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Review

A short note on inequalities of interval-valued intuitionistic fuzzy matrices

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Inequality, also known as inequation, but unlike equation does not have rich history. After the introduction of equation, inequation captures its popularity in different fields namely algebra, geometry, trigonometry, probability, set theory, fuzzy set theory, logic, calculus etc. Whereas inequalities used in algebra are called algebraic inequalities, the ones used in geometry are called geometric inequalities etc. However, if the same techniques used in solving an equation are used for an inequation, wrong results may be obtained. Unlike equation, it has limited applications. When two quantities or expressions are not the same, then we use inequality and it is written by cross-out equal sign ($\neq$) or < or >. From a logical point of view, there is a difference between $x+1<5$ and $2+1<5$. Sometimes we explain inequalities in linguistic form to describe the social values. All the above discussions on inequalities have been done according to the classical sense. In view of the present situation, it is necessary to extend this concept to fuzzy sense. Many researchers and mathematicians have shown the use of inequalities in fuzzy set, intuitionistic fuzzy set, soft set, rough set etc to describe the imprecise data. In this paper, some results related to inequalities of interval-valued intuitionistic fuzzy matrices with respect to algebraic sum and algebraic product were studied and proven.

Key words: Fuzzy matrix, Interval-valued intuitionistic fuzzy matrix, algebraic sum, algebraic product.

INTRODUCTION

A careful observation of everything around us will help us realize that we basically use equations (knowingly or unknowingly) more often everyday to determine the unknown quantity when another quantity associated to the former one is given. While there is a huge application of equations, many of the simpler equation describe financial concepts, although mathematics is not all about equations only. In many real life situations there exist some problems where equalities between two physical quantities do not hold, leading to the introduction of inequalities, which is a relation between two values when they are different. Inequalities are also used for comparison in the sense of size, area sides of a polygon etc also. A common use of inequalities is to find the minimum or maximum needed to achieve a given goal from a given starting point. It has wide application in social science, economics, probability theory etc. A system of linear inequalities is used in linear programming problem to get the optimal solution. Since inequalities may have infinitely many solutions, to find the solution of a system of inequalities we also use graph. Earlier, inequalities are used in classical sense, but now we extend classical sense into fuzzy sense due to the various types of uncertainties (that is, vague concepts)

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present in real life situation. Zadeh (1965) and Zimmermann (2011) introduced the concept of fuzzy set which is a generalization of classical set (crisp set). Fuzzy set theory is based on fuzzy membership function $\mu : X \to [0,1]$. By the fuzzy membership function, the degree of belongingness of each and every element of a set can be determined. Sometimes though, it is difficult to assign fuzzy membership function which leads to the introduction of L-fuzzy set (Goguen, 1967), intuitionistic fuzzy set (Atanassov, 1986; Shyamal and Pal, 2004), interval-valued fuzzy set (Gorzalczany, 1987) etc. Atanassov (1989) and Zhenhua et al. (2011) introduced the concept of interval-valued intuitionistic fuzzy set (IVIFS) which is an extension of intuitionistic fuzzy set (IFS). Garg and Kumar (2019) proposed linguistic interval-valued Atanassov intuitionistic fuzzy sets and their application in group decision making problems. Garg and Kumar (2018) focused on multi attribute decision-making for interval-valued intuitionistic fuzzy set environment based on set pair analysis (SPA). Garg (2018) proposed some new interactive geometrical aggregator operator under the interval-valued intuitionistic fuzzy environment for multi-criteria decision making problems. Garg and Arora (2018) proposed a nonlinear-programming methodology for multi-attribute decision-making problem by using interval-valued intuitionistic fuzzy soft sets. Kumar and Garg (2018) discussed the TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment.

Matrices play important roles in different fields like engineering, economics, game theory, computer science etc. However, we cannot successfully use classical matrices because various uncertainties are present in daily life problems. Those types of problems can be solved using fuzzy matrix (Thomason, 1977) which deals with only membership values and each membership value belonging to the interval [0,1]. As there is no scope of non-membership value in fuzzy matrix, it has been extended to intuitionistic fuzzy matrix (Khan et al., 2002) and interval-valued fuzzy matrix (Shyamal and Pal, 2006). Practically, it is difficult to measure the membership or non-membership value as a point. We need an interval for a membership value and also for a non-membership value which leads to the introduction of interval-valued intuitionistic fuzzy matrix (Pal and Khan, 2005). This paper studied and proved some inequalities connected with algebraic operations of interval-valued intuitionistic fuzzy matrices.

