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Full Length Research Paper

A novel analytical solution of steady flow over a rotating disk in porous medium with heat transfer by DTM-PADÉ

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In this study, the differential transform method (DTM) was applied to steady flow over a rotating disk in porous medium with heat transfer. The governing equations can be written as a system of nonlinear ordinary differential equations. The approximate solutions of these equations were obtained in the form of series with easily computable terms. Then, Padé approximant was applied to increase the convergence radius of the series. The results obtained in this study were compared with the numerical results (fourth-order Runge-Kutta method).

Key words: Padé approximant, differential transform method, rotating disk, porous medium.

INTRODUCTION

Nonlinear differential equations are usually arising from mathematical modeling of many physical systems. Some of them are solved using numerical methods and some are solved using the analytic methods such as perturbation (Nayfeh, 1979; Rand et al., 1987). The numerical methods such as Runge - Kutta method are based on discretization techniques, and they only permit us to calculate the numerical solutions for some values of time and space variables, which cause us to overlook some important phenomena, in addition to the intensive computer time required to solve the problem. Thus it is often costly and time-consuming to get a complete curve of results and so in these methods, stability and convergence should be considered so as to avoid divergence or inappropriate results. Numerical difficulties additionally appear if a nonlinear problem contains singularities or has multiple solutions. Perturbation techniques are based on the existence of small/large parameters, the so-called perturbation quantity. Unfortunately, many nonlinear problems in science and engineering do not contain such kind of perturbation quantities at all. Some non-perturbative techniques, such as the artificial small parameter method (Lyapunov, 1892), the $\delta-$expansion method (Karmishin, 1990) and the Adomian's (1994) decomposition method have been developed. Different from perturbation techniques, these non-perturbative methods are independent upon small parameters. However, both of the perturbation techniques and the non-perturbative methods themselves cannot provide us with a simple way to adjust and control the convergence region and rate of given approximate series.

One of the semi-exact methods which do not need small parameters is the DTM. This method first proposed by Zhou (1986), who solved linear and nonlinear problems in electrical circuit problems. Chen et al. (1999) developed this method for partial differential equations and Ayaz (2004) applied it to the system of differential equations, this method is very powerful, Abdel-Halim Hassan (2008). This method constructs an analytical solution in the form of a polynomial. It is different from the traditional higher order Taylor series method. The Taylor series method is computationally expensive for large orders. The differential transform method is an alternative procedure for obtaining analytic Taylor series solution of the differential equations. In recent years, the differential transform method has been successfully employed to solve many types of nonlinear problems (Ravi Kanth et al., 2008; Arikoglu et al., 2006; Arikoglu et al., 2005; Bildik et al., 2006; Ayaz, 2004; Rashidi, 2009 a,b,c; Rashidi et al., 2009 a,b). The fluid flow due to an infinite

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rotating disk was first considered by von Karman (1921). He introduced the similarity transformations which reduced the governing partial differential equations to ordinary differential equations. Cochran (1934) obtained asymptotic solutions for the steady problem formulated by von Karman and Benton (1966) solved the unsteady state of this problem. Millsaps et al. (1952) considered heat transfer from a rotating disk maintained at a constant temperature for different values of Prandtl numbers in the steady state. Attia (1998; 2002) and Attia et al. (2001) studied the influence of an external uniform magnetic field on the flow due to a rotating disk. The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was investigated (Stuart, 1954; Kuiken, 1971; Ockendon, 1972). The main goal of the present study is to find the totally analytic solution for steady flow over a rotating disk in porous medium with heat transfer by differential transform method. This problem studied first by Attia (2009) and exerted the similarity solution. We will extend the DTM-Padé for it. In this way, the Letter has been organized as follows. In Section 2, the flow analysis and mathematical formulation are presented. In Section 3, we extend the application of the DTM to construct the approximate solutions for the governing equations. The Padé approximant is analyzed in Section 4. Section 5 contains the results and discussion. The conclusions are summarized in Section 6.

