016) defined the notion of the fuzzy number intuitionistic fuzzy soft sets. However, for solving the decision making problems, various researchers have utilized different aggregation operators (Garg and Arora, 2018a, b, c; Arora and Garg, 2018a, b) and information measures (Garg and Arora, 2017a, b; Mukherjee and Sarkar, 2014; Rajarajeswari and Dhanalakshmi, 2014; Arora and Garg, 2018c) under the soft set environment. In this paper, a new approach for decision making problem is introduced by extending the definition of fuzzy soft set for multiple parameter sets and is called as extended fuzzy soft set. Also, few operations such as "AND" and "$MaxMin$" are defined on extended fuzzy soft sets and illustrated with examples. Finally an algorithm for decision making was presented and the decision making process includes construction of comparison matrix and ranking strategy based on the row sum of comparison matrix (Roy and Maji, 2007; Kong et al., 2009).

BASIC DEFINITIONS

Definition 1

A fuzzy set A of a non empty set X is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ represents "degree of membership" of x in A, for $x \in X$ and I^X represents family of all fuzzy sets on X (Zadeh, 1965).

Definition 2

Let X be an initial universe and E be a set of parameters, a pair (F, A) denoted by F_A for $A \subseteq E$ is called fuzzy soft set, where F is mapping given by $F : A \to I^X$ (Maji et al., 2001).

Definition 3

The Cartesian "AND" product of two fuzzy soft sets F_A and F_B over a common universe X denoted by $H_C = F_A \wedge F_B$, is defined as $H_C : A \times B \to I^X$ and $H_C(a,b) = F_A(a) \wedge F_B(b)$, where $(a,b) \in A \times B$ (Zhao and Jia, 2015).

Definition 4

Comparison matrix is a square matrix (c_{ij}) in which rows and columns are labeled by the object names of the universe and the entries $c_{ij} = \sum_{k=1}^{m} (\alpha_{ik} - \alpha_{jk})$ where α_{ik} is the membership value of i^{th} object for k^{th} parameter (Roy and Maji, 2007; Kong et al., 2009).

EXTENDED FUZZY SOFT SETS

Here, fuzzy soft set definition is extended for two parameter sets and named it as extended fuzzy soft set. Suppose X is an initial universe and E and K are primary and secondary set of parameters. Let I^X denote family of all fuzzy sets over X and E^X denote family of all fuzzy soft sets over X with respect to the parameter set E. For any $A \subseteq K$, a pair (F^*, A) denoted by F_A^* is called extended fuzzy soft set over X, where F^* is a mapping given by $F^* : A \to E^X$ defined by $F_A^*(k) = F_{E_A}(k)$ defined by $F_{E_A}(k) = \phi$ if $k \notin A$ and $F_{E_A}(k) \neq \phi$ if $k \in A$.

Example 1

Consider a universal set $X = \{x_1, x_2, x_3, x_4\}$, primary parameter set $E = \{e_1, e_2, e_3\}$ and secondary parameter set $K = \{k_1, k_2, k_3, k_4\}$ and let $A = \{k_1, k_3\}$ and $B = \{k_2, k_3\}$. Define extended fuzzy soft sets as

$F_A^* = \left\{(k_1, F_{E_A}(k_1)), (k_3, F_{E_A}(k_3))\right\}$ and

$F_B^* = \left\{(k_2, F_{E_B}(k_2)), (k_3, F_{E_B}(k_3))\right\}$, where

$$F_{E_A}(k_1) = \left\{ e_1 = \left\{ \frac{0.1}{x_1}, \frac{0.3}{x_2}, \frac{0.4}{x_3}, \frac{0.7}{x_4} \right\}, e_2 = \left\{ \frac{0.2}{x_1}, \frac{0.4}{x_2}, \frac{0.6}{x_3}, \frac{0.4}{x_4} \right\}, e_3 = \left\{ \frac{0.8}{x_1}, \frac{0.7}{x_2}, \frac{0.9}{x_3}, \frac{0.1}{x_4} \right\} \right\}$$

