African Journal of Mathematics and Computer Science Research Vol. 3(8), pp. 179-185, August 2010
Available online at http://www.academicjournals.org/AJMCSR
ISSN 2006-9731 ©2010 Academic Journals

Full Length Research Paper

Appraising of Monte Carlo forecast in nonlinear econometric models

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Accepted 4 June, 2010

In the study, Monte Carlo (stochastic) and deterministic forecast have been carried out. Though, as would be expected, there is reasonable gain in efficiency in the Monte Carlo forecast over the deterministic procedure. The extent of this gain depends on the relative dispersion criterion used.

Key words: Monte Carlo forecast, deterministic forecast, efficiency.

INTRODUCTION

The principal foci in econometric modeling are structural analysis, forecasting and policy analysis. Interests may be in some or all of these areas in one study. If interest is in forecasting, then we are required to make estimate (or a set of estimates) about the likelihood of occurrence of future events using past and current information. The forecast is more useful to users and researchers if the forecast uncertainty is kept as minimal as possible. One of the ways to achieve this, especially when the model is nonlinear is to use stochastic simulation. After obtaining stochastic simulations, we need to go ahead to appraise the gain in efficiency of the Monte Carlo procedure over deterministic simulation. The outcome of this appraisal is largely dependent on the set of measures of dispersion used. These set of measures will be mentioned in section 2 of this paper and section 3 is methodology and data. The empirical results are mentioned in section 4. The performance and choice of which measure to use are elaborately considered in section 5. We state our conclusion in section 6.

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

Solving and forecasting with an econometric model by stochastic simulation is not yet a common procedure. According to Kolsrud (1993), there are some reasons behind this assertion. One fact is that a standard deterministic simulation with only expectation values of the stochastic input variables is seen to be sufficient. The belief is that the deterministic result value approximates well the expectation value of the stochastic model. The second reason is that a stochastic simulation will be unnecessarily more demanding than a deterministic simulation, both on human and computer resources, in addition to being time consuming. Thirdly, Kolsrud observes that researchers often find it easier to relate to the usual “one solution point or path (trajectory)” of forecast than to an interval or distributional statistics. Much quantitative measures arising from a simulation experiment are perceived as not being necessary or relevant.

Kolsrud (1993) makes strong points against deterministic simulation and forecast, following recent developments in economics, numerical algorithms, computer hardware and software. Kolsrud advances first of all that, even though the majority of operational econometric models are linear or only weakly nonlinear, increasingly more nonlinearity is being built into the models. And since more models are given a highly dynamic specification, Kolsrud reasons, one cannot simply assume an insignificant deterministic bias anymore. Secondly, shunning stochastic simulation because of high demand on human and computer resources is not justified. Efficiency weighs far more than computational inconveniences as there are adequate computer hardware and software to handle stochastic simulation. Monte Carlo simulation of econometric models is nowadays a very feasible and exciting task, even though it is inherently and will always be more

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costly than a deterministic simulation. Another attraction for stochastic simulation, according to Kolsrud is that it supplies relevant and interesting distributional properties of the response variables; that can be used to improve forecasts or influence policy analysis. Ignoring this kind of information, Kolsrud warns, may lead to sub-optimal results and in extreme cases of policy experiments even wrong conclusions.

Some papers and books on stochastic simulation include Ahlstedt (1986), Fair (1984, 1986), Kolsrud (1993, 1996) and Sowey (1973) and Essi et al. (2007).

The anatomy of a nonlinear econometric model

A modern nonlinear econometric model, like the one used in this study has the structural form:

$$y_t = f_t\left(x, \theta\right) + u \tag{1}$$

where,
y = response variable, consisting of n values
$f\ \left(x, \theta\right)$: nonlinear function
θ : structural parameters
x : row vector of k explanatory variables
u = column vector of random shocks.

