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Full Length Research Paper

Approximation formulae for phase shifts of s–wave Schrödinger equation due to binomial potential and their applications

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In this paper, we derive the approximation formulae for phase shifts of s-wave Schrödinger equation on introducing binomial potential function and then make their applications to study the fluctuations in the phase shift difference from the s-wave with respect to different values of the arbitrary parameter occurring in a given binomial potential function.

Key words: s-wave Schrödinger equation, phase shifts, binomial potential function.

INTRODUCTION

The general theories of scattering and experimental studies in the nuclear and atomic collision have great deal with development in the modern science and technology. Many of these theoretical and experimental advances have been the result of their mutual stipulation. The problem of deducing a potential from the observed phase shifts has led to many mathematical investigations. However, it may be of interest to obtain phase shifts from a given potential function. Recently, a number of research scholars (Mahajan and Verma, 1975; Raghuwanshi and Sharma, 1979) have utilized Tietz (1963) method for finding the phase shifts. The theoretical value of cross-section is determined with the help of phase shifts η_L, where 'L' is the angular momentum quantum number. Bhattacharjie and Sudarshan (1962) have evaluated a solvable potential function in the form for the s-wave Schrödinger equation by transforming second order Gauss equation, where B and $\propto$ are the parameters.

$$U(r) = B\,\frac{e^{\propto r}}{1-e^{\propto r}} \tag{1}$$

Agrawal and Kumar (1999) have also studied the phase shift difference for binomial potential function for s-wave Schrödinger equation. Again, Kumar et al. (1999) have determined the phase shifts involving Srivastava and Daoust function (1969). Furthermore, Chandel and Kumar (1999) have determined the phase shift difference for binomial potential function on applying different methods to Agrawal and Kumar (1999) for s-wave Schrödinger equation. Chandel and Kumar (1999) have introduced the binomial potential in the form:

$$U(r) = \lambda\sum_{n=1}^{\infty}(1-n\propto r)^{-b},$$

$0 < b < 1$, λ is arbitrary constant and $\propto > 0$ (2)

From Equation (2) we have

$$\frac{dU}{dr} = \sum_{n=1}^{\infty}\lambda_n(1-n\propto r)^{-b-1} \tag{3}$$

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where λ_n is independent of r.For b = $\frac{\beta}{\gamma}$ and $\propto = \propto' \gamma$, 0 < β < 1 and $\propto'$ > 0, as setting $\gamma \to 0$ in Equation (3), we get following identical potential derivative of the potential function introduced by Bhattacharjie and Sudarshan (1962);

$$\frac{dU}{dr} = \sum_{n=1}^{\infty} \lambda_n e^{n \propto br} \quad (4)$$

Here, in our investigation, we derive the approximation formulae for phase shifts of s-wave Schrödinger equation on introducing binomial potential function of Chandel and Kumar (1999) and then make their application to obtain the fluctuations in phase shift difference of the s-wave (Schrödinger equation) with respect to different values of the arbitrary parameter b occurring in binomial potential function (Equation 2).

The Schrödinger equation is taken as

$$\frac{d^2\psi(r)}{dr^2} + [K^2 - U(r) - \frac{L(L+1)}{r^2}]\,\psi(r) = 0 \quad (5)$$

where K stands for total energy and U(r) is the potential energy. L is the angular momentum.

PHASE SHIFTS INVOLVING GENERALIZED HYPERGEOMETRIC FUNCTION

For binomial potential function given in Equation (2), Chandel and Kumar (1999) have determined following phase shift difference, formulae involving generalized hypergeometric function (Figures 1a-6b):

$$\eta_L - \eta_{L+1} = \frac{\sqrt{\pi}\, K^{2L+1}\,\Gamma(L+2)\,\Gamma(b-2L-3)}{\Gamma\left(L+\frac{3}{2}\right)\Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{\lambda_n}{(-n\propto)^{2L+4}}]\ {}_2F_3\left[\begin{matrix}(L+2),(L+2); \\ (2L+3),\ \left(L+2-\frac{b}{2}\right),\ \left(L+\frac{5}{2}-\frac{b}{2}\right);\end{matrix}\ \frac{-K^2}{n^2\propto^2}\right]$$

Provided that (2L+4–b) > 0 and 0 < b < 1

Re($n \pm$ iK) > 0 (6)

