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Full Length Research Paper

A catastrophic-cum-restorative queuing model with correlated input for the cell traffic generated by new broadband services

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A Catastrophic-cum-Restorative Queuing Model has been developed for the cell traffic generated by New Broadband services. Transient analysis of the queuing model has been carried out. Some particular cases of the model have been derived and discussed.

Key words: Catastrophes, correlation, broadband services, restoration.

INTRODUCTION

New Broadband Communication Networks are playing a pivotal role in providing a variety of multimedia services such as voice, video and data etc. The amount of information per unit time generated by these services vary along the connection duration. There are certain periods in which the information rate increases and others in which it decreases or becomes null. This sort of variation exhibits correlation between different time instants, for example, in video, each frame varies little with respect to the previous one so the amount of information needed to represent it will depend on the amount needed for the previous one. With this type of traffic, the Poisson arrivals assumption, in which only the mean cell arrival rate was needed to describe it, is not valid any more. Thus, the arrival process in New Broadband Communication Networks is correlated in nature by Andrade Parra (1993).

Further, the arrival of infected cells (viruses) and noise bursts etc. may annihilate all the cells in the buffer of the server (computer) and leave it momentarily inactivated until the new cell arrival occurs. Such infected cells may be modeled by catastrophes. The notion of catastrophes occurring at random, leading to annihilation of all the customers there and the momentary inactivation of service facility until a new arrival of a customer is not uncommon in many practical problems. Chao (1995) studied a queuing network model with catastrophes. Crescendo et al. (2003) studied an M/M/1 queue with catastrophes and derived its heavy traffic approximation. Recently, Jain and Kumar (2005) have obtained the time-dependent solution of a catastrophic queuing model having correlated input.

The concept of catastrophe has tremendous applications in a wide variety of areas particularly in computer-communication, biosciences, population studies and industries. Crescenzo et al. (2003) consider that the effect of each catastrophe is to make the queue instantly empty provided the system is not empty and simultaneously the system becomes ready to accept new customers. In our case, the catastrophe makes the queue instantly empty whenever the system is not empty but the system takes its own time to be ready to accept new customers; this time is referred to as ‘ the restoration time’. Practically any system suffering from catastrophe must take some time for its restoration.

In the present example, with the occurrence of catastrophe all the cells in the buffer of the server are destroyed immediately. But the server can work properly after it is free from the viruses and noise bursts. Thus, some sort of recovery / restoration time is needed. Keeping in view this important and practically valid aspect, a catastrophic-cum-restorative queuing model with correlated input has been developed for the cell traffic generated by new broadband services.

In this paper, the effects of catastrophe and restoration have been incorporated in the correlated input queue arising in New Broadband Communication Networks. The transient analysis of the model has been carried out. Some particular cases of the model have been derived and discussed.

This paper has been organized as follows: In section 2, we develop the queuing model. In section 3, the transient analysis of the model has been done by using probability generating function technique. The paper is concluded in section 4.

QUEUING MODEL

The queuing model investigated in this paper is based on the following assumptions: -

(1) The customers (cells) arrive at a service facility and form a queue. The arrivals can occur only at the transition marks t_0, t_1, t_2…where $\theta_r = t_r - t_{r-1}$; r = 1, 2, 3,……….are random variables with P [$\theta_r \leq x$] = 1 – exp (-λx); λ > 0, r = 1, 2,……….

(2) The arrivals of customers (cells) at the two consecutive transition marks t_{r-1}, t_r; r = 1,2,3,……..are governed by the following transition probability matrix:

$$\begin{array}{ll} & \quad\quad\quad \text{To } t_r \\ & \quad \text{arrival} \quad \text{no arrival} \\ \begin{array}{r} \text{arrival} \\ \text{From } t_{r-1} \\ \text{no arrival} \end{array} & \left\| \begin{array}{cc} p_{11} & p_{10} \\ & \\ p_{01} & p_{00} \end{array} \right\| \begin{array}{l} \text{where } p_{11}+p_{10}=1 \\ \quad\quad \text{and} \\ p_{01}+p_{00}=1 \end{array} \end{array}$$

Thus, the arrivals at two consecutive transition marks are correlated.

(3) The capacity of the system is infinite.

(4) There is a single server and the customers are served one by one on FCFS basis. The service time distribution is exponential with parameter μ.

(5) When the system is not empty, the catastrophes occur at the service facility according to a Poisson process with rate ξ. The catastrophes annihilate all the customers (cells) in the system instantaneously and the system starts working after the restoration time is over. No arrival can occur during the restoration time.

(6) The restoration times are independently, identically and exponentially distributed with parameter η.

