African Journal of Mathematics and Computer Science Research Vol. 5(3), pp. 48-54, 9 February, 2012
Available online at http://www.academicjournals.org/AJMCSR
DOI: 10.5897/AJMCSR11.125
ISSN 2006-9731 ©2012 Academic Journals

Full Length Research Paper

Cross efficiency by using common weights for fuzzy data

Sahand Daneshvar[1], Mojtaba Ramezani[2]*, Mozhgan Mansouri Kaleibar[1] and Sharmin Rahmatfam[3]

[1]Islamic Azad University, Tabriz Branch, Tabriz, Iran.
[2]Islamic Azad University, Bonab Branch, Bonab, Iran.
[3]Islamic Azad University, Tehran Central Branch, Tehran, Iran.

Accepted 29 August, 2011

This paper firstly revists the cross efficiency evaluation method which is an extention tool of data envelopment analysis (DEA), then analyzes the potential flawes which happens when the ultimate average cross efficiency scores are used. In this paper, we consider the decision making units (DMUs) as the players in a cooperative fuzzy game, where the characteristic function values of coalitions are defined to compute the Shapley value of each DMU with fuzzy data, and the common weights associate with the imputation of the Shapley values are used to determine the ultimate cross efficiency scores. In this paper cross efficiency defined with fuzzy data for solving fuzzy parameters problems.

Key words: Cross efficiency, cooperative game, common weights, data envelopment analysis (DEA), fuzzy game, fuzzy linear programming, Shapley value.

INTRODUCTION

As a non-parametric programming efficiency-rating technique for a set of decision making units (DMUs) with multiple inputs and multiple outputs, data envelopment analysis (DEA) (Cooper et al., 2000) is receiving more and more importance for evaluating and improving the performance of manufacturing and service operations. It has been extensively applied in performance evaluation and benchmarking of schools, hospitals, bank branches, production plants, etc (Charnes et al., 1994). However, traditional DEA models are not very appropriate for ranking DMUs since they simply classify the units into two groups: efficient and inefficient (Charnes et al., 1978). Moreover, it is often possible in DEA that some inefficient DMUs are in fact better overall performers than some efficient ones. This is because of the unrestricted weight flexibility problem in DEA by being involved in an unreasonable self-rated scheme (Dyson and Thannassoulis, 1988). The DMU under evaluation heavily weighs few favorable measures and completely ignores other inputs and outputs in order to maximize its own DEA efficiency. In the fuzzy game, each DMU will be a player, the characteristic function value of each coalition is defined, and the solution of Shapley value is computed to determine the ultimate cross efficiency of each DMU by Despotis (2002). However, there are still several limitations for utilizing the average cross efficiency measure to evaluation, like the losing association with the weights by averaging among the cross efficiencies. In most real-world situations, the possible values of parameters of mathematical models are often only imprecisely or ambiguously known to the experts. It would be certainly more appropriate to interpret the experts understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers. In this study, we deal with problems with fuzzy parameters from the viewpoint of experts imprecise of the nature of

*Corresponding author. E-mail: Mojtaba.Ramezani.53@gmail.com.

parameters in a problem- formulation process. The fuzzy parameters issue can thus be solved in this paper.

CROSS EFFICIENCY EVALUATION

Adopting the conventional nomenclature of DEA, assume that there are *n* DMUs that are to be evaluated in terms of *m* inputs and *s* outputs of triangular fuzzy number. We denote the *i*th input and *r*th output for $\mathrm{DMU}j\,(j=1,2,...,n)$ as $\tilde{x}_{ij}\,(i=1,2,...,m)$ and $\tilde{y}_{rd}\,(r=1,2,...,s)$, respectively. The efficiency rating for any given DMU_d, can be computed using the following CCR model in the form of fuzzy linear programming:

$$
\begin{aligned}
&Max \sum_{r=1}^{s} \mu_r \tilde{y}_{rd} = \theta_d \\
&\text{s.t.} \sum_{i=1}^{m} w_i \tilde{x}_{ij} - \sum_{r=1}^{s} \mu_r \tilde{y}_{rj} \geq 0,\ j=1,2,\ldots,n\ ,\ \sum_{i=1}^{m} w_i \tilde{x}_{id} = 1 \\
&\quad w_i \geq 0, i=1,2,\ldots,m\ , \mu_r \geq 0, r=1,2,\ldots,s \qquad (1)
\end{aligned}
$$