MATHEMATICAL FORMULATION

Set the disk in the plane $z=0$ and the space $z>0$ is equipped by a viscous incompressible fluid. An insulated infinite disk rotates about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model is assumed (Khaled et al., 2003). Otherwise the rest fluid is under pressure P_∞. The equations of steady motion are given by:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)+\frac{\partial P}{\partial r}=\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu}{K}u, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}-\frac{uv}{r}\right)=\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\mu}{K}v, \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)+\frac{\partial P}{\partial z}=\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\mu}{K}w, \tag{4}$$

Where u,v and w are velocity components in the directions of r,φ and z respectively. P is the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid and K is the Darcy permeability. Attia (2009) introduced von Karman transformations

$$u=r\omega f,\ \ v=r\omega g,\ \ w=\sqrt{\omega\upsilon}h,\ \ z=\sqrt{\upsilon/\omega}\eta,\ \ p-p_\infty=-p\upsilon\omega P, \tag{5}$$

Where η a non-dimensional boundary layer thickness is measured along the axis of rotation, F,G,H and P are non-dimensional functions of η and υ is the kinematic viscosity of the fluid, $\upsilon=\mu/\rho$ (Attia, 2009). Upon substitution of Equation (5) into the Navier - Stokes equations, a set of similarity non-linear ordinary differential equations are obtained thus:

$$\frac{dh}{d\eta}+2f=0, \tag{6}$$

$$\frac{d^2 f}{d\eta^2}-h\frac{df}{d\eta}-f^2+g^2-M\ f=0, \tag{7}$$

$$\frac{d^2 g}{d\eta^2}-h\frac{dg}{d\eta}-2fg-M\ g=0, \tag{8}$$

$$\frac{d^2 h}{d\eta^2}-h\frac{dh}{d\eta}+\frac{dP}{d\eta}-M\ h=0, \tag{9}$$

$M=\upsilon/K\omega$ is the porosity parameter. The boundary conditions are as follows:

$$f(0)=0,\quad g(0)=1,\quad h(0)=0, \tag{10a}$$

$$f(\infty)=0,\quad g(\infty)=0,\quad P(\infty)=0, \tag{10b}$$

Equation (10a) indicates the no-slip condition of viscous flow on the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Equation (10b). Equation (9) can be used to compute the pressure distribution. The energy equation without the dissipation terms is as follows

$$\rho c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)-k\left(\frac{d^2 T}{\partial z^2}+\frac{d^2 T}{\partial r^2}+\frac{1}{r}\frac{dT}{\partial r}\right)=0, \tag{11}$$

Where T is the temperature of the fluid. c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The non-dimensional variable θ introduced as follows:

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{12}$$

Where T_w and T_∞ are the temperature of the surface of the disk and the temperature at large distances from the disk, respectively. The energy equation is reduced by using Equations (5) and (12)

$$\frac{1}{\mathrm{Pr}}\frac{d^2\theta}{d\eta^2}-h\frac{d\theta}{d\eta}=0, \tag{13}$$

Where Pr is the Prandtl number, $\mathrm{Pr}=c_p\mu_k/k.$ the boundary conditions are

$$\theta(0)=1, \quad \theta(\infty)=0, \tag{14}$$

THE DIFFERENTIAL TRANSFORM METHOD

Differential transformation of the function $f(\eta)$ is defined as follows:

$$F(k)=\frac{1}{k!}\left[\frac{d^k f(\eta)}{dt^k}\right]_{\eta=\eta_0}, \tag{15}$$

In Equation (15) $f(\eta)$ is the original function and $F(k)$ is transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta=\eta_0$). The differential inverse transformation of $F(k)$ is defined as

$$f(\eta)=\sum_{k=0}^{\infty} F(k)(\eta-\eta_0)^k, \tag{16}$$

Combining equations (15) and (16), one can obtain

$$f(\eta)=\sum_{k=0}^{\infty}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \frac{(\eta-\eta_0)^k}{k!}, \tag{17}$$

Equation (17) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by iterative procedures that are described by the transformed equations of the original functions. From the definitions of Equations (15) and (16), it is easily proven that the transformed functions comply with the basic mathematical operations shown. In real applications, the function $f(\eta)$ in Equation (16) is expressed by a finite series and can be written as

$$f(\eta)\approx\sum_{k=0}^{N} F(k)(\eta-\eta_0)^k, \tag{18}$$

Equation (18) implies that $\sum_{k=N+1}^{\infty} F(k)(\eta-\eta_0)^k$ is negligibly small, where N is series size.

Theorems to be used in the transformation procedure, which can be evaluated from Equations. (15) and (16) are given.

Theorem 1

If $f(\eta)=c\,g(\eta)$, then $F(k)=c\,G(k)$, where, c is a constant.

Theorem 2.

If $f(\eta)=\dfrac{d^n g(\eta)}{d\eta^n}$, then $F(k)=\dfrac{(k+n)!}{k!}G(k+n)$.

Theorem 3

If $f(\eta)=g(\eta)\pm h(\eta)$, then $F(k)=G(k)\pm H(k)$.

Theorem 4

If $f(\eta)=g(\eta)\,h(\eta)$, then $F(k)=\sum_{l=0}^{k} G(l)\,H(k-l)$.