(7) Initially, the system starts with the arrival of a customer that makes the queue length (the number of customers waiting excluding the one in service) equal to zero, so that

$P_{0,1}(0) = 1$

Define, $P_{n,i}(t)$ = the probability that at time t , the queue length is equal to n, the service channel is not idle and i (= 0, 1) arrival has occurred at the previous transition mark.

$Q_{0,i}(t)$ = the probability that at time t , the queue length is equal to 0 without the occurrence of catastrophe, the service channel is idle and i(= 0, 1) arrival has occurred at the previous transition mark .

$C_{0,i}(t)$ = the probability that at time t, the queue length is equal to zero with the occurrence of catastrophe, the service channel is idle and i (= 0, 1) arrival has occurred at the previous transition mark.

$R_n(t)$ = the probability that at time t, the queue length is equal to n.

TRANSIENT ANALYSIS OF THE MODEL

The equations governing the model are:-

$$R_n(t) = P_{n,0}(t) + P_{n,1}(t) \; ; \; n = 1, 2, 3 \ldots\ldots(1)$$

$$R_0(t) = P_{0,0}(t) + P_{0,1}(t) + Q_{0,0}(t) + Q_{0,1}(t) \ldots\ldots(2)$$

$$\frac{d}{dt}Q_{0,0}(t) = -\lambda Q_{0,0}(t) + \mu P_{0,0}(t) + \lambda[p_{00} Q_{0,0}(t) + p_{10} Q_{0,1}(t)] + \eta C_{0,0}(t) \ldots\ldots(3)$$

$$\frac{d}{dt}C_{0,0}(t) = -\eta C_{0,0}(t) + \xi\left[\sum_{n=0}^{n=\infty} P_{n,o}(t)\right] \ldots\ldots(4)$$

$$\frac{d}{dt}Q_{0,1}(t) = -\lambda Q_{0,1}(t) + \mu P_{0,1}(t) + \eta C_{0,1}(t) \ldots\ldots(5)$$

$$\frac{d}{dt}C_{0,1}(t) = -\eta C_{0,1}(t) + \xi\left[\sum_{n=0}^{n=\infty} P_{n,1}(t)\right] \ldots\ldots(6)$$

$$\frac{d}{dt}P_{0,0}(t) = -(\lambda+\mu+\xi) P_{0,0}(t) + \mu P_{1,0}(t) + \lambda[p_{00} P_{0,0}(t) + p_{10} P_{0,1}(t)] \ldots\ldots(7)$$

$$\frac{d}{dt}P_{n,0}(t) = -(\lambda+\mu+\xi)P_{n,0}(t) + \mu P_{n+1,0}(t) + \lambda[p_{00} P_{n,0}(t) + p_{10} P_{n,1}(t)];\; n = 1,2,3,\ldots\ldots(8)$$

$$\frac{d}{dt}P_{0,1}(t) = -(\lambda+\mu+\xi)P_{0,1}(t) + \mu P_{1,1}(t) + \lambda[p_{01} Q_{0,0}(t) +$$

$p_{11}\, Q_{0,1}(t)$]…………………………………………………......(9)

$\frac{d}{dt} Pn_{,1}(t) = - (\lambda + \mu + \xi)Pn_{,1}(t) + \mu P_{n+1,1}(t) + \lambda[\ p_{01}\ P_{n-1,0}(t)+p_{11}\ P_{n-1,1}(t)]$; n = 1, 2, 3,……………..(10)

Define, the Laplace Transform of f (t) by

$f^*(s) = \int_0^\infty e^{-st}\ f(t)dt$ ……………………………………..(11)

Taking L.T.'s of (1) to (10), we have $R^*_n(s) = P^*_{n,0}(s) + P^*_{n,1}(s)$; n = 1, 2, 3,… ………………………………………………………………(12)

$R^*_0(s) = P^*_{0,0}(s) + P^*_{0,1}(s) + Q^*_{0,0}(s) + Q^*_{0,1}(s)$…………………………………………………...(13)

$(s + \lambda)\ Q^*_{0,0}(s) = \mu\ P^*_{0,0}(s) + \lambda[\ p_{00}\ Q^*_{0,0}(s) + p_{10}\ Q^*_{0,1}(s)\ ]+ \eta C^*_{0,0}(s)$ ………………………………………………………………(14)

$(s+\eta)C^*_{0,0}(s) = \xi\left[\sum_{n=0}^{n=\infty} P^*_{n,0}(s)\right]$……………………..(15)

$(s + \lambda)Q^*_{0,1}(s) = \mu\ P^*_{0,1}(s) + \eta C^*_{0,1}(s)$ ……..(16)

