Mathematical model of ecological disturbances in the oil rich Niger Delta region of Nigeria

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An important class of interactions between biological species is the stability. We investigate the ecological, biological and economical impacts of petroleum exploration on the Niger Delta region of Southern Nigeria. A modified reaction-diffusion model describing the spread of the predators into the prey populations is adopted to establish the interaction between the parties involved. An attempt was made to explore dispersion and patchiness of the residents resulting from these issues. Evidence from this study showed that continual invasions of the prey species by the predator species could lead to extinction of the former in the long run. Strategies are suggested on how to ensure peace and coexistence in the region.

Key words: Reaction-diffusion, dispersion, patchiness, stability.

INTRODUCTION

Some of the most severe ecological problems being responsible for the extinction of indigenous species, sustainable disturbances of ecosystems and economic damage are biological invasions. Recently, many factors have been identified that affect the spread and the pattern of the spread of an introduced species, either empirically and/or theoretically such as resource availability, spatial heterogeneity (Murray, 2003), environmental borders (Kellett et al., 2001), predation (Fagan and Bishop, 2000; Owen and Lewis, 2001), dispersal/transport effects (Hengeveld, 1989; Clark et al., 2001; Hastings et al., 2005).

The effect of age structure on the dynamics of a predator-prey system has been studied by many investigators, among them are Auslander et al. (1974), Bennett and Free (1978), Gurtin and Levine (1979), Cushing and Saleem (1982).

The main goal of this paper is to find conditions under which aggressive oil exploration, spillage and sea incursion can be responsible for decline or extinction of indigenous species. Therefore, a reaction-diffusion model describing the spread of the predators into the prey populations is introduced in the next section. Reaction-diffusion models are essential analytically tractable tools for understanding invasion dynamics (Okubo and Levin, 2001; Hilker et al., 2005). The well-known and largely used reaction-diffusion models have been used to handle variables. In this paper, we formulate these models by taking into account the random economical, ecological and biological effects of oil exploration on various ages of the dwellers in the Niger Delta region of Southern Nigeria.

The oil prospecting companies in the Niger Delta region of Nigeria that use weaker people and the resources of the area for their own advantage are regarded as the predators, while the people of Niger region, who we wish to protect from foreign predators are considered the prey species. How can we determine mathematically the coexistence equilibrium state of the predator-prey model? Should the prey species become extinct, what kinds of biotic environment shall we then have?

The outline of this paper is as follows:- First the mathematical model governing the interaction between the predators and prey populations are described in section 2. Section 3 motivates and introduces the age-structured population model and its ecological significance to this work. Some mathematical concepts and results useful in the study of age-structured models are discussed. In section 4, we explore the relationships between dispersion and patchiness of the prey species. We show that under certain conditions both concepts can induce instability. Finally, the results are applied in section 5 to the study of instability being experienced in the Niger Delta Region of Southern Nigeria.

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The model

Let $u(t, a) \geq 0, v(t, a) \geq 0$, denote the age-dependent densities of prey and predator species respectively as functions of time, $t$, and age $a$. Then the basic conservation law of population is given by the parabolic partial differential equation:

$$
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial a} &= \varepsilon \frac{\partial^2 u}{\partial x^2} + ru(x,t,a) f(u(x,t,a)) h(u(x,t,a)) \\
\frac{\partial v}{\partial t} + \frac{\partial v}{\partial a} &= D \frac{\partial^2 v}{\partial x^2} + \gamma v(x,t,a) h(u(x,t,a)) - d v(x,t,a)
\end{align*}
$$

Where $D$ is the diffusion coefficient ($m^2/yr$), $r$ is the intrinsic rate of increase per year; $u$ and $v$ are invariant under an age translation. That is to say, that $u$ and $v$ have no explicit age dependence. Equivalently, we say that a change to a reference frame in which all particle position coordinates are unchanged but age coordinates are changed ($a \to a + \tau$) leaves the values of $u, v$ unchanged (Killingbeck and Cole, 1971).

The model arises from adding an age variable to the non-linear reaction-diffusion predator-prey model.

$$
\begin{align*}
\frac{\partial u}{\partial t} &= \varepsilon \frac{\partial^2 u}{\partial x^2} + ru(x,t,a) f(u(x,t,a)) - f v(x,t,a) h(u(x,t,a)) \\
\frac{\partial v}{\partial t} &= D \frac{\partial^2 v}{\partial x^2} + \gamma v(x,t,a) h(u(x,t,a)) - d v(x,t,a)
\end{align*}
$$

(2.2)

which described the classical predator-prey dynamics (Owen and Lewis, 2001). The per capita growth rate is $f(u)$. The strictly monotone functional and numerical response $f(h(u))$ and $\gamma v(h(u))$ have identical form with $g(h(u))$ describing conversion efficiently (namely, a conversion factor transforming each unit of biomass into predator newborn). We define $K$ to be the carrying capacity for the prey in the absence of predation and we assume that, in the absence of prey, predator individual needs to adapt (Aiyelo and Tchuenche, 2005) to the new environment, so that $f(K) = h(0) = 0$. The parameters $r$ and $\varepsilon$ allow us to scale the continuous functions $f$ and $h$ so that $\max_{u \in [0,K]} f(u) = h(K) = 1$. $f$ is the growth function while $h$ represents the functional response. The parameter $\varepsilon$, $0 < \varepsilon < 1$, indicates that the prey disperse much more slowly than predators. More details on the biological interpretation of the parameters can be found in Dercole et al. (2003).