Where $\tilde{x}_{ij}, \tilde{y}_{rd}$ are triangular fuzzy numbers, in this paper, we employ a parametric approach to solving the linear programming problem with fuzzy parameters in order to construct the values of coalitions Sakawa (1993). First we introduce the α-level of the fuzzy numbers $\tilde{x}_{ij}, \tilde{y}_{rd}$ defined as the set $(\tilde{x}_{ij})_\alpha, (\tilde{y}_{rd})_\alpha$ in which the degree of their membership functions exceeds the level α:

$$
\begin{aligned}
&(\tilde{x}_{ij})_\alpha = \{(x_{ij})\ \ \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha,\ k=1,\ldots,n\ ,i=1,\ldots,m\} \\
&(\tilde{y}_{rd})_\alpha = \{(y_{rd})\ \ \mu_{\tilde{y}_{rd}}(y_{rd}) \geq \alpha,\ k=1,\ldots,n\ ,i=1,\ldots,m\} \quad (2)
\end{aligned}
$$

Now suppose that all players consider that the degree of all the membership functions of the fuzzy number involved in the linear programming problem should be greater than or equal to a certain degree α. Then, for such a degree α, the problem can be interpreted as the following non fuzzy linear programming problem which depends on a coefficient vector $(x_{ij}) \in (\tilde{x}_{ij})_\alpha, (y_{rd}) \in (\tilde{y}_{rd})_\alpha$ Sakawa (1993).

$$
\begin{aligned}
&Max \sum_{r=1}^{s} \mu_r y_{rd} = \theta_d \\
&\text{s.t.} \sum_{i=1}^{m} w_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0,\ j=1,2,\ldots,n\ ,\sum_{i=1}^{m} w_i x_{id} = 1 \\
&\quad w_i \geq 0, i=1,2,\ldots,m\ , \mu_r \geq 0, r=1,2,\ldots,s \qquad (3)
\end{aligned}
$$

Observe that there exists an infinite number of such a problem (3) depending on the coefficient vector $(x_{ij}) \in (\tilde{x}_{ij})_\alpha, (y_{rd}) \in (\tilde{y}_{rd})_\alpha$ and the values of $(x_{ij}), (y_{rd})$ are arbitrary for any $(x_{ij}) \in (\tilde{x}_{ij})_\alpha, (y_{rd}) \in (\tilde{y}_{rd})_\alpha$ in the sense that the degree of all the membership functions for the fuzzy number in the problem (3) exceeds the level α. However, if the players think that the problem should be solved by taking an optimistic view, the coefficient vector $(x_{ij}) \in (\tilde{x}_{ij})_\alpha, (y_{rd}) \in (\tilde{y}_{rd})_\alpha$ in the problem (4) would be chosen so as to maximize the objective functions under the constraints. From such a point of view, for a certain degree α, it seems to be quite natural to have understood the linear programming problem with fuzzy parameters as the following non fuzzy α-linear programming problem:

$$
\begin{aligned}
&M\ \text{ax} \sum_{r=1}^{s} \mu_r y_{rd} = \theta_d \\
&\text{s.t.} \sum_{i=1}^{m} w_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} \geq 0,\ j=1,2,\ldots,n\ ,\ \sum_{i=1}^{m} w_i x_{id} = 1 \\
&\quad w_i \geq 0, i=1,2,\ldots,m\ , \mu_r \geq 0, r=1,2,\ldots,s \\
&\quad (x_{ij}) \in (\tilde{x}_{ij})_\alpha, (y_{rd}) \in (\tilde{y}_{rd})_\alpha \qquad (4)
\end{aligned}
$$