Theorem 5

If $f(\eta)=\eta^n$, then $F(k)=\delta(k-n)$, where, $\delta(k-n)=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}$.

Taking differential transform of Equations (6) (8) and (13), this can be obtained:

$$(k+1)\,H(k+1)+2\,F(k)=0, \tag{19}$$

$$(k+2)(k+1)F(k+2)-\sum_{r=0}^{k}\left[(k+1-r)H(r)F(k+1-r)+F(r)F(k-r)-G(r)G(k-r)\right]-M\,F(k)=0, \tag{20}$$

$$(k+2)(k+1)G(k+2)-\sum_{r=0}^{k}\left[(k+1-r)H(r)G(k+1-r)+2F(r)G(k-r)\right]-M\,G(k)=0, \tag{21}$$

$$(k+2)(k+1)\Theta(k+2)-\Pr\sum_{r=0}^{k}\left[(k+1-r)H(r)\Theta(k+1-r)\right]=0, \tag{22}$$

Where $H(k), F(k), G(k)$ and $\Theta(k)$ are the differential transforms of $h(\eta), f(\eta), g(\eta)$ and $\theta(\eta)$. The transformed boundary conditions are

$$\begin{aligned}
&H(0)=0,\\
&F(0)=0, \quad F(1)=\alpha,\\
&G(0)=1, \quad G(1)=\beta,\\
&\Theta(0)=1, \quad \Theta(1)=\gamma,
\end{aligned} \tag{23}$$

that α,β and γ are constants. These constants are computed from the boundary conditions. For $M=0.5$ and $N=20$, the solutions of Equations (23) (using the DTM) are as follows:

$$\begin{aligned}
h(\eta) = {} & -0.385114\,\eta^{2}+0.333333\,\eta^{3}-0.1575501\,\eta^{4}+0.0490113\,\eta^{5}-0.0099158\,\eta^{6}\\
& +0.000865784\,\eta^{7}+0.000212323\,\eta^{8}-0.0000990884\,\eta^{9}+9.52323\times10^{-6}\,\eta^{10}\\
& +0.0000104593\,\eta^{11}-9.34131\times10^{-6}\,\eta^{12}+5.61169\times10^{-6}\,\eta^{13}-2.96571\times10^{-6}\,\eta^{14}\\
& +1.47697\times10^{-6}\,\eta^{15}-7.12997\times10^{-7}\,\eta^{16}+3.37929\times10^{-7}\,\eta^{17}-1.5797\times10^{-7}\,\eta^{18}\\
& +7.29451\times10^{-8}\,\eta^{19}-3.33205\times10^{-8}\,\eta^{20},
\end{aligned} \tag{24}$$

$$f(\eta) = 0.385114x - 0.5\eta^2 + 0.315002\eta^3 - 0.122$$
$$- 0.000849294\eta^7 + 0.000445898\eta^8 - 0.00$$
$$+ 0.0000560479\eta^{11} - 0.000036476\eta^{12} + 0.$$
$$+ 5.70397\times10^{-6}\eta^{15} - 2.87239\times10^{-6}\eta^{16} + 1.$$
$$+ 3.33205\times10^{-7}\eta^{19} - 1.58357\times10^{-7}\eta^{20},$$

$$528\eta^4 + 0.0297474\eta^5 - 0.00303025\eta^6$$
$$)0476162\eta^9 - 0.000057526\eta^{10}$$
$$)0002076\eta^{13} - 0.0000110773\eta^{14}$$
$$.42173\times10^{-6}\eta^{17} - 6.92978\times10^{-7}\eta^{18} \quad (25)$$

$$g(\eta) = 1 - 0.848728\eta + 0.25\eta^2 + 0.0576441\eta^3$$
$$- 0.0267231\eta^6 + 0.00981887\eta^7 - 0.0034$$
$$- 0.000547841\eta^{10} + 0.000244233\eta^{11} - 0.$$
$$- 0.0000223331\eta^{14} + 0.000010005\eta^{15} -$$
$$- 8.61473\times10^{-7}\eta^{18} + 3.74805\times10^{-7}\eta^{19} -$$
$$0.100155\eta^4 + 0.0612322\eta^5$$
$$196\eta^8 + 0.00130378\eta^9$$
$$00110257\eta^{12} + 0.0000496787\eta^{13}$$
$$45315\times10^{-6}\eta^{16} + 1.96616\times10^{-6}\eta^{17}$$
$$.61891\times10^{-7}\eta^{20}, \quad (26)$$