We impose some conditions on the growth function $f$ and response function $h$ as follows:

$$
\begin{align*}
f(1) &= 0, \quad \max_{u \in [0,1]} f(u) = 1, \quad h(0) = 0, \quad h(1) = 1.
\end{align*}
$$

The model (2.1) has an extinction steady state, $(u, v) = (0, 0)$, a prey only steady state, $(1, 0)$ -- corresponding to primary succession before predators have arrived, and a coexistence steady state,

$$
\begin{align*}
u_s &= h^{-1}(\delta), \quad v_s = \frac{u_s f(u_s)}{h(u_s)} = \frac{u_s f(u_s)}{\delta}
\end{align*}
$$

Denoting the coexistence steady state by $(u_s, v_s)$, equation (2.2) becomes

$$
\begin{align*}
u_s f(u_s) - \phi v_s h(u_s) &= 0 \quad (2.3a) \\
\gamma v_s - \delta v_s &= 0 \quad (2.3b)
\end{align*}
$$

From Equation 2.3b

$$
\gamma v_s = \delta v_s
$$

$$
h(u_s) = \frac{\delta}{\gamma} = \delta^*, \quad \frac{\delta}{\gamma} = \frac{\delta}{\gamma}^*
$$

so that,

$$
u_s = h^{-1}(\delta^*)
$$

Substituting this value into Equation 2.3a we have

$$
u_s f(u_s) - \phi v_s h(u_s) = 0$$

Therefore

$$v_s = \frac{r u_s f(u_s)}{\phi h(u_s)}$$

Let $K = \frac{\phi}{r}$, hence $v_s = \frac{u_s f(u_s)}{\phi h(u_s)}$

since $\max_{[0,K]} f(u) = h(K) = 1$.

The relevant range of predator mortality rate $\delta^*$ for a non-negative coexistence equilibrium is $0 \leq \delta^* \leq 1$.

Stability of the prey carrying capacity in the absence of predation and stability of the coexistence equilibrium require that

$$
f'(1) < 0, \quad g'(u_s) < 0 \quad (2.4)
$$

where

$$g(u) = \frac{u f(u)}{h(u)}.$$

Because $h$ is strictly monotonic with root at $u = 0$, $g$ is defined for $u > 0$ (Owen and Lewis, 2001).

For mathematical tractability and convenience let the space variable $x$ of a Cartesian coordinate system be discarded. This means we are not taking into account the space dependence of this model. We assume that the prey begins its dispersion at the origin. 

**Lemma 2.1**

A necessary and sufficient condition for stability of the
coexistence equilibrium state \((u_s, v_a)\) of the system (2.1) is that \(h(u) \neq \frac{\delta}{\gamma}\).

**Proof:** Suppose the coexistence equilibrium state \((u_s, v_a)\) of the system (2.1) is stable, then the system at equilibrium becomes

\[
\begin{align*}
\dot{u}_a &= r f(u) - \phi v h(u) \\
\dot{v}_a &= \gamma v h(u) - \delta v
\end{align*}
\]

and this yields

\[
\begin{pmatrix}
\dot{u}_a \\
\dot{v}_a
\end{pmatrix} =
\begin{pmatrix}
r f(u) & -\phi v h(u) \\
0 & \gamma v h(u) - \delta
\end{pmatrix}
\begin{pmatrix}
u \\
v
\end{pmatrix}
\]

For a nontrivial solution of this system

\[
|M| = \begin{vmatrix}
rf(u) & -\phi v h(u) \\
0 & \gamma v h(u) - \delta
\end{vmatrix} = rf(u)[\gamma v h(u) - \delta] \neq 0.
\]

This result holds if, and only if, in views of the definition of \(r\) and the fact that \(f\) and \(h\) are continuous functions

\[
rf(u) \neq 0 \Rightarrow f(u) \neq 0 \quad \text{and} \quad \gamma v h(u) - \delta \neq 0 \Rightarrow h(u) \neq \frac{\delta}{\gamma}. \quad \text{Thus, } h(u) \neq \frac{\delta}{\gamma} \text{ as was to be shown.}
\]

Assume \(h(u) = \frac{\delta}{\gamma}\), then (2.4) is a linear system of algebraic equation in the components \(u\) and \(v\). This is solvable if and only if, the matrix \(M\) is invertible, that is det \(M \neq 0\). This condition implies that

\[
(u, v)^T = M^{-1}(u_a, v_a)^T
\]

has no solution as a linear system, except, perhaps the trivial solution \((u, v)^T = (0, 0)^T\). This means that the system (2.4.1) should not possess any solution except the trivial solution. However, if \(h(u) = \frac{\delta}{\gamma}\) and \(f(u) \neq 0\), then a solution exists and stability is easily determined using its eigenvalues.

The following theorem provides existence conditions for the stability of the equilibrium state \((u_s, v_a)\).

**Theorem 2.1**

In the absence of diffusion, the coexistence equilibrium state \((u_s, v_a)\) of equation (2.4) is stable.

**Proof**

We can compute the eigenvalues of the matrix \(M\) above as follows:

\[
\begin{vmatrix}
r f(u) - \lambda & -\phi v h(u) \\
0 & \gamma v h(u) - \delta - \lambda
\end{vmatrix} = (r f(u) - \lambda)[\gamma v h(u) - \delta - \lambda]
\]

\[
= -\lambda v h(u) + r g(u) h(u) - r g(u) + \lambda \delta - r f(u) + \lambda^2
\]

\[
= \lambda^2 + [\delta - r f(u) - \gamma v h(u)] \lambda + r f(u) [\gamma v h(u) - \delta]
\]

Thus, the characteristic equation is

\[
\lambda^2 + [\delta - r f(u) - \gamma v h(u)] \lambda + r f(u) [\gamma v h(u) - \delta] = 0.
\]

We assume \(f(u) \neq \frac{\delta - \gamma v h(u)}{r}\) by considering the fact that the predator only steady state is stable.

\[
\lambda_{1,2} = -\frac{[\delta - r f(u) - \gamma v h(u)] \pm \sqrt{[\delta - r f(u) - \gamma v h(u)]^2 - 4r f(u)[\gamma v h(u) - \delta]}}{2}
\]