The second phase shift difference formula is given by;

$$\eta_{L-1} - \eta_{L+1} = \frac{\sqrt{\pi}\, K^{2L-1}\,\Gamma(L+1)\,\Gamma(b-2L-1)}{\Gamma\left(L+\frac{1}{2}\right)\Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{\lambda_n}{(\ \ n\propto)^{2L+2}}\ {}_2F_3\left[\begin{matrix}(L+1),\ (L+1); \\ (2L+2),\ \left(L+1-\frac{b}{2}\right),\ \left(L+\frac{3}{2}-\frac{b}{2}\right);\end{matrix}\ \frac{-K^2}{n^2\propto^2}\right]$$

provided that Re(2L+2–b) > 0 and 0 < b < 1 and

Re($n \pm iK$) (7)

Here, in our work we use above phase shift difference formulae (6) and (7) to evaluate approximation formulae.

Approximation formulae for phase shifts

In this section, on making an appeal to the phase shift difference formulae (6 and 7), we derive approximation formulae for phase shifts through following theorems:

Theorem A: For phase shifts given in Equation (6) there exists an approximation formula.

$$\eta_L - \eta_{L+1} \le \frac{\sqrt{\pi}\, K^{2L+1}\,\Gamma(L+2)\,\Gamma(b-2L-3)}{\Gamma\left(L+\frac{3}{2}\right)\Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{\lambda_n}{(-n\propto)^{2L+4}} \left\{1 + \frac{K^2}{n^2\propto^2\left(L+2-\frac{b}{2}\right)\left(L+\frac{5}{2}-\frac{b}{2}\right)}\right\}^{-(L+2)} \quad (8)$$

Proof: From the Pochhammer symbol defined by

$(\lambda)_n$ = λ (λ+1) … (λ+n-1), n ϵ N (N =1, 2, 3,….)

and $(\lambda)_0 = 1$, λ ≠ 0, (Rainville, 1960), (9)

We have the inequality

$(\lambda)_n \ge (\lambda)^n$, n ϵ N ∪ {0} (N = 1, 2, 3,…..) (10)

Again, it is well known that

(2L+3) > (L+2) ∀ L = 0, 1, 2 (11)

From Equation (6), we find that:

$$\eta_L - \eta_{L+1} \le \frac{\sqrt{\pi}\, K^{2L+1}\,\Gamma(L+2)\,\Gamma(b-2L-3)}{\Gamma\left(L+\frac{3}{2}\right)\Gamma(b+1)} \sum_{n=1}^{\infty} \frac{\lambda_n}{(-n\propto)^{2L+4}} \times \left\{\sum_{m=0}^{\infty} \frac{(L+2)_m}{\left(L+2-\frac{b}{2}\right)_m \left(L+\frac{5}{2}-\frac{b}{2}\right)_m} \frac{\left(\frac{-K^2}{n^2\propto^2}\right)^m}{m!}\right\} \quad (12)$$

Figure 1(a). Column diagram.

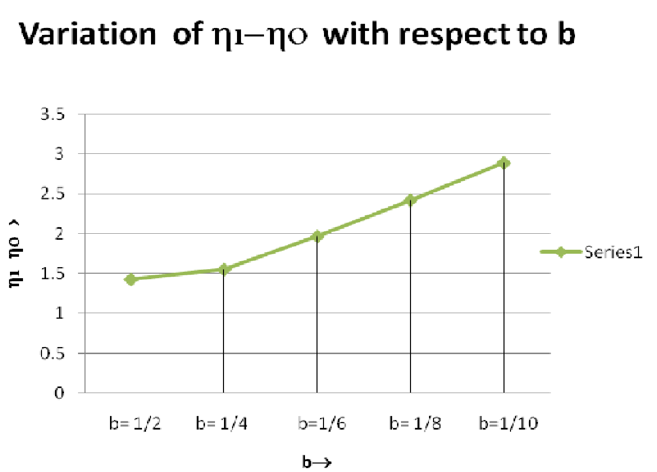

Figure 1(b). Line diagram.

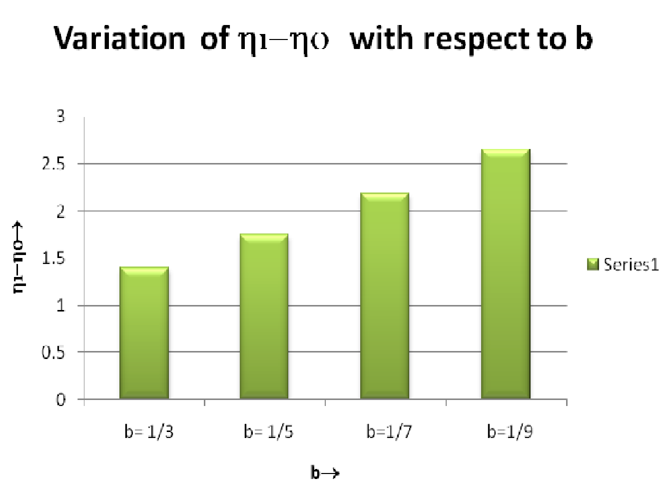

Figure 2a. Column diagram.