$$\theta(\eta) = 1 - 0.235808\eta + 0.00529742\eta^4 - 0.0027$$
$$- 0.000328628\eta^7 + 0.000183736\eta^8 - 0.00$$
$$- 0.0000191719\eta^{11} + 8.48983\times10^{-6}\eta^{12} -$$
$$- 7.96415\times10^{-7}\eta^{15} + 3.46118\times10^{-7}\eta^{16} - 1$$
$$- 2.83739\times10^{-8}\eta^{19} + 1.21859\times10^{-8}\eta^{20}.$$
$$75109\eta^5 + 0.000866599\eta^6$$
$$)000984777\eta^9 + 0.0000449095\eta^{10}$$
$$- 3.92213\times10^{-6}\eta^{13} + 1.79632\times10^{-6}\eta^{14}$$
$$- 1.50253\times10^{-7}\eta^{17} + 6.54282\times10^{-8}\eta^{18} \quad (27)$$

PADÉ APPROXIMATION

Some techniques exist to increase the convergence radius of a given series. Among them, the so-called Padé technique is widely applied. Suppose that a function $f(\eta)$ is represented by a power series $\sum_{i=0}^{\infty} c_i\eta^i$, so that $f(\eta) = \sum_{i=0}^{\infty} c_i\eta^i$. (28). This expansion is the fundamental starting point of any analysis using Padé approximants. The notation $c_i, i = 0,1,2,\dots$ is reserved for the given set of coefficients and $f(\eta)$ is the associated function. Padé approximant $[L,M]$ is a rational fraction

$$\frac{a_0 + a_1\eta + \cdots + a_L\eta^L}{b_0 + b_1\eta + \cdots + b_M\eta^M}, \quad (29)$$

This has a Maclaurin expansion which agrees with Equation (28) as far as possible. Notice that in Equation (29) there are $L+1$ numerator coefficients and $M+1$ denominator coefficients (Baker, 1981). So there are $L+1$ independent numerator coefficients and M independent denominator coefficients, making $L+M+1$ unknown coefficients in all. This number suggests that normally $[L,M]$ ought to fit the power series Equation (28) through the orders $1,\eta,\eta^2,\dots,\eta^{L+M}$. In the notation of formal power series

$$\sum_{i=0}^{\infty} c_i\eta^i = \frac{a_0 + a_1\eta + \cdots + a_L\eta^L}{b_0 + b_1\eta + \cdots + b_M\eta^M} + O(\eta^{L+M+1}), \quad (30)$$

Baker (1981) found that

$$(b_0 + b_1\eta + \cdots + b_M\eta^M)(c_0 + c_1\eta + \cdots) = a_0 + a_1\eta + \cdots + a_L\eta^L + O(\eta^{L+M+1}). \quad (31)$$

Equating the coefficients of $\eta^{L+1}, \eta^{L+2}, \dots, \eta^{L+M}$

$$\begin{aligned} b_M c_{L-M+1} + b_{M-1}c_{L-M+2} + \cdots + b_0 c_{L+1} &= 0,\\ b_M c_{L-M+2} + b_{M-1}c_{L-M+3} + \cdots + b_0 c_{L+2} &= 0,\\ &\vdots\\ b_M c_L + b_{M-1}c_{L+1} + \cdots + b_0 c_{L+M} &= 0. \end{aligned} \quad (32)$$

If $j < 0$, we define $c_j = 0$ for consistency. Since $b_0 = 1,$ Equation (32) become a set of M linear equations for the M unknown denominator coefficients

$$\begin{pmatrix} c_{L-M+1} & c_{L-M+2} & c_{L-M+3} & \cdots & c_L \\ c_{L-M+2} & c_{L-M+3} & c_{L-M+4} & \cdots & c_{L+1} \\ c_{L-M+3} & c_{L-M+4} & c_{L-M+5} & \cdots & c_{L+2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_L & c_{L+1} & c_{L+2} & \cdots & c_{L+M-1} \end{pmatrix} \begin{pmatrix} b_M \\ b_{M-1} \\ b_{M-2} \\ \vdots \\ b_1 \end{pmatrix} = - \begin{pmatrix} c_{L+1} \\ c_{L+2} \\ c_{L+3} \\ \vdots \\ c_{L+M} \end{pmatrix}, \quad (33)$$

From these equations, b_i may be found. The numerator coefficients $a_0, a_1, \dots, a_L,$ follow immediately from Equation (31) by equating the coefficients of $1,\eta,\eta^2,\dots,\eta^{L+M}$

$$\begin{aligned} a_0 &= c_0,\\ a_1 &= c_1 + b_1 c_0,\\ a_2 &= c_2 + b_1 c_1 + b_2 c_0,\\ &\vdots\\ a_L &= c_L + \sum_{i=1}^{\min\{L,M\}} b_i c_{L-i}. \end{aligned} \quad 34$$