It should be noted that the coefficient vectors $(x_{ij}), (y_{rd})$ are treated as decision variables rather than constants. Therefore, the problem (4) is not a linear programming problem. However, from the properties of the α-level set for the vectors of fuzzy number $\tilde{x}$ and $\tilde{y}$ it follows that the feasible regions for $\tilde{x}$ and $\tilde{y}$ can be denoted respectively by the closed intervals $[x^L, x^R], [y^L, y^R]$. Thus, we can obtain an optimal solution to the problem (4) by solving the following linear programming problem by Sakawa (1993).

$$
\begin{aligned}
&Max \sum_{r=1}^{s} \mu_r y_{rd}^{R} = \theta_d \\
&\text{s.t.} \sum_{i=1}^{m} w_i x_{ij}^{R} - \sum_{r=1}^{s} \mu_r y_{rj}^{R} \geq 0,\ j=1,2,\ldots,n\ ,\sum_{i=1}^{m} w_i x_{id}^{R} = 1 \\
&\quad w_i \geq 0, i=1,2,\ldots,m\ , \mu_r \geq 0, r=1,2,\ldots,s \qquad (5)
\end{aligned}
$$

Conversely the players may think that the problem should be solved by taking a pessimistic view. Then taking opposite extreme points of the closed intervals $[x^L, x^R], [y^L, y^R]$, we can formulate the following problem which yields a value of the objective function smaller than that the problem (5):

$$
\begin{aligned}
&Max \sum_{r=1}^{s} \mu_r y_{rd}^{L} = \theta_d \\
&\text{s.t.} \sum_{i=1}^{m} w_i x_{ij}^{L} - \sum_{r=1}^{s} \mu_r y_{rj}^{L} \geq 0,\ j=1,2,\ldots,n\ ,\ \sum_{i=1}^{m} w_i x_{id}^{L} = 1 \\
&\quad w_i \geq 0, i=1,2,\ldots,m\ , \mu_r \geq 0, r=1,2,\ldots,s \qquad (6)
\end{aligned}
$$

We obtain a set of optimal weights $w_{1d}^*, \ldots, w_{md}^*, \mu_{1d}^*, \ldots, \mu_{sd}^*$. Then the cross efficiency of any $DMU_j\ (j=1,2,\ldots,n)$ for fuzzy data, using the weights that DMU_d has chosen in

Table 1. A generalized cross efficiency matrix (CEM) for first and end of interval.

Rating DMU	Rated DMU			
	1	2	…	*n*
1	E_{11}^{L} E_{11}^{R}	E_{12}^{L} E_{12}^{R}	…	E_{1n}^{L} E_{1n}^{R}
2	E_{21}^{L} E_{21}^{R}	E_{22}^{L} E_{22}^{R}	…	E_{2n}^{L} E_{2n}^{R}
3	E_{31}^{L} E_{31}^{R}	E_{32}^{L} E_{32}^{R}	…	E_{3n}^{L} E_{3n}^{R}
…	… …	… …	…	…
n	E_{n1}^{L} E_{n1}^{R}	E_{n2}^{L} E_{n2}^{R}	…	E_{nn}^{L} E_{nn}^{R}
Mean	$\bar{E}_{1}^{L}$ $\bar{E}_{1}^{R}$	$\bar{E}_{2}^{L}$ $\bar{E}_{2}^{R}$	…	$\bar{E}_{n}^{L}$ $\bar{E}_{n}^{R}$

models (5) and (6) can be calculated as:

$$E_{dj}^{L} = \sum_{r=1}^{s} \mu_{rd}^{*} y_{rj}^{L} / \sum_{i=1}^{m} w_{id}^{*} x_{ij}^{L}, \; j = 1, 2, \ldots, n$$

$$E_{dj}^{R} = \sum_{r=1}^{s} \mu_{rd}^{*} y_{rj}^{R} / \sum_{i=1}^{m} w_{id}^{*} x_{ij}^{R}, \; j = 1, 2, \ldots, n \tag{7}$$

For DMU the average of all E_{dj}^{L}, E_{dj}^{R} $(d = 1, \ldots, n)$, namely $\bar{E}_{j} = 1/n \sum_{d=1}^{n} E_{dj}$ with same weights can be used as a new efficiency measure for DMU_j and will be referred to as the cross efficiency score for DMU_j.