But

\[
[\delta - r f(u) - \gamma v h(u)]^2 = [\delta - r f(u)]^2 - 2r h(u)[\delta - r f(u)] + \gamma^2 [h(u)]^2
\]

\[
= \delta^2 - 2rf(u)\delta + r^2[f(u)]^2 - 2\gamma h(u)\delta + 2\gamma r h(u)f(u) + \gamma^2[h(u)]^2
\]

\[
= \delta^2 - 2rf(u)\delta + 2\gamma h(u)\delta + 2\gamma r h(u)f(u) + \gamma^2[h(u)]^2 + r^2[f(u)]^2
\]

and

\[
-4rf(u)[\gamma h(u) - \delta] = -4rf(u)[\gamma h(u) + 4rf(u)]
\]

Let \(\Delta = \delta^2 - 2rf(u)\delta + 2\gamma h(u) + 2\gamma r h(u) + \gamma^2[h(u)]^2 + r^2[f(u)]^2\)

\[
= \delta^2 + 2\gamma h(u) - 2\gamma h(u) + \gamma^2[h(u)]^2 + r^2[f(u)]^2
\]

\[
\Delta = \delta^2 + 2[r f(u) - \gamma h(u)] \delta - 4rf(u)[\gamma h(u) + 2rf(u) + \gamma^2 h(u) + r^2[f(u)]^2
\]

\[
-2\gamma r f(u) h(u)
\]

\[
= \delta^2 + 2[r f(u) - \gamma h(u)] \delta - 4rf(u)[\gamma h(u) + \gamma^2 h(u) + r^2[f(u)]^2
\]

\[
= [\delta + (\gamma h(u) + r f(u)]^2 - 2\delta(\gamma h(u) + r f(u))
\]

\[
+ 2[r f(u) - \gamma h(u)] \delta - 4rf(u)[\gamma h(u)
\]

\[
= [\delta + \gamma h(u) + r f(u)]^2 - 2\delta(\gamma h(u) + r f(u))
\]

\[
= -2\delta h(u) - 4rf(u) h(u)
\]

\[
= [\delta + \gamma h(u) + r f(u)]^2 - 4\delta h(u) - 4rf(u) h(u)
\]
Assume that $\Delta > 0$, then the solution is stable if the $\lambda_i$s, for $i = 1, 2$ are negative.

$$\Rightarrow - \frac{[\delta - r f(u) - \gamma h(u)] \pm \sqrt{[\delta - r f(u) - \gamma h(u)]^2 - 4rf(u)[\gamma h(u) - \delta]}}{2} < 0 $$

By a simple algebraic manipulation, we distinguish two cases.

**Case (i)**

$\left[\delta - rf(u) - \gamma h(u)\right] > 0 \left[\delta - rf(u) - \gamma h(u)\right] - 4rf(u)[\gamma h(u) - \delta]

Which yield?

$h(u) > \frac{\delta}{\gamma}$

**Case (ii)**

$\left[\delta - rf(u) - \gamma h(u)\right] < 0 \left[\delta - rf(u) - \gamma h(u)\right] - 4rf(u)[\gamma h(u) - \delta]

\Rightarrow \gamma h(u) < \delta$ or $h(u) > \frac{\delta}{\gamma}$

Thus, if $h(u) < \frac{\delta}{\gamma}$, or $h(u) > \frac{\delta}{\gamma}$, the necessary and sufficient condition, stated in Lemma 2.1 is satisfied and thus $h(u) = \frac{\delta}{\gamma}$ which proves the theorem.

This result shows that in the absence of diffusive terms, the region $0 \leq u \leq 1, v \geq 0$ is an invariant set for the system in (2.4). The prey density will remain at carrying capacity for all time $t$ and space $x$ values.

**An age-structured population model**

A general predator-prey model in which the prey population is assumed to have an age structure which significantly affects its fecundity is considered. The model equations are derived from the general McKendrick equations for age structured populations. In this theory, it is assumed that the population of reproducing individuals (here taken as the prey population) can be described by a density $n(a,t)$ of time $t$ and age $a$, measured on the same scale, the integral $\int_{a_1}^{a_2} n(a,t) \, da$ of which yields the total population between ages $a_1$ and $a_2$ at time $t$. Thus, the total population (of prey species) of all ages at time $t$ is

$$P(t) = \int_{a_1}^{a_2} n(a,t) \, da.$$ 

Since the population under consideration in the Niger Delta region did not move, so we discard the diffusion parameter in equation 2.1.

If the initial conditions are ignored, the McKendrick model for a one species form is

$$n_a + n_t = - \mu n, \quad a > 0$$

$$n(0,t) = \int_0^\infty f(a,t)n(a,t)\, da$$

for $-\infty < t < +\infty$ where $\mu$ is the (per unit prey density) death rate and $f$ is the (per unit prey density) fecundity rate.

The first order partial differential equation (3.1) accounts for removals from the prey populations, which are assumed to be by death only. We take $\mu$ to be a positive constant independent of $a$, $t$ and $n$.

Equation (3.2) accounts for births into the prey population (which is assumed to be the only way to enter that population) by means of age-specific fecundity rate function $f(a,t)$ which is assumed to be a function of time only implicitly through a dependence on the total population size, $P$, of the prey and also $R$ of a predator species. In particular, we assume that $f$ has the form $f = b \beta(a)h(P,R)$.

The positive constant $b > 0$ is called the birth modulus and the function $\beta(a)$ is called maturation function. The maturation function $\beta$ clearly describes the effects of age on fecundity. It will be assumed to be continuously differentiable and bounded for $a \geq 0$ and to satisfy

$$\beta(a) \geq 0, \beta(0) = 0, \beta^*(\mu) = \int_0^\infty \beta(a)e^{-\mu a} \, da < +\infty \quad (3.3)$$

The condition (3.3) implies that the net reproductive rate at equilibrium

$$\int_0^\infty f(a,t)e^{-\mu a} \, da = bh(P,R)\beta^*(\mu)$$

in view of lack of a $t$ function here is finite. The function $h$, which will be referred to as the fecundity response function, is assumed to be twice continuously differentiable for $P,R \geq 0$ and to satisfy

$$h(P,R) \geq 0, \quad h(0,0) = 0,$

that is the nonnegative prey fecundity rate drops to zero when both population disappear. We will derive an integro differential equation for $P = P(t)$.