PRELIMINARIES

Here we will discuss some definitions and results.

Definition 1: Let X be a non-empty set. Then a *fuzzy set* A is a set having the form $A=\{(x,\mu_A(x)) : x \in X\}$, where the function $\mu_A : X \to [0,1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x \in X$ (Zadeh, 1965; Zimmermann, 2011).

Definition 2: Let X be a non-empty set. An *intuitionistic fuzzy set* A in X is an object $A=\{(x,\mu_A(x),\gamma_A(x)) : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x)+\gamma_A(x) \le 1$ for each $x \in X$ (Atanassov, 1986; Shyamal and Pal, 2004).

Definition 3: An interval-valued fuzzy set A over X is given by a function $\mu_A(x)$ where $\mu_A : X \to Int([0,1])$, the set of all sub-intervals of unit interval, that is, for every $x \in X$, $\mu_A(x)$ is an interval within $[0,1]$ (Gorzalczany, 1987).

Definition 4: An interval- valued intuitionistic fuzzy set A over universe set X is defined as $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x) : X \to Int([0,1])$ and $\gamma_A(x) : X \to Int([0,1])$ (where $Int([0,1])$ is the set of all closed intervals of $[0,1]$) are functions such that the condition: $\forall x \in X$, $0 \le sup\mu_A(x)+sup\gamma_A(x) \le 1$ is satisfied (Atanassov, 1989; Zhenhua et al., 2011).

FUZZY MATRICES AND THEIR GENERALIZATIONS

Definition 1: Let $A = [a_{ij}]$ and $B= [b_{ij}]$ be two fuzzy matrices of order (Hashimoto, 1983; Thomason, 1977). Then

1. $A \oplus B = [a_{ij}+b_{ij}-a_{ij}.b_{ij}]$
2. $A \Box B = [a_{ij}.b_{ij}]$
3. $A \vee B = [\max(a_{ij}, b_{ij})]$
4. $A \wedge B = [\min(a_{ij}, b_{ij})]$
5. $A^{(\alpha)} = [a_{ij}^{(\alpha)}]$ (upper α-cut fuzzy matrix)

6. $A_{(\alpha)} = \left[a_{(\alpha)ij} \right]$ (lower α-cut fuzzy matrix)

7. $A \le B$ *iff* $a_{ij} \le b_{ij}, \forall\, i, j$

8. $A^{C} = \left[1 - a_{ij} \right]$ (complement of A)

9. $A^{T} = \left[a_{ji} \right]$ (transpose of A)

Definition 2: An intuitionistic fuzzy matrix (IFM) is a matrix of pairs A= $\left(\left\langle \mu_{a_{ij}}, \nu_{a_{ij}} \right\rangle \right)$ of a non negative real numbers satisfying $\mu_{a_{ij}} + \nu_{a_{ij}} \le 1$, for all i, j (Khan et al., 2002),.

Definition 3: An interval-valued intuitionistic fuzzy matrix (IVIFM) A of order $m \times n$ is defined as A= $\left[x_{ij}, \left\langle a_{ij\mu}, a_{ij\nu} \right\rangle \right]_{m \times n}$ where $a_{ij\mu}$ and $a_{ij\nu}$ are both subsets of[0,1] and denoted by $a_{ij\mu} = \left[a_{ij\mu L}, a_{ij\mu U} \right]$ and $a_{ij\nu} = \left[a_{ij\nu L}, a_{ij\nu U} \right]$ which maintains the condition $a_{ij\mu U} + a_{ij\nu U} \le 1$ for $i = 1, 2, \ldots\ldots, m$ and $j = 1, 2, \ldots\ldots, n$ (Pal and Khan, 2005).

