Vol. 13(1), pp. 24-38, January-June 2020
DOI: 10.5897/AJMCSR2019.0805
Article Number: 6AF95AE62948
ISSN: 2006-9731

http://www.academicjournals.org/AJMCSR

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African Journal of Mathematics and Computer Science Research

Full Length Research Paper

An internal heat source in temperature rate dependent thermoelastic medium subjected to the effect of rotation and gravity field

F. S. Bayones

Mathematics Department, Faculty of Science, Taif University, Saudi Arabia.

Received 18 September 2019; Accepted 3 January 2020

The present work is devoted to study the effect of rotation and gravity field in a homogeneous, isotropic elastic semi- infinite, which is at initial temperature, and subjected to heat source and load moving with finite velocity. In view of calculating general problems, a numerical solution technique is to be used. For this purpose, the normal mode analysis method is used. The results for the displacement, stress components and temperature distribution are illustrated graphically with some comparisons. The numerical results are given and presented graphically for two different theories, L-S theory and G-L theory. Influence of rotation and gravity on temperature, displacement and stresses components is observed through a numerical example.

Key words: Internal heat source, isotropic, thermoelasticity, two thermal relaxation times, gravity, rotation.

INTRODUCTION

The thermoelasticity theories, which admit a finite speed for thermal signals, have received a lot of attention for the past four decades in contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation (Biot, 1956). Lord and Shulman (1967) have developed a theory on the basis of a modified heat conduction law which involves heat-flux rate. Green and Lindsay (1972) have developed a theory by including temperature-rate among the constitutive variables. Effect of rotation and initial stress on generalized thermoelastic problem in an infinite circular cylinder are discussed by Abd-Alla and Bayones (2011). The problem of transient coupled thermoelasticity of an annular fin was discussed by Abed-Alla et al. (2012). Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section was studied by Ponnusamy (2007). Rayleigh waves in generalized magneto-thermo-viscoelastic granular medium under the influence of rotation, gravity field, and initial stress was studied by Abd-Alla et al. (2011). There are some of the research that have studied effect of rotation, effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffusing body with a spherical cavity studied by Abd-Alla and Abo-Dahab (2012). Effect of hydrostatic initial stress and rotation in Green-Naghdi (type III) thermoelastic half-space with two temperatures was studied by Ailawalia

E-mail: f.s.bayones@hotmail.com

and Budhiraja (2010). An analytical solution for effect of rotation and magnetic field on the composite infinite cylinder in non-homogeneous viscoelastic media are discussed by Bayones and Hussien (2015a). The effect of magnetic field and non-homogeneity on the radial vibration in the hollow rotating elastic cylinder are discussed by Abd-Alla et al. (2013). Radial vibration of wave propagation in an elastic medium of a non-homogeneous isotropic material under the influence of rotation was studied by Abed-Alla and Yahya (2011). Abd-Alla and Bayones (2011) discussed the effect of rotation and initial stress on generalized thermoelastic problem in an infinite circular cylinder. Abd-Alla et al. (2011a) discussed the effect of the rotation on an infinite generalized magneto-thermoelastic diffusing body with a spherical cavity. Abd-Alla et al. (2011b) discussed the effect of the rotation and initial stress on generalized thermoelastic problem in an infinite circular cylinder. Effects of rotation and hydrostatic initial stress on propagation of Raylegh in waves in an elastic solid half-space under the GN theory was studied by Bayones (2012). Effect of rotation and non-homogeneity on the radial vibration in the orthotropic hollow sphere was studied by Abd-Alla et al. (2012a). Abo-Dahab and Abd-Alla (2014) discussed effects of voids and rotation on plane waves in generalized thermoelasticity. Bayones (2015) discussed the effect of rotation and initial magnetic field in fiber-reinforced anisotropic elastic media. Hussein et al. (2015) studied the effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material with the external magnetic field. Also, there are many researchers interested in studying the effect of gravity field. Abd-Alla et al. (2011c) studied the propagation of Rayleigh wave in the generalized granular elastic medium in the presence of initial stress and gravity. Such a procedure should derive an appropriate explicit dispersion equation using the fundamental relations from Abd-Alla et al. (2013a) and simple algebraic mathematical tools. Abd-Alla et al. (2013b) discussed Influence of the rotation and gravity field on stonely waves in a non-homogeneous orthotropic elastic medium. Bayones and Hussien (2015b) discussed fibre-reinforced generalized thermoelastic medium subjected to gravity field. Abd-Alla et al. (2015) discussed wave propagation in fibre-reinforced anisotropic thermoelastic medium subjected to gravity field. Abd-Alla et al. (2011d) discussed propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field.

FORMULATION OF THE PROBLEM

We consider an infinite isotropic generalized thermoelastic medium with the dependence of modulus of elasticity and thermal conductivity on the reference temperature under the effect of rotation and gravity field.

The initial stress is given as Datta (1986)

$$\sigma_{xx} = \sigma_{yy} = \tau, \quad \sigma_{xy} = 0 \tag{1}$$

where τ is a function of depth. The equilibrium equations of the initial stress is in the form

$$\frac{\partial \tau}{\partial x} = 0, \quad \frac{\partial \tau}{\partial y} - \rho g = 0. \tag{2}$$

The elastic medium is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega \underline{n}$ where $\underline{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame has two additional term centripetal acceleration, viz; $\overline{\Omega} \times \left(\overline{\Omega} \times \overline{u}\right)$ is the centripetal acceleration due to time varying motion only and $2\overline{\Omega} \times \bar{\dot{u}}$ is the Cariole's acceleration, and $\overline{\Omega} = \left(0\,,0\,,\Omega\right)$.

The equation of motion takes the form:

$$\sigma_{ij,i} + F_j = \rho[\ddot{u} + \left\{\vec{\Omega} \times (\vec{\Omega} \times \vec{u})\right\} + (2\Omega \times \dot{\vec{u}})]_j \,, \quad i, j = 1,2,3 \tag{1}$$

The constitutive equation takes the form

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu e_{ij} - \gamma \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \delta_{ij} \tag{2}$$

where, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ the components of strain, $\omega_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j})$ the stress tensor, $\lambda,\ \mu$ are elastic constants.

The energy equation is given by:

$$K\left(n^* + t_1 \frac{\partial}{\partial t}\right)\nabla^2 T = \rho c_e \left(n_1 \frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}\right) + \left(3\lambda + 2\mu\right)\alpha_t T_0 \left(n_1 \frac{\partial}{\partial t} + \tau_0 n_0 \frac{\partial^2}{\partial t^2}\right)\underline{\nabla}.\vec{u} \tag{3}$$

where τ_0, τ_1 are the two relaxation time, n^*, n_1 and n_0 the parameters helps to make the above mentioned fundamental equations possible for two different theories as:

i) L-S theory due to internal heat source, when:

$$n^* = n_1 = n_0 = 1\ ,\ t_1 = \tau_1 = 0,\ \tau_0 > 0$$

ii) G-L theory due to internal heat source, when:

$$n^* = n_1 = 1,\ n_0 = t_1 = 0,\ \tau_1 \ge \tau_0 > 0$$

In the absence of body forces F_j, the dynamic equation of motion under magnetic field g and rotation in two-dimensions (x,y) reduces to:

$$(\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\nabla^2 u + \rho g\frac{\partial v}{\partial x} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\left\{\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right\} \tag{4}$$

$$(\lambda+\mu)\frac{\partial\theta}{\partial y} + \mu\nabla^2 v - \rho g\frac{\partial u}{\partial x} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y} = \rho\left\{\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t}\right\} \tag{5}$$

The stress–displacement relations with incremental isotropy are given by

$$\begin{aligned}
\sigma_{xx} &= (\lambda+2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,\\
\sigma_{yy} &= (\lambda+2\mu)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,\\
\sigma_{xy} &= \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right).
\end{aligned} \tag{6}$$

Where $\gamma = (3\lambda+2\mu)\alpha_t$, $\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$,

$\underline{\nabla} = \frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j}$, T is the temperature, c_e is the specific heat, ρ is the density, α_t is the coefficient of thermal expansion, τ_0 is relaxation time, K is the thermal conductivity and $\vec{u} = (u, v, 0)$ is displacement vector, T_0 is the initial temperature.