Assume $n(+) = 0$ for all $t$, that is the density in age classes a drops to zero as $a \rightarrow +\infty$. It must be noted that when the death rate does not depend explicitly on age, this assumption is interpreted to represent temporal uniformity or homogeneity of the environment, and is re-
ferred to as age invariant. Integration of (3.1) yields

\[ n(a, t) = B(t - a) e^{-\mu a} \]  

where \( B(t) := n(0, t) \) which, when substituted into (3.2), yields

\[ B(t) = \int_0^\infty f(a, t) B(t - a) e^{-\mu a} da \]  

On the other hand, an integration of (3.1) from \( a = 0 \) to \( +\infty \) using Laplace Transform method and rearrangement gives

\[ P'(t) + \mu P(t) = B(t) \]  

Which, after substitution into (3.5) and integration by parts, gives

\[ P'(t) + \mu P(t) = \int_0^\infty f(a, t) P(t - a) e^{-\mu a} da . \]  

A solution of (3.7) defines the density \( n \) by means of (3.6) and (3.4). It is obvious that there exists a temporal uniformity since the death rate does not depend explicitly on age but on congestion and insufficient food due to limited available productive areas.

**Dispersion and Patchiness**

There are two problems of primary importance associated with the colonization of the Niger Delta region by prey species, namely, dispersion and patchiness. Because we are interested in a stability problem with a subsequently introduced predator in the wake of the invasion, we will show in this section that dispersion in the region gives rise to instability.

**Theorem 4.1 (Sufficient condition for stability)**

The equilibrium point \( y \equiv 0 \) of the system

\[ y'(t) = f(t, y(t)), \quad y(\tau) = \xi \]  

is stable if there exists a \( C^1 \) function \( P \) which is positive definite and such that its derivative along the solution of (4.1.1) is negative semi-definite or identically zero that is \( P'(t, y) \leq 0 \) or \( P'(t, y) \equiv 0 \).

**Proof**

Let \( \tau \geq 0 \) and \( \epsilon > 0 \) be given. Since \( P \) is continuous and \( P(\tau, 0) = 0 \) there exists a \( \delta = \delta(\tau, \epsilon) > 0 \) such that

\[ P(\tau, \xi) < \psi(\epsilon) \]  

for \( \psi \in K(\text{class } \mathcal{K} \text{ in the sense of W. Hahn}) \)

and \( P(t, y) \geq \psi(\|y\|) \quad \forall \quad 0 < \|y\| < \tau \) whenever \( \|\xi\| < \delta \), since

\[ P'(t, y) \leq 0, \quad \text{we have} \]

\[ P(t, y(t, \tau, \xi)) \leq P(t, y(t, \tau, \xi)) \leq P(t, y(t, \tau, \xi)) < \psi(\epsilon) \]

\[ \Rightarrow \quad \|y(t, \tau, \xi)\| < \epsilon \]

\[ \Rightarrow \quad \text{the trivial solution is stable.} \]

**Effects of dispersal on the stability of predator-prey system**

In this section, we investigate the effects of dispersal on the linear stability of a predator-prey system with functional response. Consider the interactions of two dispersive species constituting a predator-prey system in a linear one-dimensional finite habitat \( (0 \leq x \leq L, L \text{ being the length}) \) with interspecific interaction of prey and functional response. The dynamical equation governing their evolution with dispersal along the \( x \)-direction can be written as follows (Nallaswamy and Shukla, 1982).

\[
\frac{\partial N_1}{\partial t} = N_1 \left( a_1 - a_{11}N_1 - \frac{a_{12}N_2}{1 + \alpha N_1} \right) + \frac{\partial}{\partial x} \left( D_1 \frac{\partial N_1}{\partial x} \right) \]

\[
\frac{\partial N_2}{\partial t} = N_2 \left( -a_2 + \frac{a_{21}N_1}{1 + \alpha N_1} \right) + \frac{\partial}{\partial x} \left( D_2 \frac{\partial N_2}{\partial x} \right) \]

(4.2.1)

Where \( N_1(x, t), N_2(x, t) \) are the population distributions of prey and predator species respectively at time \( t \), and \( D_1, D_2 \) the variable coefficients of dispersal, which are positive. The interaction coefficients \( a_1, a_2, a_{11}, a_{12}, a_{21}, \) and \( \alpha \) are also positive constants. The factor \((1 + \alpha N_1)^{-1}\) represents the Type II functional response in the model, and \( \alpha \) determines the strength of the response.

Suppose we neglect the space variable \( x \), and assume that interactions of the prey and predators begin at the origin. Then, system (4.2.1) at equilibrium becomes

\[ 0 = N_1 \left( a_1 - a_{11}N_1 - \frac{a_{12}N_2}{1 + \alpha N_1} \right) \]

\[ 0 = N_2 \left( -a_2 + \frac{a_{21}N_1}{1 + \alpha N_1} \right) \]

(4.2.2)

The nontrivial positive equilibrium state \((N_1^*, N_2^*)\) of the system (4.2.2) is obtained by solving
\begin{align*}
a_1 - a_1 N_1^* + \frac{a_{12} N_2^*}{1 + \alpha N_1^*} &= 0 \quad (4.2.3)_1 \\
- a_2 + \frac{a_{21} N_1^*}{1 + \alpha N_1^*} &= 0 \quad (4.2.3)_2
\end{align*}

\begin{align*}
a_{i1} N_1^* + \frac{a_{12} N_2^*}{1 + \alpha N_1^*} &= a_1 \quad (4.2.4)_1 \\
\frac{a_{21} N_1^*}{1 + \alpha N_1^*} &= a_2 \quad (4.2.4)_2
\end{align*}