We can also write the model as:

$$y_t = f_t\left(x_t, \theta\right) + u_t, \quad u_t \sim N\left(0, \Sigma\right) \tag{2}$$

Fitting the structural model (2) above to observations x_t and y by nonlinear least squares, returns the model to

$$\hat{y}_t = f_t\left(x_t, \hat{\theta}\right) + \hat{u}_t,\ \ \hat{\theta} \sim N\left(\theta, \hat{\Omega}\right), \quad u_t \sim N\left(0, \hat{\Sigma}\right) \tag{3}$$

With the estimated parameter values $\hat{\theta}$ and the estimated empirical residuals $\hat{u}_t$. The estimated model is then used to make forecast for the response variable conditional on the estimated parameter values and the historic or anticipated values of the explanatory variables. The common way to forecast with the estimated model is to perform a simple deterministic dynamic simulation (Equation 4) with zero expectation values

$$y_d = f\left(x\ , \hat{\theta}\right) \tag{4}$$

for the stochastic residuals; where y_d gives the value of the deterministic forecast. We use hats to denote estimated or simulated values. The model (2) satisfies all the classical assumptions including absence of serial correlation in the disturbance u_t, so that $\hat{\sum}$ is the estimated contemporaneous variance-covariance matrix. The quantity $\hat{\Omega}$ is the variance-covariance matrix for the vector of estimated parameters. We assume that if the nonlinear specification is correct, then the expectations of $\hat{\theta}$ equals θ. We also assume that $\theta \in \Theta$, where Θ is a compact subset of the Euclidean space IR^p.

The specification of the model (2) clearly incorporates a stochastic disturbance process u_t that is an unexplained random component of the response variable y. Since any estimator is a function of the stochastic residuals, the parameter estimates $\hat{\theta}$ are stochastic variables too. Since we are using the estimated model (3) and not the specified model (2) whose parameters, though unknown are fixed in the space Θ, we need to take into account this source of uncertainty in our simulation and forecasting. Most of the times this is ignored. Also, the response is also stochastic since the input variables as shown by (3) are now x_t and $\hat{\theta}$ with only x_t being regarded as deterministic. The estimated model (3) is stochastic and should be estimated, as such as it is different from the model (4) with deterministic variables. Here we can state boldly one outstanding reason for stochastic simulation and that is: y_t has unknown distribution which is not well represented by a single deterministic simulation.

A nonlinear and dynamic model does not have general analytical solution in terms of input variables. Solving (3) by numerical simulation, let us envision an implicit form of the model in the form

$$\hat{y}_t = g\left(x_t, \hat{\theta}, \hat{u}_t\right) \tag{5}$$

The second strong reason for stochastic simulation of nonlinear model seems to be the fact that the expectation values of y_t are generally not equal to the deterministic value ($\hat{y}_t$). This can be demonstrated mathematically (Kolsrud 1993) as follows:

$$E\left(y_t\right) = E\left[g\left(x_t, \hat{\theta}, \hat{u}_t\right)\right] \qquad \text{(x is deterministic).}$$

$$\neq g\left[x_t,\ E\left(\hat{\theta}\right),\ E\left(\hat{u}_t\right)\right] \quad \text{(g is nonlinear)}$$

$$\neq g\left[x_t, \hat{\theta}, E\left(u_t\right)\right]$$

$$= g\left[x_t,\ \hat{\theta},\ 0\right] = \hat{y}_t \quad \text{(deterministic-simulation)} \tag{6}$$

With an unbiased parameter estimator. According to Kolsrud (1993), the estimated deterministic bias $\hat{y} - \hat{E}[y_t]$ in (mildly) nonlinear operative macro econometric models tend to be small and without serious implication. Useful references are Fisher and Salmon (1986), Hall and Henry (1988) and Kolsrud (1993). The implication here is that with stochastic simulation, $\hat{y}_t - \hat{E}[y_t]$ is reduced further by replacing $\hat{y}_t$ with $\hat{y}_s$.
If we denote the estimated deterministic bias by D, then

$$D = \hat{y}_t - \hat{E}[y_t] \tag{7}$$

Setting $\hat{E}[y_t]$ to be equal to the noiseless part of y_t, and denoting it by y_o, then we can at the period, s consider the quantities

$$D_1 = y_s - y_o \tag{8}$$

and

$$D_2 = y_d - y_o \tag{9}$$

where y_s = mean stochastic simulation
y_d = deterministic simulation and y_d is used instead of $\hat{y}$ for convenience and

$$y_s = \frac{1}{N}\sum_{r=1}^{N} y_r \quad (= \hat{y} \text{ for a linear model}) \tag{10}$$