Introducing following non-dimensional variables as

$$x_i' = \frac{c_1}{k_1}x_i\ ,\ \vec{u}' = \frac{c_1^3}{k_1}\frac{\rho}{(3\lambda+2\mu)\alpha_t T_0}\vec{u}\ ,\ \Omega' = \frac{k_1}{c_1^2}\Omega,$$

$$\tau_{ij}' = \frac{1}{(3\lambda+2\mu)\alpha_t T_0}\tau_{ij}\ ,\ T' = \frac{T}{T_0}$$

$$g' = \frac{\rho k_1 g}{(3\lambda+2\mu)\alpha_t T_0 c_1}\vec{u},$$

$$(t_1', \tau_i') = \frac{c_1^2}{k_1}(t_1, \tau_i)\ ,\ Q' = \frac{k_1^2}{K\ c_1^2 T_0}Q \tag{7}$$

where, $c_1^{\ 2} = \frac{\lambda+2\mu}{\rho}$ and $k_1 = \frac{K}{\rho c_e}$.

Introducing the non-dimension variables (7) into (3)-(5), we get

$$\left(n^* + t_1'\frac{\partial}{\partial t'}\right)\nabla'^2 T' = \left(n_1\frac{\partial}{\partial t'} + \tau_0'\frac{\partial^2}{\partial t'^2}\right)T' + \varepsilon\left(\frac{\partial}{\partial t'} + \tau_0'\frac{\partial^2}{\partial t'^2}\right)\underline{\nabla}.\vec{u'} - \left(n_1 + n_0\tau_0'\frac{\partial}{\partial t'}\right)Q' \tag{8a}$$

$$(\lambda+\mu)\frac{\partial\theta'}{\partial x'} + \mu\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) - (\lambda+2\mu)\left(1+\tau_1'\frac{\partial}{\partial t'}\right)\frac{\partial T'}{\partial x'} + g'(\lambda+2\mu)\frac{\partial v'}{\partial x'} = (\lambda+2\mu)\left\{\frac{\partial^2 u'}{\partial t'^2} - \Omega'^2 u' - 2\Omega'\frac{\partial v'}{\partial t'}\right\} \tag{8b}$$

$$(\lambda+\mu)\frac{\partial\theta'}{\partial y'} + \mu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) - (\lambda+2\mu)\left(1+\tau_1'\frac{\partial}{\partial t'}\right)\frac{\partial T'}{\partial y'} - g'(\lambda+2\mu)\frac{\partial u'}{\partial x'} = (\lambda+2\mu)\left\{\frac{\partial^2 v'}{\partial t'^2} - \Omega'^2 v' + 2\Omega'\frac{\partial u'}{\partial t'}\right\} \tag{8c}$$

$$\begin{aligned}
\sigma_{x'x'}' &= \frac{\partial u}{\partial x'} + (1-2c^2)\frac{\partial v}{\partial y'} - \left(1+\tau_1'\frac{\partial}{\partial t'}\right)T',\\
\sigma_{y'y'}' &= \frac{\partial v'}{\partial y'} + (1-2c^2)\frac{\partial u'}{\partial x'} - \left(1+\tau_1'\frac{\partial}{\partial t'}\right)T',\\
\sigma_{x'y'}' &= c^2\left(\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right),
\end{aligned} \tag{8d}$$

where,

$$\varepsilon = \frac{(3\lambda+2\mu)^2\alpha_t^{\ 2}T_0}{\rho c_e(\lambda+2\mu)}\ ,\quad c^2 = \frac{\mu}{\lambda+2\mu},\ \ \theta' = \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'}$$

For ease work, we all put $x_i' = x_i$ in above equations , we have

$$\left(n^* + t_1\frac{\partial}{\partial t}\right)\nabla^2 T = \left(n_1\frac{\partial}{\partial t'} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \varepsilon\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\underline{\nabla}.\vec{u} - \left(n_1 + n_0\tau_0\frac{\partial}{\partial t'}\right)Q \tag{9a}$$

$$(\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - (\lambda+2\mu)\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + g(\lambda+2\mu)\frac{\partial v}{\partial x} = (\lambda+2\mu)\left\{\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right\} \tag{9b}$$

$$(\lambda+\mu)\frac{\partial\theta}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-(\lambda+2\mu)\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}-g(\lambda+2\mu)\frac{\partial u}{\partial x}$$
$$=(\lambda+2\mu)\left\{\frac{\partial^2 v}{\partial t^2}-\Omega^2 v+2\Omega\frac{\partial u}{\partial t}\right\} \tag{9c}$$

$$\sigma_{x'x'}=\frac{\partial u}{\partial x}+(1-2c^2)\frac{\partial v}{\partial y}-\left(1+\tau_1\frac{\partial}{\partial t}\right)T,$$
$$\sigma_{y'y'}=\frac{\partial v}{\partial y}+(1-2c^2)\frac{\partial u'}{\partial x'}-\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{9d}$$
$$\sigma_{x'y'}=c^2\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right),$$

SOLUTION OF THE PROBLEM

Using Helmholt's theorem (Morse and Feshbach, 1953), and introducing the potential ϕ and $\vec{\psi}$ by the equation

$$\vec{u}=grad\,\phi+curl\vec{\psi}\ ,\ \vec{\psi}=(0,0,\psi) \tag{10}$$

From Equation 10, the displacement components u, v obtained as

$$u=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}\ ,\ \ v=\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}\ , \tag{11}$$

Substituting from Equation 11 into Equations (9a-9d), we get

$$\left(\left(n^*+t_1\frac{\partial}{\partial t}\right)\nabla^2-n_1\frac{\partial}{\partial t}-\tau_0\frac{\partial^2}{\partial t^2}\right)T=\varepsilon\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi-\left(n_1+n_0\tau_0\frac{\partial}{\partial t'}\right)Q\ ,$$
(12a)

$$\left(\nabla^2+g\frac{\partial}{\partial y}-\frac{\partial^2}{\partial t^2}-2\Omega\frac{\partial}{\partial t}+\Omega^2\right)\phi=\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{12b}$$

$$\left(\nabla^2+\frac{1}{c^2}\left(g\frac{\partial}{\partial y}-\frac{\partial^2}{\partial t^2}+\Omega^2-2\Omega\frac{\partial}{\partial t}\right)\right)\psi=0\ . \tag{12c}$$

BOUNDARY CONDITIONS

The boundary conditions at the interface $y=0$ subjected to an arbitrary normal force P_1 are:

1- $\sigma_{yy}(x,0,t)=-P_1e^{(\omega t+iax)}$
2- $\sigma_{xy}(x,0,t)=0$
3- $T=0$ (13)

where P_1 is the magnitude of mechanical force.

Normal mode analysis

We assume the solution of Equations (12a-12c) to take the form:

$$[\phi,\psi,T](x,y,t)=[\bar{\phi},\bar{\psi},\bar{T},\overline{\sigma_{ij}}](y)e^{(\omega t+iax)},\quad Q'=Qe^{(\omega t+iax)},Q'=Q_0 \tag{15}$$

where ω is the complex time constant, and a is the wave number in x- direction and Q_0 is the magnitude of the stable internal heat source.