From (4.2.4)_2

\begin{align*} 
a_{21} N_1^* &= a_2 + a_2 \alpha N_1^* \\
N_1^* (a_21 - a_2 \alpha) &= a_2
\end{align*}

which yields

\[ N_1^* = \frac{a_2}{a_21 - a_2 \alpha}, \quad a_21 - a_2 \alpha \neq 0 \quad (4.2.5) \]

Since only positive solutions are biologically relevant, \( a_{21} - a_2 \alpha \) must be strictly greater than zero. From (4.2.3)_1,

\[
\frac{a_{12} N_2^*}{1 + \alpha N_1^*} = a_1 - a_{i1} N_1^*
\]

That is,

\[
a_{12} N_2^* = a_1 \left(1 + \alpha N_1^*\right) - a_{i1} N_1^* \left(1 + \alpha N_1^*\right) \\
= a_1 + a_1 \alpha N_1^* - a_{i1} N_1^* - a_{i1} \alpha N_1^2
\]

By substituting equation (4.2.5) into (4.2.6) we get

\[
a_{12} N_2^* = a_1 + \frac{a_{12} a_1 \alpha}{a_21 - a_2 \alpha} - \frac{a_{12} a_2}{a_21 - a_2 \alpha} - \frac{a \alpha a_1 a_2}{(a_21 - a_2 \alpha)^2} \\
= a_1 (a_21 - a_2 \alpha)^3 + a_1 a_2 (a_21 - a_2 \alpha) - a_{i1} a_2 (a_21 - a_2 \alpha - \alpha a_1 a_2)^2 \\
= a_1 (a_21 - a_2 \alpha)^3 + a_1 a_2 (a_21 - a_2 \alpha)^2 + a_1 a_2 (a_21 - a_2 \alpha) - a_{i1} a_2 (a_21 - a_2 \alpha - \alpha a_1 a_2) \\
= a_1 a_2 (a_21 - a_2 \alpha)^3 + a_1 a_2 (a_21 - a_2 \alpha)^2 + a_1 a_2 (a_21 - a_2 \alpha) - a_{i1} a_2 (a_21 - a_2 \alpha - \alpha a_1 a_2)
\]

Provided \( a_1 (a_21 - a_2 \alpha) > a_2 \alpha > 0 \quad (4.2.7) \)

It is noted from (4.2.5) and (4.2.7) that both \( N_1^* \) and \( N_2^* \) are positive provided

\[ a_1 (a_21 - a_2 \alpha) > a_2 \alpha > 0 \] implying that \( a_21 - a_2 \alpha > 0 \).

We consider the perturbation of the equilibrium state \((N_1^*, N_2^*)\) by writing

\[
N_1(x, t) = N_1^* + n_1(x, t) > 0 \\
N_2(x, t) = N_2^* + n_2(x, t) > 0
\]

in the system (4.2.1) and making use of (4.2.3), we obtain the following system of nonlinear equations:

\[
(i)^* \frac{\partial n_1}{\partial t} = \left( N_1^* + n_1 \right) \\
\left( a_{i1} N_1^* + \frac{a_{12} N_2^*}{1 + \alpha N_1^*} - a_{i1} \left( N_1^* + n_1 \right) \right) - a_{i2} \left( N_2^* + n_2 \right) \\
+ \frac{\partial}{\partial x} \left( D_1 \frac{\partial n_1}{\partial x} \right)
\]

\[
(ii)^* \frac{\partial n_2}{\partial t} = \left( N_2^* + n_2 \right) \left( - \frac{a_{21} N_2^*}{1 + \alpha N_1^*} + \frac{a_{22} N_2^* + n_2}{1 + \alpha N_1^* + \alpha n_1} \right)
\]

* Since by adopting (4.2.8)

\[
\frac{\partial n_1}{\partial t} = \frac{\partial N_1}{\partial t} = 0 \quad \text{and} \quad \frac{\partial n_2}{\partial t} = \frac{\partial N_2}{\partial t} = 0
\]

Simplifying (i)
where \( R = (1 + \alpha N_1^* \right) (1 + \alpha N_1^* + \alpha n_1) > 0 \)

\[
\begin{align*}
\frac{\partial n_2}{\partial t} &= (N_2^* + n_2) \\
&= \frac{N_2^* + n_2}{R} \left[ -a_2 N_1^* + a_2 (n_1 + N_1^*) \right] + \frac{\partial}{\partial x} \left( D_2 \frac{\partial n_2}{\partial x} \right) \\
&= \frac{N_2^* + n_2}{R} \left[ -a_2 N_1^* - a_2 R n_1 + a_2 (n_1 + N_1^*) \right] + \frac{\partial}{\partial x} \left( D_2 \frac{\partial n_2}{\partial x} \right)
\end{align*}
\]

where \( R = (1 + \alpha N_1^* \right) (1 + \alpha N_1^* + \alpha n_1) > 0 \)

\[
\begin{align*}
\frac{\partial n_1}{\partial t} &= \frac{N_1^* + n_1}{R} \left[ -a_1 R N_1^* + a_1 R n_1 + a_1 R (n_1 + N_1^*) \right] + \frac{\partial}{\partial x} \left( D_1 \frac{\partial n_1}{\partial x} \right) \\
&= \frac{N_1^* + n_1}{R} \left[ -a_1 R N_1^* - a_1 R n_1 + a_1 R (n_1 + N_1^*) \right] + \frac{\partial}{\partial x} \left( D_1 \frac{\partial n_1}{\partial x} \right)
\end{align*}
\]