We will use the quantities $\sum D_1^2$ and $\sum D_2^2$ to measure the justification for stochastic simulation and gain in efficiency of one procedure (stochastic simulation) over deterministic simulation. The quantities D_1 and D_2 are dispersions as defined in equations (8) and (9).
Letting $D_3 = y - y_s$ and $D_4 = y - y_d$, then

$$\sum D_3^{\,2} = \sum (y - y_s)^2 \tag{11}$$

and

$$\sum D_4^{\,2} = \sum (y - y_d)^2 \tag{12}$$

These are also measures of dispersions that can be used to appraise efficiency of simulations.

METHODOLOGY AND DATA

The modified Gauss-Newton algorithm is used in estimating the intrinsically non-linear model (13). The choice of model parameters $(\theta_1, \theta_2, \theta_3)$ is such that $\theta_2 + \theta_3 < 1, \theta_2 + \theta_3 = 1$ and $\theta_2 + \theta_3 > 1$ but $\theta_2 + \theta_3 < 2$ while the value of θ_1 is arbitrary and kept constant at $\theta = 10.0$. We use the set of parameters $V = (10, 0.45, 0.50)$ in our estimation and simulation.

The input matrix is made of two variables K (capital) and L (Labour) and are randomly generated and normally distributed independent variables such that they are typical of data set on capital and labour as that of Zarembka (1966). The noisy Y's are obtained according to the relation (13). The Monte Carlo study uses sample sizes of 20 with each experiment replicated 20 times.

Forecast under additive error specification

Step 1: Generate K and L, sample size 25.
Step 2: Make the order statistics of K and L
Step 3: Specify the model

$$y = \theta_1 K^{\theta_2} L^{\theta_3} + u \tag{13}$$

and assign values $\theta_1, \theta_2, \theta_3$ and σ_u^2 for u

Step 4: Generate y (n = 25); using step 2
Step 5: Estimate the model:

$y_t = \theta_1 K_t^{\theta_2} L^{\theta_3} + u_t$ Take note of the estimates $\theta_1, \theta_2, \theta_3$ and σ_u^2 (variance of u)

Step 6: Call the standard errors of $\theta_1, \theta_2, \theta_3$ and u; e_1, e_2, e_3 and e_4
Step 7: A forecast of y is given by

$$\hat{y}_{t+1} = \hat{\theta}_1 K_{t+1}^{\hat{\theta}_2} L_{t+1}^{\hat{\theta}_3}$$

After the equation has been estimated and a forecast $\hat{y}_{t+1}$ has been computed, the standard error of forecast would be computed as follows:

Step 8: Rewrite the equation as

$$\hat{y}_{t+1} = \left(\hat{\theta}_1 + e_1\right) K_{t+1}^{\left(\hat{\theta}_2 + e_2\right)} L_{t+1}^{\left(\hat{\theta}_3 + e_3\right)} + e_4$$

where e_1, e_2, e_3, e_4 are assumed to be normally distributed random variables with mean 0 and standard deviation equal to the computed standard errors from the nonlinear regression corresponding to the last iteration of the estimated process in step 5.
Step 9: Generate random numbers (from the appropriate normal distributions) for e_1, e_2, e_3 and e_4 and use for the forecast of $\hat{y}_{t+1}$ Compute the forecast accordingly.
Step 10: Repeat step 9 some 100 or 200 times. Use the sample standard deviation of the resulting distribution of values for as $\hat{y}_{t+1}$ the standard error of forecast. This approximate standard error of forecast can then be used to calculate confidence intervals.

Table 1. Result of deterministic simulation and stochastic simulation.