$(s+\eta)C^*_{0,1}(s) = \xi\left[\sum_{n=0}^{n=\infty} P^*_{n,1}(s)\right]$……………………… (17)

$(s + \lambda + \mu + \xi)\ P^*_{0,0}(s) = \mu P^*_{1,0}(s) + \lambda[\ p_{00}\ P^*_{0,0}(s\ ) + p_{10}\ P^*_{0,1}(s)]$ ………………………………………………………….(18)

$(s + \lambda + \mu + \xi)P^*n_{,0}(s) = \mu P^*_{n+1,0}(s) + \lambda[\ p_{00}\ P^*n_{,0}(s) + p_{10}\ P^*_{n,1}(s)]$;n = 1, 2, 3,…………………………...........(19)

$(s + \lambda + \mu + \xi)\ P^*_{0,1}(s)-1= \mu P^*_{1,1}(s) + \lambda[\ p_{01}\ Q^*_{0,0}(s) + p_{11}\ Q^*_{0,1}(s)\ ]$ ……………………………......................................(20)

$(s + \lambda + \mu + \xi)\ P^*_{n,1}(s) = \mu P^*_{n+1,1}(s) + \lambda[\ p_{01}\ P^*_{n-1,0}(s)+p_{11}\ P^*_{n-1,1}(s)]$;n = 1, 2, 3,… …………………………...(21)

Define, the following probability generating functions by

P^*_i (s,□) $=\sum_{n=0}^{n=\infty} \alpha^n P^*_{n,i}(s)$; i =0,1. … ……………………………………………………………(22)

$R^*(s,\theta) = \sum_{n=0}^{n=\infty} \alpha^n R^*_n(s)$ ……………………………………(23)

Multiplying (18) and (19) by appropriate powers of □ and adding, we have

$(h-\lambda\ p_{00})\ P^*_0(s,\alpha) = \lambda\ p_{10}\ P^*{}_1(s,\alpha)- \frac{\mu}{\alpha} P^*_{0,0}(s)$ ……………………………………………………………(24)

Where $h = (s + \lambda + \mu + \xi) - \frac{\mu}{\alpha}$ …………………………(25)

Similarly (20) and (21) give

$(h-\lambda\alpha\ p_{11})\ P^*_1(s,\alpha) = \lambda\alpha\ p_{01}\ P^*_0(s,\alpha) - \frac{\mu}{\alpha} P^*_{0,1}(s) + \lambda\ [p_{01}\ Q^*_{0,0}(s) + p_{11}\ Q^*_{0,1}(s)] +1$ ………………………………..(26)

and combine (12) and (13) give

$R^*(s,\alpha) = P^*_0(s,\alpha) + P^*_1(s,\ \alpha) + Q^*_{0,0}(s) + Q^*_{0,1}(s)$ ....(27)

Solving (24) and (26) simultaneously, we have

$P^*_i(s,\alpha)= \frac{N_i(\alpha)}{D(\alpha)}$; I = 0,1………………………………..(28)

Where

$$N_0(\alpha) = \frac{1}{\alpha^2}\left\{\lambda\alpha p_{10}\left[\lambda\alpha\left(p_{01}Q^*_{0,0}(s) + p_{11}Q^*_{0,1}(s)\right) + \alpha - \mu P^*_{0,1}(s)\right] - \mu\left[g - \lambda\alpha^2 p_{11}\right]P^*_{0,0}(s)\right\}$$

.... (29)

$$N_1(\alpha) = \frac{1}{\alpha^2}\left\{\left(g - \lambda\alpha p_{0,0}\right)\left[\lambda\alpha\left(p_{01}Q^*_{0,0}(s) + p_{11}Q^*_{0,1}(s)\right) + \alpha - \mu P^*_{0,1}(s)\right] - \left[\lambda\mu\alpha^2 p_{01}\right]P^*_{0,0}(s)\right\}$$

…………………………………………………………… (30)

Combining (27) - (30) we get

$R^*(s,\alpha) = Q^*_{0,0}(s) + Q^*_{0,1}(s)+ \frac{N(\alpha)}{D(\alpha)}$ ……………………(31)

Where

$N(\alpha)=N_0(\alpha)+N_1(\alpha)= \frac{1}{\alpha^2}$ {[g+λα($p_{10-}p_{00}$)][α-$\mu P^*_{0,1}(s)+\lambda\alpha[p_{01}Q^*_{0,0}(s)+ p_{11}Q^*_{0,1}(s)] -\mu[g + \lambda\ \alpha^2\ (p_{01\ -}p_{11})]\ P^*_{0,0}(s)\}$ ……………………………………………………..........(32)

