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Full Length Research Paper

On discussion of the definition of probability

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Mathematicians and scientists generally come to know the meaning of probability at a much latter stage of learning and research instead of getting it within the first two years in college or university. This work involving some random experiments with coins and dice, is an attempt to give first or second year students in the university a classroom demonstration of the approaches to the definition of probability and some of the possible drawbacks in these definitions. We have also used outcomes of the experiments to test for unbiasedness of the die and coin. All results have shown that the devices are unbiased.

Key words: Probability, random experiments, unbiasedness.

INTRODUCTION

The popular Wikipidia has asserted that the word probability has no consistent definition. The spectrum of approaches to probability is wide and complex. Each writer on the subject according to Barnett (1973) has his own focus often developed in detail. There are two broad groups of theories on the approach to probability. One group come out with volumes devoted to a single viewpoint and its implications. Some of those in this group include von Mises (1957), Reichenbach (1949), Jeffreys (1961) and Savage (1954). The second group, some of whom are Bartlett (1962), Koopman (1940), Carnap (1962) hold on to the view that probability has different roles to play in different circumstances and that no single approach is adequate. Some other useful literature include De Finnetti (1937), Von Mises (1941), Kolmogorov (1933/1956), De Finnetti (1968), Savage (1961a, b, c). We shall look at probability through a wide spectrum of views. We shall also give theoretical illustrations, in the form of worked examples in class and state areas of application for each view. Finally, each approach of definition shall consider its merits and demerits. In addition to references just cited, Essi (2009) will help the reader to understand more definitions and concepts in probability.

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REVIEW OF DEFINITIONS OF PROBABILITY

There are many approaches to the definition of the word probability. We shall consider four of these approaches and they are the classical, frequency, subjective and axiomatic approaches.

Classical or a priori approach

The classical definition is traceable to the close association of probability to games of chance in the seventeenth century. Games of chance include throwing a die, tossing a coin and drawing a card. For instance, without experiment, it is assumed that since a fair coin has two sides, probability of a head is one out of two and probability of a tail is the same. This argument is similarly extended to a fair die when it is cast. Using the classical approach, probability of an event A, denoted by P(A) is defined as follows: If A can happen in m ways out of a total of n equally likely ways, then

$$P(A) = \frac{m}{n}$$

The probability of non-occurrence of the event is

$$P(A^C) = 1 - \frac{m}{n}$$

so that $P(A) + P(A^C) = 1$. If $p = P(A)$ and $q = P(A^C)$, then $p + q = 1$

Theoretical Illustration 1

Let a fair die be tossed once. What is the probability that an even number appears?

Solution

The sample space Ω is Ω = {1, 2, 3, 4, 5, 6} showing three even numbers 2, 4 and 6. Therefore the probability that an even number turns up is equal to

$$\frac{Three\ \ possible\ ways}{Total\ number\ of\ \ outomes(po\operatorname{int}s)\ \text{in}\ the\ sample\ space}$$

= 3/6 = ½

Though, we can be quick in estimating probability using classical approach, since (probability can be calculated before experimentation), this approach suffers from a few shortcomings, some of which are circularity, symmetry and perpetual impossibility.

Circularity: The classical definition of probability is essentially circular since the notion of "equally likely" is the same thing as "with equal probability" which is yet to be defined. We use probability to define probability. This is an aspect of circularity.

Symmetry: The inherent symmetry in the classical definition does not augur well in the real world of decision making. The probability of heads equal that of tail and equals ½. Does it mean that in 30 independent tosses of a fair coin, we will have 15 heads? Or if we use head/tail to denote rise/fail of stock prices, does it mean that out of the next 20 days, stock will rise for 10 days?

Perpetually impossible event: The definition of the classical approach creates or holds some events as being impossible to happen perpetually. For instance, this approach, rules out the possibility of a coin standing upright on its edge when cast. One has to recall that it is not so always. One day, as the leading author was teaching probability in the class, a coin dropped from his hand and after rolling to a certain point, stood erect on its edge. The definition is already making people to believe that some events though seemingly unlikely, will not happen at all. It is possible for stock prices to be the same for 10 days consecutively without falling or rising.

Relative frequency approach (posterior definition)

In this approach, we define probability of an event A as the observed relative frequency of occurrence of the event in a very large number of trials. We can further explain the notion of this definition by using a numerical illustration.

Theoretical Illustration 2

The first 6,000 tosses of a fair die result in 1092 threes. Further 6,000 tosses of the same die result in another 1,006 threes. Calculate the probability of a "3" showing up for

(a) The first 6000 tosses
(b) The total of 12000 tosses.
Comment on your results in (a) and (b).

Solution

(a) The probability of a three, showing up in 6000 tosses of the die is equal to the relative frequency in the tossing experiment. Therefore the required probability is

1092/6000 = 0.182

(c) Probability of a three in 12000 tosses

$$= \frac{\text{3's in 12000 tosses}}{\text{12000 tosses}}$$

= (1092 + 1006)/(12000)
= 0.174

Comments

The relative frequency of a three showing up approaches 1/6 = 0.167, as the number of trials get larger. This is why the relative frequency of 0.174 in (b) is closer to 0.167 than 0.182 in (a). The relative frequency, (also known as empirical or posterior) probability suffers from at least three drawbacks. Some of them are as follows:

1. We must make enough time possible and environment conducive to repeat an experiment a great number of times.
2. The phrase "large number of trials" is vague as one cannot precisely answer the question "how large is large".
3. The cumulative frequency f = x/n (where x is the number of heads in n tosses, as n gets large) may not converge after all.

Subjective probability

Subjective probability is based on the personal belief or

Table 1. Relative cumulative frequency, f , of heads out of n tosses.