$$F_{E_A}(k_3) = \left\{ e_1 = \left\{ \frac{0.2}{x_1}, \frac{0.6}{x_2}, \frac{0.5}{x_3}, \frac{0.8}{x_4} \right\}, e_2 = \left\{ \frac{0.1}{x_1}, \frac{0.7}{x_2}, \frac{0.3}{x_3}, \frac{0.6}{x_4} \right\}, e_3 = \left\{ \frac{0.8}{x_1}, \frac{0.6}{x_2}, \frac{0.4}{x_3}, \frac{0.5}{x_4} \right\} \right\}$$

Table 1. Tabular representation of extended fuzzy soft sets.

$F_{E_A}(k_1)$	x_1	x_2	x_3	x_4
e_1	0.1	0.3	0.4	0.7
e_2	0.2	0.4	0.6	0.4
e_3	0.8	0.7	0.9	0.1
$F_{E_A}(k_3)$	x_1	x_2	x_3	x_4
e_1	0.2	0.6	0.5	0.8
e_2	0.1	0.7	0.3	0.6
e_3	0.8	0.6	0.4	0.5
$F_{E_B}(k_2)$	x_1	x_2	x_3	x_4
e_1	0.3	0.5	0.6	0.8
e_2	0.1	0.9	0.4	0.7
e_3	0.1	0.6	0.4	0.6
$F_{E_B}(k_3)$	x_1	x_2	x_3	x_4
e_1	0.2	0.4	0.6	0.7
e_2	0.2	0.5	0.6	0.8
e_3	0.9	0.8	0.4	0.2

$$F_{E_B}(k_2)=\left\{e_1=\left\{\frac{0.3}{x_1},\frac{0.5}{x_2},\frac{0.6}{x_3},\frac{0.8}{x_4}\right\},e_2=\left\{\frac{0.1}{x_1},\frac{0.9}{x_2},\frac{0.4}{x_3},\frac{0.7}{x_4}\right\},e_3=\left\{\frac{0.1}{x_1},\frac{0.6}{x_2},\frac{0.4}{x_3},\frac{0.6}{x_4}\right\}\right\}$$
$$F_{E_B}(k_3)=\left\{e_1=\left\{\frac{0.2}{x_1},\frac{0.4}{x_2},\frac{0.6}{x_3},\frac{0.7}{x_4}\right\},e_2=\left\{\frac{0.2}{x_1},\frac{0.5}{x_2},\frac{0.6}{x_3},\frac{0.8}{x_4}\right\},e_3=\left\{\frac{0.9}{x_1},\frac{0.8}{x_2},\frac{0.4}{x_3},\frac{0.2}{x_4}\right\}\right\}$$

The above extended fuzzy soft sets can also be represented in a tabular form as shown below and throughout the paper following representation has been used for representing extended fuzzy soft sets and also similar representation for fuzzy soft as well as fuzzy sets shown in Table 1.

Definition 5

The Cartesian "AND" product of two extended fuzzy soft sets F_A^* and F_B^* over a common universe X denoted by $H_C^*=F_A^*\wedge F_B^*$, is defined as $H_C^*:A\times B\to E^X$ and $H_C^*(a,b)=F_{E_A}(a)\wedge F_{E_B}(b)$, where $(a,b)\in A\times B$.

Example 2

The Cartesian "AND" product of F_A^* and F_B^* (as defined in Example 1) is given by

$$H_C^*=F_A^*\wedge F_B^*=\left\{\begin{matrix}\{(k_1\,k_2)(F_{E_A}(k_1)\wedge F_{E_B}(k_2))\},\{(k_1\,k_3)(F_{E_A}(k_1)\wedge F_{E_B}(k_3))\},\\ \{(k_3\,k_2)(F_{E_A}(k_3)\wedge F_{E_B}(k_2))\},\{(k_3\,k_3)(F_{E_A}(k_3)\wedge F_{E_B}(k_3))\}\end{matrix}\right\} \quad (1)$$

where Table 2 shows extended fuzzy soft set H_C^*.