Using Equation 14 in (12a-12c), we obtain

$$\left(\frac{d^2}{dy^2}-\ell_1\right)\bar{T}=\left(\ell_2+\ell_3\frac{d^2}{dy^2}\right)\bar{\phi}+\ell_4 Q_0 \tag{16}$$

$$\left(\frac{d^2}{dy^2}+g\frac{d}{dy}+\ell_5\right)\bar{\phi}=(1+\tau_1\omega)\,\bar{T} \tag{17}$$

$$\left(\frac{d^2}{dy^2}+\frac{g}{c^2}\frac{d}{dy}+\ell_6\right)\bar{\psi}=0\ . \tag{18}$$

Eliminating T from Equations (16) and (17), we obtain

$$\left(\frac{d^4}{dy^4}+g\frac{d^3}{dy^3}+M_1\frac{d^2}{dy^2}-g\ell_1\frac{d}{dy}-M_2\right)\bar{\phi}=M_3Q_0 \tag{19}$$

where

$$\ell_1=a^2+\frac{\omega(n_1+\tau_0\omega)}{(n^*+t_1\omega)}\ ,\ \ell_2=-\frac{\omega\varepsilon a^2(1+\tau_0\omega)}{(n^*+t_1\omega)}\ ,\ \ell_3=\frac{\omega\varepsilon(1+\tau_0\omega)}{(n^*+t_1\omega)}\ ,\ell_4=\frac{-(n_1+n_0\tau_0\omega)}{(n^*+t_1\omega)}$$
$$\ell_5=\Omega^2-2\Omega\omega-\omega^2-a^2,\ \ell_6=\frac{1}{c^2}\left(\Omega^2-\omega^2-2\Omega\omega\right)-a^2,$$
$$M_1=\ell_5-\ell_1-(1+\tau_1\omega)\,\ell_3\ ,\ M_2=\ell_1\ell_5+\ell_2(1+\tau_1\omega)\ ,\ M_3=(1+\tau_1\omega)\,\ell_4.$$

Equation 19 is an ordinary differential equation of the fourth degree, its roots can be calculated using a program, the solution of Equation 19 is given by:

$$\phi = \left\{ \left(Ae^{-\lambda_1 y} + Be^{-\lambda_2 y}\right) + \frac{M_3 Q_0}{M_2} \right\} e^{(\omega t + iax)} \quad (20)$$

In a similar way, we get

$$T = \left\{ \left(h_1 Ae^{-\lambda_1 y} + h_2 Be^{-\lambda_2 y}\right) + h_3 \right\} e^{(\omega t + iax)} \quad (21)$$

The solution of Equation 18 is given by:

$$\psi = \left(Ce^{-qy}\right)\ e^{(\omega t + iax)} \quad (22)$$

where

$$h_1 = \frac{\lambda_1^2 + g\lambda_1 + \ell_5}{(1+\tau_1\omega)}, \quad h_2 = \frac{\lambda_2^2 + g\lambda_2 + \ell_5}{(1+\tau_1\omega)},$$

$$h_3 = \frac{\ell_5 M_3 Q_0}{M_2(1+\tau_1\omega)}, \quad q = \frac{\sqrt{\left(\frac{g}{c^2}\right)^2 - 4\ell_5} + \frac{g}{c^2}}{2}$$

Putting Equations (20-22) into (11) and (9d), we get the expressions of displacement, stress for isotropic thermoelastic medium as:

$$u = \left\{ ia\left[\left(Ae^{-\lambda_1 y} + Be^{-\lambda_2 y}\right) + \frac{M_3 Q_0}{M_2}\right] - Cqe^{-qy} \right\} e^{\omega t + iax} \quad (23)$$

$$v = -\left\{ \lambda_1 Ae^{-\lambda_1 y} + \lambda_2 Be^{-\lambda_2 y} + ia\ Ce^{-q_1 y} \right\} e^{\omega t + iax} \quad (24)$$

$$\sigma_{yy} = \left\{\left(H_1 Ae^{-\lambda_1 y} + H_2 Be^{-\lambda_2 y}\right) + H_3 Ce^{-qy} + H_4\right\} e^{\omega t + iax} \quad (25)$$

$$\sigma_{xx} = \left\{\left(H_5 Ae^{-\lambda_1 y} + H_6 Be^{-\lambda_2 y}\right) - 2iaqc^2 Ce^{-qy} + H_7\right\} e^{\omega t + iax} \quad (26)$$

$$\sigma_{xy} = c^2\left\{-2ia\ \left(Ae^{-\lambda_1 y} + Be^{-\lambda_2 y}\right) + \left(q^2 + a^2\right)Ce^{-qy}\right\} e^{\omega t + iax} \quad (27)$$

where

$$H_1 = \lambda_1^2 - a^2\left(1-2c^2\right) - h_1(1+\tau_1\omega),$$

$$H_2 = \lambda_2^2 - a^2\left(1-2c^2\right) - h_2(1+\tau_1\omega), \qquad H_3 = 2iac^2 q,$$

$$H_4 = -\left[\frac{a^2 M_3 Q_0(1-2c^2)}{M_2} + h_3(1+\tau_1\omega)\right],$$

$$H_5 = -a^2 + \left(1-2c^2\right)\lambda_1^2 - h_1(1+\tau_1\omega),$$

$$H_6 = -a^2 + \left(1-2c^2\right)\lambda_2^2 - h_2(1+\tau_1\omega),$$

$$H_7 = \frac{iaM_3 Q_0}{M_2} - h_3(1+\tau_1\omega).$$

Invoking the boundary conditions (13) at the surface, we obtain a system of three equations, and applying the inverse of matrix method, we obtain the values of three constants:

$$A = \frac{\Delta_1}{\Delta}, \quad B = \frac{\Delta_2}{\Delta}, \quad C = \frac{\Delta_3}{\Delta},$$

where

$$\Delta = -(q^2 + a^2)\ (h_2 H_1 + h_1 H_2) - 2iaH_3(h_2 - h_1),$$

$$\Delta_1 = (q^2 + a^2)\ (h_2(P + H_4) - h_3 H_2) - 2iaH_3 h_3,$$

$$\Delta_2 = (q^2 + a^2)\ (h_3 H_1 - h_1(P + H_4)) + 2iaH_3 h_3,$$

$$\Delta_3 = 2ia\ (h_3(H_1 - H_2) - (P + H_4)(h_1 - h_2)),$$

From Equations (21) and (23-26), we can determine the heat conduction, displacement and stress components in two theory L-S theory and G-L theory.

i) L-S theory due to internal heat source, when:

$$n^* = n_1 = n_0 = 1, \ t_1 = \tau_1 = 0, \ \tau_0 > 0$$

ii) G-L theory due to internal heat source, when:

$$n^* = n_1 = 1, \ n_0 = t_1 = 0, \ \tau_1 \geq \tau_0 > 0$$

iii) L-S theory due to internal heat source, when:

$$n^* = n_1 = n_0 = 1, \ t_1 = \tau_1 = 0, \ \tau_0 > 0$$

iv) G-L theory due to internal heat source, when:

$$n^* = n_1 = 1, \ n_0 = t_1 = 0, \ \tau_1 \geq \tau_0 > 0$$

SPECIAL CASES

a) If neglected, the gravity field g=0

i) L-S theory due to internal heat source, when:

$$n^* = n_1 = n_0 = 1 \ , \ t_1 = \tau_1 = 0, \ \tau_0 > 0$$

ii) G-L theory due to internal heat source, when:

$$n^* = n_1 = 1, \ n_0 = t_1 = 0, \ \tau_1 \geq \tau_0 > 0$$

b) If neglected, the rotation $\Omega = 0$

i) L-S theory due to internal heat source, when:

$$n^* = n_1 = n_0 = 1 \ , \ t_1 = \tau_1 = 0, \ \tau_0 > 0$$

ii) G-L theory due to internal heat source, when:

$$n^* = n_1 = 1, \ n_0 = t_1 = 0, \ \tau_1 \geq \tau_0 > 0$$

NUMERICAL RESULTS AND DISCUSSION

The material chosen for this purpose of stainless steel comprised the physical data given below (Abouelregal, 2011)

