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Full Length Research Paper

Probabilistic analysis of two warm standby system with instructions at need

G. S. Mokaddis[1]*, Y. M. Ayed[2] and H. S. AL-Hajeri[3]

[1]Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt.
[2]Department of Mathematics, Faculty of Science, Suez University, Suez, Egypt.
[3]College of Business Administration, Kuwait University, Kuwait.

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This paper is concerned with two unit warm standby systems having one expert repairman and one assistant repairman. The assistant is called only if the expert repairman is busy. The assistant repairman who may repair the failed unit after getting instructions by his master; but sometimes the assistant repairman may repair it without having the need of instructions for doing repair with probability *p* and *q*, respectively. Techniques of the semi-Markov processes and regenerative processes are used to obtain various measures of system effectiveness mean time to system failure (MTSF), availability analysis, busy period of an expert repairman, busy period of an assistant repairman and profit incurred.

Key words: Mean time to system failure (MTSF), availability, warm standby.

INTRODUCTION

In reliability analysis of redundant repairable systems, most of the research workers have concentrated their attention on cold standby systems, and have emphasized the importance of warm standby (Goel et al., 1985; Kumar et al., 1995; Pour and Singh, 1996; Sridharan and Mohanavadivu, 1998; Singh et al., 2000). And in most of these papers one of the basic assumptions is that only one repairman facility (Kuo-Hsiung and Wen-Lea, 2006; Labib, 1991; Mokaddis et al., 2009a; 2009b). However, it may not be true in all situations. There may be situations when we should call two repairmen who may do the job at the same time. So when an expert repairman is busy for repairing a failed unit and another unit fails. Then the assistant repairman who may repair the failed unit after getting instructions by his master; but sometimes the assistant repairman may repair it without having the need of instructions because the faults in the failed unit are minor. Thus, in this paper, a two-unit warm standby system is studied incorporating the above idea, with some usual assumptions and the following additional assumptions:

*Corresponding author. E-mail: gmokaddis@yahoo.com.

(i) The system consists of two identical units. Initially one unit is operative and the other unit is warm standby,
(ii) When failure of a unit is detected and is attended by the expert repairman,
(iii) System is down when both units are fail,
(iv) When the expert repairman is called on to do his job, it takes negligible time to reach the system,
(v) The expert repairman leaves the system after completing repair the failed unit. The other unit which fails will be attended to by the assistant repairman,
(vi) After repair, the unit work as good as new,
(viii) There is a single repair facility attended by the two repairmen (expert and assistant).

NOTATION

λ: Constant failure rate of the operative unit.
μ: Constant failure rate of the standby unit.
$W(t),w(t)$: Cdf and pdf of time to repair of failure standby.
$G(t),g(t)$: Cdf and pdf of time to repair by expert repairman.
$I(t),i(t)$: Cdf and pdf of time to when expert gives instructions.