Definition 6

The $MaxMin$ operators on "AND" products of two extended fuzzy soft sets F_A^* and F_B^* are defined as:

$$Max_a Min_b[H_C^*(a,b)]=\vee_{a\in A}\{\wedge_{b\in B}(F_A^*(a)\wedge F_B^*(b))\} \quad (2)$$

$$Max_b Min_a[H_C^*(a,b)]=\vee_{b\in B}\{\wedge_{a\in A}(F_A^*(a)\wedge F_B^*(b))\} \quad (3)$$

Example 3

The $MaxMin$ operations on "AND" product

Table 2. Extended fuzzy soft set H_C^*.

$(k_1\ k_2)$	x_1	x_2	x_3	x_4	$(k_1\ k_3)$	x_1	x_2	x_3	x_4
$(e_1\ e_1)$	0.1	0.3	0.4	0.7	$(e_1\ e_1)$	0.1	0.3	0.4	0.7
$(e_1\ e_2)$	0.1	0.3	0.4	0.7	$(e_1\ e_2)$	0.1	0.3	0.4	0.7
$(e_1\ e_3)$	0.1	0.3	0.4	0.6	$(e_1\ e_3)$	0.1	0.3	0.4	0.2
$(e_2\ e_1)$	0.2	0.4	0.6	0.4	$(e_2\ e_1)$	0.2	0.4	0.6	0.4
$(e_2\ e_2)$	0.1	0.4	0.4	0.4	$(e_2\ e_2)$	0.2	0.4	0.6	0.4
$(e_2\ e_3)$	0.1	0.4	0.4	0.4	$(e_2\ e_3)$	0.2	0.4	0.4	0.2
$(e_3\ e_1)$	0.3	0.5	0.6	0.1	$(e_3\ e_1)$	0.2	0.4	0.6	0.1
$(e_3\ e_2)$	0.1	0.7	0.4	0.1	$(e_3\ e_2)$	0.2	0.5	0.6	0.1
$(e_3\ e_3)$	0.1	0.6	0.4	0.1	$(e_3\ e_3)$	0.8	0.7	0.4	0.1
$(k_3\ k_2)$	x_1	x_2	x_3	x_4	$(k_3\ k_3)$	x_1	x_2	x_3	x_4
$(e_1\ e_1)$	0.2	0.5	0.5	0.8	$(e_1\ e_1)$	0.2	0.4	0.5	0.7
$(e_1\ e_2)$	0.1	0.6	0.4	0.7	$(e_1\ e_2)$	0.5	0.5	0.5	0.8
$(e_1\ e_3)$	0.1	0.6	0.4	0.6	$(e_1\ e_3)$	0.2	0.6	0.4	0.2
$(e_2\ e_1)$	0.1	0.5	0.3	0.6	$(e_2\ e_1)$	0.1	0.4	0.3	0.6
$(e_2\ e_2)$	0.1	0.7	0.3	0.6	$(e_2\ e_2)$	0.1	0.5	0.3	0.6
$(e_2\ e_3)$	0.1	0.6	0.3	0.6	$(e_2\ e_3)$	0.1	0.7	0.3	0.2
$(e_3\ e_1)$	0.3	0.5	0.4	0.5	$(e_3\ e_1)$	0.2	0.4	0.4	0.5
$(e_3\ e_2)$	0.1	0.6	0.4	0.5	$(e_3\ e_2)$	0.2	0.5	0.4	0.5
$(e_3\ e_3)$	0.1	0.6	0.4	0.5	$(e_3\ e_3)$	0.8	0.6	0.4	0.2