Bringing together the derived equations we have:

\[
\begin{align*}
\frac{\partial n_1}{\partial t} &= -e n_1 - f n_2 + \frac{\partial}{\partial x} \left( D_1 \frac{\partial n_1}{\partial x} \right), \\
\frac{\partial n_2}{\partial t} &= g n_1 + \frac{\partial}{\partial x} \left( D_2 \frac{\partial n_2}{\partial x} \right)
\end{align*}
\] (4.2.11)

Where

\[
\begin{align*}
e &= \frac{N_1^*}{1 + \alpha N_1^*} \left( a_1 (a_{21} + a_2 \alpha) - a_1 \alpha (a_{21} - a_2 \alpha) \right) \\
f &= \frac{a_2 N_1^*}{1 + \alpha N_1^*} \left( (1 + \alpha N_1^*) \right)
\end{align*}
\] (4.2.12)

Depend upon the strength of the functional response and \( n_1(x, t) \), \( n_2(x, t) \) are small perturbed population distributions.

The systems (4.2.9) and (4.2.11) are associated with the following boundary conditions (Rosen, 1975).

(i) Flux conditions:

\[
\begin{align*}
\frac{\partial n_1}{\partial x} (0, t) &= \frac{\partial n_1}{\partial x} (L, t) = 0 \\
\frac{\partial n_2}{\partial x} (0, t) &= \frac{\partial n_2}{\partial x} (L, t) = 0
\end{align*}
\] (4.2.13a)

These conditions imply that there is no migration of the species across the boundary of the habitat.

(ii) Reservoir conditions:

\[
\begin{align*}
n_1(0, t) &= n_1(L, t) = 0 \\
n_2(0, t) &= n_2(L, t) = 0
\end{align*}
\] (4.2.13b)

These conditions suggest that the species populations remain at equilibrium level on the boundary of the habitat.

**Prey patchiness**

Primary succession is an inherently spatial process because successful early colonists give rise to local patches of vegetation that facilitates colonization via dispersal of their progeny (Fagan and Bishop, 2000). In this section we consider the case when prey is spatially patchy in the distribution. Here, we are concerned with the growth of prey species from infancy through premature stage before metamorphosis into mature (adult) human. The aim of the mathematical model to be presented here is to investigate whether small-scale patchiness can influence the spread/growth of prey species which rely on farming and fishing for food.

In ecology the structure of dispersion of biotic population is conventionally classified into three categories:
frequency distributions may be random, uniform (regular), or clumped (contiguous). This classification is based on the frequency distribution of individual organisms in the statistical sense. If the organisms disperse according to a purely random process their frequency distribution should be represented by the binomial distribution (Ito, 1963).

**Predator-prey encounter model in a patchy environment**

Motivated by the work of Beyer and Nielsen (1996), our main assumptions are that predators encounter patches with a Poisson distribution while encounter with individual prey within patches are considered as binomial processes. Assume that prey species are found only in disjoint identical patches, and that each individual predator not within a patch encounters patches according to a Poisson process with rate parameter \( \alpha \), \( P(\alpha) \). This implies that a predator not inside a patch at time \( t \) has a probability \( \alpha \Delta t + o(\Delta t)^2 \) of encountering a patch in the small time interval \([t, t + \Delta t] \). As the rate of patches entering any given patch is the average rate at which predators leave it, and define \( F \) as the fraction of volume devoid of prey. Suppose the whole volume taken up by prey patches, so that \( 1 - F \) is the proportion of volume devoid of prey. Suppose that predators entering any given patch equals the average rate at which predators leave it, and define \( \beta \) as parameter of the Poisson process governing the event of a predator within a patch, characterized by

\[
1 - F = F \beta \quad (4.3.1)
\]

That is

\[
\beta = \frac{1 - F}{F} \alpha \quad (4.3.2)
\]

Now, consider interactions between predator and prey within the patch. Assume that a predator, once inside a patch, encounters prey as a Poisson process with rate \( \gamma \), \( P(\gamma) \). If the predator encounters a prey, that is if a prey falls within the predators’ perceptive range then it attempts to capture it. This process takes a finite amount of time, which will be assumed to be a constant \( \tau \) in this model. While within a patch, however, a predator which does not encounter prey has a probability of leaving the patch, characterized by \( \beta \), \( p(\beta) \) as above. In this way, the event of a predator being within a patch, while encounter is not taking place is \( p(\gamma+\beta) \). A predator-prey encounter occurs with probability \( \frac{\gamma}{\gamma+\beta} \) and probability \( \frac{\beta}{\gamma+\beta} \) of no encounter and an exit of predator from the patch. The occurrence of an event within the patch can be described using a binomial model of ‘success’ probability of \( \frac{\gamma}{\gamma+\beta} \) corresponding to prey encounter, and probability of failure \( \frac{\beta}{\gamma+\beta} \) corresponding to leaving the patch. The mean number of prey encountered during one encounter with a patch is then \( E[K] \), where

\[
E[K] = \sum_{k=0}^{n} k p(K = k) \quad \text{where no. of prey encountered} = k
\]

\[
= \sum_{k=0}^{n} \binom{n}{k} \left( \frac{\gamma}{\gamma+\beta} \right)^k \left( \frac{\beta}{\gamma+\beta} \right)^{n-k}
\]

\[
= \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \left( \frac{\gamma}{\gamma+\beta} \right)^k \left( \frac{\beta}{\gamma+\beta} \right)^{n-k}
\]

\[
= \sum_{k=0}^{n} \frac{n!}{(k-1)!(n-k)!} \left( \frac{\gamma}{\gamma+\beta} \right)^k \left( \frac{\beta}{\gamma+\beta} \right)^{n-k}
\]

Let \( y = (k-1) \). Then

\[
E[K] = \sum_{y=0}^{n-1} \frac{(n-1)!}{y!(n-y-1)!} \left( \frac{\gamma}{\gamma+\beta} \right)^{y+1} \left( \frac{\beta}{\gamma+\beta} \right)^{n-y-1}
\]

\[
= n \left( \frac{\gamma}{\gamma+\beta} \right) \quad (4.3.3)
\]