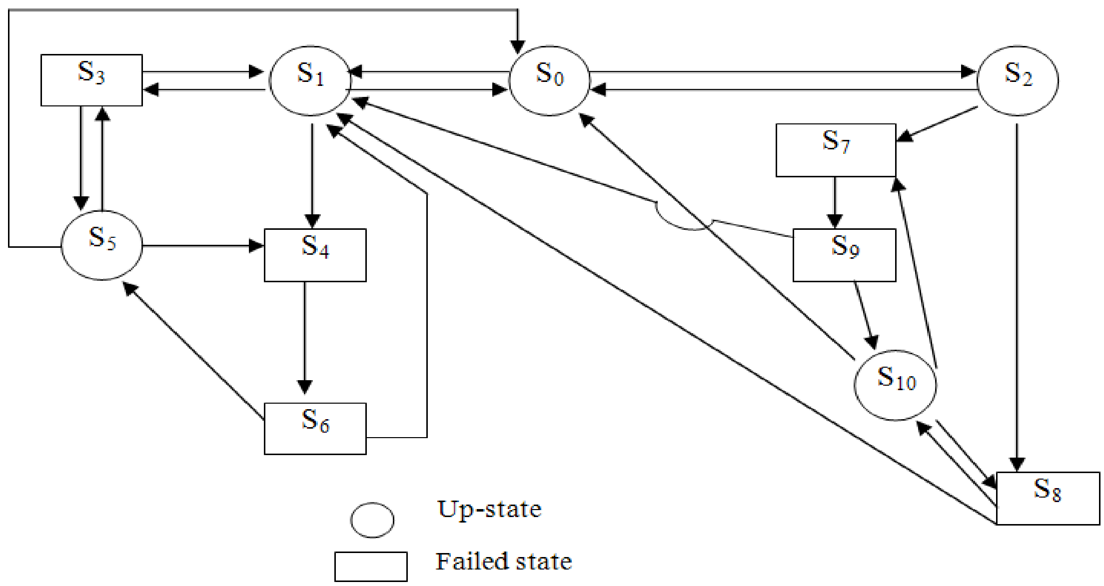

Figure 1. State transition diagram.

U(t),u(t): Cdf and pdf of time to repair by to his assistance.
p/q: Probability that the assistant repairman needs/ does not need instructions before starting the repair.
p_{ij} : One step transition probability from state i to state j.
p_{ij}^{k} : Probability that the system in state i goes to state j through state k; i,j $\in$ E and k $\in$ $\overline{E}$.
E_0 : State of the system at epoch t = 0.
E: Set of regenerative states { $S_0, S_1, S_2, S_4, S_5, S_6, S_7, S_9, S_{10}$ }.
$\overline{E}$: State of non regenerative states :{S_3,S_8}.

TRANSITION PROBABILITIES AND SOJOURN TIMES

The system can take one of the following states (Figure 1):

$S_0(O,ST)$: One unit is operative and the other unit is kept as warm standby.
$S_1(r_e,O)$: One unit is operative and the other is under expert repairman.
$S_2(S_{re},O)$: One unit is operative and warm standby unit under repair by the expert repairman.
$S_3(R_e,r_a)$: One unit continuous to repair by expert repairman and the other unite under assistant repairman.
$S_4(W_{re},W_{ra})$: One unit waiting for expert repairman and other unit waiting for assistant repairman.
$S_5(O,R_e)$: One unite is operative and the standby unit continuous to repair by expert repairman.
$S_6(r_e,r_a)$: One unit is under repair by the expert repairman and the other unit under repair by the assistant repairman.
$S_7(Sw_{re},w_{ra})$: One unit waiting for expert repairman and other unit waiting for assistant repairman.
$S_8(SR_e,r_a)$: One unit continuous to repair by expert repairman and the other unite under assistant repairman.
$S_9(Sr_e,r_a)$: One unit is under repair by the expert repairman and the other unit under repair by the assistant repairman.
$S_{10}(O,SR_e)$: One unite is operative and the standby unit continuous to repair by expert repairman.

Mean time to system failure

Time to system failure can be regarded as the first passage to any of the failed states S_3, S_4, S_6, S_7, S_8, S_9, S_{10} which is considered as absorbing. Employing the arguments used for regenerative process the following recursive relations for $\tilde{\pi}_i$ t are obtained when E$_o$=S$_i$

$$\pi_0(t) = Q_{01} \quad \pi_1(t) + Q_{02} \quad \pi_2$$
$$\pi_1(t) = Q_{10} \quad \pi_0(t) + Q_{13} + Q_{14}$$
$$\pi_2(t) = Q_{20} \quad \pi_0(t) + Q_{27} + Q_{28}$$

$$\pi_5(t) = Q_{50} \quad \pi_0(t) + Q_{53} + Q_{54}$$

$$\pi_{10}(t) = Q_{10\,0} \quad \pi_0(t) + Q_{10\,7} + Q_{10\,8}$$

Taking Laplace-Stieltjes transform for this equations and solving for $\tilde{\pi}_0(s)$ we get the mean time to system failure (MTSF) which is given by:

$$\frac{-d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)},$$

Where,

$$\text{MTSF} = \frac{\mu_0 + P_{01}\mu_1 + P_{02}\mu_2}{1 - P_{01}P_{10} + P_{02}P_{20}}$$

AVAILABILITY ANALYSIS

The following probabilistic arguments

$$\mu_0(t) = \overline{F}(t)\overline{R}(t) \qquad \mu_1(t) = p\overline{F}(t)q\overline{F}(t)G(t)$$
$$\mu_2(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t) \qquad \mu_5(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$$
$$\mu_{10}(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$$