Then, the inequality (12) immediately gives the approximation formula (8)

Theorem B: For phase shifts given in Equation (7), there exists an approximation formula

$$\eta_{L-1}-\eta_{L+1} \le \frac{\sqrt{\pi}\, K^{2L-1}\,\Gamma(L+1)\,\Gamma(b-2L-1)}{\Gamma\left(L+\frac{1}{2}\right)\Gamma(b+1)} \times\sum_{n=1}^{\infty}\frac{\lambda_n}{(-n\alpha)^{2L+2}} \left\{1+\frac{K^2}{n^2\alpha^2\left(L+1-\frac{b}{2}\right)\left(L+\frac{3}{2}-\frac{b}{2}\right)}\right\}^{-(L+1)} \quad (13)$$

Proof: On making an appeal to Equations (7), (10) and with the inequality (L+1) < (2L+2), ∀ L = 0, 1, 2,…….
we find

$$\eta_{L-1}-\eta_{L+1} \le \frac{\sqrt{\pi}\, K^{2L-1}\,\Gamma(L+1)\,\Gamma(b-2L-1)}{\Gamma\left(L+\frac{1}{2}\right)\Gamma(b+1)} \sum_{n=1}^{\infty}\frac{\lambda_n}{(-n\alpha)^{2L+2}} \times\left\{\sum_{m=0}^{\infty}\frac{(L+1)_m}{\left(L+1-\frac{b}{2}\right)_m\left(L+\frac{3}{2}-\frac{b}{2}\right)_m}\,\frac{\left(\frac{-K^2}{n^2\alpha^2}\right)^m}{m!}\right\} \quad (14)$$

The inequality (14) on solving immediately gives the formula (13)

Theorem C: For L = 0, 1, 2,…………, unit total energy and unit ∝ (in binomial potential), there exists the following inequalities

$$\eta_{L+1}-\eta_L \ge -f(L), \quad (15)$$

and

$$\eta_{L+2}-\eta_L \ge -f(L), \quad (16)$$

where

$$f(L)=\frac{\sqrt{\pi}\,\Gamma(L+2)\,\Gamma(b-2L-3)}{\Gamma\left(L+\frac{3}{2}\right)\Gamma(b+1)} \times\sum_{n=1}^{\infty}\frac{1}{(n)^{2L+5}}\left\{1+\frac{1}{n^2\left(L+2-\frac{b}{2}\right)\left(L+\frac{5}{2}-\frac{b}{2}\right)}\right\}^{-(L+2)} \quad (17)$$

Proof: In the theorems A and B, putting K = 1, ∝ = 1, $\langle\lambda_n\rangle_{n=0}^{\infty}=\langle\frac{1}{n}\rangle_{n=0}^{\infty}$ and making some rearrangements we find the results Equations (15) and (16).

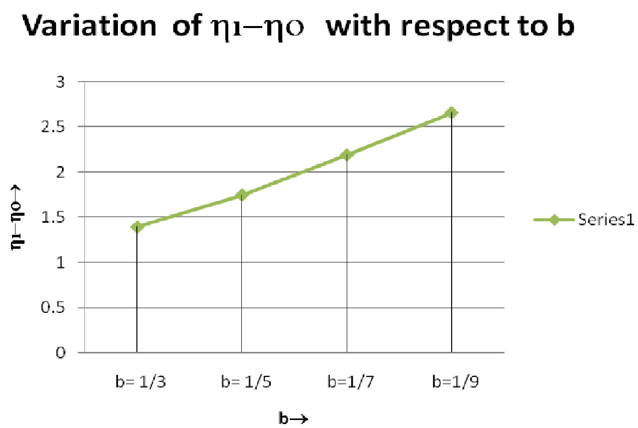

Figure No. 2 (b) (Line Diagram).

Figure No. 3 (a) (Column Diagram).

