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Full Length Research Paper

Asymptotic behavior of solutions of nonlinear delay differential equations with impulse

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This paper studies the asymptotic behavior of solutions of the second-order nonlinear delay differential equations with impulses: $(r(t)x'(t))' - p(t)x'(t) + \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) + f(t) = 0, \quad t \neq t_k,$ $x(t_k^+) - x(t_k) = a_k x(t_k), x'(t_k^+) - x'(t_k) = b_k x'(t_k), \quad k \in Z^+$ **and some sufficient conditions are obtained.**

Key words: Asymptotic behavior, second-order nonlinear delay differential equation, impulses.

INTRODUCTION

Liu and Shen (1999) studied the asymptotic behavior of solution of the forced nonlinear neutral differential equation with impulses:

$$[x(t) - px(t-\tau)]' + \Sigma_{i=1}^{n} q_i(t) f(x(t-\sigma_i)) = h(t), \quad t \neq t_k,$$

$$x(t_k^+) - x(t_k) = b_k x(t_k), \qquad k \in Z^+.$$

Zhao and Yan (1996) the authors researched the effective sufficient conditions for the asymptotic stability of the trivial solution of impulsive delay differential equation:

$$x'(t) + \Sigma_{i=1}^{n} p_i(t) x(t-\tau_i) = 0, t \neq t_k,$$

$$x(t_k^+) - x(t_k) = b_k x(t_k), k = 1, 2, \cdots.$$

In this paper, we discuss the asymptotic behavior of a class of second-order nonlinear delay differential equation with impulses. The equation is:

$$(r(t)x'(t))' - p(t)x'(t) + \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) + f(t) = 0, \quad t \neq t_k, \tag{1}$$

$$x(t_k^+) - x(t_k) = a_k x(t_k), x'(t_k^+) - x'(t_k) = b_k x'(t_k), \quad k \in Z^+. \tag{2}$$

where $0 \leq t_0 < t_1 < t_2 < \cdots, \lim_{k\to+\infty} t_k = +\infty$, and

$a_k, b_k, k = 1, 2, \cdots$ are constant.

$$x'(t_k) = \lim_{h\to 0^-} \frac{x(t_k+h) - x(t_k)}{h}, \quad x'(t_k^+) = \lim_{h\to 0^+} \frac{x(t_k+h) - x(t_k^+)}{h}, k = 1, 2, \cdots.$$

$$r(t), p(t), q_i(t), h(t) \in C([0,\infty), R^+), i = 1, 2, \cdots, n; 0 \leq \sigma_1 < \sigma_2 < \cdots < \sigma_n.$$

Let PC_{t_0} denotes the set of function $\phi : [t_0 - \sigma_n, t_0] \to R$, which is continuous in the set $[t_0 - \sigma_n, t_0] \backslash \{t_k : k = 1, 2, \cdots\}$ and may have discontinuities of the first kind and is continuous from left at the points t_k situated in the interval $(t_0 - \sigma_n, t_0]$. For

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any $t_0 \geq 0, \phi \in PC_{t_0}$, a function x is said to be a solution of (1) and (2) and satisfying the initial value condition:

$$x(t)=\phi(t), x(t_0^+)=x_0, x'(t)=\phi'(t), x'(t_0^+)=x_0', t\in[t_0-\sigma_n, t_0], \quad (3)$$

in the interval $[t_0-\sigma_n,\infty)$, if $x:[t_0-\sigma_n,\infty)\to R$ satisfies (3) and (i) for

$t\in(t_0,\infty), t\neq t_k, t\neq t_k+\sigma_i, i=1,2,\cdots,n, k=1,2,\cdots, x(t), x'(t)$

is continuously differential and satisfies (1);

(ii) for $t_k\in[t_0,\infty), x(t_k^+), x'(t_k^+), x(t_k^-)$ and $x'(t_k^-)$ exist,

$x(t_k^-)=x(t_k), x'(t_k^-)=x'(t_k)$ and satisfies (2).