Year (T)	K	L	Y	Y_d	Y_s	Y_0	$(Y_d-Y_0)^2$	$(Y_s-Y_0)^2$
1	0.042989	37.84368	15.60766	15.02096	15.02096	14.9282	0.008604	0.008604
2	0.056616	39.2892	18.14622	17.33493	17.33493	17.21707	0.01389	0.01389
3	0.065594	68.01032	23.4381	24.3155	24.3155	24.20339	0.012567	0.012567
4	0.086804	76.26497	30.2531	29.21666	29.21666	29.07404	0.020338	0.020338
5	0.123174	90.77083	37.09047	37.31769	37.31769	37.12844	0.035813	0.035813
6	0.218497	95.80353	49.5472	49.68976	49.72272	49.36751	0.103843	0.126174
7	0.265526	243.744	86.09495	86.1873	86.1873	85.96371	0.049992	0.049992
8	0.296224	534.0287	133.1104	133.5502	133.5637	133.6631	0.012739	0.009873
9	0.377132	545.9177	150.5103	150.6192	150.6192	150.6557	0.001331	0.001331
10	0.383181	568.3063	155.7575	154.7585	154.7585	154.8186	0.003613	0.003613
11	0.394106	571.8151	156.3225	157.2201	157.2201	157.2728	0.002782	0.002782
12	0.424784	686.6588	178.1312	178.0798	178.0798	178.2568	0.031344	0.031344
13	0.448396	704.6885	184.6847	184.8544	184.8544	185.0318	0.031481	0.031481
14	0.479156	730.4341	193.6885	193.9091	193.9091	194.0909	0.033039	0.033039
15	0.509991	816.6073	210.5419	210.79	210.79	211.0619	0.07393	0.07393
16	0.523374	831.8268	216.0429	215.2368	215.2368	215.5172	0.0786	0.0786
17	0.803683	845.2429	263.0052	263.4672	263.4672	263.499	0.001008	0.001008
18	0.805179	884.1231	269.5333	269.6276	269.6276	269.7168	0.007958	0.007958
19	0.823236	914.2897	277.7866	276.9125	276.9125	277.0307	0.01397	0.01397
20	0.83709	932.8829	281.4654	281.8091	281.8942	281.9428	0.01788	0.002364
21	0.858723	943.2417	286.8319	286.648	286.648	286.7777	0.016832	0.016832
22	0.989826	1021.829	317.5849	318.0594	318.0594	318.193	0.017838	0.017838
23	1.546828	1098.346	403.2296	403.5101	403.5101	403.291	0.047994	0.047994
24	1.779118	1289.088	464.8901	465.3866	465.3866	465.3004	0.007427	0.007427
25	1.799437	1922.948	571.5314	570.2389	570.3267	571.2093	0.941732	0.778983
						Sum=	1.586542	1.427742

EMPIRICAL RESULTS

Altogether we estimated 20 equations. Some of the numerical results obtained are summarized and presented in Tables 1 - 4.

RESULTS AND DISCUSSION

We would compare the results of deterministic simulation with that of stochastic (Monte Carlo) simulation of the output. In all, we have estimated at least 20 equations in this study. Essentially this section focuses on the discussion of the results of Monte Carlo simulations and forecast presented in Section 4.

The model considered is:

$$Y = \theta_1 K^{\theta_2} L^{\theta_3} + u \quad (14)$$

where u_1 follow $N\left(0,\sigma^2\right)$.

In all the tables, N stands for the number of replications. The specification (5.1) is the model where an additive error generated data is filled with an additive based model.

Deterministic simulation (y_d), stochastic simulation (y_s) and dispersion from the noiseless output (y_0)

We simulate with the model (14) to obtain the results seen in Table 1. K and L are values of capital and labour, y is the noisy value of the output. The symbol y_0 is the output without the noise (stochastic disturbance) whereas y_d and y_s respectively stand for the deterministic and stochastic simulated values of the output. Table 1 is to enable us construct the measures of dispersions.

$$\sum D_1^2 = \sum \left(y_s - y_0 \right)^2 \quad (15)$$

$$\sum D_2^2 = \sum \left(y_d - y_0 \right)^2 \quad (16)$$

Equation (15) gives the dispersion of stochastic simulated output y_s from y_o. Equation (16) gives the dispersion of deterministic simulation y_d from the noiseless responses y_0. The period for sample estimated is T = 1 to T = 20.

Table 2. Dispersion of y from y_s and y_d.