Thus Equations (33) and (34) normally determine the Padé numerator and denominator and are called the Padé equations. The Padé approximant $[L,M]$ is constructed which agrees with $\sum_{i=0}^{\infty} c_i\eta^i$, through order η^{L+M}. for more details the reader is referred to (Baker, 1981). The [11, 11] Padé approximants of Equation (24) - (27) are as follows:

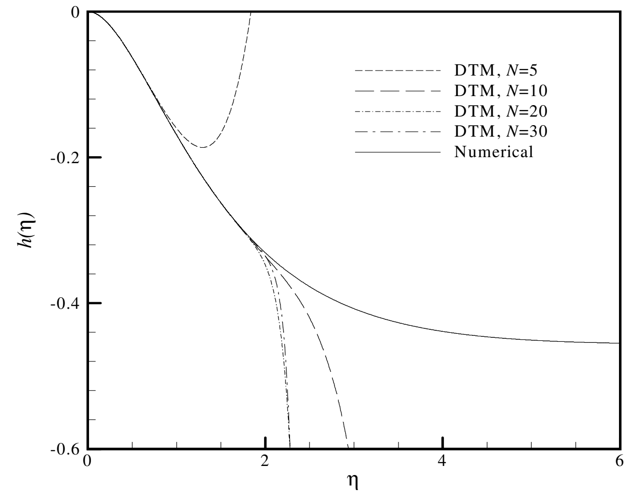

Figure 1. The profile of $h(\eta)$ obtained by the DTM for different value of N in comparison with the numerical solution, when $M=0.5$.

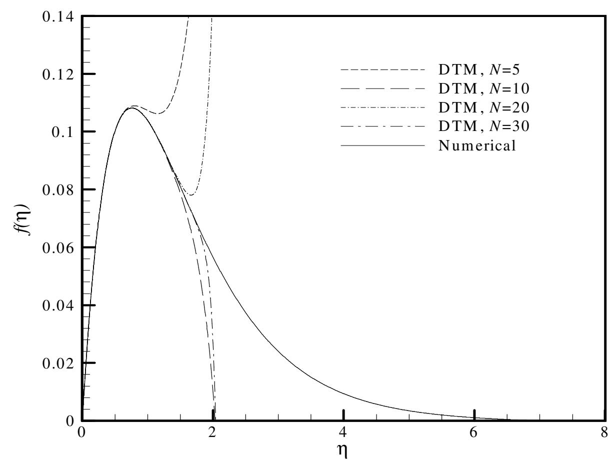

Figure 2. The profile of $f(\eta)$ obtained by the DTM for different value of N in comparison with the numerical solution, when $M=0.5$.

$$h(\eta)_{[9,9]} = (-0.385114\eta^2 - 0.437609\eta^3 - 0.208$$
$$-0.00141477\eta^7 - 0.000201321\eta^8 - 4.$$
$$+1.08178\eta^3 + 0.434213\eta^4 + 0.125016$$
$$+0.000339609\eta^8 + 0.0000149153\eta^9),$$
$$084\eta^4 - 0.0615456\eta^5 - 0.0120185\eta^6$$
$$06828\times10^{-6}\eta^9)/(1+2.00185\eta+1.86404\eta^2$$
$$\eta^5 + 0.025878\eta^6 + 0.00372426\eta^7 \quad (35)$$

$$f(\eta)_{[9,9]} = (0.385114\eta + 0.280441\eta^2 + 0.0783372\eta^3$$
$$-0.00130115\eta^6 + 0.000510628\eta^7 - 0.0000$$
$$(1+2.02652\eta + 2.01653\eta^2 + 1.31424\eta^3 + 0$$
$$+0.00906905\eta^7 + 0.00105022\eta^8 + 0.00006$$
$$+0.0136951\eta^4 - 0.00541265\eta^5$$
$$593906\eta^8 + 2.42524\times10^{-6}\eta^9)/$$
$$.610344\eta^4 + 0.206984\eta^5 + 0.0513544\eta^6$$
$$23236\eta^9), \quad (36)$$