Can be used in the theory of regenerative process in order to find the point wise availabilities $A_i(t)$ as shown in following recessive relations:

$$A_0(t) = M_0\ t + Q_{01}(t)©A_1(t) + Q_{02}(t)©A_2(t)$$
$$A_1(t) = M_1\ t + Q_{10}(t)©A_0(t) + Q_{11}^3(t)©A_1(t) + Q_{15}^3(t)©A_5(t) + Q_{14}©A_4(t)$$
$$A_2(t) = M_2\ t + Q_{20}(t)©A_0(t) + Q_{27}(t)©A_7(t) + Q_{2.10}^8(t)©A_{10}(t) + Q_{21}^8(t)©A_1(t)$$
$$A_4(t) = Q_{46}(t)©A_6(t)$$
$$A_5(t) = M_5\ t + Q_{50}(t)©A_0(t) + Q_{54}(t)©A_4(t) + Q_{55}^3(t)©A_5(t) + Q_{51}^3(t)©A_1(t)$$
$$A_6(t) = Q_{61}(t)©A_1(t) + Q_{65}(t)©A_5(t)$$
$$A_7(t) = Q_{79}(t)©A_9(t)$$
$$A_9(t) = Q_{9\,10}(t)©A_{10}(t) + Q_{91}(t)©A_1(t)$$
$$A_{10}(t) = M_{10}\ t + Q_{10\,0}(t)©A_0(t) + Q_{10\,7}(t)©A_7(t) + Q_{1010}^8(t)©A_{10}(t) + Q_{101}^8(t)©A_1(t).$$

After using Laplace transform for equations and solving for $A_0^*(s)$ we can calculate the steady state availability such that:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)}$$