No of tosses (n)	50	100	150	200	250	300	350	400	450	500
f = x/n	0.44	0.48	0.48	0.49	0.50	0.50	0.49	0.49	0.49	0.49

Table 2. Result of 500 tosses of a coin.

Replications	1	2	3	4	5
Head	45	50	49	50	51
Tail	55	50	51	50	49
Total	100	100	100	100	100

Table 3. Result of 1200 tosses of a die.

Outcome	1	2	3	4	5	6	Total
Observed frequency	212	202	164	204	211	207	1200
Expected frequency	200	200	200	200	200	200	1200

interpretation. Suppose we have to choose one out of five candidates for a professorial chair in environmental statistics. All five have to be assessed using as parameters, personality, communication skill, numerical ability and technical knowledge in their areas of specialization. The process of selection may involve assigning subjectively, probability to each person's potential. At times, subjective probability could be nothing more than a good guess. For instance, one may ask "what is the probability that it will rain tomorrow"? One person may say the answer is 0.5 and another may believe that it is 0.7, all estimates emanating from personal feelings. This method of estimating probability is popular among decision makers in business because of its flexibility and ease of assigning probability values.

Axiomatic probability

The classical definition involves using the expression "equally likely" which is the same as "equally probable". This makes us guilty of circular reasoning. In addition, the definition is also plagued with symmetry and holds the view that seemingly impossible event cannot happen. In the frequency approach, there is also vagueness in the use of the word "large". The subjective approach is equally not reliable since it is based on personal feelings of those assigning probabilities. Due to these drawbacks mathematicians and other researchers have resorted to the axiomatic approach as the only logically satisfactory way to define probability. In the axiomatic approach, we simply state what probability is by enumerating the rules (axioms) that it follows. We state axiomatic probability.

Definition

The function P(.): $\Omega \rightarrow [0,1]$. is a probability function if

(i) P(A) ≥ 0 for all A$\in \mathcal{F}$
(ii) P(Ω) = 1
(iii) $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

where A_i, i = 1.2.3…. are mutually disjoint sets in $\mathcal{F}$ and $\mathcal{F}$ *is a* σ-algebra, which we define as a collection of all the subsets of Ω satisfying certain conditions.

METHODOLOGY AND DATA

A coin was tossed n= 100 times and the outcomes recorded on a tally sheet. This was replicated 5 times, bringing the total number of tosses to 500. From this, we draw a frequency table of heads in every 100 tosses and draw a cumulative distribution of the number of heads x in n tosses in steps of 50 tosses. That is, we calculate f = x/n, for n=50, 100, 150, … , 450, 500 tosses. We also roll a die 1200 times and the frequencies of the numbers 1, 2, 3 , 4, 5 and 6 were recorded.

EMPIRICAL RESULTS AND DISCUSSION

The results of our experiments with the coin and die are shown in Tables 1, 2 and 3.

We also plot the relative cumulative frequency f against

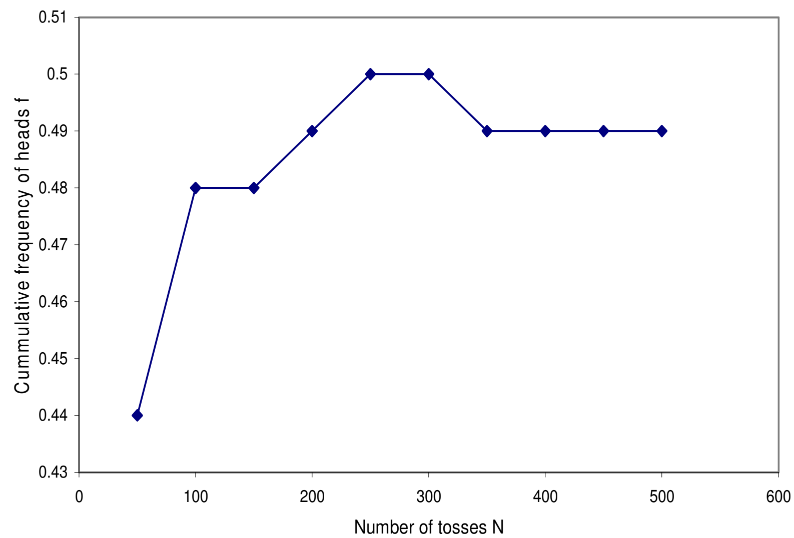

Figure 1. Cummulative frequency of heads in 500 tosses of a coin.

the number of tosses n for the coin in Figure 1. Somehow we observe that the frequency rises from 0.44 to about 0.50 and then goes down to 0.49. We then test the following hypotheses:

(a) H_0: P = 0.5(the coin is unbiased) Versus H_1: P ≠ 0.5 (the coin is biased) and

(b) H_0: P = 1/6 (the die is unbiased) Versus H_1: P ≠ 1/6 (the die is biased).

If the coin is fair the probability P of head in a single toss must be 0.5 and for every 100 tosses the number of heads must be 50. This same symmetry reasoning is applied to the die. For instance for a total number of 1200 tosses, each of the six possible outcomes must appear 200 times if the die is fair. This can only happen if P = 1/6. With these in mind we use χ^2 analysis. For the coin χ^2 = 0.88 with tail probability p = 0.93 >0.05, the result with the die give 8.15 for the value of χ^2 with p = 0.15 > 0.05. These results all show that the die and coin are all fair.

Conclusion

The authors have used theoretical illustrations to demonstrate approaches to the definition of probability. We have also backed up these illustrations with experimental examples using the coin and die. One approach to the definition of probability may not be adequate in some circumstances and other approaches can be better.

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