$H_C^* = F_A^* \wedge F_B^*$ obtained in Example 2 are shown below:

$$Max_a Min_b[H_C^*(a,b)] = \underset{a\in A}{\vee}\{\underset{b\in B}{\wedge}(F_A^*(a)\wedge F_B^*(b))\}$$

$$= \vee \begin{Bmatrix} \wedge\left\{F_{E_A}(k_1)\wedge F_{E_B}(k_2), F_{E_A}(k_1)\wedge F_{E_B}(k_3)\right\} \\ \wedge\left\{F_{E_A}(k_3)\wedge F_{E_B}(k_2), F_{E_A}(k_3)\wedge F_{E_B}(k_3)\right\} \end{Bmatrix}$$

$$= \vee \begin{Bmatrix} \wedge\left\{(k_1\ k_2),(k_1\ k_3)\right\} \\ \wedge\left\{(k_3\ k_2),(k_3\ k_3)\right\} \end{Bmatrix} = F_C(say) \tag{4}$$

where Table 3 show fuzzy soft set F_C.

$$Max_b Min_a[H_C^*(a,b)] = \underset{b\in B}{\vee}\{\underset{a\in A}{\wedge}(F_A^*(a)\wedge F_B^*(b))\}$$

$$= \vee \begin{Bmatrix} \wedge\left\{F_{E_A}(k_1)\wedge F_{E_B}(k_2), F_{E_A}(k_3)\wedge F_{E_B}(k_2)\right\} \\ \wedge\left\{F_{E_A}(k_1)\wedge F_{E_B}(k_3), F_{E_A}(k_3)\wedge F_{E_B}(k_3)\right\} \end{Bmatrix}$$

$$= \vee \begin{Bmatrix} \wedge\left\{(k_1\ k_2),(k_3\ k_2)\right\} \\ \wedge\left\{(k_1\ k_3),(k_3\ k_3)\right\} \end{Bmatrix} = F_D(say) \tag{5}$$

where Table 4 shows fuzzy soft set F_D.

APPLICATION IN DECISION MAKING PROBLEM

Suppose a set of projects are to be evaluated in two stages based on a certain set of parameters. In each stage two evaluators evaluate the projects and assign the marks between 0 and 100 and

Table 3. Fuzzy soft set F_C.

F_C	x_1	x_2	x_3	x_4
$(e_1\, e_1)$	0.2	0.4	0.5	0.7
$(e_1\, e_2)$	0.1	0.5	0.4	0.7
$(e_1\, e_3)$	0.1	0.6	0.4	0.2
$(e_2\, e_1)$	0.2	0.4	0.6	0.6
$(e_2\, e_2)$	0.1	0.5	0.4	0.6
$(e_2\, e_3)$	0.1	0.6	0.4	0.2
$(e_3\, e_1)$	0.2	0.4	0.6	0.5
$(e_3\, e_2)$	0.1	0.5	0.4	0.5
$(e_3\, e_3)$	0.1	0.6	0.4	0.2

Table 4. Fuzzy soft set F_D.

F_D	x_1	x_2	x_3	x_4
$(e_1\, e_1)$	0.1	0.3	0.4	0.7
$(e_1\, e_2)$	0.1	0.3	0.4	0.7
$(e_1\, e_3)$	0.1	0.3	0.4	0.6
$(e_2\, e_1)$	0.1	0.4	0.3	0.4
$(e_2\, e_2)$	0.4	0.4	0.3	0.4
$(e_2\, e_3)$	0.1	0.4	0.3	0.4
$(e_3\, e_1)$	0.3	0.5	0.4	0.1
$(e_3\, e_2)$	0.2	0.6	0.4	0.1
$(e_3\, e_3)$	0.8	0.6	0.4	0.1

problem here is to rank the projects based on the evaluation. Here, we present an algorithm to solve this decision making problem for which marks allotted are converted on the scale of 0 to 1 to get extended fuzzy soft sets and these sets will be the input for the proposed algorithm.