Because (1) can be transformed to one-order differential equations with impulses, so the existence and sole of solutions of (1) can be deduced by Wen and Chen (1999)

A solution of (1) and (2) is called eventually positive (negative) if it is positive (negative) for all t sufficiently large, and it is called oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is called nonoscillatory.

Main Lemmas

Throughout this paper, we assume that the following conditions hold:

(H_1) $r(t)\geq r, \int_0^\infty p(t)dt\leq p, q_i(t)\leq q_i, i=1,2,\cdots,n. r, p, q_i\in R^+$.

(H_2) for all $t\in[0,\infty)$, the intergration $H(t)=\int_t^\infty f(s)ds$ converges; $\Sigma_{k=1}^\infty b_k^+<\infty$ where $b_k^+=\max\{b_k,0\}$;

(H_3) $\lim_{n\to\infty}\sum_{m=0}^{n-1}\prod_{k=m+1}^{n-1}\prod_{l=0}^{m}(a_k+1)(b_l+1)\int_{t_m}^{t_{m+1}}\frac{1}{r(u)}\exp[\int_{t_0}^{u}\frac{p(s)}{r(s)}ds]du=+\infty.$

(H_4) $\prod_{k=0}^{n-1}(b_{j+k}+1)\frac{r(t_j)}{r(t_{j+n})}\exp[-\int_{t_j}^{t_{j+n}}\frac{p(s)}{r(s)}ds]>1.$

Lemma 1. Suppose that $x(t)$ is a solution of equations(1) and (2), and there exists $T\geq t_0$ such that $x(t)>0, t\geq T$, If (H_3) hold, then $x'(t_k)>0, x'(t)>0$, where $t\in(t_k,t_{k+1}], k=1,2,\cdots$.

Proof. First, we prove $x'(t_k)>0$, for all $t_k\geq T$. Otherwise, there exists some j such that $t_j\geq T, x'(t_j)<0$, then $x'(t_j^+)=(1+b_j)x'(t_j)$ from (1), we get

$$[r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]]'=-\sum_{i=1}^{n}q_i(t)x(t-\sigma_i)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]-f(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$$
$$=[-\sum_{i=1}^{n}q_i(t)x(t-\sigma_i)-f(t)]\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]<0.$$

Hence, $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$ and

$$r(t_{j+1})x'(t_{j+1})\exp[-\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds]\leq r(t_j)x'(t_j^+)\leq r(t_j)(b_j+1)x'(t_j).$$

$$x'(t_{j+1})\leq(b_j+1)\frac{r(t_j)}{r(t_{j+1})}x'(t_j)\exp[\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds].$$

on $(t_{j+1},t_{j+2}]$,

$$x'(t_{j+2})\leq(b_{j+1}+1)\frac{r(t_{j+1})}{r(t_{j+2})}x'(t_{j+1})\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds]$$
$$\leq(b_{j+1}+1)\frac{r(t_{j+1})}{r(t_{j+2})}(b_j+1)\frac{r(t_j)}{r(t_{j+1})}x'(t_j)\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds]$$
$$=(b_{j+1}+1)(b_j+1)\frac{r(t_j)}{r(t_{j+2})}x'(t_j)\exp[\int_{t_j}^{t_{j+2}}\frac{p(s)}{r(s)}ds].$$

By induction, we have, for all $n\geq 2$.

$$x'(t_{j+n})\leq\prod_{k=0}^{n-1}(b_{j+k}+1)\frac{r(t_j)}{r(t_{j+n})}x'(t_j)\exp[\int_{t_j}^{t_{j+n}}\frac{p(s)}{r(s)}ds].$$

Because $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$, so,

$$x'(t)\leq(b_j+1)\frac{r(t_j)}{r(t)}x'(t_j)\exp[\int_{t_j}^{t}\frac{p(s)}{r(s)}ds],\quad t\in(t_j,t_{j+1}].$$

Integrating the above inequality from s to t, we have

$$x(t)\leq x(s)+(b_j+1)r(t_j)x'(t_j)\int_s^t\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du,\quad t_j<s<t\leq t_{j+1},$$