$\alpha_t = 13.2\ 10^{-6} \deg^{-1}$., $c_e = 0.560\ 10^3$., $T_0 = 293.1K$,
$\lambda = 9.3\ 10^{10}\ Nm^{-1}$., $\mu = 8.4\ 10^{10}\ Nm^{-1}$., $P = 2$., $P_1 = 1$., $m = 4.25$,
$\rho = 7.97\ 10^3$.,
$K = 1.13\ 10^2$., $Q_0 = 10$., $n_1 = 1$., $n_0 = 1$., $n_s = 1$., $a = 2.1$.

The results are displayed in Figures 1 to 8.

General cases

Figure 1 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in L-S theory due to internal heat source. It is observed that the u, v are decreasing with increasing of the rotation, while the mean values of σ_{yy}, σ_{xy} and T increase with the increasing of rotation.

Figure 2 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in G-L theory due to internal heat source. It is observed that the u, v are decreasing with increasing of the rotation, while the mean values of σ_{yy}, σ_{xx}, σ_{xy} and T are increased with the increasing of rotation.

Figure 3 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in L-S theory due to internal heat source. It is observed that the u, v, σ_{xx} and T are decreased with increasing of the rotation, while the mean values of σ_{yy} and σ_{xy} are increased with increasing of gravity field.

Figure 4 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in G-L theory due to internal heat source. It is observed that the u, v, σ_{xx} and T are decrease with increasing of the rotation, while the mean values of σ_{yy} and σ_{xy} are increased with increasing of gravity field.

Special cases

i) If neglected, the gravity field g=0, we have

Figure 5 shows the variations of the non-dimensional values of displacement vectors u,v ., normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in L-S theory due to internal heat source. It is observed that the v , σ_{yy} and σ_{xy} are decreasing with increasing of the rotation, while the mean values of u, σ_{xx}, and T are increased with the increasing of rotation.

Figure 6 shows the variations of the non-dimensional values of displacement vectors u,v ., normal stress σ_{yy}, σ_{xx}., shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in G-L theory due to internal heat source. It is observed that the σ_{yy}, σ_{xy} are decreased with increasing of the rotation, while the mean values of u, v , σ_{xx} and T increased with increasing of

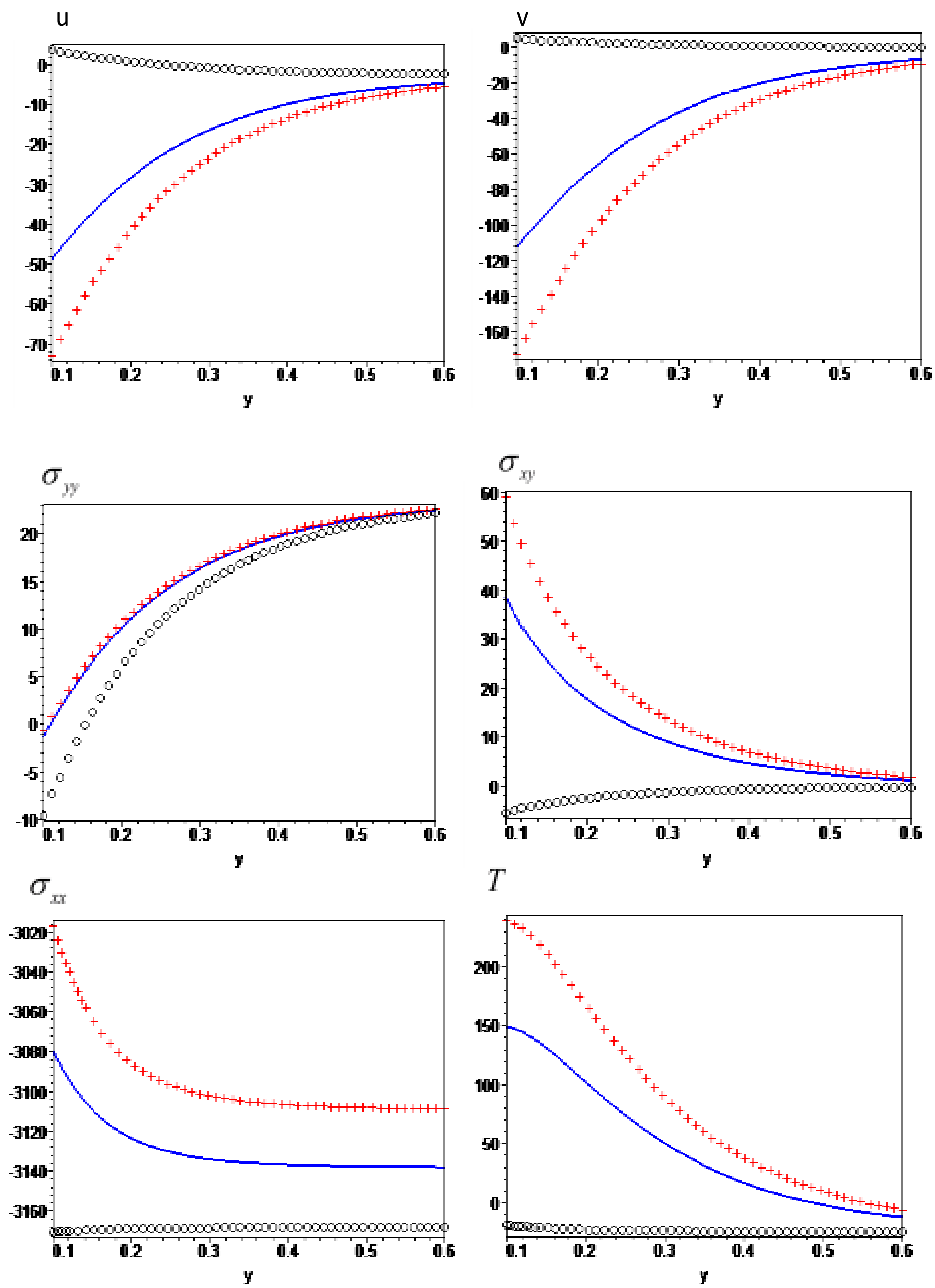

Figure 1. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of rotation with respect to y.
$\Omega = 0.3$ooooooooooooooo $\Omega = 0.4$ ------------------------------ $\Omega = 0.5$ ++++++++++++++++, $g = 5$., $\omega = 8.$, $\tau_0 = 3.5$.

rotation.

ii) If neglected, the rotation $\Omega = 0$***, we have***

Figure 7 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect to axial y for moving heat source in L-S theory due to internal heat source. It is observed that the u, v, σ_{xx} and