$$g(\eta)_{[9,9]} = (1+1.14505\eta + 0.384017\eta^2$$
$$-0.000450242\eta^6 + 0.000308$$
$$(1+1.99378\eta + 1.8262\eta^2 + 1.$$
$$+0.0228601\eta^6 + 0.00326927$$
$$+0.0375617\eta^3 - 0.00197482\eta^4 - 0.00378612\eta^5$$
$$258\eta^7 - 0.0000411259\eta^8 + 1.70699\times10^{-6}\eta^9)/$$
$$03142\eta^3 + 0.402092\eta^4 + 0.112812\eta^5$$
$$\eta^7 + 0.000308109\eta^8 + 0.0000159253\eta^9), \quad (37)$$

$$\theta(\eta)_{[9,9]} = (1+1.46155\eta + 0.945797\eta^2 + 0.36354$$
$$+0.0000921723\eta^6 - 0.00018088\eta^7 - 0.$$
$$/(1+1.69736\eta + 1.34605\eta^2 + 0.680949$$
$$+0.0112414\eta^6 + 0.00142344\eta^7 + 0.000$$
$$1\eta^3 + 0.0864312\eta^4 + 0.0106365\eta^5$$
$$0000230614\eta^8 + 7.32582\times10^{-7}\eta^9)$$
$$\eta^3 + 0.241707\eta^4 + 0.0613923\eta^5$$
$$113107\eta^8 + 5.9919\times10^{-6}\eta^9). \quad (38)$$

RESULTS AND DISCUSSION

In this paper, the DTM was applied successfully to find analytical solution of steady flow over a rotating disk in porous medium with heat transfer. Graphical representation of results is very useful to demonstrate the efficiency and accuracy of the differential transform method for the problem stated in this work. Figures 1 – 4 show the velocity components $h(\eta), f(\eta), g(\eta)$ and the dimensionless temperature distribution $\theta(\eta)$ obtained by the DTM for different value of series size. Figures 5 - 8 show $h(\eta)$, $f(\eta)$, $g(\eta)$ and $\theta(\eta)$ obtained by the DTM and the DTM-Padé in comparison with the numerical solutions obtained by the fourth-order Runge–Kutta method. In Figure 5 - 8, one can see a very good agreement between the DTM and the numerical results, but these series diverge around infinity. One Padé approximant solve this problem and increase the convergence of given series. So, the solutions are obtained by DTM-Padé are more accurate than the DTM. In Figures 9 -12, the velocity components and the dimensionless temperature distribution are represented for different values of M.

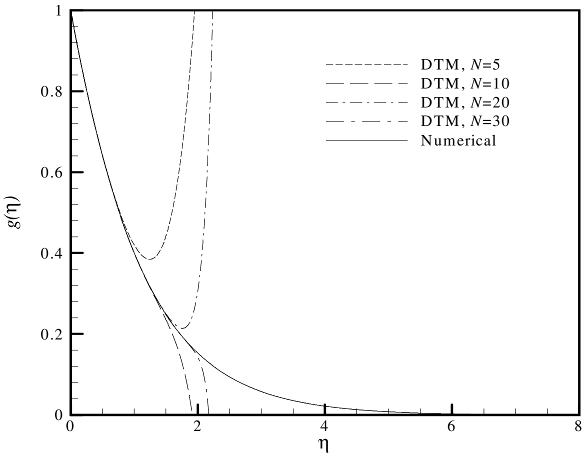

Figure 3. The profile of $g(\eta)$ obtained by the DTM for different value of N in comparison with the numerical solution, when $M = 0.5$.

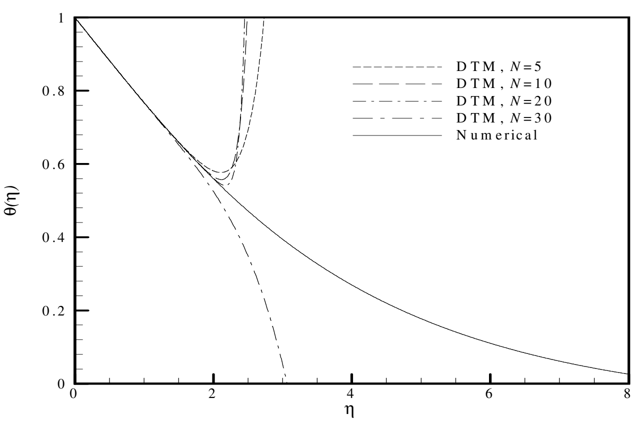

Figure 4. The profile of $\theta(\eta)$ obtained by the DTM for different value of N in comparison with the numerical solution, when $M = 0.5$ and $\Pr = 0.7$.