Let $t\to t_{j+1}, s\to t_j^+$, we get

$$x(t_{j+1})\le x(t_j^+)+(b_j+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$
$$\le(a_j+1)x(t_j)+(b_{j+1}+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$
$$x(t_{j+2})\le(a_{j+1}+1)(a_j+1)x(t_j)+(a_{j+1}+1)(b_j+1)r(t_j)x'(t_j)\int_{t_j}^{t_{j+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du$$
$$+(b_{j+1}+1)(b_j+1)r(t)x'(t_j)\int_{t_{j+1}}^{t_{j+2}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du.$$

By induction, we get, for all n

$$x(t_{j+n})\le\prod_{k=0}^{n-1}(a_{j+k}+1)x(t_j)+r(t_j)x'(t_j)(\sum_{m=0}^{n-1}\prod_{k=m+1}^{n-1}\prod_{l=0}^{m}(a_{j+k}+1)(b_{j+l}+1)\int_{t_{j+m}}^{t_{j+m+1}}\frac{1}{r(u)}\exp[\int_{t_j}^{u}\frac{p(s)}{r(s)}ds]du).$$

because of $x(t)>0, x'(t_j)<0(t_j\ge T)$, it is contraction to the condition (H_3). Hence, $x'(t_k)>0$ for all $t_k\ge T$ and $r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]$ is decreasing on $(t_j,t_{j+1}]$, thus,

$$r(t)x'(t)\exp[-\int_{t_j}^{t}\frac{p(s)}{r(s)}ds]\ge r(t_{j+1})x'(t_{j+1})\exp[-\int_{t_j}^{t_{j+1}}\frac{p(s)}{r(s)}ds]\ge 0.$$

therefore, $x'(t)\ge 0, t\in(t_k,t_{k+1}]$. The proof is complete.

Theorem 1. Let $(H_1)-(H_3)$ hold. Suppose that

$$\sum_{i=1}^{n}q_i(t+\sigma_i)\ge 0,\qquad \int_0^\infty\sum_{i=1}^{n}q_i(s+\sigma_i)ds=\infty, \tag{4}$$

and there exists constant $\lambda>0$ such that for sufficiently large t

$$\sum_{i=1}\int_{t-\sigma_i}^{t-r}q_i(s+\sigma_i)ds\le\lambda<r+p. \tag{5}$$

where

$$r\in[0,\sigma_n], q_i^+(t)=\max\{q_i(t),0\}, q^-(t)=\max\{-q_i(t),0\}.$$

Then every nonoscillatory solution of (1) and (2) tends to zero as $t\to\infty$.

Proof: Choose a positive integer N such that (5) holds for $t\ge t_N$ and $\sum_{k=N}^{\infty}b_k^+<r-p-\lambda$. let $x(t)$ be a nonoscillatory solution of (1) and (2). We will assume that $x(t)$ is eventually positive, the case where $x(t)$ is eventually negative is similar and omitted. Let $x(t)>0$ for $t\ge t_N$, By **Lemma 1,** we know that $x'(t)>0$, for $t\ge t_N$. Define

$$y(t)=r(t)x'(t)-\int_{t_N}^{t}p(s)x'(s)ds-\sum_{i=1}^{n}\int_{t-\sigma_i}^{t-r}q_i(s+\sigma_i)x(s)ds-H(t)-\sum_{t_N<t_k\le t}b_k^+x'(t_k). \tag{6}$$