Algorithm:

Step 1: *Input extended fuzzy soft sets* F_A^* *and* F_B^*

Step 2: *Perform Cartesian AND product of* F_A^* *and* F_B^* *to obtain* H_C^*

Step 3: *Apply* $MaxMin$ *operators on* H_C^* *to obtain fuzzy soft sets* F_C *and* F_D

Step 4: *Apply* $MaxMin$ *operators on* F_C *and* F_D *to obtain four fuzzy sets*

Step 5: *Construct comparison matrix (Roy and Maji, 2007) of fuzzy sets obtained in step 4, in which both rows and columns are labeled by project names and the entries are (Kong et al., 2009)* $c_{ij} = \sum_{k=1}^{4}(\alpha_{ik} - \alpha_{jk})$ *where* α_{ik} *is the membership value of* i^{th}

Table 5. Marks allotted by evaluators k_1, k_2, k_3 and k_4.

k₁	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	80	60	40	60	80	70	60	20	50	80
e_2	70	50	60	70	70	70	70	20	50	70
e_3	60	90	50	80	40	40	40	30	40	70
k_2	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	60	80	70	70	70	40	60	20	10	90
e_2	10	50	70	60	50	30	80	40	10	70
e_3	80	60	60	50	80	70	10	30	10	80
k_3	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	80	60	60	20	90	70	80	40	20	70
e_2	60	30	80	80	70	60	10	40	70	60
e_3	80	40	50	50	60	50	30	80	30	40
k_4	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	50	40	70	50	80	80	80	60	70	20
e_2	60	70	50	30	80	70	80	40	90	60
e_3	70	60	50	90	60	50	80	30	50	70

project for k^{th} *fuzzy set and then compute row sum* $r_i = \sum_{j=1}^{4} c_{ij}$

Step 6: *The decision is* $rank(x_i) > rank(x_j)$ *if* $r_i > r_j$ *and* $rank(x_i) = rank(x_j)$ *if* $r_i = r_j$

Example 4

Suppose ten projects need to be evaluated based on a certain set of parameters by four evaluators in a pair in two different stages and projects to be ranked. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ be a list of projects and parameter (primary) set $E = \{e_1, e_2, e_3\}$. Let secondary parameter (evaluators) set $K = \{k_1, k_2, k_3, k_4\}$ and $A = \{k_1, k_2\}$ & $B = \{k_3, k_4\}$ be two pairs of evaluators. The marks allotted by the four evaluators are presented in Table 5.

Implementation of Algorithm

Step 1: Based on the evaluation of projects with respect to set parameters by the four evaluators, let the corresponding extended fuzzy soft sets be $F_A^* = \{(k_1, F_{E_A}(k_1)), (k_2, F_{E_A}(k_2))\}$ and $F_B^* = \{(k_3, F_{E_B}(k_3)), (k_4, F_{E_B}(k_4))\}$, where Table 6 shows the extended fuzzy soft sets F_A^* and F_B^*.

Step 2: Perform Cartesian *AND* Product of F_A^* and F_B^*:

$$H_C^* = F_A^* \wedge F_B^* = \left\{ \begin{array}{l} \{(k_1 k_3)(F_{E_A}(k_1) \wedge F_{E_B}(k_3))\}, \{(k_1 k_4)(F_{E_A}(k_1) \wedge F_{E_B}(k_4))\}, \\ \{(k_2 k_3)(F_{E_A}(k_2) \wedge F_{E_B}(k_3))\}, \{(k_2 k_4)(F_{E_A}(k_2) \wedge F_{E_B}(k_4))\} \end{array} \right\} \quad (6)$$

Step 3: $MaxMin$ *operators on "AND" products that is,* F_C = $Max_a Min_b[H_C^*(a,b)]$ *and* F_D = $Max_b Min_a[H_C^*(a,b)]$ *(Tables 7 and 8).*

Step 4: *Apply* $MaxMin$ *operators on* F_C *and* F_D *to get various fuzzy sets (Table 9).*

Step 5 and 6: *The Comparison table of the above fuzzy sets, row sum and ranking (Table 10).*

Table 6. Extended fuzzy soft sets F_A^* and F_B^*.