Where,

$$D_2'(0) = \mu_0\ 1 - p_{107}p_{79}p_{910} - p_{1010}^8 \quad p_{50} \quad p_{14}p_{46}p_{65}$$
$$+ p_{15}^3 \quad + p_{10}(1 - p_{46}p_{54}p_{65} \quad - p_{55}^3))$$
$$+ \mu_1(p_{01} \quad (1 \quad - p_{107}p_{79}p_{910} \quad - p_{1010}^8) \quad + p_{02} \quad (p_{27}p_{79}$$
$$(p_{910}p_{101}^8 \quad + p_{91}(1 \quad - p_{1010}^8)) \quad + (1 - p_{107}p_{79}p_{910}$$
$$- p_{1010}^8)p_{21}^8 \quad + (p_{107}p_{79}p_{91} \quad + p_{101}^8) \quad p_{210}^8))$$
$$(1 - p_{46}p_{54}p_{65} - p_{55}^3)$$
$$+ \mu_2 p_{02}(1 \quad + p_{107}p_{79}p_{910} \quad + p_{1010}^8) \quad + (p_{50} \quad (p_{14}p_{46}p_{65}$$
$$+ p_{15}^3) \quad + p_{10}\ (1 \ - p_{46}p_{54}p_{65}p_{55}^3)$$
$$+ \mu_4 \quad p_{54} \quad p_{61}p_{15}^3 \quad 1 - p_{107}p_{79}p_{910} \quad - p_{1010}^8 - p_{02} \quad p_{20}$$
$$1 - p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad + p_{100} \quad p_{27}p_{79}p_{910} \quad + p_{210}^8$$
$$+ p_{65} \quad 1 \quad - p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad 1 - p_{11}^3 \quad - p_{01}p_{10}$$
$$1 - p_{107}p_{79}p_{910} - p_{1010}^8 \quad - p_{100} \quad 1 \quad - p_{11}^3$$
$$p_{27}p_{79}p_{910} + p_{210}^8 \quad - p_{10} \quad p_{27}p_{79} \quad p_{91} \quad + p_{910} \quad + p_{1010}^8$$
$$p_{21}^8 \quad + \quad p_{107}p_{79}p_{91} + p_{101}^8 \quad p_{210}^8 \quad + p_{14}$$
$$p_{65}p_{65} \quad p_{01}p_{50} \quad -1 \quad + p_{1010}^8 \quad + p_{107}p_{79}p_{910} \quad - 1$$
$$- p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad p_{51}^3 \quad + p_{02} \quad p_{50}$$
$$p_{21}^8 \quad 1 - p_{107}p_{79}p_{910} - p_{1010}^8 \quad + \quad p_{107}p_{79}p_{91} \quad + p_{101}^8$$
$$p_{210}^8 \quad - p_{20} \quad 1 - p_{107}p_{79}p_{910} - p_{1010}^8 \quad - p_{100}p_{210}^8 \quad p_{51}^3$$
$$+ p_{27}p_{79} \quad p_{50} \quad p_{910}p_{101}^8 \quad + p_{91} \quad 1 - p_{1010}^8$$
$$- p_{100}p_{910}p_{51}^3 \quad + p_{61} \quad 1 \quad - p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad - p_{02}$$
$$p_{20} \ 1 - p_{107}p_{79}p_{910} - p_{1010}^8 \quad + p_{100} \quad p_{27}p_{79}p_{910} + p_{210}^8 \quad 1 - p_{55}^3$$
$$+ \mu_5 \quad p_{14}p_{46}p_{65} \quad + p_{15}^3 \quad 1 \quad - p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad - p_{02}$$
$$p_{20} \quad 1 \quad - p_{1010}^8 \quad - p_{107}p_{79}p_{910} \quad + p_{100} \quad p_{107}p_{79}p_{910}$$
$$+ p_{210}^8$$
$$+ \mu_6 p_{46} \quad 1 \ - p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad - p_{02} \quad p_{20} \quad 1$$
$$- p_{107}p_{79}p_{910} \quad - p_{1010}^8 \quad + p_{100} \quad p_{107}p_{79}p_{910} \quad + p_{210}^8$$
$$p_{54}p_{15}^3 + p_{14} \ 1 - p_{55}^3$$
$$+ \mu_7 \quad p_{107} \quad p_{910} \quad 1 \ - p_{01}p_{10} \quad - p_{02}p_{20} \quad - p_{11}^3 \ 1$$
$$- p_{02}p_{20} \quad - p_{02}p_{10}p_{21}^8 \quad 1 \ - p_{46}p_{54}p_{65} \ - p_{55}^3 \quad - p_{14}p_{46}$$
$$1 \ - p_{02}p_{20} \quad p_{61} \quad 1 \ - p_{55}^3 \quad + p_{65}p_{51}^3 \quad - \quad p_{01}p_{50}$$

$+p_{02}p_{50}p_{21}^{8} \quad +p_{51}^{3} \quad 1-p_{02}p_{20} \qquad p_{46}p_{54}p_{65} \quad +p_{15}^{3}$
$+p_{02}p_{210}^{8}p_{91} \quad p_{10} \quad 1 -p_{46}p_{54}p_{65} -p_{55}^{3} \quad +p_{50} \quad p_{15}^{3}$
$+p_{14}p_{46}p_{65} \qquad +p_{02}p_{27}$
$p_{91}p_{10} \; 1-p_{46}p_{54}p_{65} \; +p_{14}p_{46}p_{50}p_{65} \quad +p_{910}p_{100} \quad 1$
$-p_{46}p_{54}p_{65} \quad -p_{14}p_{46}p_{61} \quad - \quad p_{11}^{3} \quad +p_{55}^{3} \quad 1$
$-p_{46}p_{54}p_{65} \quad -p_{11}^{3}p_{55}^{3} \quad -p_{15}^{3} \quad p_{51}^{3} \quad +p_{14}p_{46}p_{65}$
$+p_{46}p_{54}p_{61} \quad p_{91} \quad 1 \quad -p_{1010}^{8} \quad +p_{910}p_{101}^{8}$
$p_{50}p_{15}^{3}-p_{10}p_{55}^{8} \quad +\mu_{9}p_{02}p_{79} \quad (p_{27} \quad (1 \quad -p_{1010}^{8})$
$+p_{107}p_{210}^{8}) \quad (p_{50} \quad (p_{14}p_{46}p_{65} \quad +p_{15}^{3})$
$+p_{10} \quad (1 \quad -p_{46}p_{54}p_{65} \quad -p_{55}^{3})) \quad +\mu_{10}p_{02} \quad (p_{27}p_{79}p_{910}$
$+p_{210}^{8}) \quad (p_{50} \quad (p_{14}p_{46}p_{65} \quad +p_{15}^{3}) \quad +p_{10} \quad (1 \quad -p_{46}p_{54}p_{65}$
$-p_{55}^{3}))$