Year (T)	y	Y_d	Y_s	$(y-y_s)^2$	$(y-y_d)^2$
1	15.60766	15.02096	15.02096	0.344221	0.344221
2	18.14622	17.33493	17.33493	0.658201	0.658205
3	23.4381	24.3155	24.3155	0.769828	0.769828
4	30.2531	29.21666	29.21666	1.074214	1.074215
5	37.09047	37.31769	37.31769	0.051625	0.051625
6	49.5472	49.68976	49.72272	0.030806	0.020322
7	86.09495	86.1873	86.1873	0.008527	0.008527
8	133.1104	133.5502	133.5637	0.20546	0.193432
9	150.5103	150.6192	150.6192	0.011877	0.011877
10	155.7575	154.7585	154.7585	0.998106	0.998106
11	156.3225	157.2201	157.2201	0.805684	0.805686
12	178.1312	178.0798	178.0798	0.002646	0.002646
13	184.6847	184.8544	184.8544	0.028783	0.028783
14	193.6885	193.9091	193.9091	0.048678	0.048678
15	210.5419	210.79	210.79	0.061514	0.061514
16	216.0429	215.2368	215.2368	0.649673	0.649672
17	263.0052	263.4672	263.4672	0.21348	0.21348
18	269.5333	269.6276	269.6276	0.00889	0.00889
19	277.7866	276.9125	276.9125	0.764075	0.764075
20	281.4654	281.8091	281.8942	0.183862	0.118152
21	286.8319	286.648	286.648	0.033847	0.033847
22	317.5849	318.0594	318.0594	0.225221	0.225221
23	403.2296	403.5101	403.5101	0.078644	0.078644
24	464.8901	465.3866	465.3866	0.246531	0.24653
25	571.5314	570.2389	570.3267	1.451328	1.670691
			Sum=	8.955724	9.086867

Table 3. Monte Carlo and deterministic forecast of output using dispersions of y from y_s and y_d.

Monte Carlo forecast						
T	**Y**	**y_s**	**Standard deviation**	**Standard error**	**$(y - y_s)^2$**	**95% confidence interval**
21	286.8319	286.648	20.3	4.54	0.033847	286.65 ± 8.90
22	317.5849	318.0594	23.29	5.21	0.225221	318.06 ±10.21
23	403.2296	403.5101	30.19	6.75	0.078644	403.51± 10.80
24	464.8901	465.3866	35.19	7.87	0.24653	465.39± 15.43
25	571.5314	570.3267	43.7	9.77	1.451302	570.33 ±19.15
	RMSE = 0.64			Sum =	2.035545	
Deterministic forecast						
T	**Y**	**y_d**	**Standard deviation**	**Standard error**	**$(y - y_d)^2$**	**95% confidence interval**
21	286.8319	286.648	Na	na	0.033847	na
22	317.5849	318.0594	Na	na	0.225221	na
23	403.2296	403.5101	Na	na	0.078644	na
24	464.8901	465.3866	Na	na	0.24653	na
25	571.5314	570.2389	Na	na	1.670691	na
	RMSE = 0.67			Sum=	2.254933	

Table 4. Monte Carlo and deterministic forecast of output using dispersions of y_o from y_s and y_d.

			Monte Carlo forecast			
T	**y_o**	**y_s**	**Standard deviation**	**Standard error**	**$(y_0 - y_s)^2$**	**95% confidence interval**
21	286.7777	286.648	20.3	4.54	0.016832	286.65 ± 8.90
22	318.1930	318.0594	23.29	5.21	0.017838	318.06 ±10.21
23	403.291	403.5101	30.19	6.75	0.047994	403.51± 10.80
24	465.3004	465.3866	35.19	7.87	0.007427	465.39± 15.43
25	571.2093	570.3267	43.7	9.77	0.778983	570.33 ±19.15
	RMSE = 0.42			Sum =	0.869074	
			Deterministic forecast			
T	**y_o**	**y_d**	**Standard deviation**	**Standard error**	**$(y_0 - y_d)^2$**	**95% confidence interval**
21	286.7777	286.648	Na	na	0.016832	na
22	318.1930	318.0594	Na	na	0.017838	na
23	403.291	403.5101	Na	na	0.047994	na
24	465.3004	465.3866	Na	na	0.007427	na
25	571.2093	570.2389	Na	na	0.941732	na
	RMSE = 0.45			Sum =	1.031823	

The period T = 21 to T = 25 is the forecast period.

