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Full Length Research Paper

A class of collocation methods for general second order ordinary differential equations

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In this paper, self starting hybrid block method of order $(4,3,3)^T$ is proposed for the solution of general second order initial value problem of the form $Y'' = f(x,y,y')$ directly without reducing it to first systems of odes. The continuous formation of the integrator enable us to differentiate and evaluate at some grids and off-grid points to take care of y' in the method. The schemes compare favorably with optimal order four (Fatunla Based) proposed in Yahaya (2004). There is anticipated speed up of computation as a result of admissible parallelism across the method.

Key words: General second order, initial value problems, parallel/block method, hybrid, self starting.

INTRODUCTION

Linear multistep methods constitute a powerful class of numerical procedures for showing a second order equation of the form

$$y'' = f(x,y,y'), y(a) = y_0, y'(a) = \beta \qquad (1)$$

It has been well known that an analytical solution to this equation is of little value because many of such problems cannot be solved by analytical approach. In practice, the problems are reduced to systems of first order equations and any methods for first order equations are used to solve them. Awoyemi (1999); Fatunla (1998); Lambert (1973) extensively discussed that due to dimension of the problem after it has been reduced to a system of first order equations, the approach waste a lot of computer time and human efforts.

Some attempts has been made to solve problem (1) directly without reduction to a first order systems of equations; Brown (1977) and Lambert (1991) independently proposed a method known as Multi derivative to solve second order initial value problems type (1) directly. In a recent paper of Onumanyi et al. (2008), they proposed direct block Adam Molton Method (BAM) and hybrid block Adam Molton method (IBAM) for accurate approximation to y' appearing in equation (1) to be able to solve problem (1) directly. The aim of this paper is to demonstrate using the present hybrid block method of order $(4, 3, 3)^T$ derived to solve equation (1) directly and compare its performance with the optimal order four schemes (Fatunla) based proposed in Yahaya (2004).

DERIVATION OF THE SCHEMES

3-step optimal order 4 (Funtula) based (Yahaya 2004)

We consider a powerful series of a single variable x in the form:

$$P(x) = \sum_{j=0}^{\infty} a_j$$

Is used as the basis or trial function to produce our approximate solution to (1) as

$$P(x) = \sum_{j=0}^{\infty} a_j x^j \qquad (2)$$

$$P'(x) = \sum_{j=0}^{m+t-1} j a_j x^{j-1} \qquad (3)$$

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$$P''(x) = \sum_{j=0}^{m+t-1} j(j-1)a_j x^{j-2} \qquad (4)$$

From equations (2) and (4)

$$P''(x) = \sum_{j=0}^{m+t-1} j(j-1)a_j x^{j-2} = f(x,y,y') \qquad (5)$$

Where the a_j are the parameters to be determined, t and m are points of interpolation and collocation points. We collocate equation 5 at x_{n+j}, j = o(1), k+1 and interpreting 2 at x_{n+j}, j = o(1), k-1. Specifically K ≥ 2 yields the following systems of non linear equations

$$\sum_{j=0}^{m+t-1} a_j x^j = y_{n+i}\,, \quad i=0(1)k-1 \qquad (6)$$

$$\sum_{j=0}^{m+t-1} j(j-1)a_j x^{j-2} = f_{n+i, i=0,(1),k+1} \qquad (7)$$

After some algebraic manipulation, we obtained continuous of the form

$y(x) = \frac{[-(x-x_{n+1})]}{h} y_n + \frac{[h+(x-x_{n+1})]}{h} y_{n+1} + [-3(x - x_{n+1})^5 - 15h(x - x_{n+1})^4 - 20h^2(x - x_{n+1})^3 - 38h^4(x - x_{n+1})]f_n/360h^3 + [3(x - x_{n+1})^5 - 10h(x - x_{n+1})^4 - 10h^2(x - x_{n+1})^3 + 60h^3(x - x_{n+1})^2 + 57h^4(x - x_{n+1})] f_{n+1}/120h^3 + [-3(x - x_{n+1})^5 + 5h(x - x_{n+1})^4 + 20h^2(x - x_{n+1})^3 - 12h^4(x - x_{n+1})]f_{n+2}/120h^3 + [3(x - x_{n+1})^5 - 10h^2(x - x_{n+1})^3 + 7h^4(x - x_{n+1})]f_{n+3}/360h^3$ (8)

Evaluating 2.8 at $x = x_{n+2}$ and x_{n+3} respectively yields

$y_{n+2} - 2y_{n+2} + y_2 = h^2/12[f_{n+2} + 10f_{n+1} + f_n]$.
$y_{n+3} - 3y_{n+1} + 2y_n = h^2/12[f_{n+3} + 12f_{n+2} + 21f_{n+1}\ 2f_n]$ (9)

Which is of order $(4, 4)^T$ and error constants $(-1/240, -1/80)^T$. The first derivative of equation 2.8 at $x = x_0$ gives

$Z_0 = -1/360(360y_n - 360y_{n+1} + 97h^2y_n + 114h^2y_{n+1} - 39h^2y_{n+2} + 8h^2y_{n+3})/h$ (10)

Which is of order 4 and error constant -7/480.