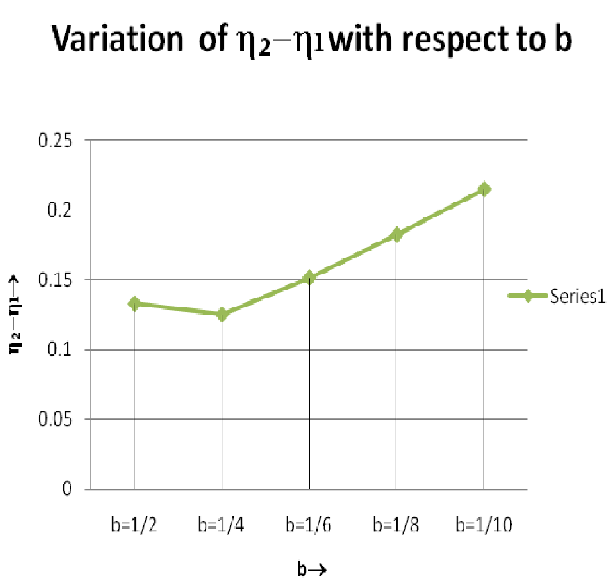

Figure No. 3 (b) (Line Diagram).

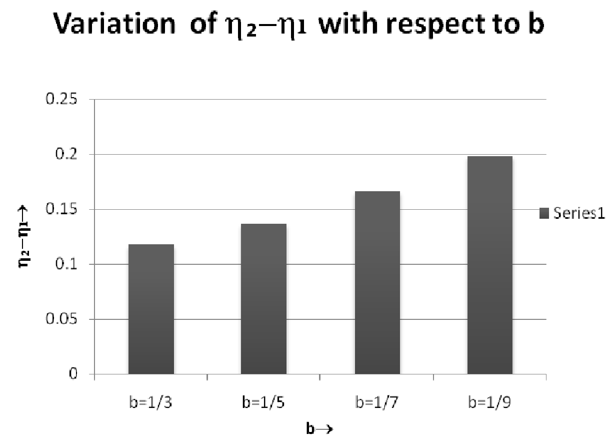

Figure No. 4 (a) (Column Diagram**).**

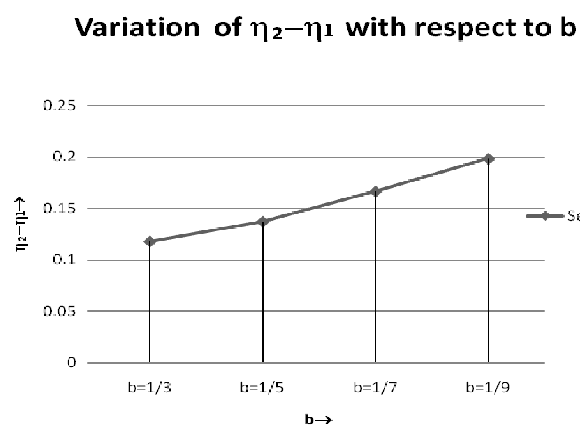

Figure No. 4 (b) (Line Diagram).

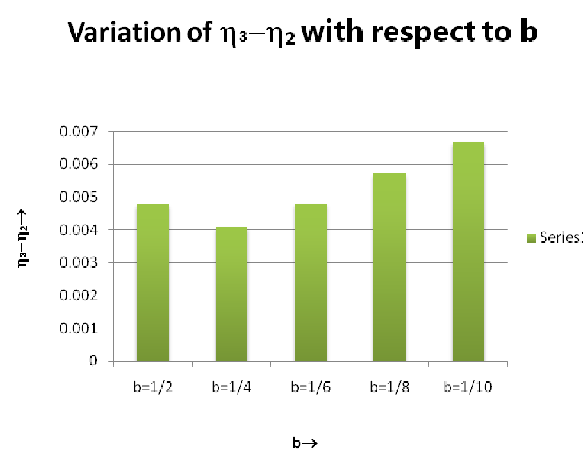

Figure No. 5 (a) (Column Diagram).