As seen in Table 1, when we move along the *d*th row of the matrix E of cross efficiencies, each element E_{dj} is the efficiency that DMU_d accords to DMU_j, given the computed weighting scheme described above. The leading diagonal is the special case where DMU_d rates itself. Each of the columns of the cross efficiency matrix (CEM) in Table 1 is then averaged to get a mean cross efficiency measure for each DMU. In fuzzy environment for DMU_j $(j = 1, 2, \ldots, n)$, the average of all $E_{dj} (d = 1, \ldots, n)$, namely $\overline{E_{j}^{L}} = \frac{1}{n} \sum_{d=1}^{n} E_{dj}^{L}$ and $\overline{E_{j}^{R}} = \frac{1}{n} \sum_{d=1}^{n} E_{dj}^{R}$ can be used as a new efficiency measure for DMU_j, and will be referred to as the cross efficiency score for DMU_j (Table 1).

DETERMINATION OF ULTIMATE CROSS EFFICIENCY USING FUZZY DEA GAME MODEL

Here, we will use the fuzzy DEA game model and Shapley value in, for determine the ultimate cross efficiency of each DMU.

Definition of DEA game

As defined in Table 1, matrix $E^{L} = (E_{dj}^{L}) \in R_{+}^{n \times n}$ and $E^{R} = (E_{dj}^{R}) \in R_{+}^{n \times n}$ are the cross efficiency matrix (CEM), and the elements of E_{dj}^{L}, E_{dj}^{R} represents the efficiency that DMU_d accords to DMU_j. Analogously to the models in, we normalize the data set E^{L}, so that it is row-wise normalized,that is $\sum_{p=1}^{n} E_{dp}^{L} = 1, \; \sum_{p=1}^{n} E_{dp}^{R} = 1$. For this purpose, we divide the row $(E_{d1}^{L}, \ldots E_{dn}^{L})$ by the row-sum $\sum_{p=1}^{n} E_{\mathrm{dp}}^{L}$ for $d = 1, \ldots, n$, and denote the *d*th row after row-wise normalizing as $(E_{d1}'^{L}, \ldots E_{dn}'^{L})$. Similarly we can normalize data set E^{R}. After using Charnes-Cooper transformation scheme, the linear program to select the most preferable weights for each DMU can be expressed as follows:

$$\begin{aligned} c^{L}(j) = & \; Max \sum_{d=1}^{n} w_{d}^{j} E_{dj}'^{L} \\ s.t. \; & \sum_{d=1}^{n} w_{d}^{j} = 1 \\ & w_{d}^{j} \geq 0 (\forall d) \end{aligned} \tag{8}$$

$$c^L(j) = Max \sum_{d=1}^{n} w_d^j E'^R_{dj}$$
$$s.t. \sum_{d=1}^{n} w_d^j = 1$$
$$w_d^j \geq 0 (\forall d) \qquad (9)$$

Let the coalition S be a subset of player set N=(1,…,n). The record of the coalition S is defined by

$$E'^L_d(S) = \sum_{j \in s} E'^L_{dj}, \quad d = 1,...,n.$$
$$E'^R_d(S) = \sum_{j \in s} E'^R_{dj}, \quad d = 1,...,n. \quad (10)$$

This coalition aims at obtaining the maximal outcome $c(S)$:

$$c^L(S) = Max \sum_{d=1}^{n} w_d E'^L_d(S)$$
$$s.t. \sum_{d=1}^{n} w_d = 1$$
$$w_d \geq 0 (\forall d) \qquad (11)$$

$$c^R(S) = Max \sum_{d=1}^{n} w_d E'^R_d(S)$$
$$s.t. \sum_{d=1}^{n} w_d = 1$$
$$w_d \geq 0 (\forall d) \qquad (12)$$