Since by the binomial probability distribution theorem the summation on the right hand side equals 1. The solution given by Equation (4.3.3) above differs from that obtained by Pitchford and Brindley (2001) in that, instead of

\[
M_p = \sum_{n=0}^{m} np \quad \text{(no. of prey encountered = n)}
\]

\[
= \frac{\gamma}{\beta}
\]

Used in Pitchford and Brindley (2001) the following procedure was utilized:

The probability that a predator encounters a patch is assumed to be Poisson with parameter \( p(\gamma) \), while, once a predator enters a patch, the probability that an individual prey is consumed by the predator is further assumed to be binomial, \( b(k; n, \frac{\gamma}{\gamma+\beta}) \) hence, in this model if we
let $n \to \infty$ and $\frac{\gamma}{\gamma + \beta}$ be very small, then the binomial probability distribution above tends to the Poisson probability distribution.

The mean time for a predator to encounter a patch, encounter prey within it, and then exit the patch is $T_p$, made up of the mean time to encounter a patch, the mean time $n \left( \frac{\gamma}{\gamma + \beta} \right)$ taken to encounter and attempt to capture prey before leaving the patch;

$$T_p = \frac{1}{\alpha} + n \left( \frac{\gamma}{\gamma + \beta} \right) \tau + \frac{1}{\beta}$$

$$= \beta(\gamma + \beta) + n \alpha \beta \gamma + \alpha(\gamma + \beta)$$

$$= \frac{(\alpha + \beta)(\gamma + \beta) + n \alpha \beta \gamma \tau}{\alpha \beta (\gamma + \beta)}$$

The mean encounter rate for an individual predator is therefore $R$ prey per unit time, where

$$R = \frac{E[K]}{T_p}$$

$$= \frac{\alpha \beta (\gamma + \beta)}{(\alpha + \beta)(\gamma + \beta) + n \alpha \beta \gamma \tau} \cdot \frac{n \gamma}{\gamma + \beta}$$

$$= \frac{n \alpha \beta \gamma}{(\alpha + \beta)(\gamma + \beta) + n \alpha \beta \gamma \tau}$$

By adopting equation (4.3.2), we obtain

$$R = \frac{n \alpha \gamma \left( \frac{1 - F}{F} \right) \alpha}{[\alpha + \left( \frac{1 - F}{F} \right) \alpha]\left[ \gamma + \left( \frac{1 - F}{F} \right) \alpha \right] + n \alpha \gamma \tau \left( \frac{1 - F}{F} \right) \alpha}$$

$$= \frac{n \alpha \gamma \left( \frac{1 - F}{F} \right)}{\left[ \alpha F + (1 - F) \alpha \right] \left[ \gamma F + (1 - F) \alpha \right] + n \alpha \gamma \tau \left( \frac{1 - F}{F} \right)}$$

$$= \frac{n \alpha \gamma \left( \frac{1 - F}{F} \right)}{(\alpha F + \alpha - \alpha F)(\gamma F + (1 - F) \alpha) + n \alpha \gamma \tau \left( \frac{1 - F}{F} \right)}$$

$$= \frac{n \alpha \gamma \left( \frac{1 - F}{F} \right)}{\alpha F + \alpha(1 - F)[1 + n \gamma \tau F]}$$

Observe that (4.3.4) depends on $\alpha$, so the overall predator-prey encounter rate is dependent upon the rate at which patches are encountered in this model: the patchiness manifests itself also in its effect on the parameter $F$.

Expression (4.3.4) shows that $R \to \frac{n \gamma \alpha(1 - F) F}{\gamma F + \alpha(1 - F)}$ as $\tau \to 0$, as one would expect in the case where predator-prey encounters are instantaneous. According to this model, (4.3.4) shows that the prey encounter rate depends on the number of prey encountered $n$, patches encounter rate $\alpha$, predator-prey encounter rate within patches $\gamma$, fraction of the whole volume taken up by prey patches $F$, and the handling time $\tau$.

An alternative approach to obtain mean encounter rate in a homogeneous environment using spatially averaged prey density is given in Pitchford and Brindley (2001).

The above model is, of course, rather idealized. Prey patches are assumed to exist as coherent entities on timescales longer than those associated with predator foraging, while the mechanisms acting to form the patches are assumed not to affect the predators.

Niger Delta origin populations are well known to exhibit spatial patchiness at length scales ranging from tens of kilometer to less than 100 metres. Whilst the main driving force for these people’s patchiness is probably physical, and can be cultural, there are other biological reasons why patchiness may be favourable. For instance, by observation, it promotes nucleus family awareness among different age groups.

Of course, an oil exploration company would ideally wish to remain within an oil rich patch indefinitely, but its difficulty lies in detecting whether or not the inhabitants within a patch, together with legislative restrictions, would
make this impossible ideal realizable. The interaction between the prey and predator populations has been negative due largely to the ecological impact of oil spill and sea incursion arising from the activities of the oil exploration companies. This has resulted in compelling dispersal of the prey species searching for conducive habitat yielding continuous instability in the oil rich region. The kidnapping of expatriate Shell workers who were subsequently released on Monday, January 30, 2006 points to the potential dangers in the oil rich Niger Delta region of Nigeria.

Since the prey species disperse according to a purely random process their distribution was represented by the binomial distribution. It was observed, however, that patchiness in the region favourably promotes cultural balances among different age groups.

**DISCUSSION**

Most of the research on predator-prey models are based on insects, plankton, cannibalism, bacteria species and many others (Blasio et al., 1982; Levis and Segel, 1976; Okubo, 1980; Fagan and Bishop, 2000; Levin, 1981; Thompson et al., 1982) but the ecological impacts of the Multinational Corporations activities on the dwellers of the oil producing areas, Niger Delta region of Southern Nigeria is basically untouched and provides the motivation for this paper. This section is concerned with the best possible ways of eradicating restiveness and kidnapping of expatriate oil workers arising from acute shortage of social amenities, and disruption of fishing and farming activities in the region. These problems stemmed from occasional oil spill (causing pollution) and sea incursion (rendering fishing activities impossible).