Then for $t\ne t_k, t\ne t_k+\sigma_i, i=1,2,\cdots,n; k=1,2,\cdots.$

$$y'(t)=-\sum_{i=1}^{n}q_i(t-r+\sigma_i)x(t-r) \tag{7}$$

and

$$y(t_k^+)-y(t_k)=(b_k-b_k^+)x'(t_k)\le 0, k=N,N+1,\cdots.$$

Thus, $y(t)$ is nonincreasing on $[t_N,\infty)$. Set $L=\lim_{t\to\infty}y(t)$, we claim that $L\in R$. Otherwise, $L=-\infty$, then $x'(t)$ must be unbounded by virtue of (H_1) and (4). Hence, it is possible to choose $t^*>t_N+\sigma_n$ such that $y(t^*)+H(t^*)<0$ and $x'(t^*)=\max\{x'(t):t_N\le t\le t^*\}$. Thus, we have:

$$0>y(t^*)+H(t^*)$$
$$\ge r(t^*)x'(t^*)\int_{t_N}^{t^*}p(s)x'(s)ds-\sum_{i=1}\int_{t^*-\sigma_i}^{t^*-r}q_i(s+\sigma_i)x(s)ds-\sum_{t_N<t_k\le t^*}b_k^+x'(t_k)$$
$$\ge x'(t^*)(r-p-\lambda-\sum_{k=N}^{\infty}b_k^+)>0,$$

which is a contradiction and so $L\in R$. By integrating both sides of (7) from t_N to t, we have:

$$\int_{t_N}^{t}\sum_{i=1}^{n}q_i(s-r-\sigma_i)x(s-r)ds=-\int_{t_N}^{t}y'(s)ds$$
$$=y(t_N^+)+\sum_{t_N<t_k\le t}[y(t_k^+)-y(t_k)]-y(t)<y(t_N^+)-L.$$

which, together with (4) implies that $x(t)\in L^1([t_N,\infty),R)$ and so $\lim_{t\to\infty}x(t)=0$. The proof is then complete.

Lemma 2. Let $x(t)$ be an oscillatory solution of equation (1) and (2), suppose that there exists some $T \geq t_0$, if (H_4) hold, then $|x'(t_k)| \geq |x(t_k)|, |x'(t)| \geq |x(t)|$, where

$$t \in (t_k, t_{k+1}], k = 1, 2, \cdots.$$

Proof: From the result of Lemma 1, we know that, if $x(t) > 0$ then, $x'(t_k) > 0, x'(t) > 0$, where, $t \in (t_k, t_{k+1}]$. we will assume that when $x(t) > 0$ we have $x'(t_k) \geq x(t_k), x'(t) \geq x(t), t \in (t_k, t_{k+1}]$, the case $x(t)$ is negative is similar and omitted. From Lemma 1, we have $x'(t_k) > 0, x'(t) > 0, t \in (t_k, t_{k+1}]$, then the $x(t)$ is increased. We also obtained

$$[r(t)x(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)} ds]]' < [r(t)x'(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)} ds]]' < 0.$$

Hence, $r(t)x(t)\exp[-\int_{t_j}^{t} \frac{p(s)}{r(s)} ds]$ is decreasing on $(t_j, t_{j+1}]$ and

$$x(t_{j+1}) \leq (b_j + 1)\frac{r(t_j)}{r(t_{j+1})} x(t_j)\exp[-\int_{t_j}^{t_{j+1}} \frac{p(s)}{r(s)} ds],$$

for all n, we obtain

$$x(t_{j+n}) \leq \prod_{k=0}^{n-1} (b_{j+k} + 1)\frac{r(t_j)}{r(t_{j+n})} x(t_j)\exp[-\int_{t_j}^{t_{j+n}} \frac{p(s)}{r(s)} ds].$$

By the condition (H_4), we get $x(t_{j+n}) < x(t_j)$, which is a contraction. The proof is complete.

Theorem 2. Let $(H_1), (H_2)$ and (H_4) holds. Suppose that

$$\sum_{k=1}^{\infty} |b_k| < \infty, \tag{8}$$

and there exists positive constant λ and $r \in (0, \sigma_n]$ such that

$$\limsup_{t \to \infty} Q_1(t) + \limsup_{t \to \infty} Q_2(t) \leq \lambda < r - 2p, \tag{9}$$

$$\sum_{i=1}^{n} q_i(t + \sigma_i) \neq 0, \qquad for\, large \quad t, \tag{10}$$

where

$$Q_1(t) = \sum_{i=1}^{n} \int_{t-\sigma_i}^{t} q_i(s + \sigma_i) ds, \tag{11}$$

$$Q_2(t) = \sum_{i=1}^{n} \int_{t-r}^{t-\sigma_i} sgn(r - \sigma_i) q_i(s + \sigma_i) ds, \tag{12}$$

Then every oscillatory solution (1) and (2) tends to zero as $t \to \infty$.