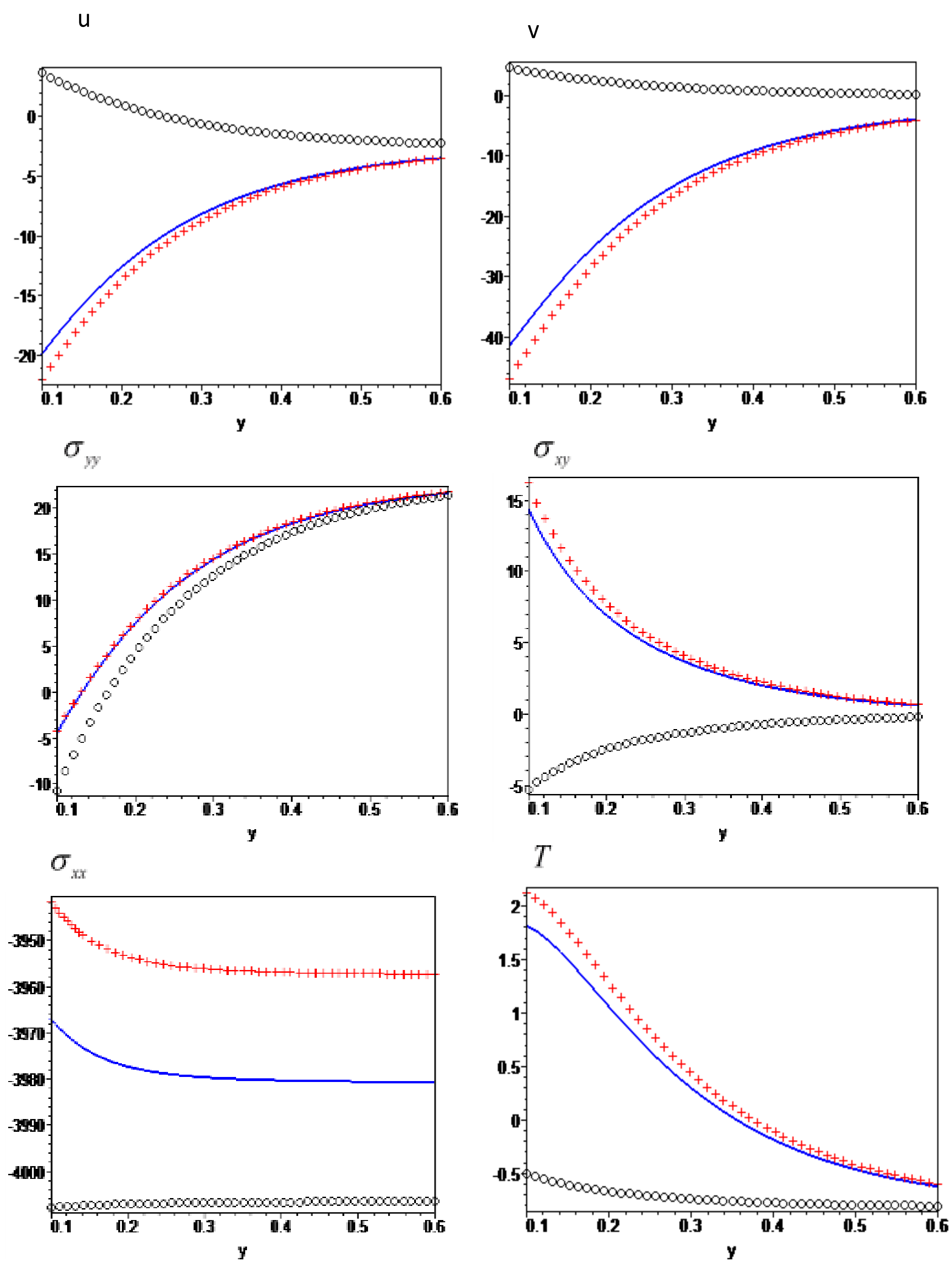

Figure 2. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of rotation with respect to y.

$\Omega = 0.3$ oooooooooooooo $\Omega = 0.4$ ------------------------------ $\Omega = 0.5$ +++++++++++++++,
$g = 5$., $\omega = 8.$, $\tau_0 = 3.5$.

T are decreased with increasing of the rotation, while the mean values of σ_{yy} and σ_{xy} are increased with increasing of gravity field.

Figure 8 shows the variations of the non-dimensional values of displacement vectors u, v, normal stress σ_{yy}, σ_{xx}, shear stress σ_{xy} and temperature T with respect

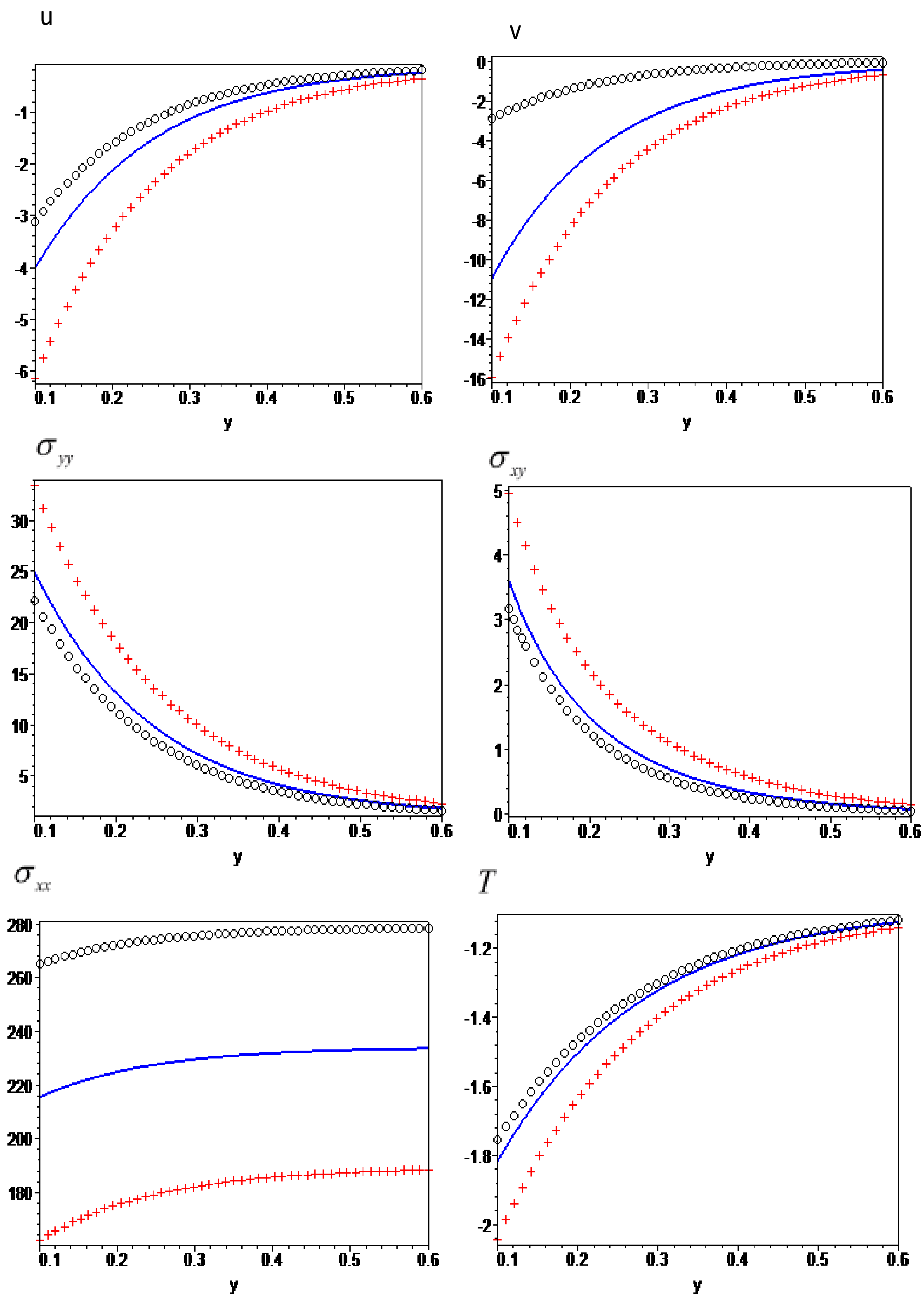

Figure 3. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of gravity field with respect to y.
$g = 3$ooooooooooooooo $g = 4$ ------------------------------- $g = 5$ ++++++++++++++++,
$\Omega = 0.5$., $\omega = 8.$, $\tau_0 = 0.4$.