Definition 4: Let A and B be two IVIFMs such that A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U} \right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U} \right\rangle \right]$ and B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U} \right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U} \right\rangle \right]$ (Pal and Khan, 2005). Then

$$A \vee B\,[\text{or } (A+B)] = \left\langle \left[\max\left(a_{ij\mu L}, b_{ij\mu L} \right), \max\left(a_{ij\mu U}, b_{ij\mu U} \right) \right], \left[\min\left(a_{ij\nu L}, b_{ij\nu L} \right), \min\left(a_{ij\nu U}, b_{ij\nu U} \right) \right] \right\rangle$$

$$A \wedge B\,[or\ (A.B)] = \left\langle \left[\min\left(a_{ij\mu L}, b_{ij\mu L} \right), \min\left(a_{ij\mu U}, b_{ij\mu U} \right) \right], \left[\max\left(a_{ij\nu L}, b_{ij\nu L} \right), \max\left(a_{ij\nu U}, b_{ij\nu U} \right) \right] \right\rangle$$

Definition 5: Let A and B be two IVIFMs such that A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U} \right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U} \right\rangle \right]$ and B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U} \right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U} \right\rangle \right]$ (Pal and Khan, 2005). Then

$(i) A \oplus B = \left\langle \left[a_{ij\mu L} + b_{ij\mu L} - a_{ij\mu L}.b_{ij\mu L}, a_{ij\mu U} + b_{ij\mu U} - a_{ij\mu U}.b_{ij\mu U} \right], \right|$ is called the algebraic sum of A and B.

$(ii)\ A \odot B = \left\langle \left[a_{ij\mu L}.b_{ij\mu L}, a_{ij\mu U}.b_{ij\mu U} \right], \left[a_{ij\nu L} + b_{ij\nu L} - a_{ij\nu L}.b_{ij\nu L}, a_{ij\nu U} + b_{ij\nu U} - a_{ij\nu U}.b_{ij\nu U} \right] \right\rangle$ is called the algebraic product of A and B.

Definition 6: Complement of an IVIFM A is denoted by A^{c} and is defined by $A^{c} = \left[\left\langle a_{ij\nu L}, a_{ij\nu U} \right\rangle, \left\langle a_{ij\mu L}, a_{ij\mu U} \right\rangle \right]$ (Pal and Khan, 2005),

Definition 7: Let A and B be two IVIFMs such that A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U} \right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U} \right\rangle \right]$ and B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U} \right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U} \right\rangle \right]$ (Pal and Khan, 2005). Then we write $A \le B$ if $a_{ij\mu L} \le b_{ij\mu L}$, $a_{ij\mu U} \le b_{ij\mu U}$ and $a_{ij\nu L} \ge b_{ij\nu L}$, $a_{ij\nu U} \le b_{ij\nu U}$.

Definition 8: An IVIFM is a complete null IVIFM if all the elements are $\langle [0,0],[0,0] \rangle$ and it is denoted by O (Pal and Khan, 2005).

PROOF OF SOME RESULTS OF INEQUALITIES ON IVIFMS

Here, we prove some results of inequalities using algebraic operations on IVIFMs

Theorem 1: Let A, B and C be three IVIFMs of same order. If $A \le C$ and $B \le C$, then prove that $A \vee B \le C$.

Proof: Let A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U} \right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U} \right\rangle \right]$, B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U} \right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U} \right\rangle \right]$ and C= $\left[\left\langle c_{ij\mu L}, c_{ij\mu U} \right\rangle, \left\langle c_{ij\nu L}, c_{ij\nu U} \right\rangle \right]$ be three IVIFMs of same order $m \times n$.

If $A \le C$, then $a_{ij\mu L} \le c_{ij\mu L}, a_{ij\mu U} \le c_{ij\mu U}, a_{ij\nu L} \ge c_{ij\nu L}$ and $a_{ij\nu U} \le c_{ij\nu U} \forall\, i, j$ and $B \le C$, then $b_{ij\mu L} \le c_{ij\mu L}$, $b_{ij\mu U} \le c_{ij\mu U}, b_{ij\nu L} \ge c_{ij\nu L}$ and $b_{ij\nu U} \le c_{ij\nu U} \forall\, i, j$.

Now, $\max\left[a_{ij\mu L}, b_{ij\mu L}\right] \le c_{ij\mu L}, \max\left[a_{ij\mu U}, b_{ij\mu U}\right] \le c_{ij\mu U}$

and $\min\left[a_{ij\nu L}, b_{ij\nu L}\right] \ge c_{ij\nu L}, \min\left[a_{ij\nu U}, b_{ij\nu U}\right] \ge c_{ij\nu U}$

Thus, $A \vee B \le C$

Theorem 2: Let A, B and C be three IVIFMs of same order. If $A \le B,$ then prove that $\mathrm{A} \vee \mathrm{C} \le \mathrm{B} \vee \mathrm{C}$.