PRESENT HYBRID BLOCK METHOD(Badmus 2006)

Using equations 2 and 5, we collocated equation 5 at $[x_{n+1/2}, x_{n+1}, x_{n+3/2}]$ and interpolation 2 at $[x_n, x_{n+1}]$ while $v = (y_n, y_{n+1}, f_{n+1/2}, f_{n+1}, f_{n+3/2})$, specifically k ≥ 2 yields the following systems of non linear equations

$$\sum_{j=0}^{m+t-1} a_j x^j = y_{n+i}\,, \quad i=0(1)k-1 \qquad (11)$$

$$\sum_{j=0}^{m+t-1} j(j-1)a_j x^{j-2} = f_{n+i,} \quad i=\tfrac{1}{2},1,3/2 \qquad (12)$$

After some algebraic manipulation, we obtained a continuous scheme of the form

$y(x) = \frac{-(x-x_{n+1})}{h} y_n + \frac{[h+(x-x_{n+1})]}{h} y_{n+1} + \frac{[(x-x_{n+1})^4 - h(x-x_{n+1})^3 + 2h^3(x-x_{n+1})]}{6h^2} f_{n+1/2} + [-2(x - x_{n+1})^4 + 3h^2(x - x_{n+1})^2 + h^3(x - x_{n+1})]f_{n+1}/6h^2 + [(x - x_{n+1})^4 + h(x - x_{n+1})^3]f_{n+3/2}/6h^2$ (13)

Evaluating (13) at $x = x_{n+2}$, $x_{n+1/2}$ and $x = x_{n+3/2}$ we obtained the following discrete schemes to be used for solving problem (1) directly.

$y_{n+2} - 2y_{n+1} + y_n = h^2/3[f_{n+3/2} + f_{n+1} + f_{n+1/2}]$
$2y_{n+3/2} - 3y_{n+1} + y_n = h^2/48[3f_{n+3/2} + 18f_{n+1} + 15f_{n+1/2}]$
$2y_{n+1/2} - y_{n+1} - y_n = h^2/48[-f_{n+3/2} + 2f_{n+1} + 13f_{n+1/2}]$ (14)

The hybrid block scheme of 14 is of orders $(4,3,3)^T$ with error constants $(1/960, 1/384, -1/180)^T$

The first derivative of equation 13 at $x = x_0$ is used along with the schemes in 14 to start the integration process, that is,

$hz_0 - y_{n+1} + y_2 = h^2/6[-f_{n+3/2} + 3f_{n+1}\ -5f_{n+1/2}]$

IMPLEMENTATION STRATEGIES

In optimal order 4 (Funtala) based, we obtained first derivative of equation 8 and we evaluated at some grid points $x = x_{n+j}$, j = 0(1), k+1; which is then substituted in equation 9 together with equation 10. These simultaneously provide values for y_1, $_2$, and $_3$ respectively. Similarly, for the hybrid block method, we equally obtained first derivative of equation 13 and evaluated at the points $x = x_{n+j}$, j = 0(1), k+1 and x_{n+v}, where v = [1/2, 3/2, 5/2, 7/2]. After substituting these in equations 14 and 15 also simultaneously provides solution for $y_{1/2}$, y_1, $y_{3/2}$ and y_2 at once.

In both schemes, the advancement of the integration process could be done either sequential or block form.

NUMERICAL EXPERIMENT

Both methods were demonstrated with example

$$y'' - y' = 0, \qquad y(0) = 0, y'(0) = -1$$

Analytical solution is $y(x) = 1 - e^x$.

Conclusion

This paper demonstrated a successful application of linear multi-step method to solve a general second order ordinary differential equation of the form $Y'' = f(x, y, y')$ directly without reducing it to first order odes. Numerical results show that the hybrid $(4, 3, 3)^T$ order block method converges better than optimal order 4 block method. Furthermore, the proposed block methods are self starting and does not call for special predictors to estimate y' in the integrators, all the discrete schemes used in each of the method were derived from a single continuous formula and its derivatives making use of both grid and off-grid points in the formulation.

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Appendix

Table of results and absolute errors

	Theoretical solution	Optimal order 4 block method	Hybrid bock method	Absolute error of optimal method	Absolute error of hybrid method
0.1	-0.105170918	-0.100162782	-0.10525885	5.008136E-03	8.79316E-05
0.2	-0.221402758	-0.210383758	-0.22107604	1.101918E-02	3.26718E-04
0.3	-0.349858808	-0.330817662	-0.347643243	1.9041146E-02	2.215564E-03
0.4	-0.491824698	-0.463450532	-0.48697605	2.8374166E-02	4.857093E-03
0.5	-0.648721271	-0.608679322	-0.639623537	4.0041949E-02	9.097734E-03
0.6	-0.8221188	-0.768722236	-0.807727706	5.339556E-02	1.4391394E-02
0.7	-1.01375270	0.944270975	-0.99231479	6.9481732E-02	2.1437918E-02
0.8	-1.225540928	-1.137831009	-1.195642204	8.7709919E-02	2.9898724E-02
0.9	-1.459603111	-1.350444386	-1.419302392	1.09158725E-01	4.0300719E-02
1.0	1.718281828	-1.584986115	-1.665729698	1.33295713E-01	5.255213E-02