We begin by investigating the nature of the fixed points of the interacting species model (2.3). Two similar species of animals compete with each other in an environment where their common food supply is limited. There are several possible outcomes to this competition:

i.) species 1 survives and species 2 becomes extinct.

ii.) Species 2 survives and species 1 becomes extinct.

iii.) Two species coexist.

iv.) Both species become extinct.

Each of these outcomes can be represented by an equilibrium state of populations u and v of the two species. The differential equations used to model the dynamics of u and v are therefore required to have four isolated fixed points.

In the model (2.3), a necessary condition for the coexistence of the two species is that (2.3) has a fixed point with both populations greater than zero. Such point is commonly referred to as the equilibrium point. The point at which the populations are settled, here there is neither growth nor decline. At equilibrium if the linear equations have solution, such solutions must be unique. Hence, in this case a fixed point can only arise in (2.3) if the linear equations

\[ r_u u + \phi v u = u_a \]

\[ r_v v + \delta v = v_a \]

have a solution. We will assume that this system has a unique solution in the positive quadrant of uv – plane. The fixed point is then given by

\[ \left( \frac{u_a + \phi(u)v_a}{r_f(u)(\phi(u) - \delta)}, \frac{-rf(u)v_a}{rf(u)(\phi(u) - \delta)} \right) \]

Where \( h(u) \neq \frac{\delta}{\gamma} \).

The Niger Delta region comprises nine states, namely, Abia, Akwa Ibom, Bayelsa, Cross River, Delta, Edo, Imo, Ondo and Rivers with 185 local government areas and the population of about 20 million. It is the third largest wetlands in the world after Mississippi and the Poratana. It covers an area of about 70,000 square kilometers and is noted for its peculiar difficult riverine terrain (NDDC, 2003). The region is rich in natural resources including oil and gas, cash crops including oil palm, rubber, cocoa, coconut, a diversity of aquatic resources and fertile land which supports year round agriculture. Since 1959 when the first commercial discovering of oil in Nigeria by Shell D’Arvey Exploration Parties was made, other oil producing companies have been attracted to the region.

Recently frequent oil spill occur from corroded oil pipe lines due to age. Fire outbreak have caused large scale environmental damages to the Niger Delta region resulting in loss of agricultural land through pollution, petroleum exploration installations e.g. oil rig. This problem has forced most people (who are predominantly fishermen and farmers) from the unproductive areas to search for suitable places of abode in other unaffected parts of the ecosystem.

Every age-group has its fair share of the untold difficulties ravaging the Niger Delta region. Some of these include lack of good roads, potable water, good education, electricity, health care, empowerment, jetties etc.

Great numerical impact of predators occur at oil rich areas, yet at extremely high density all prey species will grow so slowly, due to effective food limitation arising from unproductive farmland and polluted fishing areas, that they can (if there is no alternative source of livelihood) be driven into extinction. The net result is that effective selection on growth rate will occur only within certain region of the parameter space, depending upon the corresponding population density. Therefore, the question now arises; what will be the definitive solution to poverty and other problems that plague the Niger Delta region of Nigeria? The following suggestions provide lasting help.
i.) It is instructive to solve these problems by beginning with the solution of their immediate needs. The Federal Government of Nigeria (FGN) should look into the area of peace, security, development and poverty alleviation by considering a people-oriented development strategy.

ii.) A committee of environmentalist should be setup to study the area and make recommendation on bioremediation strategies and targets.

iii.) The Federal Government of Nigeria (FGN) should transform the Niger Delta region from a state of conflict to a state of continuity by enacting environmental legislation ordering the oil producing companies to operate a joint venture account. Funds from this account should be carefully managed to build schools; primary, secondary, vocational and tertiary institutions in their areas of operation. In addition these schools should provide free education at all levels to the indigenous students. Those who successfully complete their academic pursuit and are found suitable for the companies’ work should be automatically employed. This should replace the irregular offer of scholarship award to a privileged few.

iv.) However, in the sense of this research, the oil companies should establish good relationship with their host communities to forestall further crises in the region. The various oil exploration companies should honour the Global Memorandum of Understanding signed with their host communities.

v.) More ecologists are required to study and proffer workable approaches to tackle ecological ad environmental problems: prevention and control of oil spillage, gas flaring and environmental pollution. Our universities should develop a one-semester course in ecology to be taught to upper level undergraduate and graduate students. The objective should be to teach the basic mathematics of ecological models; and to teach how simulation and mathematical approaches could be combined to achieve a better understanding of an ecological system than either could achieve alone.

vi.) More enlightenment programmes should be put in place to educate the militant groups to dialogue with appointed government agencies in their quest for emancipation, empowerment and resources control. This will reduce or totally eradicate confrontational attempts to achieve this.

vii.) As a short term measure, the Federal Government of Nigeria (FGN) should institute a renewable ten year master plan of the Niger Delta area for infrastructural development. A substantial Oil Derivation Fund should be allocated and prudently managed to actualize this dream.

viii.) Further research areas should aim at the identification and development of alternative uses of crude oil fallouts (spillages) to enhance agricultural development. The Federal Government of Nigeria (FGN) should establish some designated laboratories to be used as project centres for the raw materials in the areas of food processing and pharmaceuticals. The Government can also explore strategic implication of depletion, conservation or stock-piling of our natural resources.

ix.) Finally, the rural areas can be rendered more attractive through the rural electrification scheme and the establishment of some industries in these areas. This request becomes necessary due to compelling needs for capacity building in our country, Nigeria.

REFERENCES