Proof: Let A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U}\right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U}\right\rangle\right]$, B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U}\right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U}\right\rangle\right]$ and C= $\left[\left\langle c_{ij\mu L}, c_{ij\mu U}\right\rangle, \left\langle c_{ij\nu L}, c_{ij\nu U}\right\rangle\right]$ be three IVIFMs of same order $m \times n$.

If $A \le B$, then $a_{ij\mu L} \le b_{ij\mu L}, a_{ij\mu U} \le b_{ij\mu U}$ and $a_{ij\nu L} \ge b_{ij\nu L}, a_{ij\nu U} \le b_{ij\nu U}$.

$$\text{Now, } \max\left[a_{ij\mu L}, c_{ij\mu L}\right] \le \max\left[b_{ij\mu L}, c_{ij\mu L}\right], \max\left[a_{ij\mu U}, c_{ij\mu U}\right] \le \max\left[b_{ij\mu U}, c_{ij\mu U}\right]$$
$$\min\left[a_{ij\nu L}, c_{ij\nu L}\right] \ge \min\left[b_{ij\nu L}, c_{ij\nu L}\right] \text{ and } \min\left[a_{ij\nu U}, c_{ij\nu U}\right] \le \max\left[b_{ij\nu U}, c_{ij\nu U}\right] \forall\, i, j.$$

So $\mathrm{A} \vee \mathrm{C} \le \mathrm{B} \vee \mathrm{C}$

Theorem 3: Let A, B and C be three IVIFMs of same order and if $C \le A$ and $C \le B,$ then prove that $C \le A \wedge B.$

Proof: Let A= $\left[\left\langle a_{ij\mu L}, a_{ij\mu U}\right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U}\right\rangle\right]$, B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U}\right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U}\right\rangle\right]$ and C= $\left[\left\langle c_{ij\mu L}, c_{ij\mu U}\right\rangle, \left\langle c_{ij\nu L}, c_{ij\nu U}\right\rangle\right]$ be three IVIFMs of same order $m \times n$.

If $C \le A$, then $c_{ij\mu L} \le a_{ij\mu L}, c_{ij\mu U} \le a_{ij\mu U}, c_{ij\nu L} \ge a_{ij\nu L}$ and $c_{ij\nu U} \le a_{ij\nu U} \forall\, i, j$.

and $C \le B$, then $c_{ij\mu L} \le b_{ij\mu L}, c_{ij\mu U} \le b_{ij\mu U}, c_{ij\nu L} \ge b_{ij\nu L}$ and $c_{ij\nu U} \le b_{ij\nu U} \forall\, i, j$.

Now, $c_{ij\mu L} \le \min\left[a_{ij\mu L}, b_{ij\mu L}\right], c_{ij\mu U} \le \min\left[a_{ij\mu U}, b_{ij\mu U}\right]$

and $c_{ij\nu L} \ge \max\left[a_{ij\nu L}, b_{ij\nu L}\right], c_{ij\nu U} \ge \max\left[a_{ij\nu U}, b_{ij\nu U}\right]$

Thus, $C \le A \wedge B$

Theorem 4: If A, B and C are three IVIFMs of same order and if $A \le B, A \le C$ and $B \wedge C = O,$ then $A = O.$

Proof: If $A \le B$, then $a_{ij\mu L} \le b_{ij\mu L}, a_{ij\mu U} \le b_{ij\mu U}$ and $a_{ij\nu L} \ge b_{ij\nu L}, a_{ij\nu U} \le b_{ij\nu U}$.

Again, If $A \le C$, then $a_{ij\mu L} \le c_{ij\mu L}, a_{ij\mu U} \le c_{ij\mu U}$, $a_{ij\nu L} \ge c_{ij\nu L}$ and $a_{ij\nu U} \le c_{ij\nu U} \forall\, i, j$.

By Theorem 3, $A \le B \wedge C$.

Since $B \wedge C = O$ then $A = O$.

Theorem 5: If A, B and C are three IVIFMs of same order and if $A \le B,$ then $\mathrm{A} \wedge \mathrm{C} \le \mathrm{B} \wedge \mathrm{C}$.