Proof: Let $x(t)$ be an oscillatory solution of (1) and (2). We first show that $x'(t)$ and $x(t)$ are bounded. Otherwise, $x'(t)$ is unbounded which implies that there exists positive integer N such that $\lim_{t\to\infty}\sup_{t_N \leq s \leq t} |x'(s)| = \infty$ and

$$\sup_{t_N + \sigma_n \leq s \leq t} |x'(s)| = \sup_{t_N \leq s \leq t} |x'(s)|, \qquad t \geq t_N + \sigma_n,$$

and

$$\sum_{k=N}^{\infty} |b_k| < \frac{r - 2|p| - \lambda}{2}. \tag{13}$$

Set

$$y(t) = r(t)x'(t) - \int_{t_N}^{t} p(s)x'(s)ds - \sum_{i=1}^{n} \int_{t-\sigma_i}^{t-r} q_i(s + \sigma_i)x(s)ds - H(t) - \sum_{t_N < t_k \leq t} b_k^+ x'(t_k),$$

where $b_k^+ = \max\{b_k, 0\}$. Then (7) holds. For $t \geq t_N + \sigma_n$, using **Lemma 2** we have

$$\begin{aligned} |y(t)| &\geq r|x'(t)| - p|x'(t)| - \sum_{i=1}^{n} \int_{t-\sigma_i}^{t-r} q_i(s+\sigma_i)|x(s)|ds - |H(t)| - \sum_{t_N \leq t_k \leq t} |b_k x'(t_k)| \\ &\geq (r-p)|x'(t)| - (Q_2(t) + \sum_{k=N}^{\infty} |b_k|) \sup_{t_N \leq s \leq t} |x'(s)| - |H(t)|, \end{aligned}$$

which implies

$$\sup_{t_N+\sigma_n \leq s \leq t} |y(s)| \geq (r - p - \sup_{t_N \leq s \leq t} Q_2(t) - \sum_{k=N}^{\infty} |b_k|) \sup_{t_N \leq s \leq t} |x'(s)| - \sup_{t_N+\sigma_n \leq s \leq t} |H(s)|. \tag{14}$$

Hence, $\limsup_{t\to\infty} |y(t)| = \infty$. From (7) we notice that $y'(t)$ is oscillatory, we see that there is a $\xi' \geq t_N + 2\sigma_n$ such that $|y(\xi')| = \sup_{t_N+\sigma_n \leq s \leq t} |y(s)|$ and $y'(\xi') = 0$. From (7) and (10), we get $x(\xi' - r) = 0$ by Lemma 2. We know that $x'(t)$ is oscillatory, hence, there is a $\xi > \xi' + r$ such that $x'(\xi - r) = 0$. Integrating both sides of (7) from $\xi - r$ to ξ, we obtain

$$\begin{aligned} y(\xi) &= y(\xi-r) - \int_{\xi-r}^{\xi} \sum_{i=1}^{\xi} q_i(s-r+\sigma_i)x(s-r)ds \\ &= -\int_{t_N}^{\xi-r} p(s)x'(s)ds + \sum_{i=1}^{n}\int_{\xi-2r}^{\xi-r-\sigma_i} q_i(s+\sigma_i)x(s)ds + H(\xi-r) - \sum_{t_N \le t_k \le \xi-r} b_k x'(t_k) \\ &\quad - \int_{\xi-r}^{\xi} \sum_{i=1}^{n} q_i(s-r+\sigma_i)x(s-r)ds \\ &= \int_{t_N}^{\xi-r} p(s)x'(s)ds + H(\xi-r) - \sum_{i=1}^{n}\int_{\xi-r-\sigma_i}^{\xi-r} q_i(s+\sigma_i)x(s)ds - \sum_{t_N \le t_k < \xi-r} b_k x'(t_k), \end{aligned}$$

which implies that

$$|y(\xi)| \le \left(p + Q_1(\xi - r) + \sum_{k=N}^{\infty} |b_k|\right) \sup_{t_N \le s \le \xi} |x'(s)| + |H(\xi - r)|. \tag{15}$$