APPLICATIONS

In this section, we make the applications of our approximation formulae, derived in (Approximation formulae

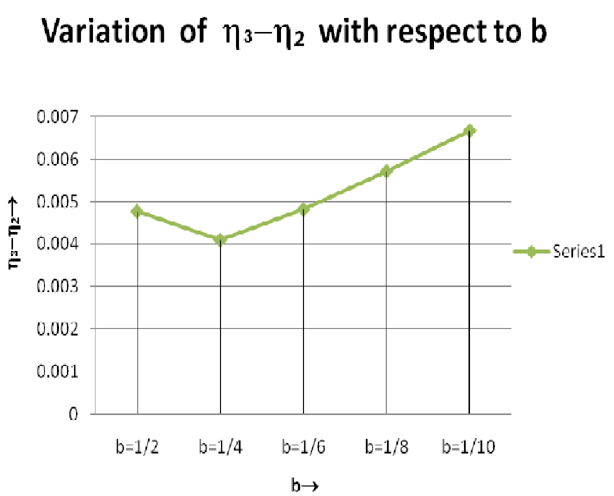

Figure No. 5 (b) (Line Diagram).

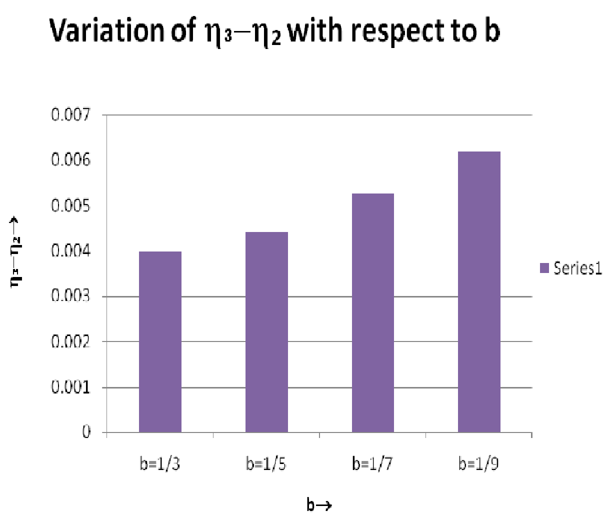

Figure No. 6(a) (Column Diagram).

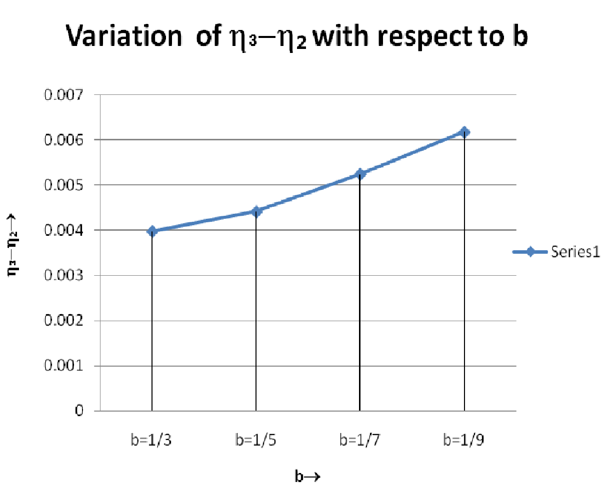

Figure 6(b) (Line Diagram).

Table 1. Variation in $\eta_1 - \eta_0$ when b is inverse of even numbers.

b	$\eta_1 - \eta_0$ **(for n = 4)**
b= 1/2	1.425972365
b= 1/4	1.550959933
b= 1/6	1.966661376
b= 1/8	2.422368128
b= 1/10	2.891265042

Table 2. . Variation in $\eta_1 - \eta_0$ when b is inverse of odd numbers

b	$\eta_1 - \eta_0$ **(for n = 4)**
b= 1/3	1.396653425
b= 1/5	1.749955078
b= 1/7	2.192025663
b= 1/9	2.655793042

for phase shifts), to obtain the fluctuations/variations of phase shifts with respect to the arbitrary parameter b given in the potential function of Chandel and Kumar (1999).

In the Equation (15) of the theorem C setting L = 0, we get

$$\eta_1 - \eta_0 \geq \frac{-\sqrt{\pi}\ \Gamma(b-3)}{\Gamma(3/2)\ \Gamma(b+1)} \sum_{n=1}^{\infty} \frac{1}{n^5} \left\{ 1 + \frac{1}{n^2 \left(2-\frac{b}{2}\right)\left(\frac{5}{2}-\frac{b}{2}\right)} \right\}^{-2} \quad (18)$$

and same of that Equation (16), we have

$$\eta_2 - \eta_0 \geq \frac{-\sqrt{\pi}\ \Gamma(b-3)}{\Gamma(3/2)\ \Gamma(b+1)} \sum_{n=1}^{\infty} \frac{1}{n^5} \left\{ 1 + \frac{1}{n^2 \left(2-\frac{b}{2}\right)\left(\frac{5}{2}-\frac{b}{2}\right)} \right\}^{-2} \quad (19)$$