Proof: If $A \le B$, then $a_{ij\mu L} \le b_{ij\mu L}, a_{ij\mu U} \le b_{ij\mu U}$ and $a_{ij\nu L} \ge b_{ij\nu L}, a_{ij\nu U} \le b_{ij\nu U}$.

$$\text{Now, } \min\left[a_{ij\mu L}, c_{ij\mu L}\right] \le \min\left[b_{ij\mu L}, c_{ij\mu L}\right], \min\left[a_{ij\mu U}, c_{ij\mu U}\right] \le \min\left[b_{ij\mu U}, c_{ij\mu U}\right]$$
$$\max\left[a_{ij\nu L}, c_{ij\nu L}\right] \ge \max\left[b_{ij\nu L}, c_{ij\nu L}\right] \text{ and } \max\left[a_{ij\nu U}, c_{ij\nu U}\right] \le \max\left[b_{ij\nu U}, c_{ij\nu U}\right] \forall\, i, j.$$

So, $\mathrm{A} \wedge \mathrm{C} \le \mathrm{B} \wedge \mathrm{C}$.

Theorem 6: Let A, B and C be three IVIFMs of same order

(i) *If* $(A \wedge B) \vee (A \wedge C) = A,$ *then* $A \le (B \vee C).$

(ii) *If* $(A \vee B) \wedge (A \vee C) = A,$ *then* $A \ge (B \wedge C).$

Proof:(i)

Let A=$\left[\left\langle a_{ij\mu L}, a_{ij\mu U}\right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U}\right\rangle\right]$, B= $\left[\left\langle b_{ij\mu L}, b_{ij\mu U}\right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U}\right\rangle\right]$ and C=$\left[\left\langle c_{ij\mu L}, c_{ij\mu U}\right\rangle, \left\langle c_{ij\nu L}, c_{ij\nu U}\right\rangle\right]$ be three IVIFMs of same order $m\times n$.

Then,

$$(A\wedge B)\vee(A\wedge C)=A$$

$$\Rightarrow\left\langle\left[\min\left(a_{ij\mu L}, b_{ij\mu L}\right), \min\left(a_{ij\mu U}, b_{ij\mu U}\right)\right], \left[\max\left(a_{ij\nu L}, b_{ij\nu L}\right), \max\left(a_{ij\nu U}, b_{ij\nu U}\right)\right]\right\rangle \vee \left\langle\left[\min\left(a_{ij\mu L}, c_{ij\mu L}\right), \min\left(a_{ij\mu U}, c_{ij\mu U}\right)\right], \left[\max\left(a_{ij\nu L}, c_{ij\nu L}\right), \max\left(a_{ij\nu U}, c_{ij\nu U}\right)\right]\right\rangle$$

$$=\left\langle\left[a_{ij\mu L}, a_{ij\mu U}\right], \left[a_{ij\nu L}, a_{ij\nu U}\right]\right\rangle$$

$$\Rightarrow\left\langle\left[\max\left(\min\left(a_{ij\mu L}, b_{ij\mu L}\right), \min\left(a_{ij\mu L}, c_{ij\mu L}\right)\right), \max\left(\min\left(a_{ij\mu U}, b_{ij\mu U}\right), \min\left(a_{ij\mu U}, c_{ij\mu U}\right)\right)\right], \left[\min\left(\max\left(a_{ij\nu L}, b_{ij\nu L}\right), \max\left(a_{ij\nu L}, c_{ij\nu L}\right)\right), \min\left(\max\left(a_{ij\nu U}, b_{ij\nu U}\right), \max\left(a_{ij\nu U}, c_{ij\nu U}\right)\right)\right]\right\rangle$$

$$=\left\langle\left[a_{ij\mu L}, a_{ij\mu U}\right], \left[a_{ij\nu L}, a_{ij\nu U}\right]\right\rangle$$

$$\Rightarrow\left\langle\left[\min\left(a_{ij\mu L}, \max\left(b_{ij\mu L}, c_{ij\mu L}\right)\right), \min\left(a_{ij\mu U}, \max\left(b_{ij\mu U}, c_{ij\mu U}\right)\right)\right], \left[\max\left(a_{ij\nu L}, \min\left(b_{ij\nu L}, c_{ij\nu L}\right)\right), \max\left(a_{ij\nu U}, \min\left(b_{ij\nu U}, c_{ij\nu U}\right)\right)\right]\right\rangle$$

$$=\left\langle\left[a_{ij\mu L}, a_{ij\mu U}\right], \left[a_{ij\nu L}, a_{ij\nu U}\right]\right\rangle$$