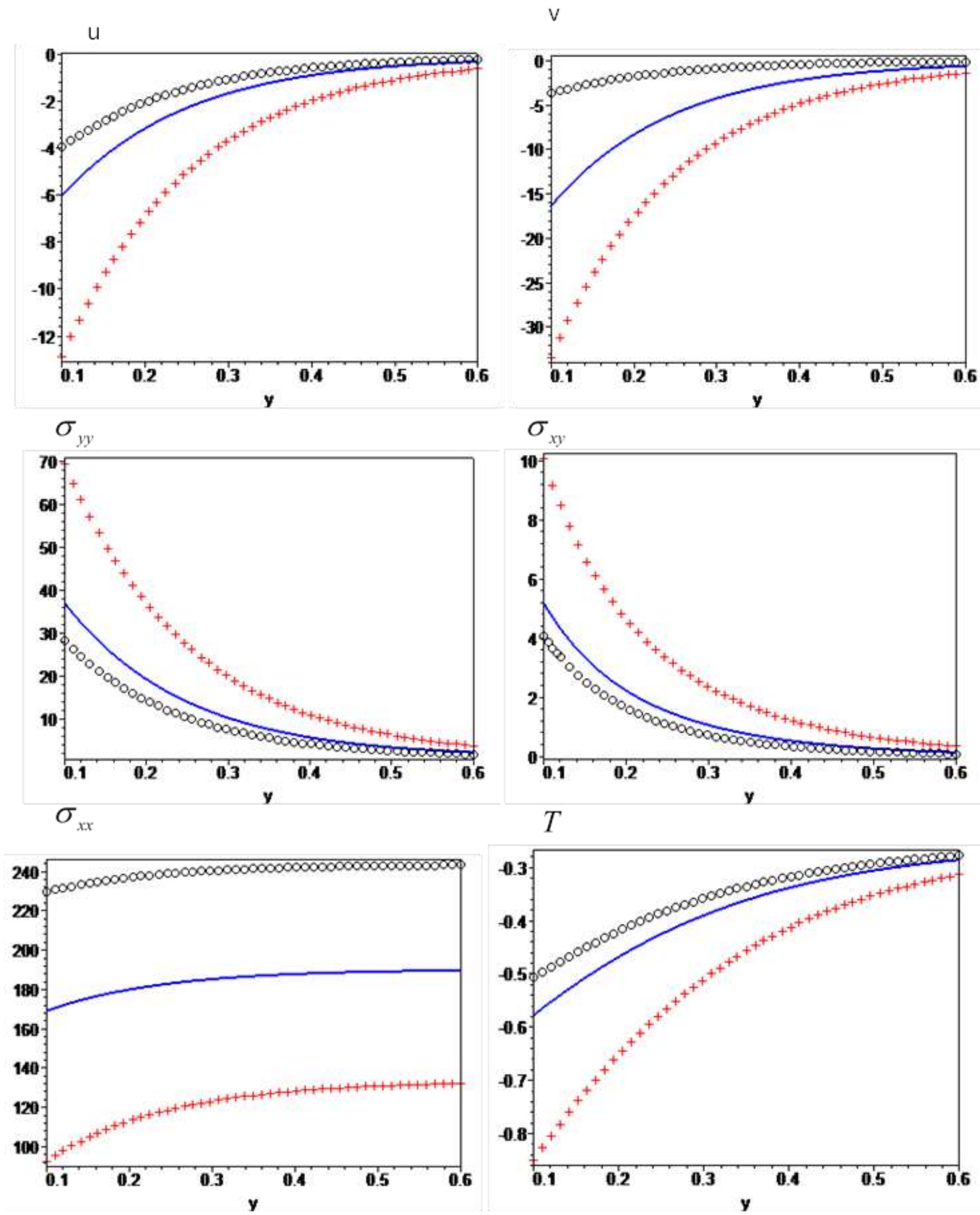

Figure 4. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of gravity field with respect to y.
$g = 3$ ooooooooooooooo, $g = 4$ --------------------------------, $g = 5$ ++++++++++++++++,
$\Omega = 0.5$., $\omega = 8.$, $\tau_0 = 0.4$.

to axial y for moving heat source in G-L theory due to internal heat source. It is observed that the u, v, σ_{xx} and