Now, different values of b that is, $\left(b = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right)$ introducing in (18), we find following variation table of $\eta_1 - \eta_0$ with respect to b (Table 1):

Again, different values of b that is, $\left(b = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right)$ using Equation (18), we find the following variation table of $\eta_1 - \eta_0$ with respect to b (Table 2):

In the theorem C, setting L = 1 in Equation (15), we get:

Table 3. Variation in $\eta_2 - \eta_1$ when b is inverse of even numbers.

b	$\eta_2 - \eta_1$ (for n = 4)
b= 1/2	0.132770555
b= 1/4	0.124900246
b= 1/6	0.151290653
b= 1/8	0.182215514
b= 1/10	0.214693411

Table 4. Variation in $\eta_2 - \eta_1$ when b is inverse of odd numbers.

b	$\eta_2 - \eta_1$ (for n = 4)
b= 1/3	0.117888807
b= 1/5	0.137090787
b= 1/7	0.166475538
b= 1/9	0.198298849

Table 5. Variation in $\eta_3 - \eta_2$ when b is inverse of even numbers.

b	$\eta_3 - \eta_2$ (for n = 4)
b= 1/2	0.004763764
b= 1/4	0.004088408
b= 1/6	0.004808100
b= 1/8	0.005707068
b= 1/10	0.006663482

Table 6. Variation in $\eta_3 - \eta_2$ when b is inverse of odd numbers

b	$\eta_3 - \eta_2$ (for n = 4)
b=1/3	0.003976628
b=1/5	0.004408342
b=1/7	0.005246698
b=1/9	0.006180853

$$\eta_2 - \eta_1 \geq \frac{-\sqrt{\pi}\ 2\ \Gamma(b-5)}{\Gamma(5/2)\ \Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{1}{n^7} \left\{1 + \frac{1}{n^2\left(3-\frac{b}{2}\right)\left(\frac{7}{2}-\frac{b}{2}\right)}\right\}^{-3} \quad (20)$$

and from Equation (16), we get

$$\eta_3 - \eta_1 \geq \frac{-\sqrt{\pi}\ 2\ \Gamma(b-5)}{\Gamma(5/2)\ \Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{1}{n^7} \left\{1 + \frac{1}{n^2\left(3-\frac{b}{2}\right)\left(\frac{7}{2}-\frac{b}{2}\right)}\right\}^{-3} \quad (21)$$

Putting the different values of b that is, $\left(b = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right)$ in Equation (20), we find the following variation table of $\eta_2 - \eta_1$ with respect to b (Table 3).

For different values of b that is, $\left(b = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right)$ using Equation (20), we find following variation table of $\eta_2 - \eta_1$ with respect to b (Table 4):

In the theorem C set L = 2, we obtain

$$\eta_3 - \eta_2 \geq \frac{-\sqrt{\pi}\ 6\ \Gamma(b-7)}{\Gamma(7/2)\ \Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{1}{n^9} \left\{1 + \frac{1}{n^2\left(4-\frac{b}{2}\right)\left(\frac{9}{2}-\frac{b}{2}\right)}\right\}^{-4} \quad (22)$$

and from Equation (16), we find

$$\eta_4 - \eta_2 \geq \frac{-\sqrt{\pi}\ 6\ \Gamma(b-7)}{\Gamma(7/2)\ \Gamma(b+1)} \times \sum_{n=1}^{\infty} \frac{1}{n^9} \left\{1 + \frac{1}{n^2\left(4-\frac{b}{2}\right)\left(\frac{9}{2}-\frac{b}{2}\right)}\right\}^{-4} \quad (23)$$

Then, putting different values of b that is, $\left(b = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right)$ in (22), we find the following variation table of $\eta_3 - \eta_2$ with respect to b (Table 5): Again, setting different values of b that is, $\left(b = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right)$ in Equation (22), we find the following variation table of $\eta_3 - \eta_2$ with respect to b (Table 6):

INTERPRETATION

From the Tables 1–6, we observe that when the value of the parameter b is reduced, the difference in phase shift is increased. This is the reason why a reduction in the value of the parameter b, lessen the potential energy of the particle; also the kinetic energy of the particle is increased which differs and grows the phase shift difference.

In other words, on reducing the value of arbitrary parameter b, we may magnify the phase shift difference and obtain the fluctuation/variations in phase shifts.

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