On comparing both sides, we have

$$\min\left(a_{ij\mu L}, \max\left(b_{ij\mu L}, c_{ij\mu L}\right)\right)=a_{ij\mu L}$$

$$\min\left(a_{ij\mu U}, \max\left(b_{ij\mu U}, c_{ij\mu U}\right)\right)=a_{ij\mu U}$$

$$\max\left(a_{ij\nu L}, \min\left(b_{ij\nu L}, c_{ij\nu L}\right)\right)=a_{ij\nu L}$$

$$\max\left(a_{ij\nu U}, \min\left(b_{ij\nu U}, c_{ij\nu U}\right)\right)=a_{ij\nu U}$$

From above we can write

$$a_{ij\mu L}\le\max\left(b_{ij\mu L}, c_{ij\mu L}\right), a_{ij\mu U}\le\max\left(b_{ij\mu U}, c_{ij\mu U}\right)$$

$$a_{ij\nu L}\ge\min\left(b_{ij\nu L}, c_{ij\nu L}\right), a_{ij\nu U}\ge\min\left(b_{ij\nu U}, c_{ij\nu U}\right)$$

which implies $A\le(B\vee C)$.

Therefore, Proof (ii) is similar to Proof (i).

Theorem 7: If A, B and C are three IVIFMs of same order and if $A\le B$ and $B\wedge C=O$, then $A\wedge C=O$.

Proof: By Theorem 5, the proof is straight forward.

Theorem 8: Let A and B be two IVIFMs of same order. Then $A\le B$ iff $B^c\le A^c$.

Proof: If $A\le B$,then $a_{ij\mu L}\le b_{ij\mu L}, a_{ij\mu U}\le b_{ij\mu U}$ and $a_{ij\nu L}\ge b_{ij\nu L}, a_{ij\nu U}\le b_{ij\nu U}$

If we rewrite the same as

$b_{ij\nu L}\le a_{ij\nu L}, b_{ij\nu U}\le a_{ij\nu U}, b_{ij\mu L}\ge a_{ij\mu L}, b_{ij\mu U}\ge a_{ij\mu U}$ which implies $B^c\le A^c$, then the converse is also true.

Theorem 9: Let A and B be two IVIFMs of same order. If $A\le B^c$, then $B\le A^c$.

Proof: Let A=$\left[\left\langle a_{ij\mu L}, a_{ij\mu U}\right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U}\right\rangle\right]$, B=$\left[\left\langle b_{ij\mu L}, b_{ij\mu U}\right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U}\right\rangle\right]$

If $A\le B^c$, then $a_{ij\mu L}\le b_{ij\nu L}, a_{ij\mu U}\le b_{ij\nu U}, a_{ij\nu L}\ge b_{ij\mu L}, a_{ij\nu U}\ge b_{ij\mu U}$

If we rewrite these, then we have $B\le A^c$.

Theorem 10: Let A and B be two IVIFMs of same order. If $A^c\le B$,then $B^c\le A$.

Proof: The proof is similar to Theorem 9.

Theorem 11 (De Morgans Law): Let A and B be two IVIFMs of same order. Then
(i) $(A\vee B)^c=A^c\wedge B^c$ (ii) $(A\wedge B)^c=A^c\vee B^c$

Proof: (i) Let A=$\left[\left\langle a_{ij\mu L}, a_{ij\mu U}\right\rangle, \left\langle a_{ij\nu L}, a_{ij\nu U}\right\rangle\right]$, B=$\left[\left\langle b_{ij\mu L}, b_{ij\mu U}\right\rangle, \left\langle b_{ij\nu L}, b_{ij\nu U}\right\rangle\right]$

$$A\vee B=\left\langle\left[\max\left(a_{ij\mu L},b_{ij\mu L}\right),\max\left(a_{ij\mu U},b_{ij\mu U}\right)\right],\left[\min\left(a_{ij\nu L},b_{ij\nu L}\right),\min\left(a_{ij\nu U},b_{ij\nu U}\right)\right]\right\rangle$$

$$\left(A\vee B\right)^{c}=\left\langle\left[\min\left(a_{ij\nu L},b_{ij\nu L}\right),\min\left(a_{ij\nu U},b_{ij\nu U}\right)\right],\left[\max\left(a_{ij\mu L},b_{ij\mu L}\right),\max\left(a_{ij\mu U},b_{ij\mu U}\right)\right]\right\rangle=A^{c}\wedge B^{c}$$

Therefore, Proof (ii) is similar to Proof (i).