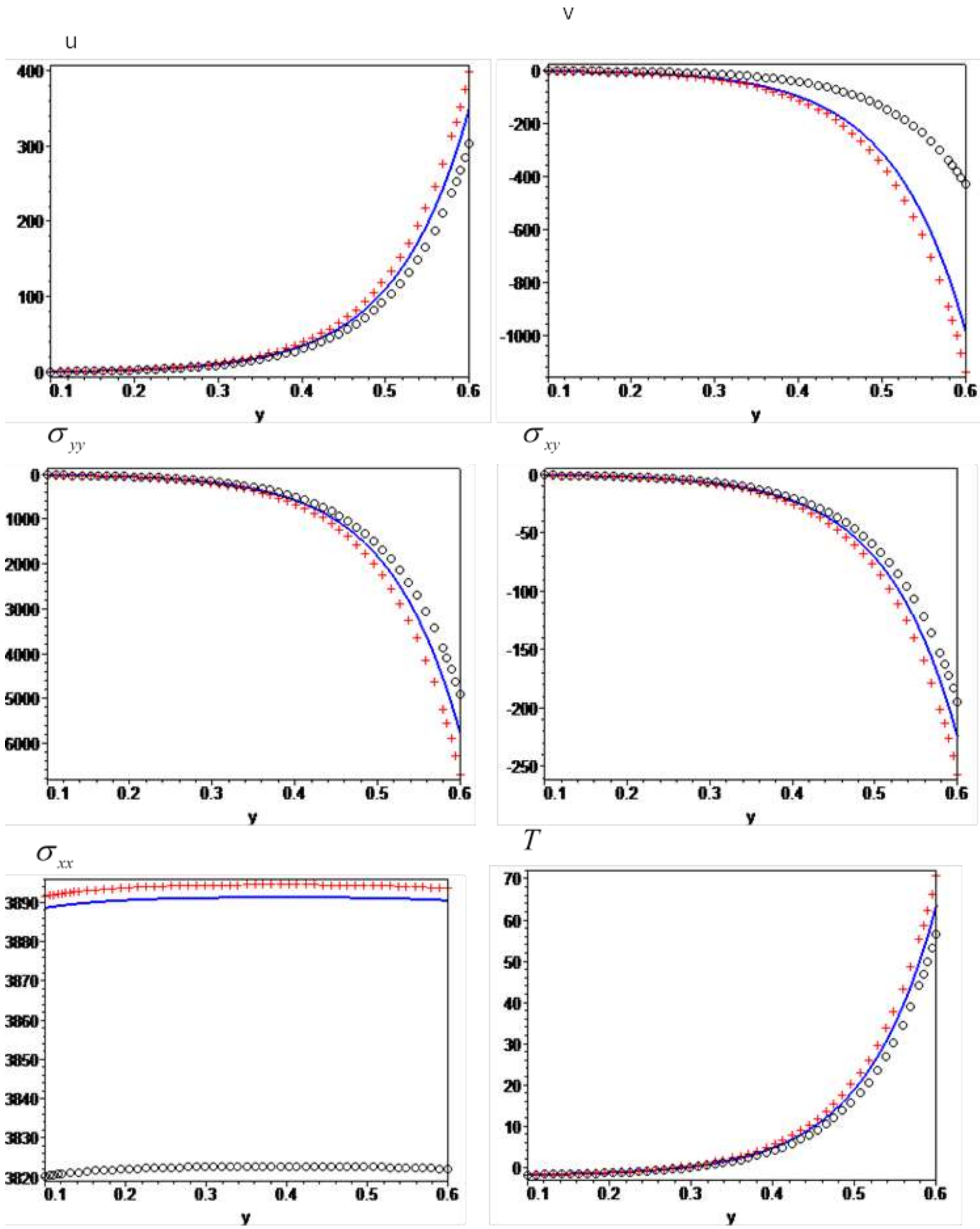

Figure 5. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of rotation with respect to y $\Omega = 0.3$ooooooooooooooo, $\Omega = 0.4$ -------------------------------, $\Omega = 0.5$ ++++++++++++++++, $g = 0$., $\omega = 8.$, $\tau_0 = 3.5$.

T are decreased with increasing of the rotation, while the mean values of σ_{yy} and σ_{xy} are increased with

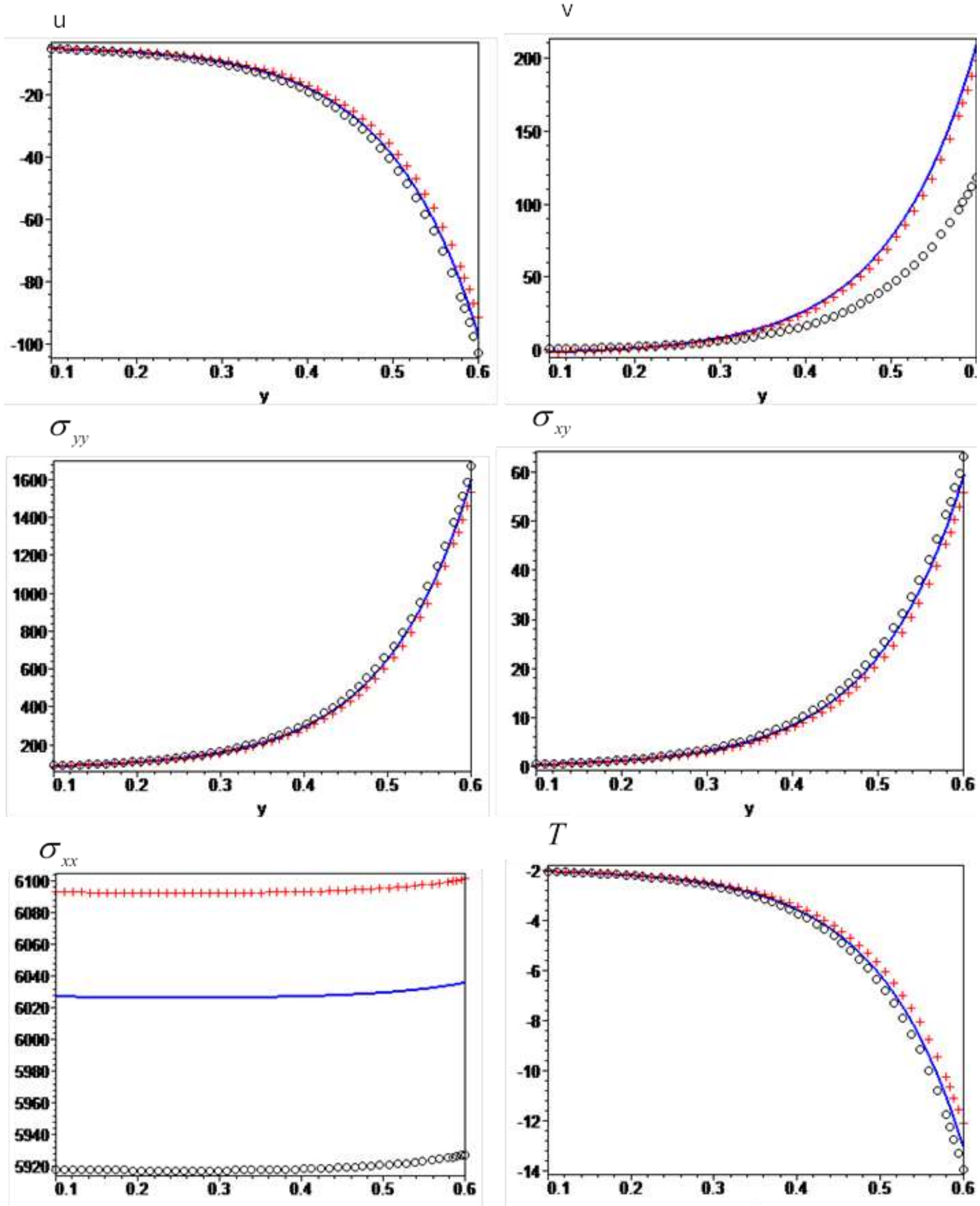

Figure 6. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of rotation with respect to y.

$\Omega = 0.3$ ooooooooooooooo, $\Omega = 0.4$ -------------------------------, $\Omega = 0.5$ ++++++++++++++++,

$g = 0$., $\omega = 8.$, $\tau_0 = 3.5.$

increasing of gravity field.

CONCLUDING REMARKS

(1) The present work examined the potential procedure to study the internal heat source in temperature rate dependent thermoelastic medium in the presence of rotation and gravity. A correct procedure available in literature uses the Helmholt's theorem formalism, which is not transparent enough for any generalization, whereas, the procedure explained in the present work is equally