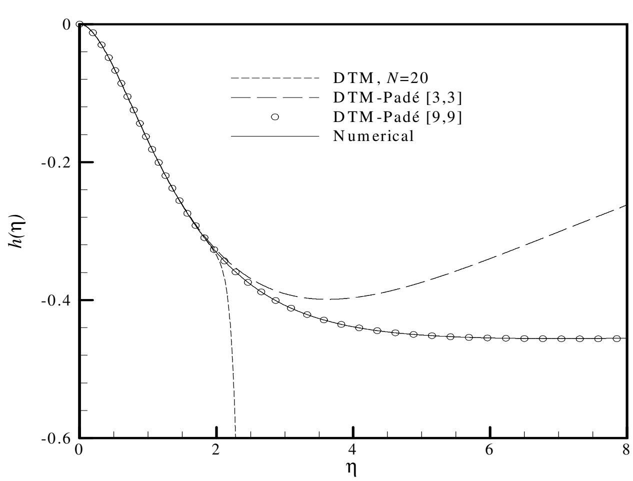

Figure 5. The profile of $h(\eta)$ obtained by the DTM$^{(N=20)}$ and the DTM-Padé in comparison with the numerical solution.

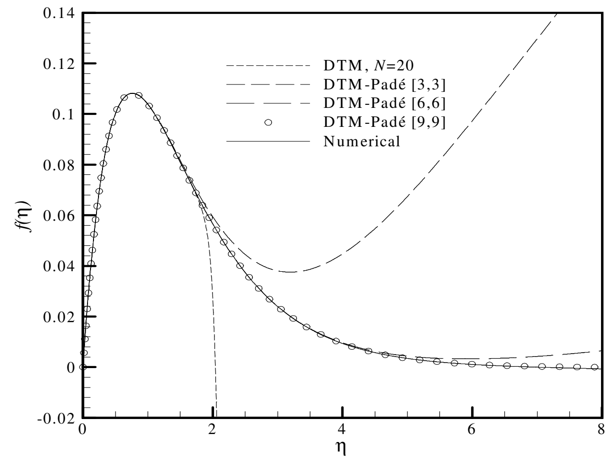

Figure 6. The profile of $f(\eta)$ obtained by the DTM$^{(N=20)}$ and the DTM-Padé in comparison with the numerical solution.

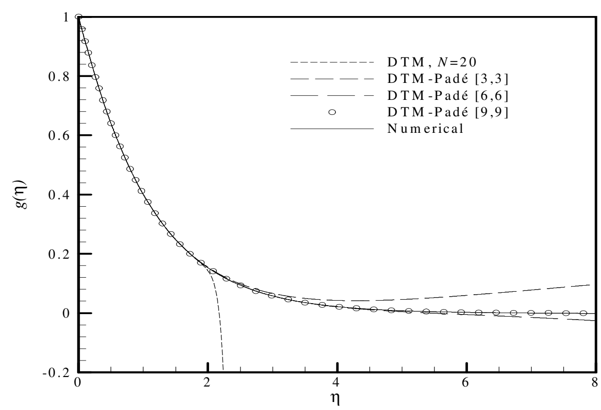

Figure 7. The profile of $g(\eta)$ obtained by the DTM$^{(N=20)}$ and the DTM-Padé in comparison with the numerical solution.

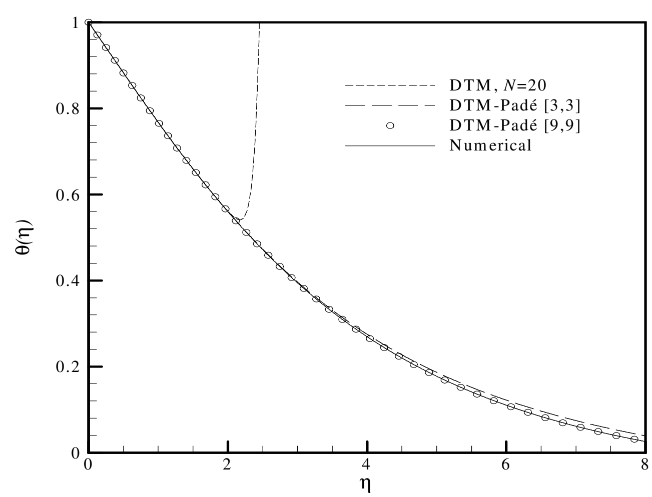

Figure 8. The profile of $\theta(\eta)$ obtained by the DTM$^{(N=20)}$ and the DTM-Padé in comparison with the numerical solution.