The $c(S)$, with $c(\varphi)=0$ defines a characteristic function of the coalition S. thus, we have a game in coalition form with transferable utility, as represented by (N,c). We can easily find that the characteristic function c is a sub-additive, so we consider the opposite side of the game (N,c), which is defined by replacing *max* in (11) and (12) by *min* as follows:

$$d^L(j) = Min \sum_{d=1}^{n} w_d^j E'^L_{dj}$$
$$s.t. \sum_{d=1}^{n} w_d^j = 1,$$
$$w_d^j \geq 0 (\forall d) \qquad (13)$$

$$d^R(j) = Min \sum_{d=1}^{n} w_d^j E'^R_{dj}$$
$$s.t. \sum_{d=1}^{n} w_d^j = 1,$$
$$w_d^j \geq 0 (\forall d) \qquad (14)$$

The optimal value $d(j)$ assures the minimum division that player j can expect from the game. Analogously to the game (N,c), for the coalition $S \subset N$ we define

$$d^L(S) = Min \sum_{d=1}^{n} w_d E'^L_d(S)$$
$$s.t. \sum_{d=1}^{n} w_d = 1,$$
$$w_d \geq 0 (\forall d) \qquad (15)$$

$$d^R(S) = Min \sum_{d=1}^{n} w_d E'^R_d(S)$$
$$s.t. \sum_{d=1}^{n} w_d = 1,$$
$$w_d \geq 0 (\forall d) \qquad (16)$$

Game (N,d) is super-additivethat is we have $d(S \bigcup T) \geq d(S) + d(T)$ for any $S \subset N$ and $T \subset N$ with $S \bigcap T = \phi$.

From the description in 'Nakabayashi and Tone (2006)' we have the following proposition between the games (N,c) and (N,d): $d(S) + c(N \setminus S) = 1, \ \forall S \subsetneq N$, so (N,c) and (N,d) are dual games.

Shapley value of the DEA game

For DEA game (N,d) above, its imputation is a vector $z = (z_1,...,z_n)$ that satisfies the following individuals and grand coalition rationalities:

Individuals rationality: $z_j \geq d(j), j = 1,...,n$

Grand coalition rationality: $\sum_{j=1}^{n} z_j = d(N) = 1$

In this paper, we consider the Shapley value as the representative imputations of the cooperative game above. The Shapley value $\phi_i(d)$ of player I for the game (N,d) is defined by (Liang and Ynag 2009):

$$\phi_i(d) = \sum_{S \setminus i \in S \subset N} \frac{(s-1)!(n-s)!}{n!} \{d(S) - d(S \setminus \{i\})\}, \qquad (17)$$

Where s is the number of members of coalition S. From the introduction above, we can get the conclusion that the games (N,c) and (N,d) are dual games, so they have the same Shapley value.

Table 2. Five DMUs, with two inputs x_1, x_2 and one output y_1 in fuzzy environment.

Variable	X_1	X_2	Y_1
DMU1	(5,7,9)	(4.7,5,5.1)	(5.2,5.4,5.7)
DMU2	(2,5,7)	(6.9,7.2,7.3)	(6.3,6.5,6.8)
DMU3	(2,3,6)	(7.8,8,8.4)	(7.5,7.9,8)
DMU4	(3,5,6)	(10.8,11,11.3)	(4.7,5,5.2)
DMU5	(4,6,7)	(8.7,8.8,9)	(9.9,10,10.2)

Table 3. Values of first input.

Variable	$\alpha=1$	$\alpha=0.5$	$\alpha=0$
DMU1	[7,7]	[6,8.5]	[5,10]
DMU2	[5,5]	[3.5,6]	[2,7]
DMU3	[3,3]	[2.5,4.5]	[2,6]
DMU4	[3,3]	[4,4.5]	[3,6]
DMU5	[5,5]	[4.5,6]	[4,7]

Determination of common weights

Now we return to the subject incorporating our knowledge of imputation $z=(z_1,...,z_n)$ induced by coalitions and allocations of the Shapley value in fuzzy environment. The weight $w=(w_1,...,w_n)\in R^n$ associates with the imputation $z=(z_1,...,z_n)\in R^n$ through $wE'\in R^n$. In an effort to determine *w* in away that wE'^L, wE'^R approximates z as close as possible, we formulate the following LP with variables