$F_{E_A}(k_1)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	0.8	0.6	0.4	0.6	0.8	0.7	0.6	0.2	0.5	0.8
e_2	0.7	0.5	0.6	0.7	0.7	0.7	0.7	0.2	0.5	0.7
e_3	0.6	0.9	0.5	0.8	0.4	0.4	0.4	0.3	0.4	0.7
$F_{E_A}(k_2)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	0.6	0.8	0.7	0.7	0.7	0.4	0.6	0.2	0.1	0.9
e_2	0.1	0.5	0.7	0.6	0.5	0.3	0.8	0.4	0.1	0.7
e_3	0.8	0.6	0.6	0.5	0.8	0.7	0.1	0.3	0.1	0.8
$F_{E_B}(k_3)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	0.8	0.6	0.6	0.2	0.9	0.7	0.8	0.4	0.2	0.7
e_2	0.6	0.3	0.8	0.8	0.7	0.6	0.1	0.4	0.7	0.6
e_3	0.8	0.4	0.5	0.5	0.6	0.5	0.3	0.8	0.3	0.4
$F_{E_B}(k_4)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
e_1	0.5	0.4	0.7	0.5	0.8	0.8	0.8	0.6	0.7	0.2
e_2	0.6	0.7	0.5	0.3	0.8	0.7	0.8	0.4	0.9	0.6
e_3	0.7	0.6	0.5	0.9	0.6	0.5	0.8	0.3	0.5	0.7
$(k_1\ k_3)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.8	0.6	0.4	0.2	0.8	0.7	0.6	0.2	0.2	0.7
$(e_1\ e_2)$	0.6	0.3	0.4	0.6	0.7	0.6	0.1	0.2	0.5	0.6
$(e_1\ e_3)$	0.8	0.4	0.4	0.5	0.6	0.5	0.3	0.2	0.3	0.4
$(e_2\ e_1)$	0.7	0.5	0.6	0.2	0.7	0.7	0.7	0.2	0.2	0.7
$(e_2\ e_2)$	0.6	0.3	0.6	0.7	0.7	0.6	0.1	0.2	0.5	0.6
$(e_2\ e_3)$	0.7	0.4	0.5	0.5	0.6	0.5	0.3	0.2	0.3	0.4
$(e_3\ e_1)$	0.6	0.6	0.5	0.2	0.4	0.4	0.4	0.3	0.2	0.7
$(e_3\ e_2)$	0.6	0.3	0.5	0.8	0.4	0.4	0.1	0.3	0.4	0.6
$(e_3\ e_3)$	0.6	0.4	0.5	0.5	0.4	0.4	0.3	0.3	0.3	0.4
$(k_1\ k_4)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.5	0.4	0.4	0.5	0.8	0.7	0.6	0.2	0.5	0.2
$(e_1\ e_2)$	0.6	0.6	0.4	0.3	0.8	0.7	0.6	0.2	0.5	0.6
$(e_1\ e_3)$	0.7	0.6	0.4	0.6	0.6	0.5	0.6	0.2	0.5	0.7
$(e_2\ e_1)$	0.5	0.4	0.6	0.5	0.7	0.7	0.7	0.2	0.5	0.2

Table 6. Contd.