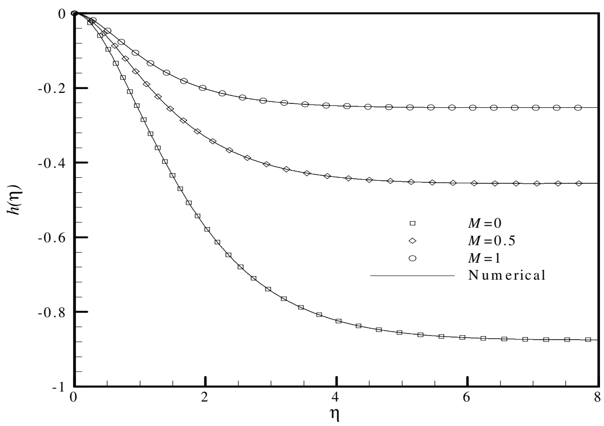

Figure 9. The profile of $h(\eta)$ obtained by the DTM-Padé (Padé approximant [9, 9]) in comparison with the numerical solution for different values of the porosity parameter M.

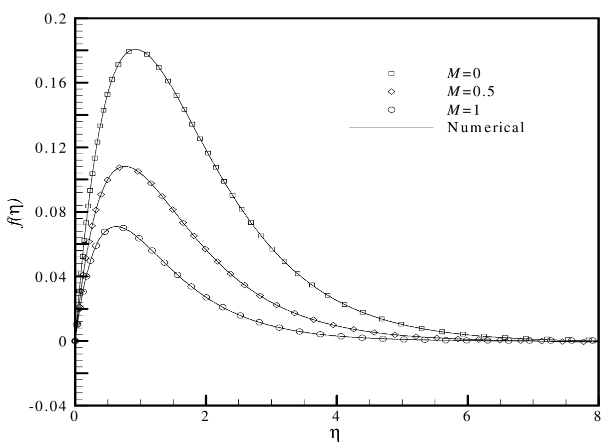

Figure 10. The profile of $f(\eta)$ obtained by the DTM-Padé (Padé approximant [9, 9]) in comparison with the numerical solution for different values of the porosity parameter M.

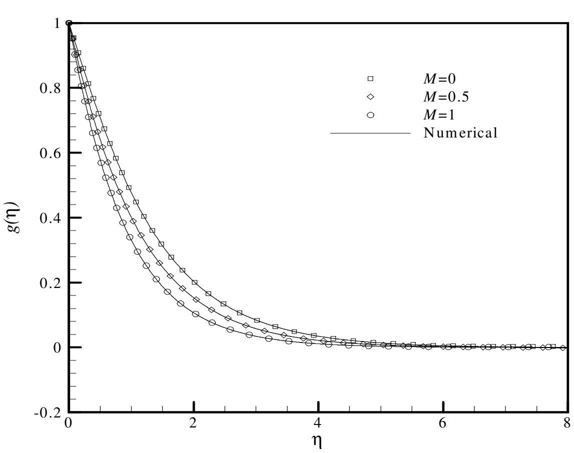

Figure 11. The profile of $g(\eta)$ obtained by the DTM-Padé (Padé approximant [9,9]) in comparison with the numerical solution for different values of the porosity parameter M.

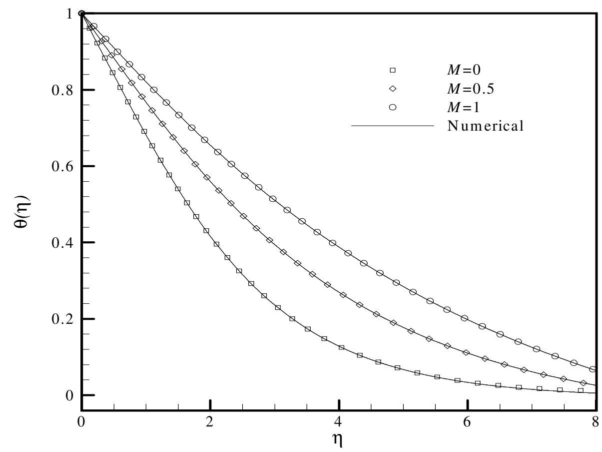

Figure 12. The profile of $\theta(\eta)$ obtained by the DTM-Padé (Padé approximant [9, 9]) in comparison with the numerical solution for different values of the porosity parameter M, when $\mathrm{Pr} = 0.7$.

Conclusion

In this paper, the DTM was applied successfully to find the analytical solution of steady flow over a rotating disk in porous medium with heat transfer. The results show that the differential transform method does not require small parameters in the equations, so the limitations of the traditional perturbation methods can be eliminated. The reliability of the method and reduction in the size of computational domain give this method a wider applicability. Therefore, this method can be applied to many nonlinear integral and differential equations without linearization, discretization or perturbation.

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