$$N_2(0) = Y_0 + Y_1 + Y_2 + Y_5 + Y_{10}$$

$Y_0 = \mu_0[\ (1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ (1 \ -p_{11}^{3} \ -p_{46}p_{54} \ (p_{65} \ (1 \ -p_{11}^{3}) \ +p_{61}p_{15}^{3}) \ -p_{15}^{3}p_{51}^{3} \ -p_{14}p_{46} \ (p_{65}p_{51}^{3} \ -p_{61} \ (1 \ -p_{55}^{3})) \ -p_{55}^{3} \ +p_{11}^{3}p_{55}^{3})]$

$Y_1 = \mu_1 \ [(1 \ -p_{46}p_{54}p_{65} \ -p_{55}^{3}) \ (p_{01} \ (1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ -p_{02} \ (p_{27}p_{79} \ (p_{910}p_{101}^{8} \ +p_{91} \ (1 \ -p_{1010}^{8})) \ +p_{21}^{8}(1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ +p_{210}^{8} \ (p_{107}p_{79}p_{91} \ +p_{101}^{8})))]$

$Y_2 = \mu_2 p_{02} \ [(1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ (1 \ -p_{11}^{3} \ -p_{46}p_{54} \ (p_{65} \ (1 \ -p_{11}^{3}) \ +p_{61}p_{15}^{3}) \ -p_{15}^{3}p_{51}^{3} \ -p_{14}p_{46} \ (p_{65}p_{51}^{3} \ +p_{61} \ (1 \ -p_{55}^{3})) \ +p_{55}^{3} -p_{11}^{3}p_{55}^{3})]$

$Y_5 = \mu_5 \ [(p_{14}p_{46}p_{65} +p_{15}^{3}) \ (p_{01} \ (1 -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ +p_{02} \ (p_{27}p_{79} \ (p_{910}p_{101}^{8} \ +p_{91} \ (1 \ -p_{1010}^{8})) \ +p_{21}^{8}(1 \ -p_{107}p_{79}p_{910} -p_{1010}^{8}) +p_{210}^{8} \ (p_{107}p_{79}p_{91} \ +p_{101}^{8})))]$

$Y_{10} = \mu_{10} \ [p_{02} \ (p_{27}p_{79}p_{910} \ +p_{210}^{8}) \ (1 \ -p_{11}^{3} \ -p_{46}p_{54} \ (p_{65} \ (1 \ -p_{11}^{3}) \ -p_{61}p_{15}^{3}) \ -p_{15}^{3}p_{51}^{3} \ -p_{14}p_{46} \ (p_{65}p_{15}^{3} +p_{61} \ (1 \ -p_{55}^{3})) \ -p_{55}^{3} \ +p_{11}^{3}p_{55}^{3})]$

BUSY PERIOD OF AN EXPERT REPAIRMAN

The following probabilistic arguments

$\mu_0(t) = \overline{F}(t)\overline{R}(t)$ $\qquad \mu_1(t) = p\overline{F}(t)q\overline{F}(t)G(t)$
$\mu_2(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$ $\qquad \mu_5(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$
$\mu_{10}(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$

Can be used in the theory of regenerative process in order to find the busy period analysis for the expert repairman $B_i(t)$ (the probability that the operative unit is under repair at time (t) as shown in following recessive relations:

$B_0(t) = Q_{01}(t) \quad B_1(t) + Q_{02}(t) \quad B_2(t)$
$B_1(t) = W_1(t) + Q_{10}(t) \quad B_0(t) + Q_{11}^{3}(t) \quad B_1(t) + Q_{15}^{3}(t) \quad B_5(t) + Q_{14}(t) \quad B_4(t)$
$B_2(t) = W_2(t) + Q_{20}(t) \quad B_0(t) + Q_{27}(t) \quad B_7(t) + Q_{2.10}^{8}(t) \quad B_{10}(t) + Q_{21}^{8}(t) \quad B_1(t)$
$B_4(t) = W_4(t) + Q_{46}(t) \quad B_6(t)$
$B_5(t) = W_5(t) + Q_{50}(t) \quad B_0(t) + Q_{54}(t) \quad B_4(t) + Q_{55}^{3}(t) \quad B_5(t) + Q_{51}^{3}(t) \quad B_1(t)$
$B_6(t) = W_6(t) + Q_{61}(t) \quad B_1(t) + Q_{65}(t) \quad B_5(t)$
$B_7(t) = W_7(t) + Q_{79}(t) \quad B_9(t)$
$B_8(t) = W_9(t) + Q_{9.10}(t) \quad B_{10}(t) + Q_{9.1}(t) \quad B_1(t)$
$B_{10}(t) = W_{10}(t) + Q_{100}(t) \& B_0(t) + Q_{107}(t) \& B_7(t) + Q_{1010}^{8}(t) \& B_{10}(t) + Q_{101}^{8}(t) \& B_1(t).$

After using Laplace transform for equations and solving for $B_0^*(s)$ we can calculate the busy period analysis steady such that:

$$B_0 = \lim_{s \to o} s\widetilde{B}_0(s) = \frac{N_3(0)}{D_2'(0)}$$

Where,

$$N_3(0) = Y_1 + Y_2 + Y_4 + Y_5 + Y_6 + Y_7$$

$Y_1 = \mu_1 \ [(1 \ -p_{46}p_{54}p_{65} -p_{55}^{3}) \ (p_{01} \ (1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ -p_{02} \ (p_{27}p_{79} \ (p_{910}p_{101}^{8} \ +p_{91} \ (1 \ -p_{1010}^{8})) \ +p_{21}^{8}(1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ +p_{210}^{8} \ (p_{107}p_{79}p_{91} \ +p_{101}^{8})))]$

$Y_2 = \mu_2 p_{02} \ [(1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ (1 \ -p_{11}^{3} \ -p_{46}p_{54} \ (p_{65} \ (1 \ -p_{11}^{3}) \ +p_{61}p_{15}^{3}) \ -p_{15}^{3}p_{51}^{3} \ -p_{14}p_{46} \ (p_{65}p_{51}^{3} \ +p_{61} \ (1 \ -p_{55}^{3})) \ +p_{55}^{3} -p_{11}^{3}p_{55}^{3})]$ $Y_4 = \mu_4 \ [(p_{01} \ (1 \ -p_{107}p_{79}p_{910} -p_{1010}^{8}) +p_{02} \ (p_{27}p_{79} \ (p_{910}p_{101}^{8} \ +p_{91} \ (1 \ -p_{1010}^{8})) \ +p_{21}^{8} \ (1 \ -p_{107}p_{79}p_{910} \ -p_{1010}^{8}) \ +p_{210}^{8} \ (p_{107}p_{79}p_{91} \ +p_{101}^{8}))) \ (p_{54}p_{15}^{3} \ +p_{14}(1-p_{55}^{3}))]$

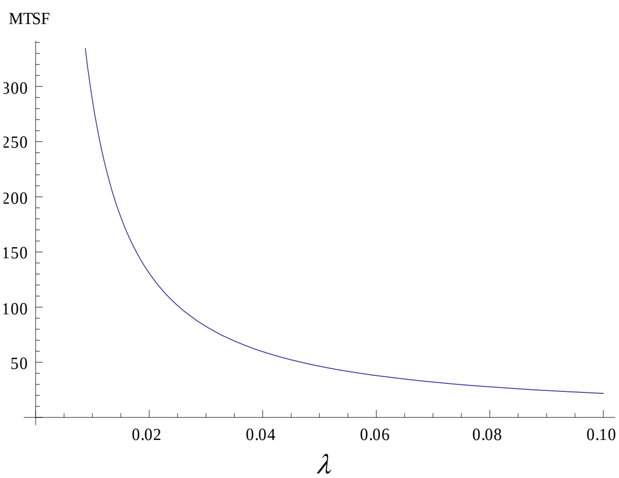

Figure 2. MTSF versus failure rate λ.