From (14) and (15), we have

$$-r + 2p + (Q_1(\xi - r) + \sup_{t_N \le s \le \xi} Q_2(s)) + 2\sum_{k=N}^{\infty}|b_k| + (\sup_{t_N+\sigma_n \le s \le \xi} H(s) + |H(\xi - r)|)(\sup_{t_N \le s \le \xi} |x'(s)|)^{-1} \ge 0.$$

Let $\xi \to \infty$ and noting that $\limsup_{\xi\to\infty} \sup_{t_N \le s \le \xi} |x'(s)| = \infty$, we have

$$-r + 2p + \lambda + 2\sum_{k=N}^{\infty} |b_k| \ge 0,$$

by (9), which contradicts (13) and so $x'(t)$ is bounded. By Lemma 2, we know that $x(t)$ is bounded.

Next we will prove that $\mu = \limsup_{t\to\infty} |x'(t)| = 0$. To this end, we define

$$z(t) = r(t)x'(t) - \int_{t_N}^{t} p(s)x'(s)ds + \sum_{i=1}^{n}\int_{t-r}^{t-\sigma_i} q_i(s+\sigma_i)x(s)ds + H(t) + \sum_{t_k \ge t} b_k x'(t_k) \tag{16},$$

then $z(t)$ is bounded and for sufficiently large t,

$$|z(t)| \ge r|x'(t)| - p|x'(t)| - Q_2(t) \sup_{t-\sigma_n \le s < t} |x'(s)| - |H(t)| - \sum_{t_k \ge t} |b_k x'(t_k)|,$$

thus, by (H_2) and (8)

$$\begin{aligned} \beta = \limsup_{t\to\infty} |z(t)| &\ge (r-p)\mu - \mu \limsup_{t\to\infty} Q_2(t) \\ &= \mu[r - p - \limsup_{t\to\infty} Q_2(t)]. \end{aligned} \tag{17}$$

on the other hand, we have by (16) for

$$t \ne t_k, t \ne t_k + \sigma_i, k = 1, 2, \cdots .i = 1, 2, \cdots,$$

$$z'(t) = -\sum_{i=1}^{n} q_i(t - r + \sigma_i)x(t - r) \tag{18}$$

From this we see that $z'(t)$ is oscillatory. Hence there exists a sequence $\{\xi_m'\}$ such that $\lim_{m\to\infty} \xi_m' = \infty, \lim_{m\to\infty} |z(\xi_m')| = \beta, z'(\xi_m') = 0.$ and $x(\xi_m' - r) = 0, m = 1, 2, \cdots$: similar to (15) we can obtain by (16) and (18), there is a $\xi_m > \xi_m'$, such that

$$|z(\xi_m)| \le (p + Q_1(\xi_m - r)) \sup_{\xi_m - 2\sigma_n \le s \le \xi_m} |x'(s)| + |H(\xi_m - r)| + \sum_{t_k \ge \xi_m - r} |b_k x(t_k)|,$$

which implies by (8) and (H_2) that

$$\beta \le \mu[p + \limsup_{t\to\infty} Q_1(t)].$$

This, together with (17), yields

$$\mu[-r + 2p + \limsup_{t\to\infty} Q_1(t) + \limsup_{t\to\infty} Q_2(t)] \ge 0.$$

Therefore, by (9) we have

$$\mu(-r + 2p + \lambda) \ge 0,$$

which implies $\mu = 0$ by (9) and so, $\lim_{t\to\infty} x'(t) = 0.$ Hence we can obtain that $\lim_{t\to\infty} x(t) = 0$. Thus, the proof is completed.

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