The gain in efficiency (Table 1) of stochastic simulation over deterministic simulation is simply:

$$Eff_1 = \frac{\sum D_2^2 - \sum D_1^2}{\sum D_1^2} x100 = \frac{1.586542 - 1.427742}{1.427742} x100 = 11\% \qquad (17)$$

Dispersion of actual output y from deterministic and stochastic simulations

Using Table 2 and letting $D_3 = y - y_s$ and $D_4 = y - y_d$, then

$$\sum D_3^{\,2} = \sum (y - y_s)^2 = 8.955724 \qquad (18)$$

$$\sum D_4^{\,2} = \sum (y - y_d)^2 = 9.086867 \qquad (19)$$

Gain in efficiency, Eff_2 is

$$Eff_2 = \frac{9.086867 - 8.955724}{8.955724} x100 = 1.5\% \qquad (20)$$

Monte Carlo and deterministic forecast of output

The forecast for each period (T = 21 to T = 25) is made according to the algorithm stated in Section 3. With reference to Table 3, y_s is the mean stochastic simulated forecast. The standard deviation and hence standard error of forecast is obtained from values of stochastic simulation over (N=20) replications. The quantity y_d is the deterministic forecast. The results are recorded in Table 3. The gain in efficiency of Monte Carlo forecast over deterministic forecast.

$$Eff_4 = \frac{2.254933 - 2.035545}{2.035545} x100 = 11\% \qquad (21)$$

The root-mean-squared error (RMSE) in both forecast are respectively 0.64 and 0.67. The result obtained from

Table 4 gives the relative efficiency as

$$Eff_5 = \frac{1.031823 - 0.869074}{0.869074} x100 = 18.7\% \qquad (22)$$

CONCLUSION AND RECOMMENDATIONS

The principal foci of the study are:

(i) To carry out Monte Carlo and deterministic simulations and forecasts of production, and
(ii) To compare forecast errors in (i) under alternative simulations.

We have seriously attempted to meet these objectives in the study. We have performed deterministic and Monte Carlo simulations with the Model (14). We have found out that there is a gain of 11% in efficiency of Monte Carlo simulation over deterministic simulation. Kolrud (1993) demonstrated this gain in efficiency using macro-econometric system of simultaneous equations having some intrinsic non linearity. Our case involves a single intrinsically nonlinear production model.

We have to state that gain in efficiency using dispersion of y_s and y_d from y_0 (Equation 18 and Table 1) is higher than using dispersion of y from y_s and y_d (Equation (19 and Table 2). The first is 11% and the second is 1.5%. In either case, stochastic simulation is superior to deterministic simulation (Essi, 2010).

We now come to dispersion criteria to be used in assessing forecasts. The gain in efficiency of Monte Carlo forecast over deterministic forecast is 11%. The root-mean-squared errors (RMSE) in both forecasts are respectively 0.64 and 0.67. The dispersion criteria used are $\sum (y-y_s)^2$ and $\sum (y-y_d)^2$, the summation starts from t = 21 to t = 25. (Table 3). If we replace $\sum (y-y_s)^2$ and $\sum (y-y_d)^2$ respectively by $\sum (y_0-y_s)^2$ and $\sum (y_0-y_d)^2$, where y_0 is noiseless respond of the output y, the gain in efficiency of Monte Carlo forecast over deterministic forecast is 18.7% (Table 4).

For the forecasts, the dispersion measures $\sum (y-y_s)^2$ and $\sum (y-y_d)^2$ gives 11% efficiency than the measures $\sum (y_0-y_s)^2$ and $\sum (y_0-y_d)^2$ which gives 18.7% efficiency.

We have to state that gain in efficiency using dispersion of y_s and y_d from y_0 is higher than using dispersion of y from y_s and y_d, both for results in simulation and forecast. The question is which relative criterion should be used?

We suggest that the criterion involving y_0 be used if we know the data generating process, otherwise we use dispersion of y from y_s and y_d.

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