$$Y_6 = \mu_6 \ [p_{46} \ (p_{01} \ (1 \ -p_{107} p_{79} p_{910} \ -p^8_{1010}) \ +p_{02} \ (p_{27} p_{79} \ (p_{910} p^8_{101} \ +p_{91} \ (1 \ -p^8_{1010})) \ +p^8_{21} \ (1 \ -p_{107} p_{79} p_{910} \ -p^8_{1010}) \ -p^8_{210} \ (p_{107} p_{79} p_{91} \ +p^8_{101}))) \ (p_{54} p^3_{15} \ +p_{14}(1- p^3_{55}))]$$

$$Y_7 = \mu_7 \ [p_{02} \ (p_{27} \ (1 \ -p^8_{1010}) \ +p_{107} p^8_{210}) \ (1 \ -p^3_{11} \ -p_{46} p_{54} \ (p_{65} \ (1 -p^3_{11}) \ +p_{61} p^3_{15}) \ -p^3_{15} p^3_{51} \ -p_{14} p_{46} \ (p_{65} p^3_{51} \ +p_{61} \ (1 \ -p^3_{55})) \ -p^3_{55} \ +p^3_{11} p^3_{55})]$$

$$Y_9 = \mu_9 \ [p_{02} p_{79} \ (p_{27} \ (1 -p^8_{1010}) \ +p_{107} p^8_{210}) \ (1 \ -p^3_{11} \ -p_{46} p_{54} \ (p_{65} \ (1 \ -p^3_{11}) \ -p_{61} p^3_{15} p^3_{51} \ -p_{14} p_{46} \ (p_{65} p^3_{51} \ -p_{61} \ (1 \ -p^3_{55})) \ -p^3_{55} \ +p^3_{11} p^3_{55})]$$

BUSY PERIOD OF AN ASSISTANT REPAIRMAN

The following probabilistic arguments,

$$\mu_0(t) = \overline{F}(t)\overline{R}(t) \qquad \mu_1(t) = p\overline{F}(t)q\overline{F}(t)G(t)$$
$$\mu_2(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t) \qquad \mu_5(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$$
$$\mu_{10}(t) = p\overline{F}(t)q\overline{F}(t)\overline{W}(t)$$

Can be used in the theory of regenerative process in order to find the busy period analysis $T_i(t)$ (the probability that the operative unit is under repair at time (t) as shown in following recessive relations:

$$T_0(t) = Q_{01}(t) \quad T_1(t) + Q_{02}(t) \quad T_2(t)$$
$$T_1(t) = Q_{10}(t) \quad T_0(t) + Q^3_{11}(t) \quad T_1(t) + Q^3_{15}(t) \quad T_5(t) + Q_{14}(t) \quad T_4(t)$$
$$T_2(t) = Q_{20}(t) \quad T_0(t) + Q_{27}(t) \quad T_7(t) + Q^8_{2.10}(t) \quad T_{10}(t) + Q^8_{21}(t) \quad T_1(t)$$
$$T_4(t) = W_4(t) + Q_{46}(t) \quad T_6(t)$$
$$T_5(t) = Q_{50}(t) \quad T_0(t) + Q_{54}(t) \quad T_4(t) + Q^3_{55}(t) \quad T_5(t) + Q^3_{51}(t) \quad T_1(t)$$
$$T_6(t) = W_6(t) + Q_{61}(t) \quad T_1(t) + Q_{65}(t) \quad T_5(t)$$
$$T_7(t) = W_7(t) + Q_{79}(t) \quad T_9(t)$$
$$T_8(t) = W_9(t) + Q_{9.10}(t) \quad T_{10}(t) + Q_{9.1}(t) \quad T_1(t)$$
$$T_{10}(t) = Q_{10\,0}(t) \quad T_0(t) + Q_{10\,7}(t) \quad T_7(t) + Q^8_{10\,10}(t) \quad T_{10}(t) + Q^8_{10\,1}(t) \quad T_1(t)$$

After using Laplace transform for equations and solving for $T_0^*(s)$ we can calculate the busy period analysis steady such that:

$$T_0 \ = \lim_{s \to o} s\tilde{T}_0 \ (s) = \frac{N_4(0)}{D_2'(0)}$$

Where,

$$N_2(0) = Y_4 + Y_6 + Y_7 + Y_9$$

COST BENEFT ANALYSIS

At steady state the net expected gain per unit of time is:

$$G = \lim_{t \to \infty} G(t)/t = C_1 A_0 - C_2 B_0 - C_3 T_0 \ ,$$

Where,

C_1 : Is revenue per unit uptime by the system.