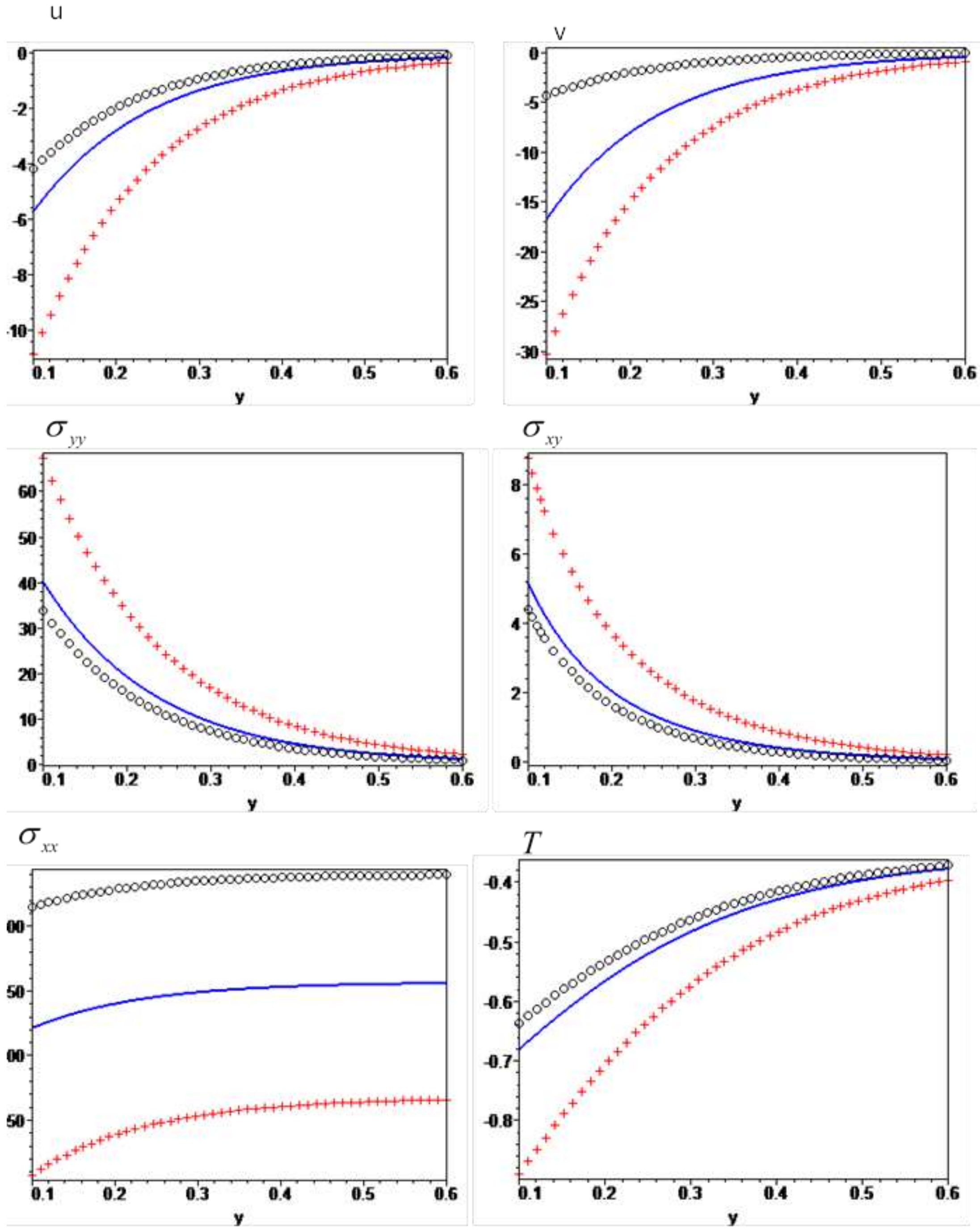

Figure 7. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of gravity field with respect to y.

$g = 3$ooooooooooooooo, $g = 4$ -------------------------------, $g = 5$ ++++++++++++++++,

$\Omega = 0$., $\omega = 8.$, $\tau_0 = 0.4$

applicable to study the effect of rotation (and/or gravity) on the displacement components, temperature and stress components in an isotropic generalized thermoelastic medium.

2) An important observation from the above numerical example is nearly 10% increase in the non-dimensional

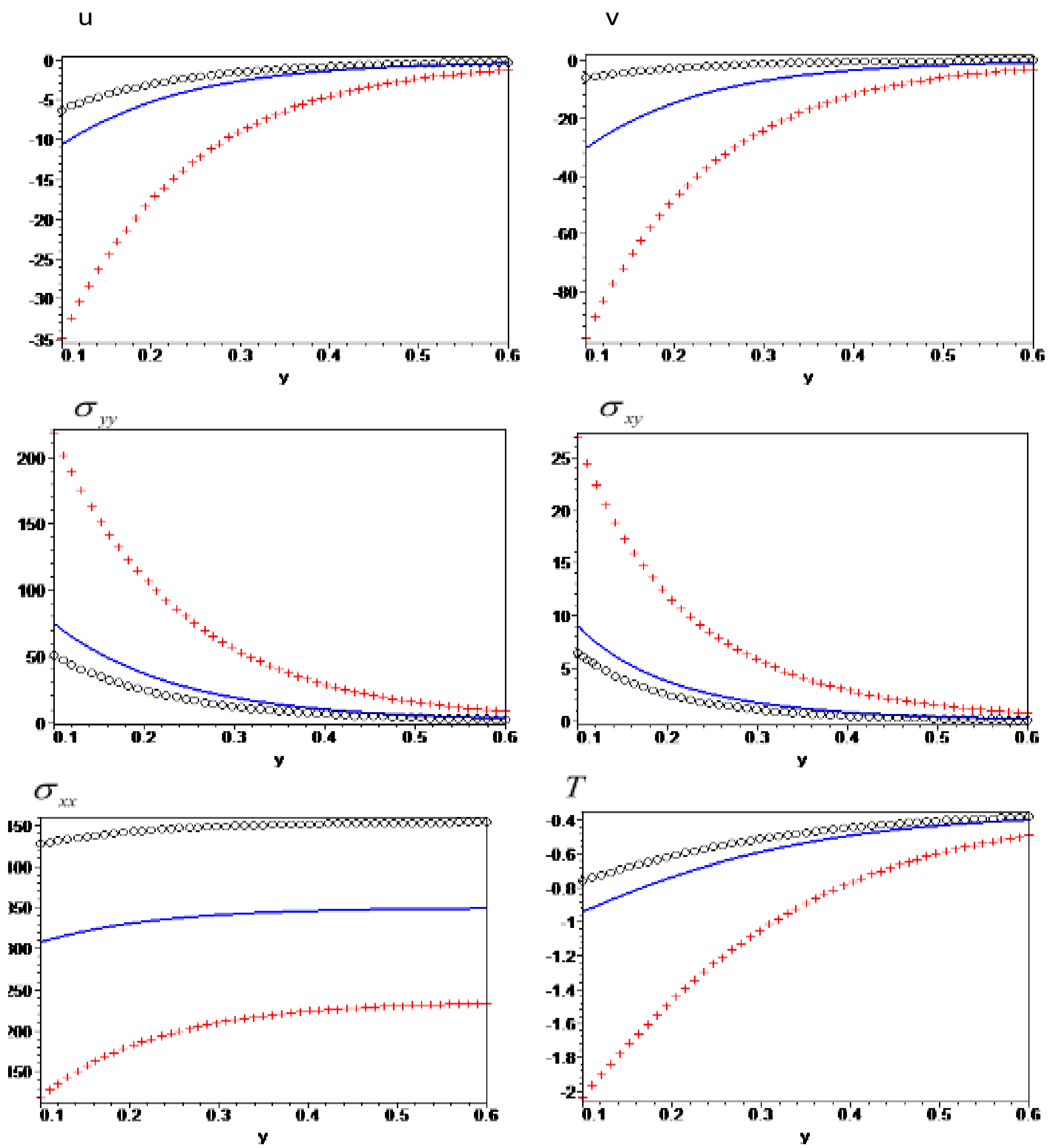

Figure 8. Variation of magnitude of $u, v, \sigma_{xx}, \sigma_{xx}, \sigma_{xx}$ and T with varying values of gravity field with respect to y.

$g = 3$ooooooooooooooo, $g = 4$ -------------------------------, $g = 5$ ++++++++++++++++,

$\Omega = 0$., $\omega = 8.$, $\tau_0 = 0.4$.

displacement components with the presence of rotation. Similarly, the enhancement in particle motion is another clear effect of rotation, for different values of y-axis. In fact, the numerical example considers only a particular elastic material with hypothetical rotation values. So, these results may not qualify for generalization. However, when supported with a real data, the mathematical model derived in this work may be used to compute the exact effects of rotation on temperature, displacement and stresses components.

3) The rotation and gravity significantly influenced the variations of the temperature, displacement and stresses components.

4) The present study provides a simple and correct alternative approach to all the researchers, who are made to believe that an isotropic body can be composed through the use of potential functions. It does not make any difference, whether such isotropy is inherent or

induced by stress, reinforcement, lamination, piezoelasticity, etc.

5) The present problem can also be studied in the absence of a heat source as well.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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