$D(\alpha) = \frac{1}{\alpha^2}\ \{(g-\lambda\alpha\ p_{00})(g-\lambda\ \alpha^2\ p_{11}) - \alpha^3\lambda^2\ \ p_{10}\ p_{01}\}$ ....(33)

Where g = α (s + λ + μ + ξ) - μ,
Also from (15) and (17), we have

$$C^*_{0,0}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,0}(s)\right]\text{.....................}(34)$$

and

$$C^*_{0,1}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,1}(s)\right]\text{.....................}(35)$$

Also

$$P^*_0(s,1)=\sum_{n=0}^{\infty}P^*_{n,0}(s) \quad \text{and}$$

$$P^*_1(s,1)=\sum_{n=0}^{\infty}P^*_{n,1}(s) \quad \text{...........................}(36)$$

From (28) for α = 1, we have

$$P^*_0(s,1)=\frac{N_0(1)}{D(1)} \quad \text{and}$$

$$P^*_1(s,1)=\frac{N_1(1)}{D(1)} \quad \text{..}(37)$$

Therefore, from (29) and (30) we get

$$\sum_{n=0}^{\infty}P^*_{n,0}(s)=\frac{N_0(1)}{D(1)} \quad \text{and} \quad \sum_{n=0}^{\infty}P^*_{n,1}(s)=\frac{N_1(1)}{D(1)}$$

$$\text{...}(38)$$

By Rouche's theorem the denominator D (α) in (27) has 2 zeros inside the unit circle $|\alpha| = 1$. Since, $R^*(s,\alpha)$ is a finite quantity, these zeros must vanish the numerator N (α) giving rise to a set of two equations. Solving these two equations together with (14) and (16) one can determine all the four unknowns $P^*_{0,0}(s)$, $P^*_{0,1}(s)$, $Q^*_{0,0}(s)$ and $Q^*_{0,1}$(s. Hence, $R^*(s,\alpha)$ can be completely determined.

PARTICULAR CASES

(i) When η = ∞ (that is, when we do not include the factor of restoration), from (34) and (35) we have

$$C^*_{0,0}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,0}(s)\right] = 0 \text{ and}$$

$$C^*_{0,1}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,1}(s)\right] = 0 \text{ respectively.}$$

And hence the system now behaves like a catastrophic queue with correlated input studied by Jain and Kumar.

(ii) When ξ = 0 and η = ∞ (that is, when both the catastrophic and restorative effects are excluded), we have from (31) - (35)

$$R^*(s,\alpha) = Q^*_{0,0}(s) + Q^*_{0,1}(s) + \frac{N(\alpha)}{D(\alpha)} \text{.....................}(39)$$

Where

$$N(\alpha) = N_0(\alpha)+N_1(\alpha) = \frac{1}{\alpha^2}\{[g+\lambda\alpha(p_{10}-p_{00})][\alpha\mu P^*_{0,1}(s)+\lambda\alpha[p_{01}Q^*_{0,0}(s)+ p_{11}Q^*_{0,1}(s)] - \mu[g + \lambda\alpha^2(p_{01}-p_{11})P^*_{0,0}(s)]\}$$
$$\text{..}(40)$$

$$D(\alpha) = \frac{1}{\alpha^2}\{(g-\lambda\alpha p_{00})(g-\lambda\alpha^2 p_{11}) - \alpha^3\lambda^2 p_{10}p_{01}\}$$
$$\text{..}(41)$$

Where g = α (s + λ + μ) - μ,

$$C^*_{0,0}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,0}(s)\right]= 0$$
$$\text{..}(42)$$

and

$$C^*_{0,1}(s)=\left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty}P^*_{n,1}(s)\right]= 0$$
$$\text{..}(43)$$

(39) is the Laplace transform of the p. g. f. of the system size of a single server queue having correlated input and exponential service time distribution.

Conclusion

Through this study, the transient solution of a Catastrophic-cum-Restorative Queuing Model with correlated input has been derived. The study carried out in this paper is very useful for the analysis of cell traffic generated by new broadband services.

REFERENCES

Crescenzo A, Giorno V, Ricciardi LM (2003). "On the M/M/1 queue with catastrophes and its continuous approximation", Queuing Syst. 43: 329- 347.
Andrade Parra J (1993). "Statistical parameters to describe cell traffic

generated by broadband services", Communicaciones de Telefonica, No.8, December.
Jain NK, Rakesh K (2005). "A catastrophic queuing model with correlated input for the cell traffic generated by new broadband services", Indian. J. Mathe. Mathe. Sci. 1(2): 99-108.
Chao X (1995), "A queueing network model with catastrophes and product form solution", O.R.Letters. 18: 75-79.