$(e_2\ e_2)$	0.6	0.5	0.5	0.3	0.7	0.7	0.7	0.2	0.5	0.6
$(e_2\ e_3)$	0.7	0.5	0.5	0.7	0.6	0.5	0.7	0.2	0.5	0.7
$(e_3\ e_1)$	0.5	0.4	0.5	0.5	0.4	0.4	0.4	0.3	0.4	0.2
$(e_3\ e_2)$	0.6	0.7	0.5	0.3	0.4	0.4	0.4	0.3	0.4	0.6
$(e_3\ e_3)$	0.6	0.6	0.5	0.8	0.4	0.4	0.4	0.3	0.4	0.7
$(k_2\ k_3)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.6	0.6	0.6	0.2	0.7	0.4	0.6	0.2	0.1	0.7
$(e_1\ e_2)$	0.6	0.3	0.7	0.7	0.7	0.4	0.1	0.2	0.1	0.6
$(e_1\ e_3)$	0.6	0.4	0.5	0.5	0.6	0.4	0.3	0.2	0.1	0.4
$(e_2\ e_1)$	0.1	0.5	0.6	0.2	0.5	0.3	0.8	0.4	0.1	0.7
$(e_2\ e_2)$	0.1	0.3	0.7	0.6	0.5	0.3	0.1	0.4	0.1	0.6
$(e_2\ e_3)$	0.1	0.4	0.5	0.5	0.5	0.3	0.3	0.4	0.1	0.4
$(e_3\ e_1)$	0.8	0.6	0.6	0.2	0.8	0.7	0.1	0.3	0.1	0.7
$(e_3\ e_2)$	0.6	0.3	0.6	0.5	0.7	0.6	0.1	0.3	0.1	0.6
$(e_3\ e_3)$	0.8	0.4	0.5	0.5	0.6	0.5	0.1	0.3	0.1	0.4
$(k_2\ k_4)$	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.5	0.4	0.7	0.5	0.7	0.4	0.6	0.2	0.1	0.2
$(e_1\ e_2)$	0.6	0.7	0.5	0.3	0.7	0.4	0.6	0.2	0.1	0.6
$(e_1\ e_3)$	0.6	0.6	0.5	0.7	0.6	0.4	0.6	0.2	0.1	0.7
$(e_2\ e_1)$	0.1	0.4	0.7	0.5	0.5	0.3	0.8	0.4	0.1	0.2
$(e_2\ e_2)$	0.1	0.5	0.5	0.3	0.5	0.3	0.8	0.4	0.1	0.6
$(e_2\ e_3)$	0.1	0.5	0.5	0.6	0.5	0.3	0.8	0.3	0.1	0.7
$(e_3\ e_1)$	0.5	0.4	0.6	0.5	0.8	0.7	0.1	0.3	0.1	0.2
$(e_3\ e_2)$	0.6	0.6	0.5	0.3	0.8	0.7	0.1	0.3	0.1	0.6
$(e_3\ e_3)$	0.7	0.6	0.5	0.5	0.6	0.5	0.1	0.3	0.1	0.7

RESULTS AND DISCUSSION

The performance of the algorithm is illustrated with an example of ranking ten different projects which are evaluated by four evaluators based on three different parameters. The ranking strategy is based on value of row sum (r_i) as depicted in Table 10. The project with highest row sum (r_i) is ranked number 1 and the project with lowest row sum (r_i) is given last rank. For the example under discussion, project x_5 has the highest row sum that is, 6.2 and is given rank 1 and the project x_9 has the least row sum and hence assigned last rank. If two or more values in the row sum are same then the corresponding projects will be assigned the same ranks

Table 7. Fuzzy soft set $F_C = Max_a Min_b[H_C^*(a,b)]$.

F_C	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.5	0.4	0.6	0.2	0.8	0.7	0.6	0.2	0.2	0.2
$(e_1\ e_2)$	0.6	0.3	0.5	0.3	0.7	0.6	0.1	0.2	0.5	0.6
$(e_1\ e_3)$	0.7	0.4	0.5	0.5	0.6	0.5	0.3	0.2	0.3	0.4
$(e_2\ e_1)$	0.5	0.4	0.6	0.2	0.7	0.7	0.8	0.4	0.2	0.2
$(e_2\ e_2)$	0.6	0.3	0.5	0.3	0.7	0.6	0.1	0.4	0.5	0.6
$(e_2\ e_3)$	0.7	0.4	0.5	0.5	0.6	0.5	0.3	0.3	0.3	0.4
$(e_3\ e_1)$	0.5	0.4	0.6	0.2	0.8	0.7	0.4	0.3	0.2	0.2
$(e_3\ e_2)$	0.6	0.3	0.5	0.3	0.7	0.6	0.1	0.3	0.4	0.6
$(e_3\ e_3)$	0.7	0.4	0.5	0.5	0.6	0.5	0.3	0.3	0.3	0.4

Table 8. Fuzzy soft set $F_D = Max_b Min_a[H_C^*(a,b)]$.