C_2 : Is per unit expert repair cost.

C_3 : Is per unit assistant repair cost.

SPECIAL CASES

The failure and repair times are exponential distribution

$$F(t) = 1 - e^{-\lambda t}, \qquad I(t) = 1 - e^{-\pi t},$$
$$G(t) = 1 - e^{-\alpha t}, \qquad X(t) = 1 - e^{-\iota t},$$
$$\mathrm{U}(t) = 1 - e^{-\beta t}, \qquad W(t) = 1 - e^{-\vartheta t}.$$

Graphical representation

The graphical representations are shown in Figures 2, 3 and 4.

Conclusion

By comparing the characteristic, MTSF, Availability and PROFIT with respect to λ, it was observing that:

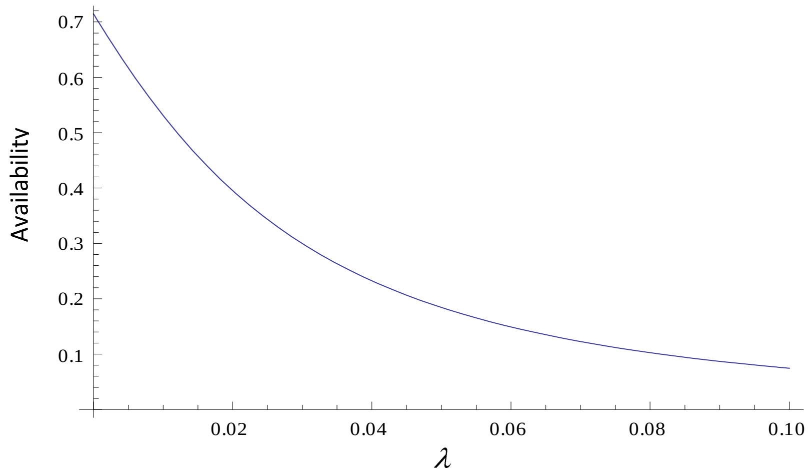

Figure 3. Availability versus failure rate λ.

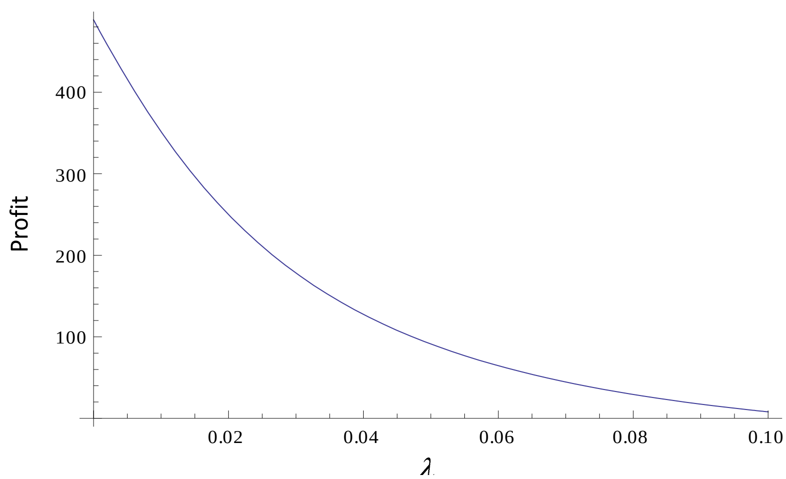

Figure 4. Profit versus failure rate λ.

The increase of failure rate λ at constant $\gamma = 0.02,$ $\alpha = 0.04,$ $\beta = 0.06,$ $\varphi = 0.05,$ $\tau = 0.02,$ $p = 0.4,$ $q = 0.6,$ $C_0 = 700$, $C_1 = 40$, $C_2 = 10$ the MTSF, Availability and PROFIT of the system decrease.

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