$w\in R^n$, $s^+\in R^n$, $s^-\in R^n$, $p\in R$

$$\begin{aligned}
&\text{Min}\, p\\
&\text{s.t. } wE'^L_j+s^+_j-s^-_j=z_j, j=1,2,...,n\\
&\quad w_1+w_2+w_3+..w_m=1,\\
&\quad 0\le s^+_j\le p,\ 0\le s^-_j\le p, j=1,2,...,n,\\
&\quad w_i\ge 0, i=1,2,...,m
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\text{Min}\, p\\
&\text{s.t. } wE'^R_j+s^+_j-s^-_j=z_j, j=1,2,...,n\\
&\quad w_1+w_2+w_3+..w_m=1,\\
&\quad 0\le s^+_j\le p,\ 0\le s^-_j\le p, j=1,2,...,n,\\
&\quad w_i\ge 0, i=1,2,...,m
\end{aligned} \tag{19}$$

Where E'^L_j, E'^R_j denotes the *j*th column vector of E'^L and E'^R respectively. Let an optimal solution of this program be (p^*,w^*,s^{+*},s^{-*}). Then we have two cases:

Case 1: $p^*=0$. In this case, it holds that $z=w^*E'^L$, $z=w^*E'^R$, and so the imputation *z* is explained by the common weight w^*.

All players will accept the solution since it represents the common value judgment corresponding to the cooperative game solution.

Case 2: $p^*>0$. In this case, we have no common weight w^* which can express *z* as $z=w^*E'^L$, $z=w^*E'^R$ perfectly, while the optimal weight vector w^* can approximate the solution of the game within the tolerance p^*.

The common weights, that is the optimal value of *w* in model (18) and (19), can be used to determine the ultimate cross efficiency of each DMU is expressed as follows:

$$\begin{aligned}
E_j^{(L)cross}&=\sum_{d=1}^{n} w_d^* E^L_{dj}, j=1,...n\\
E_j^{(R)cross}&=\sum_{d=1}^{n} w_d^* E^R_{dj}, j=1,...n
\end{aligned} \tag{20}$$

NUMERICAL EXAMPLE

To illustrate the proposed approach above, we consider a simple numerical example given in Table 2 involving five DMUs, with two inputs x_1, x_2 and one output y_1 in fuzzy environment. Suppose that the parameters are characterized by triangular fuzzy numbers as shown in Table 2. By varying parameter α from 0.0 to 1.0 at intervals of 0.5, we construct the fuzzy values of row normalizing and cross efficiency for first and end of intervals (Tables 4 and 5). Cross efficiency values calculated in Tables 6, 7 and 8 for three different parameter by using the introduced model. Comparing the cross efficiency values in three interval shows DMU5 with $\alpha=0.5$ having better efficient value.

Table 4. Values of second input.

Variable	$\alpha=1$	$\alpha=0.5$	$\alpha=0$
DMU1	[5,5]	[4.85,5.05]	[4.7,5.1]
DMU2	[7.2,7.2]	[7.05,7.25]	[2,7]
DMU3	[8,8]	[7.9,8.2]	[7.8,8.4]
DMU4	[11,11]	[10.9,11.15]	[10.8,11.3]
DMU5	[8.8,8.8]	[8.75,8.9]	[8.7,9]

Table 5. Values of output.

Variable	$\alpha=1$	$\alpha=0.5$	$\alpha=0$
DMU1	[5.4,5.4]	[5.3,5.55]	[5.2,5.7]
DMU2	[6.5,6.5]	[6.4,6.665]	[6.3,6.8]
DMU3	[7.9,7.9]	[7.7,7.95]	[7.5,8]
DMU4	[5,5]	[4.85,5.1]	[4.7,5.2]
DMU5	[10,10]	[9.95,10.1]	[9.9,10.2]

Table 6. Cross efficiency in parameter $\alpha=1$.