F_D	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$(e_1\ e_1)$	0.6	0.6	0.4	0.5	0.7	0.4	0.6	0.2	0.1	0.7
$(e_1\ e_2)$	0.6	0.6	0.4	0.6	0.7	0.4	0.6	0.2	0.1	0.6
$(e_1\ e_3)$	0.6	0.6	0.4	0.6	0.6	0.4	0.6	0.2	0.1	0.7
$(e_2\ e_1)$	0.1	0.5	0.6	0.5	0.5	0.3	0.7	0.2	0.1	0.7
$(e_2\ e_2)$	0.1	0.5	0.6	0.6	0.5	0.3	0.7	0.2	0.1	0.6
$(e_2\ e_3)$	0.1	0.5	0.5	0.6	0.5	0.3	0.7	0.2	0.1	0.7
$(e_3\ e_1)$	0.6	0.6	0.5	0.5	0.4	0.4	0.1	0.3	0.1	0.7
$(e_3\ e_2)$	0.6	0.6	0.5	0.5	0.4	0.4	0.1	0.3	0.1	0.6
$(e_3\ e_3)$	0.6	0.6	0.5	0.5	0.4	0.4	0.1	0.3	0.1	0.7

Table 9. Various fuzzy sets after applying $MaxMin$ operators on F_C and F_D.

Fuzzy sets	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
$Max_a Min_b(F_C)$	0.5	0.3	0.5	0.2	0.6	0.5	0.1	0.3	0.2	0.2
$Max_b Min_a(F_C)$	0.7	0.4	0.6	0.5	0.7	0.7	0.4	0.2	0.4	0.6
$Max_a Min_b(F_D)$	0.6	0.6	0.5	0.5	0.6	0.4	0.7	0.3	0.1	0.6
$Max_b Min_a(F_D)$	0.1	0.5	0.4	0.5	0.4	0.3	0.1	0.2	0.1	0.7

Table 10. Comparison table, row sum and ranking.

Row sum	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	Row sum (r_i)	Rank
x_1	0.0	0.1	-0.1	0.2	-0.4	0.0	0.6	0.9	1.1	-0.2	**2.2**	4
x_2	-0.1	0.0	-0.2	0.1	-0.5	-0.1	0.5	0.8	1.0	-0.3	1.2	5
x_3	0.1	0.2	0.0	0.3	-0.3	0.1	0.7	1.0	1.2	-0.1	**3.2**	3
x_4	-0.2	-0.1	-0.3	0.0	-0.6	-0.2	0.4	0.7	0.9	-0.4	0.2	6
x_5	0.4	0.5	0.3	0.6	0.0	0.4	1.0	1.3	1.5	0.2	**6.2**	1
x_6	0.0	0.1	-0.1	0.2	-0.4	0.0	0.6	0.9	1.1	-0.2	**2.2**	4
x_7	-0.6	-0.5	-0.7	-0.4	-1.0	-0.6	0.0	0.3	0.5	-0.8	-3.8	7
x_8	-0.9	-0.8	-1.0	-0.7	-1.3	-0.9	-0.3	0.0	0.2	-1.1	-6.8	8
x_9	-1.1	-0.6	-1.2	-0.9	-1.5	-1.1	-0.5	-0.2	0.0	-1.3	-8.4	9
x_{10}	0.2	0.3	0.1	0.4	-0.2	0.2	0.8	1.1	1.3	0.0	**4.2**	2

which can be seen for projects x_1 and x_6.

CONCLUSION

Here, a new approach for decision making problem is introduced by extending the definition of fuzzy soft set for multiple parameter sets called extended fuzzy soft set. Some operations such as "AND" and "$MaxMin$" are defined. Finally an algorithm for decision making was presented and the decision making process includes construction of comparison matrix and ranking strategy based on the row sum of comparison matrix. The proposed algorithm is illustrated with an example where ten different projects are evaluated and ranked based on the marks allotted by four different evaluators.

Data availability

The data used in this study is a randomly generated data to validate the algorithm presented in the paper and is not real time data.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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