Theorem 12: If A, B and C are three IVIFMs of same order, then we have the following:

(i) $A\le A$ (Reflexivity)
(ii) $A\le B, B\le C\Rightarrow A\le C$ (Transitivity)
(iii) $A\vee A=A,\ A\wedge A=A$ (Idempotency)
(iv) $A\vee B=B\vee A,\ A\wedge B=B\wedge A$ (Commutativity)
(v) $A\vee(B\vee C)=(A\vee B)\vee C,\ A\wedge(B\wedge C)=(A\wedge B)\wedge C$ (Associativity)
(vi) $A\vee(B\wedge C)=(A\vee B)\wedge(A\vee C),\ A\wedge(B\vee C)=(A\wedge B)\vee(A\wedge C)$ (Distributivity)
(vii) $A\vee\left(A\wedge B\right)=A,\ A\wedge\left(A\vee B\right)=A$ (Absorption).

Proof: Proof of above laws are obvious and we can verify these by choosing suitable A, B and C.

Theorem 13: For any two IVIFMs A and B of same order

(i) $A\vee\left(A\oplus B\right)=\left(A\oplus B\right)$
(ii) $A\wedge\left(A\oplus B\right)=A$

Proof: (i) $A\oplus B=\left\langle\left[a_{ij\mu L}+b_{ij\mu L}-a_{ij\mu L}.b_{ij\mu L},a_{ij\mu U}+b_{ij\mu U}-a_{ij\mu U}.b_{ij\mu U}\right],\left[a_{ij\nu L}+b_{ij\nu L}-a_{ij\nu L}.b_{ij\nu L},a_{ij\nu U}+b_{ij\nu U}-a_{ij\nu U}.b_{ij\nu U}\right]\right\rangle$

$A\vee\left(A\oplus B\right)=$

$$\left\langle\left[\max\left(a_{ij\mu L},a_{ij\mu L}+b_{ij\mu L}-a_{ij\mu L}.b_{ij\mu L}\right),\max\left(a_{ij\mu U},a_{ij\mu U}+b_{ij\mu U}-a_{ij\mu U}.b_{ij\mu U}\right)\right],\left[\min\left(a_{ij\nu L},a_{ij\nu L}+b_{ij\nu L}-a_{ij\nu L}.b_{ij\nu L}\right),\min\left(a_{ij\nu U},a_{ij\nu U}+b_{ij\nu U}-a_{ij\nu U}.b_{ij\nu U}\right)\right]\right\rangle$$

$$=\left\langle\left[a_{ij\mu L}+b_{ij\mu L}-a_{ij\mu L}.b_{ij\mu L},a_{ij\mu U}+b_{ij\mu U}-a_{ij\mu U}.b_{ij\mu U}\right],\left[a_{ij\nu L}+b_{ij\nu L}-a_{ij\nu L}.b_{ij\nu L},a_{ij\nu U}+b_{ij\nu U}-a_{ij\nu U}.b_{ij\nu U}\right]\right\rangle$$

$=A\oplus B$

Therefore, Proof (ii) is similar to Proof (i).

Theorem 14: For any two IVIFMs A and B of same order

(i) $A\vee(A\odot B)=A$
(ii) $A\wedge(A\odot B)=A\odot B$

Proof: Using the algebraic product property, we can easily prove the above two results.

Theorem 15: If A, B and C are three IVIFMs of same order and if *A* ≥ *B* and *C* ≥ *D,* then

$(i)\ A\vee C\ge B\vee D$
$(ii)\ A\wedge C\ge B\wedge D$

Proof: Using the algebraic product property, we can easily prove the above two results.

CONCLUSION

The main objective of this article is to generalize the concept of inequalities related to intuitionistic fuzzy matrices into inequalities related to interval-valued intuitionistic fuzzy matrices and prove some results based on it. In the future, there exists a scope for applying these results in real life situation by using authentic data. By reducing IVIFSM into IFM, different types of inequalities need to be established. Hence, the concept of inequalities is also extended for neutrosophic set.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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