E	DMU1	DMU2	DMU3	DMU4	DMU5
DMU1	0.9504, 0.9504	0.7916, 0.7916	0.869, 0.869	0.4, 0.4	1, 1
DMU2	0.0703, 0.0703	0.7943, 0.7943	0.8688, 0.8688	0.3999, 0.3999	1, 1
DMU3	0.6079, 0.6070	0.7322, 0.7322	1, 1	0.5004, 0.5004	1, 1
DMU4	0.3300, 0.3300	0.4938, 0.4938	0.5000, 0.5000	0.3165, 0.3165	0.7597, 0.7597
DMU5	0.9474, 0.9474	0.7948, 0.7948	0.8693, 0.8693	0.4001, 0.4001	1, 1

Table 7. Cross efficiency in parameter $\alpha=0.5$.

E	DMU1	DMU2	DMU3	DMU4	DMU5
DMU1	0.9605, 0.7965	0.7982,0.1000	0.8681,0.7026	0.3912,0.3315	1, 0.8225
DMU2	0.6038,0.4193	0.8076,0.7644	1,1	0.4337,0.7280	1,1
DMU3	0.6037,0.4193	0.8076,0.7644	1,1	0.4337,0.7279	1,1
DMU4	0.6044,0.4193	0.7959,0.7644	0.9676,1	0.4203,0.7279	1,1
DMU5	0.9610,0.4191	0.7982,0.7640	0.8681,1	0.3912,0.7274	1,1

Table 8. Cross efficiency in parameter $\alpha=0.5$.

E	DMU1	DMU2	DMU3	DMU4	DMU5
DMU1	0.3509,0.8099	0.9992,0.7039	0.3050,0.5003	0.1381, 0.3334	0.3609, 0.9356
DMU2	0.3314,0.8095	1, 0.7036	1, 0.5001	0.4284, 0.3333	0.5284, 0.9352
DMU3	0.3314,0.3659	1, 0.6232	0.9334,0.6205	0.4275, 0.2954	0.7324, 0.6119
DMU4	0.3315,0.3659	1, 0.6232	1, 0.6205	0.4249, 0.2954	0.7296, 0.6119
DMU5	0.3317,0.9703	1, 0.6002	1, 0.4484	0.4253, 0.6538	0.7300, 0.5226

CONCLUSIONS

In this paper, we have studied the ultimate average cross efficiency scores in fuzzy environment. We eliminate the assumption of average and consider the DMUs as the players in a cooperative game, the characteristic function value of coalitions are defined to compute the Shapley value of each DMU, and the common weights associate with the imputation of the Shapley values are used to determine the ultimate cross efficiency scores. Regarding this subject, we have proposed a method for compute cross efficiency for fuzzy data, and transform fuzzy programs to non fuzzy for solving fuzzy numbers problem by α -level set of the fuzzy numbers.

REFERENCES

Charnes A, Cooper WW, Levin AY, Seaford LM (1994). “Data envelopment analysis: Theory, methodology and applications,” Boston: Kluwer.

Charnes A, Cooper WW, Rhodes E (1978). “Measuring the efficiency of decision Making units.” Eur. J. Oper. Res., 2: 429-444.

Cooper WW, Seiford LM, Tone K (2000). “Data Envelopment Analysis.” Boston: Klawer Academic publishers.

Despotis DK (2002). “Improving the discriminating power of DEA: Focus on globally efficient units.” J. Oper. Res. Soc., 53: 314-323.

Dyson RG, Thannassoulis E (1988). “Reducing weight flexibility in data envelopment analysis.” J. Oper. Res. Soc., 39: 563-576.

Liang Liang JW, Ynag F (2009). “Determination of the weights for the ultimate cross efficiency using Shapley value in cooperative game.” Expert Syst. Appl., 36: 872-876.

Nakabayashi K, Tone K (2006). “Egoist's dilemma: A DEA game.” Int. Manage. Sci., 36: 135-148.

Sakawa M (1993). Fuzzy sets and Interactive Multiobjective Optimization, Plenum Press, New York.